MA 341

$$y'' + py' + qy = f(t)$$

Product of
- polynomials
- exponentials
- sine and cosine

$y_p(x)$

$P(x),\ Q(x)$

Quasipolynomials

Method of Undetermined Coefficients

$$P(x)\,e^{ax}$$

$$P(x)\cos bx + Q(x)\sin bx$$

$$e^{ax}\cos bx \;/\; e^{ax}\sin bx$$

$$P(x)e^{ax}\cos bx + Q(x)e^{ax}\sin bx$$

Examples

ex) $y'' + 4y' + 5y = e^x\cos x + x^2 e^x \sin x$

Polynomial degree: 0

Polynomial degree: 2

$1+i$ $\qquad$ $1+i$

bc $\overline{1-i} = 1+i$

complex #'s

i interlude

$(e^x \sin 2x)$

$1+2i$

(2 #'s, 2 terms)

$\max\{0,2\} = 2$

Start w/ these

~~$y_p(x)$~~

$$y_p(x) = P(x)\,e^x\cos x + Q(x)\,e^x \sin x$$

2 degree

3 coefficients

$$\boxed{y_p(x) = (A_0 + A_1 x + A_2 x^2)\, e^{x} \cos x + (B_0 + B_1 x + B_2 x^2)\, e^{x} \sin x}$$

→ not always what you need. Need to be in control.

Find complex conj pair associated w/ left-hand side.

Char eqn of $y'' + 4y' + 5y$

$$r^2 + 4r + 5 = 0$$

roots

$$r = -2 \pm i$$

$$\boxed{y_h(x) = C_1 e^{-2x} \cos x + C_2 e^{-2x} \sin x}$$

$$\boxed{y_h(x) + y_p(x)}$$

ex)

$$\underbrace{y'' + 4y' + 5y}_{-2 \pm i} = \underbrace{e^{-2x}\cos x + x^2 e^{-2x}\sin x}_{-2 \pm i}$$

$-2 \pm i \longleftrightarrow -2 \pm i$

~~$-2 \pm i \rightarrow -2 + i$~~

So we need a correction

$$y_p = \left[(A_0 + A_1 x + A_2 x^2)\, e^{x} \cos x + (B_0 + B_1 x + B_2 x^2)\, e^{x} \sin x \right] x^{k}$$

correction

multiplicity

→ if 2nd-order eqn,

with trig, and there's a corrective x,

k will be 1

ex] $y'' + 4y' + 5y = e^{-2x}\cos x + x^2 e^{-x} \sin x$

Find roots $\Rightarrow$ $\underline{-2 \pm i}$ $\ne$ $-2 \pm i$ $\ne$ $-1 \pm i$

ex] $y'' + py' + qy = f_1(x) + f_2(x)$

split!

$$\left[\begin{array}{l} y'' + py' + qy = f_1(x) \\ y'' + py + qy = f_2(x) \end{array}\right. \xrightarrow{\text{Solve}} \left[\begin{array}{l} y_{p,1}(x) \\ y_{p,2}(x) \end{array}\right.$$

sum

$$y_p(x) = y_{p,1}(x) + y_{p,2}(x)$$

bc $z_{left} = z_{right}$

ex] $y'' + 4y' + 5y = e^{-2x}\cos x \rightarrow y_{p,1}(x) =$

$z_1 = z_2$

$\left(A e^{-2x} \cos x + B e^{-2x} \sin x\right) x$

(bc 0-degree Polynomial)

$y'' + 4y' + 5y = x^2 e^{-x} \sin x$

$-1 \pm i$

$$y_{p,2}(x) = (C_0 + C_1 x + C_2 x^2) e^{-x}\cos x + (D_0 + D_1 x + D_2 x^2) e^{-x} \sin x$$

$\leftarrow$ no x bc $z_{left} \ne z_{right}$

bc 2-degree Polynomial

Now, $y_h + y_{p,1} + y_{p,2}$