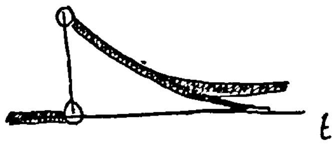
MA 341

Laplace

$f(t)$



Fourier transform

$\int_{-\infty}^{\infty} f(t)\, e^{-i\alpha t}\, dt$

$$\text{DE} \xrightarrow{\text{transform}} \text{Alg. eqn} \qquad \text{Solution(s) of the DE}$$

$$\downarrow \qquad\qquad \uparrow$$

$$\text{Sol'n of Alg. eqn} \longrightarrow \text{inverse transform}$$

$f(t)$

$F(s) = \mathcal{L}\{f(t)\}$

$\mathcal{L} = \int_0^{\infty} f(t)\, e^{-st}\, dt$

$\hookrightarrow F(s)$ Integral transforms

① $f(t)e^{at}$

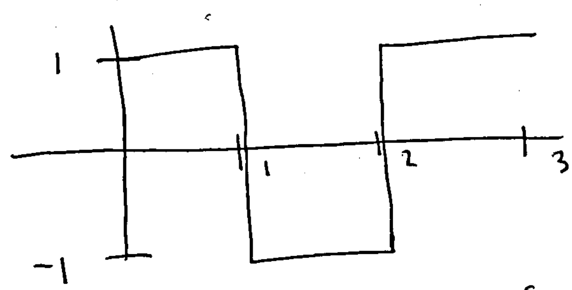
$\mathcal{L}\{f(t)e^{at}\} = F(s-a)$

$a>0$: shift to the right

$\frac{dy}{dt} + y = 0$

$\frac{dy}{dt} + y = f(t)$ input

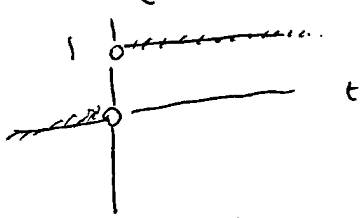
Solution (typically subject to the 0 initial condition) is viewed as output



Laplace transform of functions with discontinuities/continuous functions

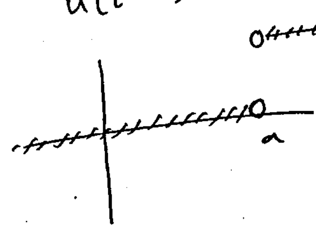
$u(t)$ unit step function

$$\eta(t) = u(t) = \begin{cases} 0 & \text{if } t<0 \\ 1 & \text{if } t>0 \end{cases}$$



$\mathcal{L}\{u(t)\} = \frac{1}{s}$

$u(t-a)$



$\mathcal{L}\{u(t-a)\}$

$f(t)\, u(t)$

$f(t-a)\, u(t-a)$

$\boxed{\text{Theorem}}$

$F(s) = \mathcal{L}\{f(t)\}$

$\equiv \{f(t)\, u(t)\}$

$F(s) = \mathcal{L}\{f(t)\}$

Then, $\mathcal{L}\{f(t-a)u(t-a)\} = F(s)e^{-as}$

$$\Pi_{a,b}(t) = \begin{cases} 0 & \text{if } t<a \\ 1 & \text{if } a<t\le b \\ 0 & \text{if } t>b \end{cases}$$

$\Pi_{a,b}(t) = u(t-a) - u(t-b)$ (A)

Then

$\mathcal{L}\{f(t-a)u(t-a)\}$

$= \int_0^{\infty} f(t-a)\, u(t-a)\, e^{-st}\, dt$

$= \int_0^{\infty} f(t-a)u(t-a)\, e^{-s(t-a)} e^{-sa}\, d(t-a)$

$e^{-sa} \int f(\tau)\, e^{-s\tau}\, d\tau$

$t-a$