



太原理工大学  
TAIYUAN UNIVERSITY OF TECHNOLOGY



太原理工大学  
大数据学院  
COLLEGE OF DATA SCIENCE  
TAIYUAN UNIVERSITY OF TECHNOLOGY

# 深度学习中的正则化

Regularization for Deep Learning

## 本章内容概况

### 主要内容

**01** 数据增强与提前终止

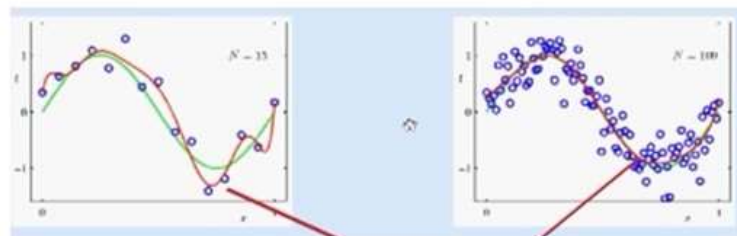
**02** Dropout

**03** 稀疏表示

# PART 数据增强与 提前终止 ONE

## 数据增强与提前终止

### 数据增强



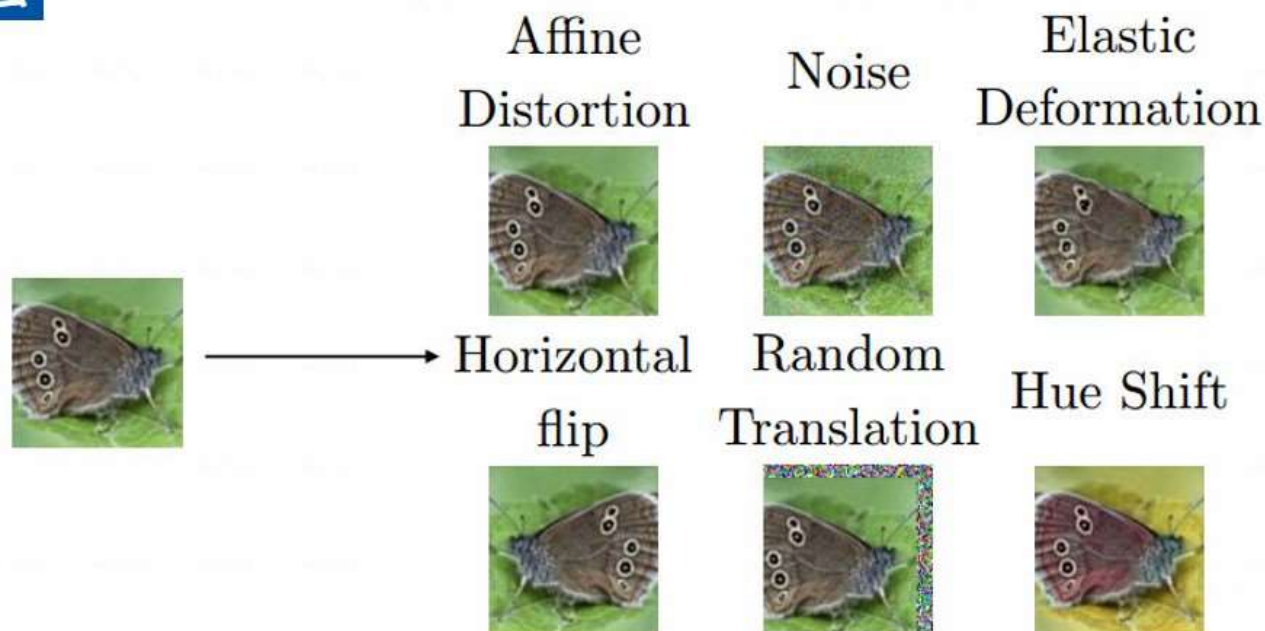
▶ 图像数据的增强主要是通过算法对图像进行转变，引入噪声等方法来增加数据的多样性。

▶ 图像数据的增强方法：

- ▶ 旋转 (Rotation) : 将图像按顺时针或逆时针方向随机旋转一定角度；
- ▶ 翻转 (Flip) : 将图像沿水平或垂直方法随机翻转一定角度；
- ▶ 缩放 (Zoom In/Out) : 将图像放大或缩小一定比例；
- ▶ 平移 (Shift) : 将图像沿水平或垂直方法平移一定步长；
- ▶ 加噪声 (Noise) : 加入随机噪声。

## 数据增强与提前终止

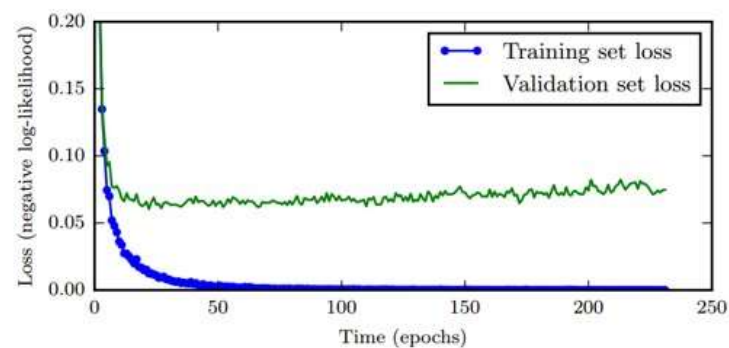
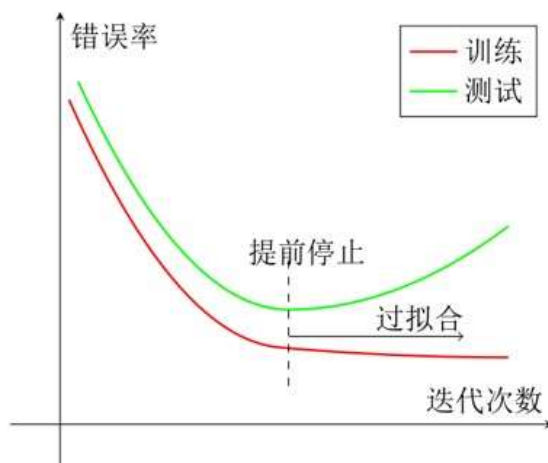
### 数据增强



## 数据增强与提前终止

### 提前终止

- ▶ 我们使用一个验证集 (Validation Dataset) 来测试每一次迭代的参数在验证集上是否最优。如果在验证集上的错误率不再下降, 就停止迭代。



## 数据增强与提前终止

### 提前终止

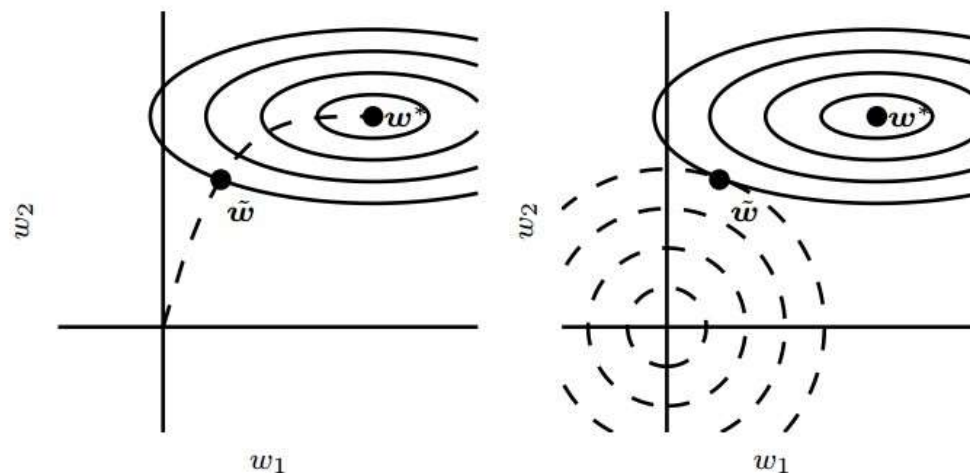


图 7.4: 提前终止效果的示意图。(左) 实线轮廓线表示负对数似然的轮廓。虚线表示从原点开始的 SGD 所经过的轨迹。提前终止的轨迹在较早的点  $\tilde{w}$  处停止, 而不是停止在最小化代价的点  $w^*$  处。(右) 为了对比, 使用  $L^2$  正则化效果的示意图。虚线圆圈表示  $L^2$  惩罚的轮廓,  $L^2$  惩罚使得总代价的最小值比非正则化代价的最小值更靠近原点。



# PART Dropout TWO

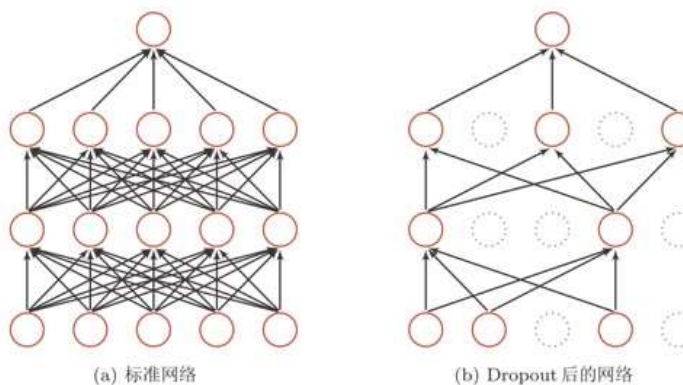


## Dropout Definition

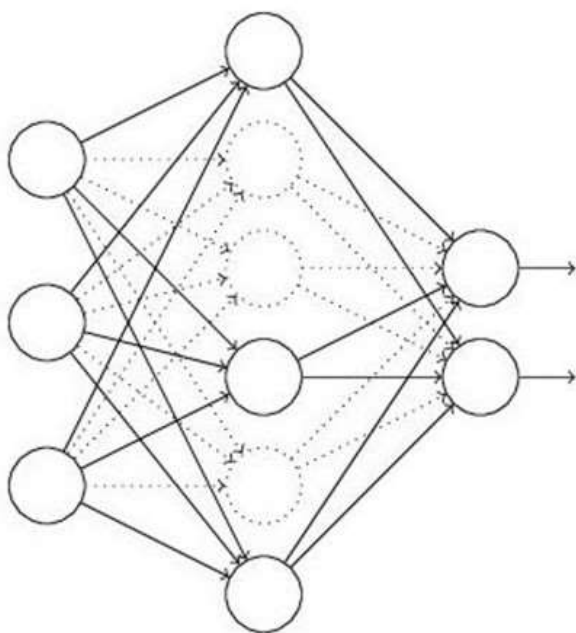
- ▶ 对于一个神经层  $y = f(Wx + b)$ ，引入一个丢弃函数  $d(\cdot)$  使得  $y = f(Wd(x) + b)$ 。

$$d(\mathbf{x}) = \begin{cases} \mathbf{m} \odot \mathbf{x} & \text{当训练阶段时} \\ p\mathbf{x} & \text{当测试阶段时} \end{cases}$$

- ▶ 其中  $m \in \{0,1\}^d$  是丢弃掩码 (dropout mask)，通过以概率为  $p$  的贝努力分布随机生成。



## Dropout Definition



首先随机（临时）删掉网络中一半的隐藏神经元，输入输出神经元保持不变（图中虚线为部分临时被删除的神经元）

然后把输入 $x$ 通过修改后的网络前向传播，然后把得到的损失结果通过修改的网络反向传播。一小批训练样本执行完这个过程后，在没有被删除的神经元上按照随机梯度下降法更新对应的参数（ $w, b$ ）。

然后继续重复这一过程：

恢复被删掉的神经元（此时被删除的神经元保持原样，而没有被删除的神经元已经有所更新）

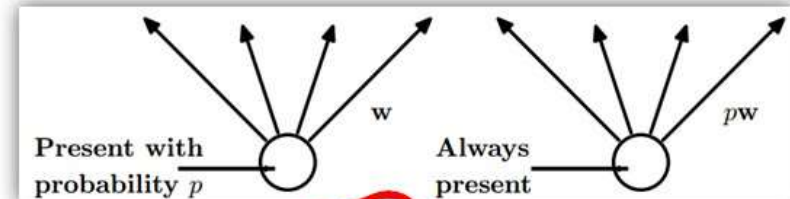
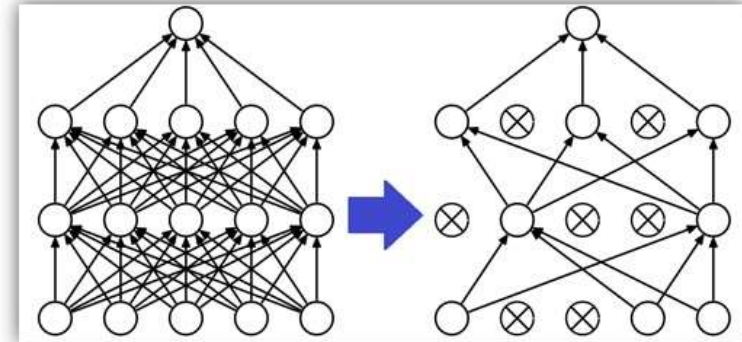
从隐藏层神经元中随机选择一个一半大小的子集临时删除掉（备份被删除神经元的参数）。

对一小批训练样本，先前向传播然后反向传播损失并根据随机梯度下降法更新参数（ $w, b$ ）（没有被删除的那一部分参数得到更新，删除的神经元参数保持被删除前的结果）。

# Dropout

## Definition

- At training (each iteration):  
Each unit is retained with a probability  $p$ .
- At test:  
The network is used as a whole.  
The weights are scaled-down by a factor of  $p$ .
- In practice, dropout trains  $2^n$  networks  
( $n$  – number of units).

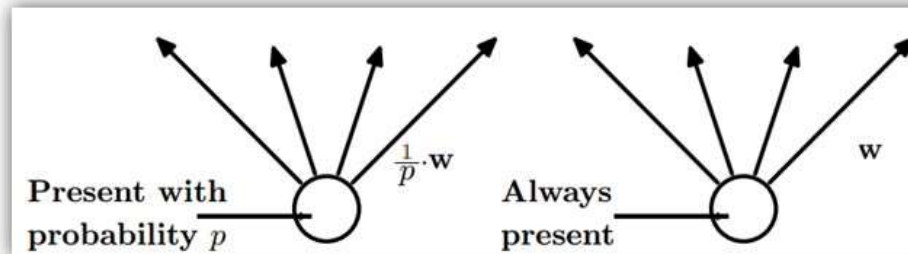


**$p = 0.5$   
MAKES THE  
CHARM!**

## Dropout Definition

- At training, weights are scaled-up by a factor of  $\frac{1}{p}$ .
- At test time, no scaling is applied.
- This method is used in Tensorflow:

`tf.nn.dropout(x, keep_prob=p)`



# Dropout

## 实验结果对比

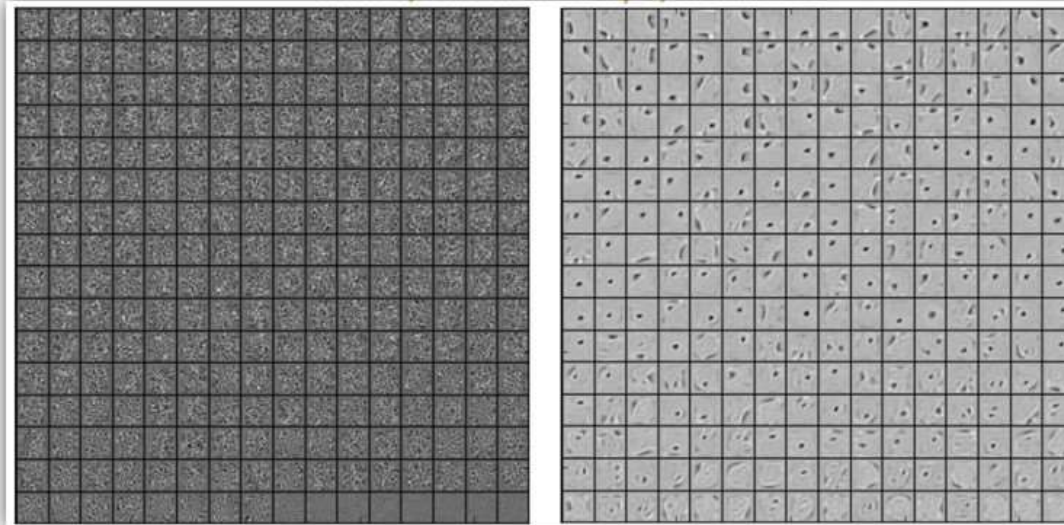
The effect of dropout on learned features:

- Without dropout, units tend to **compensate for mistakes** of other units.
- This leads to overfitting, since **these co-adaptations do not generalize to unseen data.**
- Dropout prevents co-adaptations by making the presence of other hidden units unreliable.

MNIST, one hidden layer, 256 ReLUs

### No dropout

Units have co-adapted.  
Each unit does not  
detect a meaningful  
feature.



### Dropout ( $p = 0.5$ )

Units detect edges,  
strokes and spots in  
different parts of the  
image.

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# Dropout

## 代码不同

```
# The model
XX = tf.reshape(X, [-1, 784])
Y1 = tf.nn.relu(tf.matmul(XX, W1) + B1)
Y2 = tf.nn.relu(tf.matmul(Y1, W2) + B2)
Y3 = tf.nn.relu(tf.matmul(Y2, W3) + B3)
Y4 = tf.nn.relu(tf.matmul(Y3, W4) + B4)
Ylogits = tf.matmul(Y4, W5) + B5
Y = tf.nn.softmax(Ylogits)
```

```
# The model, with dropout at each layer
XX = tf.reshape(X, [-1, 28*28])

Y1 = tf.nn.relu(tf.matmul(XX, W1) + B1)
Y1d = tf.nn.dropout(Y1, pkeep)

Y2 = tf.nn.relu(tf.matmul(Y1d, W2) + B2)
Y2d = tf.nn.dropout(Y2, pkeep)

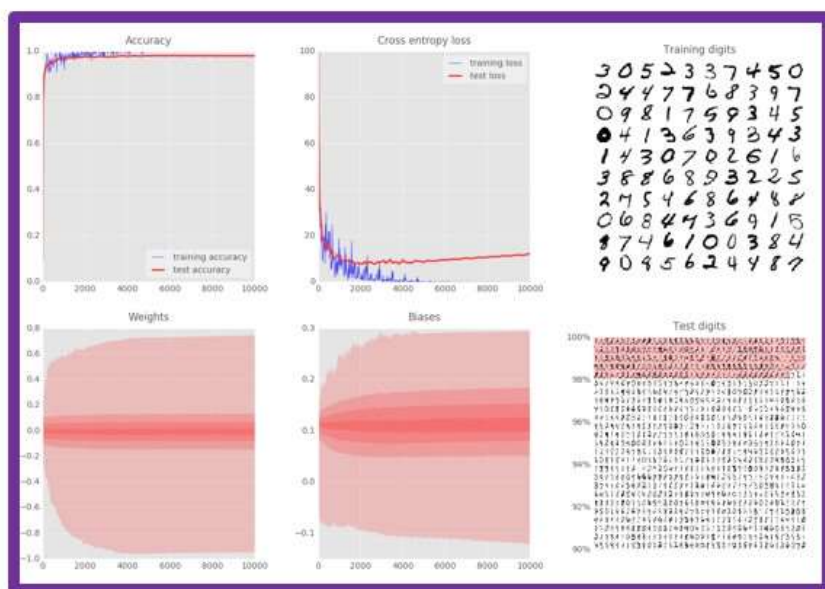
Y3 = tf.nn.relu(tf.matmul(Y2d, W3) + B3)
Y3d = tf.nn.dropout(Y3, pkeep)

Y4 = tf.nn.relu(tf.matmul(Y3d, W4) + B4)
Y4d = tf.nn.dropout(Y4, pkeep)

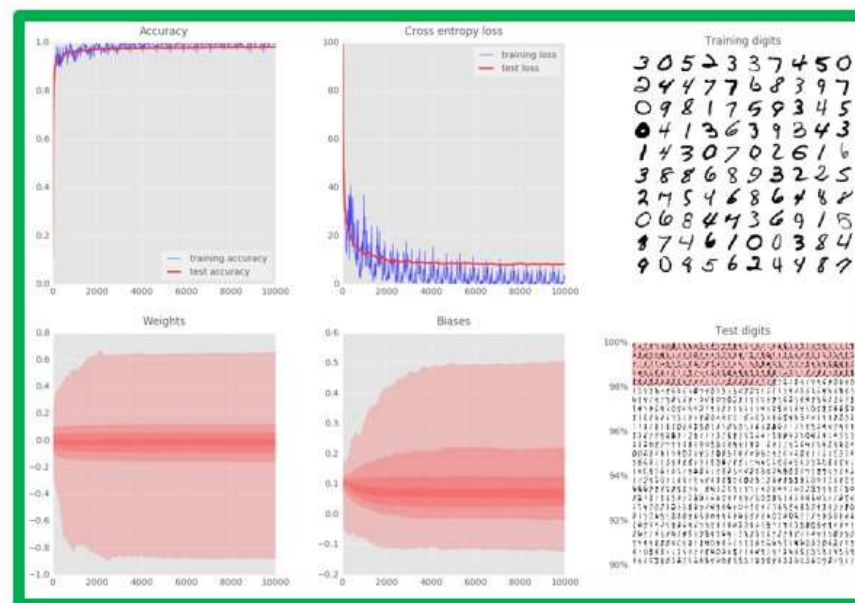
Ylogits = tf.matmul(Y4d, W5) + B5
Y = tf.nn.softmax(Ylogits)
```

# Dropout

## 性能对比



No Dropout



Dropout

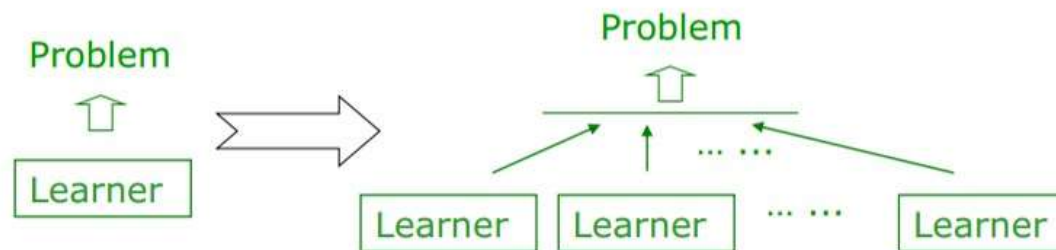


# Dropout

## 集成学习

5道题  
3个学生  
每人会做2道

“Ensemble methods” is a machine learning paradigm where multiple (homogenous/heterogeneous) individual learners are trained for the same problem  
e.g. neural network ensemble, decision tree ensemble, etc.



The more **accurate** and **diverse** the component learners, the better the ensemble

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## Dropout

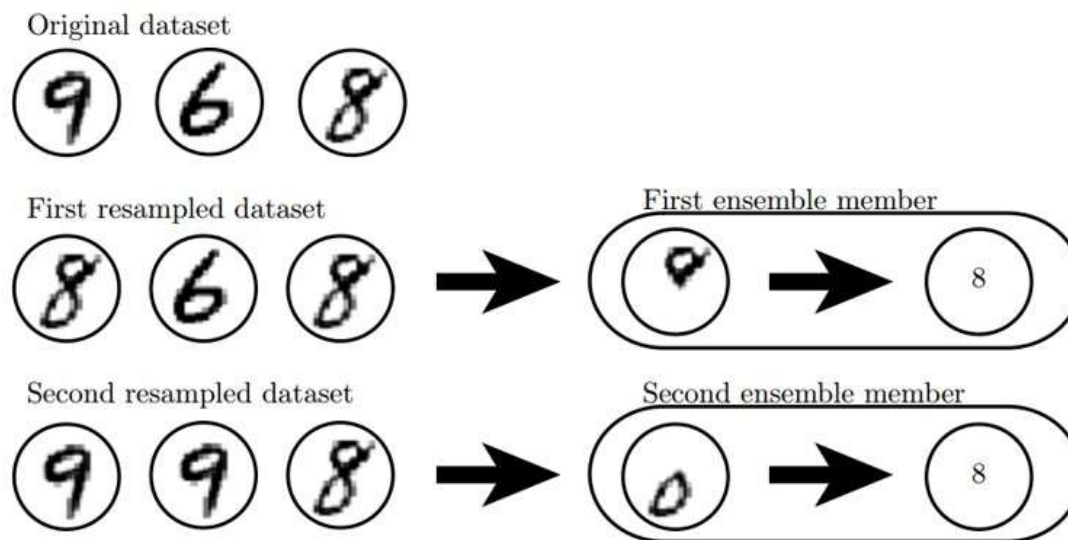
### Bagging和Dropout

## Dropout as bagging

- In bagging we define  $k$  different models, construct  $k$  different data sets by sampling from the dataset with replacement, and train model  $i$  on dataset  $i$
- Dropout aims to approximate this process, but with an exponentially large no. of neural networks

## Dropout Bagging

# Bagging

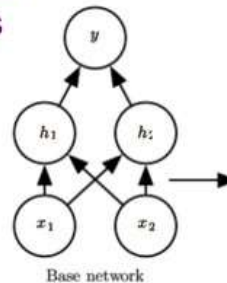


# Dropout

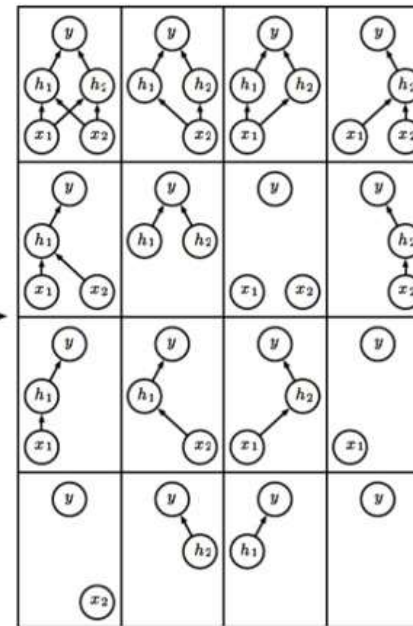
## Bagging和Dropout的联系

### Dropout as an ensemble method

- Remove non-output units from base network.
- Remaining 4 units yield 16 networks



- Here many networks have no path from input to output
- Problem insignificant with large networks



## Dropout

### Bagging和Dropout的差异

## Bagging training vs Dropout training

- Dropout training not same as bagging training
  - In bagging, the models are all independent
  - In dropout, models share parameters
    - Models inherit subsets of parameters from parent network
    - Parameter sharing allows an exponential no. of models with a tractable amount of memory
- In bagging each model is trained to convergence on its respective training set
  - In dropout, most models are not explicitly trained
    - Fraction of subnetworks are trained for a single step
    - Parameter sharing allows good parameter settings



# Dropout

## model description

### Prediction: Bagging vs. Dropout

- Bagging:
  - Ensemble accumulates votes of members
  - Process is referred to as inference
    - Assume model needs to output a probability distribution
    - In bagging, model  $i$  produces  $p^{(i)}(y|\mathbf{x})$
    - Prediction of ensemble is the mean  $\frac{1}{k} \sum_{i=1}^k p^{(i)}(y|\mathbf{x})$
- Dropout:
  - Submodel defined by mask vector  $\mu$  defines a probability distribution  $p(y|\mathbf{x}, \mu)$
  - Arithmetic mean over all masks is  $\sum_{\mu} p(y|\mathbf{x}, \mu)$ 
    - Where  $p(\mu)$  is the distribution used to sample  $\mu$  at training time

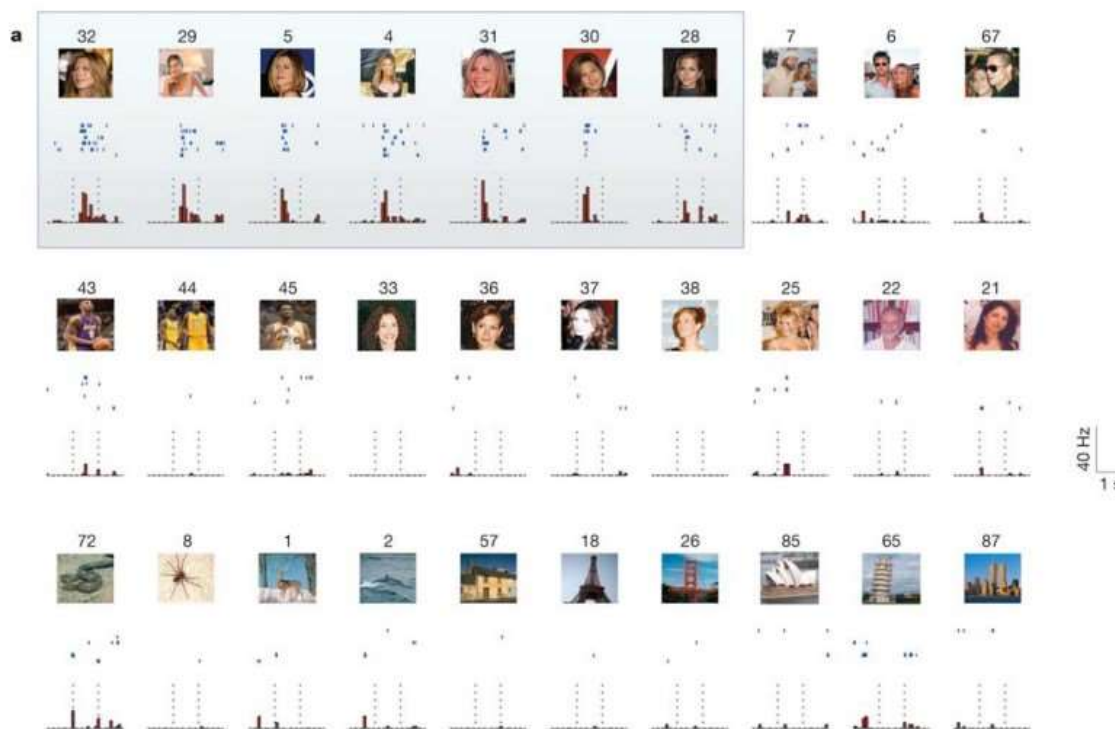
# PART 稀疏表示 THREE



概念

Definition

Grandmother  
cell

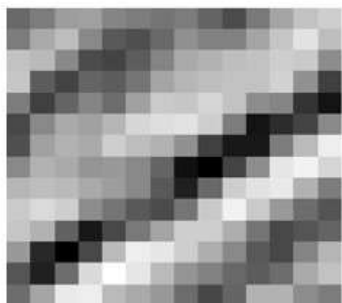


[Quiroga, Reddy, & Kreiman Nature2005]

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## 稀疏表示

### Definition

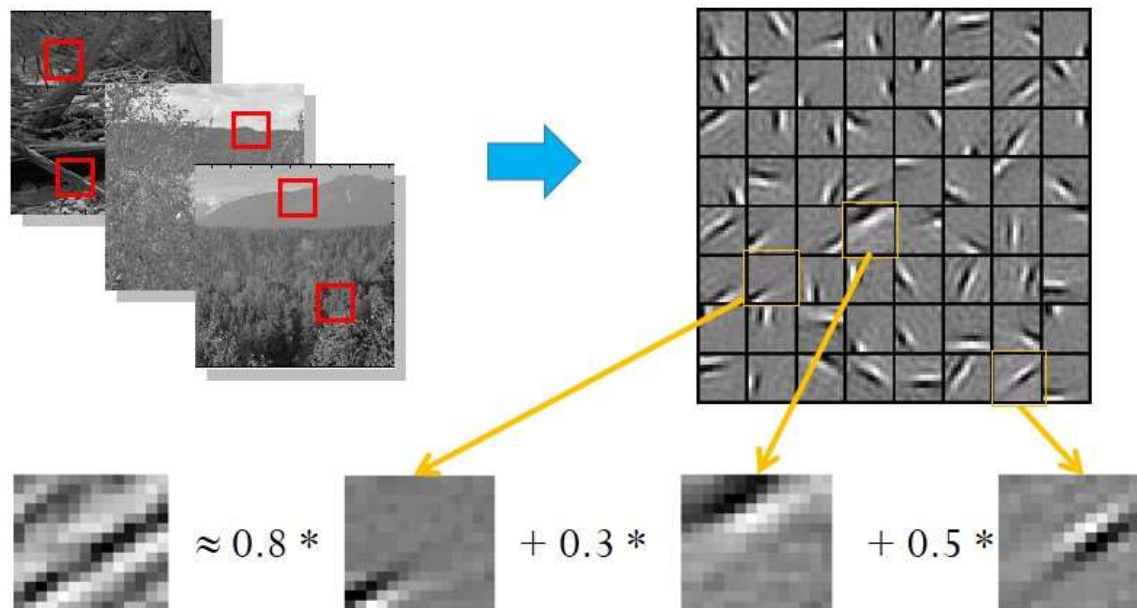


**Learn a better way to  
represent image than pixels**



## 稀疏表示

### Definition



$[a_1, \dots, a_{64}] = [0, 0, \dots, 0, \mathbf{0.8}, 0, \dots, 0, \mathbf{0.3}, 0, \dots, 0, \mathbf{0.5}, 0]$  (feature representation)

Code:

<http://web.eecs.umich.edu/~honglak/software/nips06-sparsecoding.htm>

## 稀疏表示

### Definition

**Input:** Patch  $x_i$  ( in  $\mathbb{R}^d$  ) and previously learned  $\phi_i (i=1, \dots, k)$

**Output:** Representation  $[a_{i,1}, a_{i,2}, \dots, a_{i,k}]$  of image patch  $x_i$

$$\min_{a, \phi} \sum_{i=1}^m \left( \left\| x_j - \sum_{j=1}^k a_{i,j} \phi_i \right\|^2 + \sum_{j=1}^k |a_{i,j}| \right)$$

## 稀疏表示

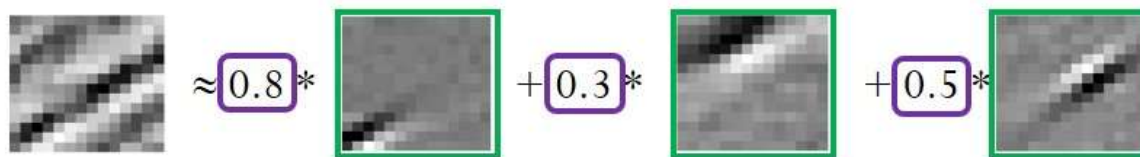
### Definition

**Input:** Patch  $x_i$  ( in  $\mathbb{R}^d$  ) and previously learned  $\phi_i (i=1, \dots, k)$

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↑      ↑



## 稀疏表示

### Definition

**Input:** Patch  $x_i$  ( in  $\mathbb{R}^d$  ) and previously learned  $\phi_i (i=1, \dots, k)$

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$l_1$  sparsity term

## 稀疏表示

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$l_1$  sparsity term

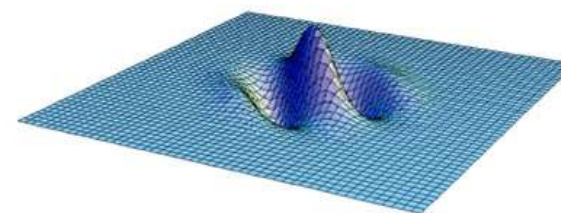
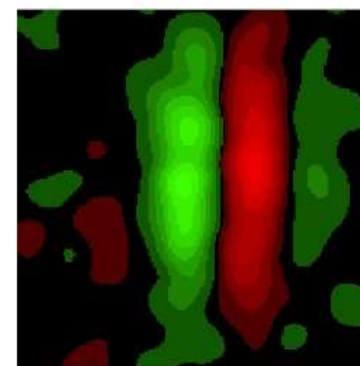
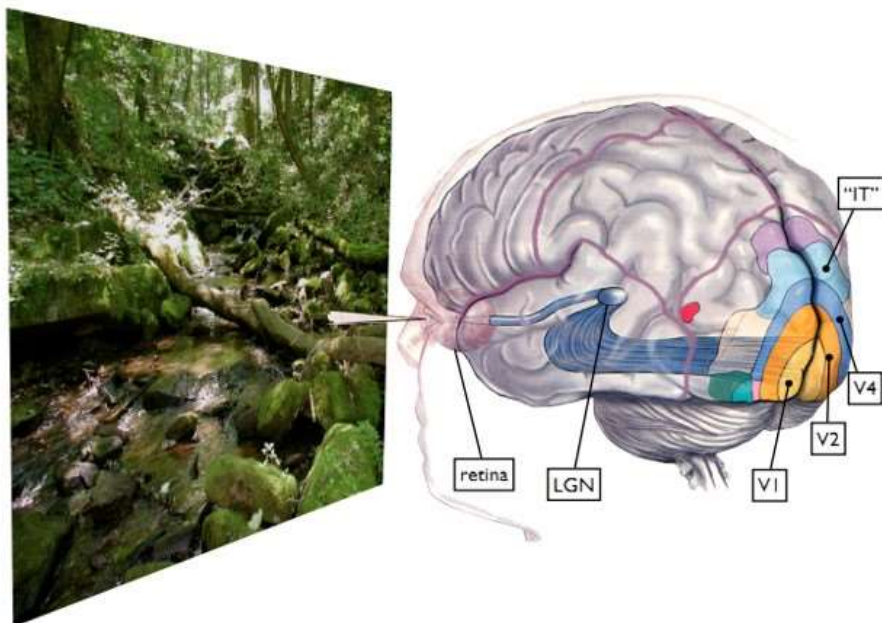
Alternating optimization:

1. Fix dictionary  $\phi$ , optimize  $a$  (LASSO problem) **Harder**
2. Fix activations  $a$ , optimize dictionary  $\phi$  (convex QP problem) **Easy**



## 稀疏表示

### Definition



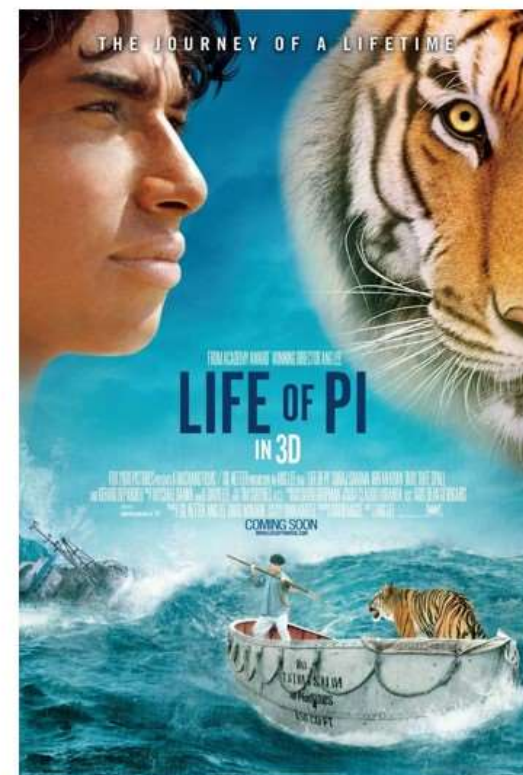
[Olshausen, Field. Nature1996]

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## 稀疏表示

### Definition

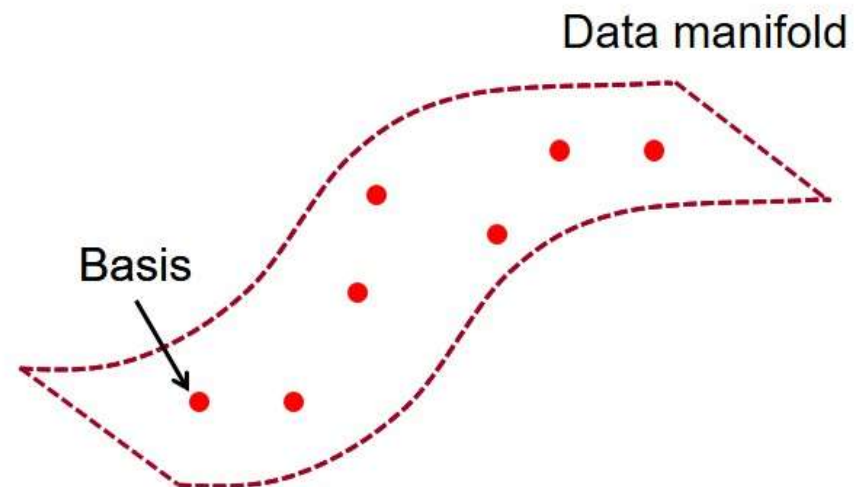
Believing in everything at  
the same time  
is the same as not believing  
in anything at all



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## 稀疏表示

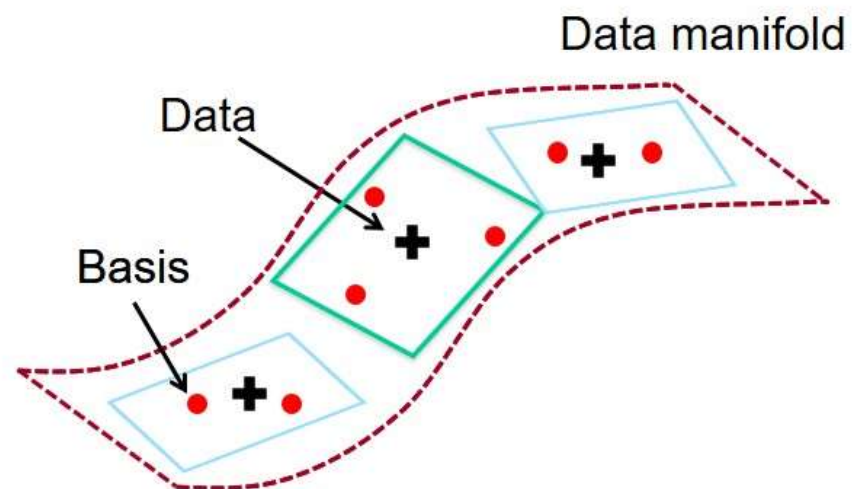
### Definition



- Each basis is somewhat like a pseudo data point – “**anchor point**”
- **Sparsity**: each datum is a sparse combination of neighbor anchors.
- The coding scheme explores the **manifold structure** of data.

## 稀疏表示

### Definition

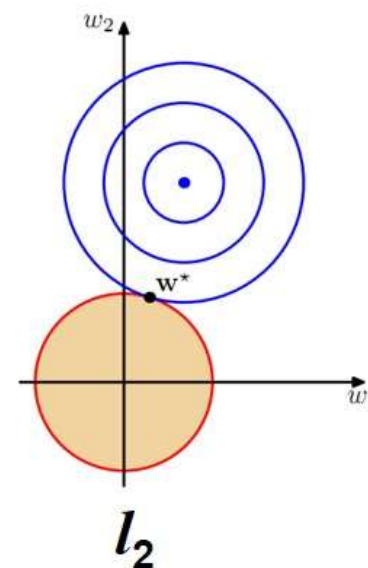
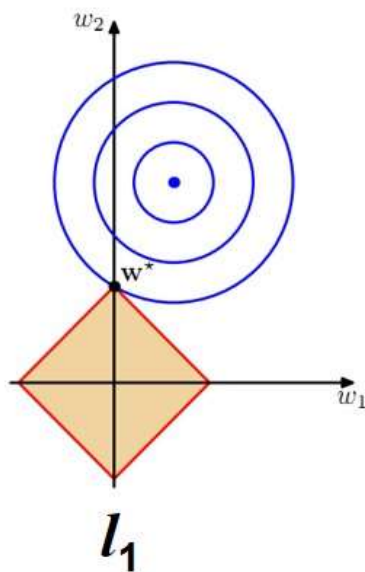


- Each basis is somewhat like a pseudo data point – “**anchor point**”
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## 稀疏表示

### Definition

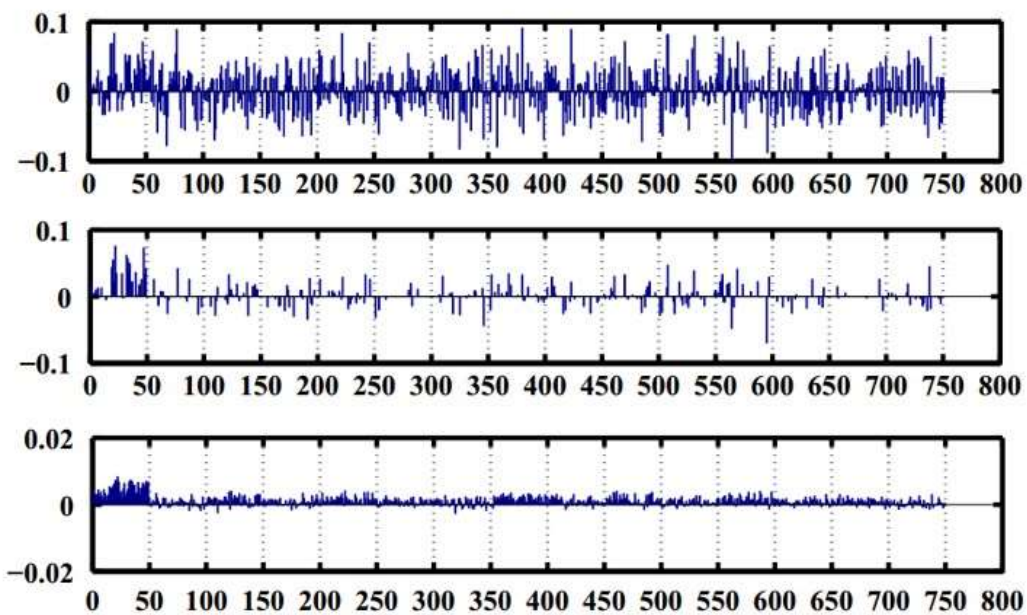
$$\min_{a, \phi} \sum_{i=1}^m \left( \left\| x_j - \sum_{j=1}^k a_{i,j} \phi_i \right\|^2 + \sum_{j=1}^k |a_{i,j}| \right)$$



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## 稀疏表示

### DEMO



LR

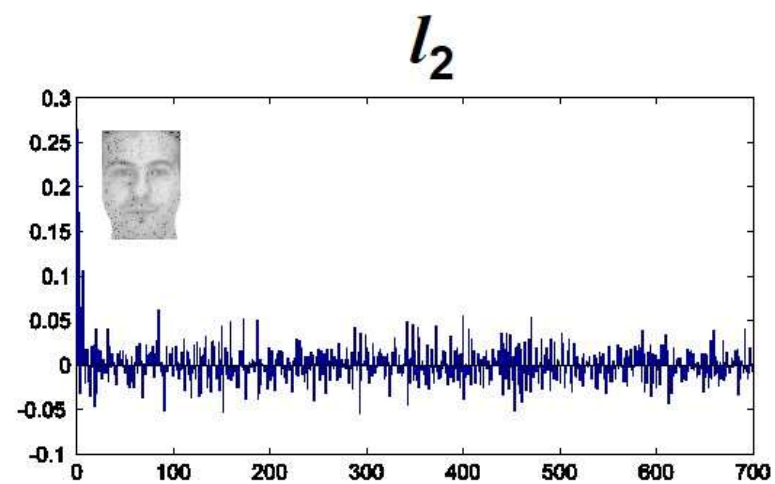
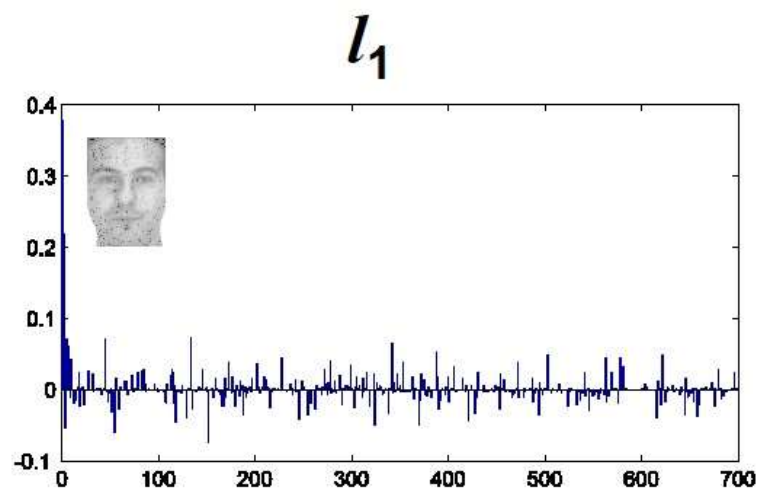
LR-L1

LR-L2



## 稀疏表示

DEMO



Demo

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## 稀疏表示

KL

相对熵是一种标准的用来测量两个分布之间差异的方法

$$\hat{\rho}_j = \frac{1}{m} \sum_{i=1}^m [a_j^{(2)}(x^{(i)})]$$

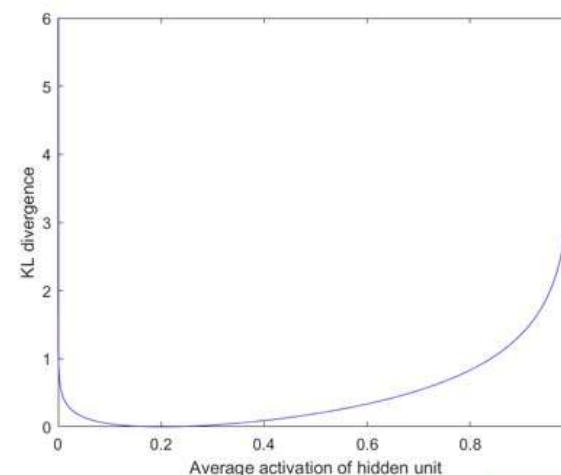
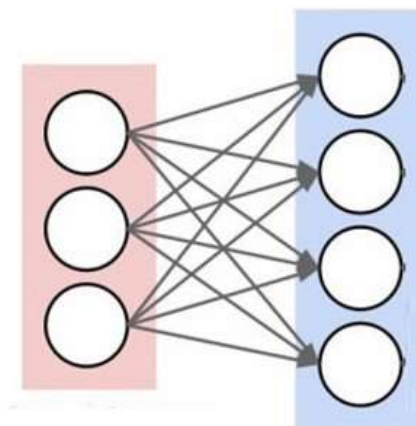
$$\sum_{j=1}^{s_2} \text{KL}(\rho || \hat{\rho}_j) = \sum_{j=1}^{s_2} \rho \log \frac{\rho}{\hat{\rho}_j} + (1 - \rho) \log \frac{1 - \rho}{1 - \hat{\rho}_j}$$

$$J_{\text{sparse}}(W, b) = J(W, b) + \beta \sum_{j=1}^{s_2} \text{KL}(\rho || \hat{\rho}_j)$$

相对熵在  $\hat{\rho}_j = \rho$  时达到它的最小值0

而当  $\hat{\rho}_j$  靠近0或者1的时候, 相对熵则变得非常大 (其实是趋向于 $\infty$ )

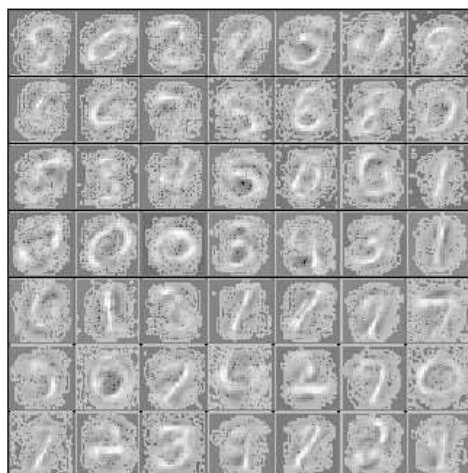
最小化这一惩罚因子具有使得  $\hat{\rho}_j$  靠近  $\rho$  的效果



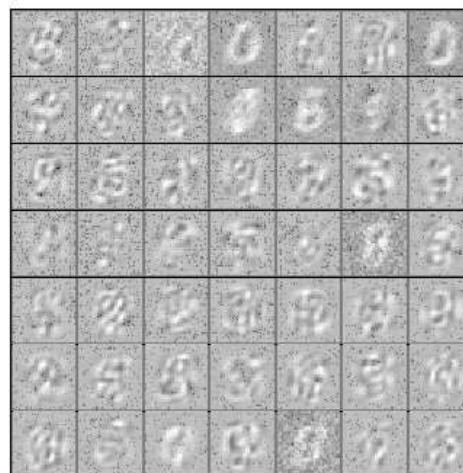
37

## 稀疏表示

KL



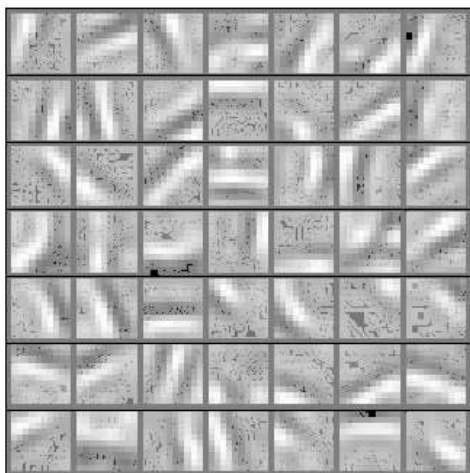
sparsity



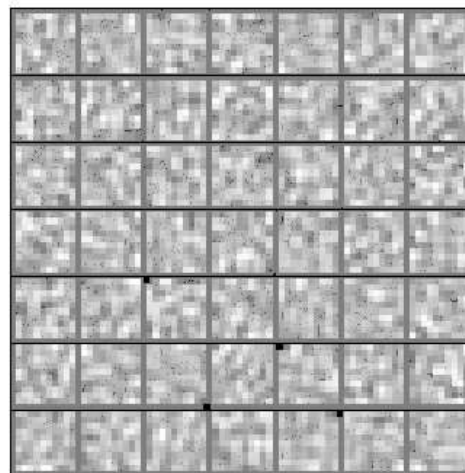
No sparsity

## 稀疏表示

KL



sparsity



No sparsity

Demo

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**THANK YOU**  
**Q&A**