Examples of Multi-Step Schemes for Solving Ordinary Differential Equations (ODEs)

1. Adams–Bashforth Second-Order Method (Explicit)

$$y_{n+1} = y_n + \frac{h}{2}\left[3f(t_n, y_n) - f(t_{n-1}, y_{n-1})\right]$$

- **Order**: 2
- **Type**: Explicit
- **Steps**: 2

2. Adams–Bashforth Fourth-Order Method (Explicit)

$$y_{n+1} = y_n + \frac{h}{24}\left[55f(t_n, y_n) - 59f(t_{n-1}, y_{n-1}) + 37f(t_{n-2}, y_{n-2}) - 9f(t_{n-3}, y_{n-3})\right]$$

- **Order**: 4
- **Type**: Explicit
- **Steps**: 4

3. Adams–Moulton Second-Order Method (Implicit, Trapezoidal Rule)

$$y_{n+1} = y_n + \frac{h}{2}\left[f(t_{n+1}, y_{n+1}) + f(t_n, y_n)\right]$$

- **Order**: 2
- **Type**: Implicit
- **Steps**: 1

4. Adams–Moulton Third-Order Method (Implicit)

$$y_{n+1} = y_n + \frac{h}{12}\left[5f(t_{n+1}, y_{n+1}) + 8f(t_n, y_n) - f(t_{n-1}, y_{n-1})\right]$$

- **Order**: 3
- **Type**: Implicit
- **Steps**: 2

5. Backward Differentiation Formula (BDF2) (Implicit)

$$y_{n+1} = \frac{4}{3}y_n - \frac{1}{3}y_{n-1} + \frac{2h}{3}f(t_{n+1}, y_{n+1})$$

- **Order**: 2
- **Type**: Implicit
- **Steps**: 2

6. Nyström Explicit Midpoint Method (Explicit)

$$y_{n+1} = y_{n-1} + 2h \cdot f(t_n, y_n)$$

- **Order**: 2
- **Type**: Explicit
- **Steps**: 2

7. Milne's Fourth-Order Predictor (Explicit)

$$y_{n+1} = y_{n-3} + \frac{4h}{3} \left[2f(t_n, y_n) - f(t_{n-1}, y_{n-1}) + 2f(t_{n-2}, y_{n-2}) \right]$$

- **Order**: 4
- **Type**: Explicit
- **Steps**: 4

8. Simpson's Fourth-Order Corrector (Implicit)

$$y_{n+1} = y_{n-1} + \frac{h}{3} \left[f(t_{n+1}, y_{n+1}) + 4f(t_n, y_n) + f(t_{n-1}, y_{n-1}) \right]$$

- **Order**: 4
- **Type**: Implicit
- **Steps**: 2

Summary

These methods span explicit and implicit families (Adams–Bashforth, Adams–Moulton, BDF, and others) and vary in order and step count.