Force ($\mathbf{F}$, $\mathbf{f}$)
$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}$ *point charge*
$\mathbf{F} = Q\mathbf{E}$ *e-field*
$\mathbf{f} = \sigma\mathbf{E}$ *per unit A*
$\mathbf{f} = \frac{1}{2}\sigma(\mathbf{E}_{abv} + \mathbf{E}_{blw})$ *general*
$\mathbf{F}_{mag} = Q(\mathbf{v}\times\mathbf{B})$ *B-field*
$\mathbf{F} = Q(\mathbf{E} + (\mathbf{v}\times\mathbf{B}))$ *E+B fields* $\mathbf{F}_{mag} = \int I(d\mathbf{l}\times\mathbf{B})$ *sgmt wire w/ I*
$\mathbf{F}_{mag} = \int I(d\mathbf{l}\times\mathbf{B})$ *sgmt wire w/ I*
$\mathbf{F}_{mag} = I\int(d\mathbf{l}\times\mathbf{B})$ *sgmt, const I*
$\mathbf{F}_{mag} = \int(\mathbf{v}\times\mathbf{B})\sigma da = \int(\mathbf{K}\times\mathbf{B})da$ *surf current, avg B*
$\mathbf{F}_{mag} = \int(\mathbf{v}\times\mathbf{B})\rho d\tau = \int(\mathbf{J}\times\mathbf{B})d\tau$ *vol I*
$\mathbf{J} = \sigma\mathbf{f} = \mathbf{f}/\rho$ *vol I dens*
$\mathbf{f} = \mathbf{f_s} + \mathbf{E}$
circuit
Electric Field ($\mathbf{E}$)
$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i$ *point charges*
$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}dq$ *continuous q-dist*
$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}dl'$ *linear q-dist*
$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}da'$ *surface q-dist*
$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}d\tau'$ *volume q-dist*
$\oint\mathbf{E}\cdot d\mathbf{a} = \frac{1}{\epsilon_0}Q_{enc}$ *G-symm q-dist*
$\mathbf{E} = -\nabla V$ *e-potential*
$\mathbf{E} = \mathbf{0}$ *inside conductor*
$\mathbf{E} \neq \mathbf{0}$ *cavity in conductor*
$\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{n}$ *just outside conductor*
$\mathbf{E_{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}(3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p})$ *dipole, coordinate-free*
$\nabla\times\mathbf{E} = 0$ *always*
$E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}$ *any surf q*
$\mathcal{E} = \oint\mathbf{E}\cdot d\mathbf{l}$ *emf*
$\oint\mathbf{E}\cdot\, d\mathbf{l} = -\int\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{a}$ *Farraday*
$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ *Farraday*
$\mathbf{E} = -\frac{1}{4\pi}\int\frac{(\partial\mathbf{B}/\partial t)\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}d\tau$ *E frm just chngng B*
Magnetic Field ($\mathbf{B}$)
$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}I\int\frac{d\mathbf{l}'\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$ *steady line I*
$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{K}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}da'$ *steady surf I*
$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}d\tau'$ *steady vol I*
$\oint\mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{enc}$ *A-symm steady I*
$\oint\mathbf{B}\cdot d\mathbf{l} = \mu_0\int\mathbf{J}\cdot d\mathbf{a}$ *A-symm steady I*
$\nabla\cdot\mathbf{B} = 0$ *always*
$\mathbf{B} = \nabla\times\mathbf{A}$ vec potential
$\mathbf{B}_{abv} - \mathbf{B}_{blw} = \mu_0(\mathbf{K}\times\hat{\mathbf{n}})$ *any at surf I*
$\mathbf{B}_{dip}(\mathbf{r}) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\theta})$ *due to dipole*
$\mathbf{B}_{dip}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}(3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m})$ *coord-free*
$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ *Farraday*
$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$ *corrective* **Magnetic Vector Potential** ($\mathbf{A}$)
$\nabla\cdot\mathbf{A} = 0$ *always*
$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}}d\tau'$ *finite vol I dens*
$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\int\frac{1}{\mathscr{r}}d\mathbf{l}'$ *vol I dens*
$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{K}}{\mathscr{r}}da'$ *vol I dens*
$\frac{\partial\mathbf{A}_{abv}}{\partial n} - \frac{\partial\mathbf{A}_{blw}}{\partial n} = -\mu_0\mathbf{K}$ *any at surf I*
$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint(r')^n P_n(\cos\alpha)d\mathbf{l}'$ alph src to field
$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}(\frac{1}{r}\oint d\mathbf{l}' + \frac{1}{r^2}\oint r'\cos\alpha d\mathbf{l}'$
$+ \frac{1}{r^3}\oint(r')^2(\frac{3}{2}\cos\alpha^2 - \frac{1}{2})d\mathbf{l}' + \cdots)$ *localized I-dist at distant pts*

$\mathbf{A}_{dip}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}$ *dipole approx*
$\mathbf{A}_{dip}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$ *dip at origin, sphrcl*
Magnetic Dipole Moment
$\mathbf{m} \equiv I\int d\mathbf{a} = I\mathbf{a}$ *I-loop w/ vec A*
$\mathbf{m} = \frac{1}{2}\int(\mathbf{r}\times\mathbf{J})d\tau$ *vol I*
Electromotive Force ($\mathcal{E}$)
$\mathcal{E} \equiv \oint\mathbf{f}\cdot d\mathbf{l} = \oint\mathbf{f}_s\cdot d\mathbf{l}$ *circuit*
$\mathcal{E} = V$ *ideal src / terminals*
$\mathcal{E} = vBh$ *wire thru B*
$\mathcal{E} = -\frac{d\Phi}{dt}$ *loop w/ chngng flx*
$\mathcal{E} = \oint\mathbf{E}\cdot d\mathbf{l}$ *E-field*
$\mathcal{E} = -L\frac{dI}{dt}$ *I in loop*
Flux ($\mathbf{\Phi}$)
$\mathbf{\Phi} \equiv \int\mathbf{B}\cdot d\mathbf{a}$ *general*
$\Phi_2 = M_{21}I_1$ *two loops*
$\Phi = LI$ *loop*
Mutual Inductance (M_{21})
Self Inductance (L)
$\Phi = LI$ *loop*
$\mathcal{E} = -L\frac{dI}{dt}$ *I in loop*
Displacement Current ($\mathbf{J}_d$)
$\mathbf{J}_d \equiv \epsilon_0\frac{\partial\mathbf{E}}{\partial t}$ *definition*
Electric Potential (V)
$V(\mathbf{r}) \equiv \int_{\mathcal{O}=\infty}^{\mathbf{r}}\mathbf{E}\cdot d\mathbf{l}$ *def*
$V = \sum_i V_i$ *linsup*
$V = \frac{1}{4\pi\epsilon_0}\sum_i\frac{q_i}{\mathscr{r}_i}$ *point charges*
$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}}dq$ *continuous q-dist*
$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}')}{\mathscr{r}}d\tau'$ *volume q-dist*
$V = \frac{1}{4\pi\epsilon_0}\int\frac{\lambda(\mathbf{r}')}{\mathscr{r}}dl'$ *linear q-dist*
$V = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma(\mathbf{r}')}{\mathscr{r}}da'$ *surface q-dist*
$V(\mathbf{r}) = W/Q$ *work*
$\nabla^2 V = -\frac{\rho}{\epsilon_0}$ *Poisson w/ bc's*
$\nabla^2 V = 0$ *no enclosed ρ, uniqueness thms*
$\pm\mathbf{q}$ q_i *above conducting plane(s)*
$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}$
$\int(r')^n P_n(\cos\alpha)\rho(\mathbf{r}')d\tau'$ *distnt q-dist (mp exp)*
$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}(\frac{1}{r}\int\rho(\mathbf{r}')d\tau'$
$+ \frac{1}{r^2}\int r'\cos\alpha\rho(\mathbf{r}')d\tau'$
$+ \frac{1}{r^3}\int(r')^2(\frac{3}{2}\cos\alpha^2))$ *distnt q-dist (mp exp)*
$V_{dip}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$ *no net charge*
$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = -\frac{\sigma}{\epsilon_0}$ *any surf q*
$V = IR$ *ohm*
$P + VI + I^2R$ Joule heating
Current, ($I, \mathbf{I}$)
$\mathbf{I} = \lambda\mathbf{v}$ *moving line charge*
$I = \int_{\mathcal{S}}\mathbf{J}\cdot d\mathbf{a}$ *vol I dens*
$I = \frac{\mathcal{E}}{R}$ *circuits*
$\Phi = LI$ *loop*
Surface Current Density ($\mathbf{K}$)
$\mathbf{K} \equiv \frac{d\mathbf{I}}{dl_{\perp}}$ *I per unit width*
$\mathbf{K} = \sigma\mathbf{v}$ *mobile surf q dens*
Volume Current Density ($\mathbf{J}$)
$\mathbf{J} \equiv \frac{d\mathbf{I}}{da_{\perp}}$ *I per unit area*
$\mathbf{J} = \rho\mathbf{v}$ *mobile vol q dens*
$\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t}$ *local q consvtn*
$\nabla\cdot\mathbf{J} = 0$ *magnetostatics*
$\nabla\times\mathbf{B} = \mu_0\mathbf{J}$ *B field*
$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}$ *vec pot*
$\mathbf{J} = \sigma\mathbf{f}$ *f per unit q*
$\mathbf{J} = \sigma\mathbf{E}$ *resistors, bad conds*
$\mathbf{J} = nfq\mathbf{v}_{ave}$ *mols, e's*
Resistance (R)
$R = \frac{\epsilon_0}{\sigma C}$ *embedded metals*

Velocity ($\mathbf{v}$)
Work (W)
$W = \int_a^b\mathbf{F}\cdot d\mathbf{l}$ *def*
$W = QV(\mathbf{r})$ *e-potential*
$W = \frac{1}{2}\sum_{i=1}^{n}q_i V(\mathbf{r}_i)$ *p-q's, V_i of others*
$W = \frac{1}{2}\int\sigma V da$ *surf-q density*
$W = \frac{1}{2}\int\rho V d\tau$ *vol-q density*
$W = \frac{\epsilon_0}{2}\int E^2 d\tau$ *e-field, all space*
$W_{mag} = 0$ *B forces don't work*
$W = \frac{1}{2}LI^2$ *cranking current*
$W = \frac{1}{2}\oint(\mathbf{A}\cdot\mathbf{I})dl$ *surf I*
$W = \frac{1}{2}\int_{\mathcal{V}}(\mathbf{A}\cdot\mathbf{J})d\tau$ *vol I*
$W = \frac{1}{2\mu_0}\int_{\Re}B^2 d\tau$ *B field*
Energy (W)
$W = \frac{1}{2}\sum_{i=1}^{n}q_i V(\mathbf{r}_i)$ *assembly*
$W = \frac{\epsilon_0}{2}\int E^2 d\tau$ *total, e-field*
$W = \frac{1}{2}\int\rho V d\tau$ *total, q-dist*
$W = \frac{1}{2}CV^2$ *capacitor*
Dipole Moment ($\mathbf{p}$)
$\mathbf{p} \equiv \int\mathbf{r}'\rho(\mathbf{r}')d\tau'$ *gen*
$\mathbf{p} = \sum_i^n q_i\mathbf{r}'_i$ *p-q's*
$\mathbf{p} = q\mathbf{d}$ *equal/opp q's*
$\int_{\mathcal{V}}\mathbf{J}d\tau = d\mathbf{p}/dt$ *confgn of q/I's in volume*
Physical Dipole ($\mathbf{d}$)
Curl of E
$\nabla\times\mathbf{E} = \mathbf{0}$ *static q-dist* **Volume Charge Density** (ρ)
$\nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\rho$ *e-field*
$\nabla^2 V = -\frac{\rho}{\epsilon_0}$ *e-potential*
$\rho = 0$ *inside conductor*
Capacitance (C)
$C \equiv \frac{Q^+}{V^+}$ *def*
Surface Charge (σ)
$\sigma = -\epsilon_0\frac{\partial V}{\partial n}$ *conductor*
e-Pressure
$P = \frac{\epsilon_0}{2}E^2$ *conductor in e-field*
Charge Enclosed (Q_{enc})
$\oint\mathbf{E}\cdot d\mathbf{a} = \frac{1}{\epsilon_0}Q_{enc}$ *G-symm q-dist*
$Q_{enc} = \int\lambda dl = \int\sigma da = \int\rho d\tau$ *general*
$Q_{enc} = Q_{surf}$ *conductor*
$Q_{enc} = -q_{induced}$ *induction*
$Q_{enc} = \sum q_i$ *conductor w/ cavity q's*
Laplace ($\nabla^2 V = 0$)
$V(x, y) = \sum_{n,m,\ldots}(Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$ *planes, "box"*
$V(r, \theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$ *sph*
Cartesian Coefficients (A, B, C, D)
$C = 0$ *term diverges*
$C = \mathcal{F}$ *orthogonal series*
$C = \mathcal{B}$ *boundary conditions*
Spherical Coefficients (A_l, B_l)
$C = 0$ *term diverges*
$C = \mathcal{F}$ *orthogonal series*
$C = \mathcal{B}$ *boundary conditions*
Method of Images ($\pm\mathbf{q}$)
Fourier Method ($\mathcal{F}$)
Legendre Polynomials (P_l, P_n)
$P_l = P_n \equiv \frac{1}{2^l l!}(\frac{d}{dx})^l(x^2 - 1)^l$ *Rodrigues*
Boundary Conditions ($\mathcal{B}$)
$\mathcal{B} = \mathcal{B}(V_{in/out})$ *(sphr) e-pot cont. crss bndry*
$\mathcal{B} = \mathcal{B}(V_0)$ *surf pot specified*
$\mathcal{B} = \mathcal{B}(\sigma_0)$ *surf charge specified*
Zero (0)

Separation vector
$\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$ *general*
Charge differential
$dq = \lambda dl'$ *linear dist*
$dq = \sigma da'$ *surface dist*
$dq = \rho d\tau'$ *volume dist*
Line differential
$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$

Volume differential
$d\tau = s'ds'd\phi dz$ *cylin*
$d\tau = r^2 \sin\theta dr d\theta d\phi$ *sphr*
Boundary Conditions
$E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}$ *any surf q*
$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = -\frac{\sigma}{\epsilon_0}$ *any surf q*
$\frac{\partial \mathbf{A}_{abv}}{\partial n} - \frac{\partial \mathbf{A}_{blw}}{\partial n} = -\mu_0 \mathbf{K}$ *any surf I*

$\mathbf{B}_{abv} - \mathbf{B}_{blw} = \mu_0(\mathbf{K} \times \hat{\mathbf{n}})$ *any surf I*
$V_{in}(R) = V_{out}(R)$ *V cont across boundry*
$V(\theta_1) = V(\theta_2)$ *conductor* $\rightarrow$ *equipotential*
Normal Derivative
$\frac{\partial V}{\partial n} = \nabla V \cdot \hat{n}$ *general*
Dictionary $(\mathcal{D})$
$\mathcal{D} = \mathcal{N}$ *see notes*