[25pts] 3. Proofs. Write a complete mathematical proof of the following statements.

(a) If S is a subspace of another subspace $V \subset \mathbb{R}^n$, i.e. $S \subset V$, then the orthogonal complement $V^\perp$ of V is a subspace of $S^\perp$, i.e. $V^\perp \subset S^\perp$.

(b) If $\mathbf{Q}_1 \in \mathbb{R}^{n\times m}$ and $\mathbf{Q}_2 \in \mathbb{R}^{m\times p}$ have orthonormal columns, then the product matrix $\mathbf{Q}_1\mathbf{Q}_2$ does also.

(c) [*Challenge*] If $\mathbf{Q} \in \mathbb{R}^{n\times n}$ has orthonormal columns and is also upper triangular, then $\mathbf{Q}$ must be diagonal. (*Bonus: Does* $\mathbf{Q} = \mathbf{I}$? *Why or why not?*)

Exam 3

MA 405 Section 001 | Vallélian

4/12/17

Directions: This exam has **THREE** (3) problems. Show your work for each problem and justify your answers unless otherwise directed. *Solve the challenge parts at the end!*

[50pts] 1. QR Decomposition and Least-Squares. *[Similar to Ch. 3 Review #14, 33]*

(a) Find the QR decomposition $\mathbf{A} = \mathbf{QR}$, where $\mathbf{Q}$ has orthonormal columns and $\mathbf{R}$ is upper triangular and invertible.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$