eqn

check:

Separable?

$$\frac{\partial y}{\partial x} = g(x)h(y)$$

① separate

$$\frac{\partial y}{h(y)} = g(x)\partial x$$

② Integrate

Bernoulli?

$$\frac{\partial y}{\partial x} + P(x)y = q(x)\,y^a$$

① divide by y^a

② introduce $u = y^{1-a}$

③ find $\partial u/\partial x$

④ Plug in u

⑤ it's linear

Linear?

$$\frac{\partial y}{\partial x} + P(x)y = q(x)$$

① integrating factor

$$\mu(x) = e^{\int P(x)\partial x}$$

② multiply eqn. by IF

③ integrate

④ Simplify

left side becomes

$$\frac{\partial}{\partial x}\left(\mu(x)\cdot y\right)$$

⑤ check for lost solutions!

Exact?

$$M(x,y)\partial x + N(x,y)\partial y = 0$$

where $M_y = N_x$

① $F_x = M, \quad F_y = N$

② $F = \int M\partial x = \int N\partial y$

③ $\int M\partial x$ = something $+ g(y)$

④ F_y = something $+ g'(y) = N$

⑤ find $g'(y) \longrightarrow$ find $g(y)$

⑥ $F = \int M\partial x + g(y)$

Not Exact?

$$M(x,y)\partial x + N(x,y)\partial y = 0$$

$M_y \neq N_x$ or $M, N,$ and P.d.'s not continuous in rectangle around IV

① integrating factor (can make exact)

② $\frac{\partial \mu}{\partial x} = \left(\frac{M_y - M_x}{N}\right)\mu$ ← if $\frac{M_y - M_x}{N}$ not ha[…] y, Simple ODE for μ

③ Solve for μ:

$$\frac{\partial \mu}{\partial x} = P(x)\mu \qquad \text{separable.}$$

Homogeneous?

$$\boxed{\begin{gathered}\frac{\partial y}{\partial x} = f(x,y) \\ f(ax,ay) = f(x,y)\end{gathered}}$$

1. rearrange to $\frac{dy}{\partial x} = f(x,y)$
2. replace x, y w/ ax, ay.
3. if $f(x,y) = f(ax,ay)$, homogeneous.
4. $y = ux$, new dependent variable u
5. $f(x, ux)$ right-hand
6. $\frac{\partial y}{\partial x} = x\frac{\partial u}{\partial x} + u$ left-hand
7. separable.

Exists? Unique?

1. Draw rectangle around IV
2. $\frac{\partial y}{\partial x} = f(x,y)$
3. are $f(x,y)$ and $f_y(x,y)$ Continuous in vicinity of IV?
4. find $f(x,y)$ and $f_y(x,y)$
5. determine continuity in vicinity of IV point. Exists?
6. Construct extra solutions if possible. If >1, not unique.

Explicit Solution?

$$\boxed{y = \phi(x)}$$

Implicit Solution?

$$\boxed{\begin{gathered}F(x,y,y') = 0 \\ \varphi(x,y) = 0\end{gathered}}$$