

then

[25pts] 3. Proofs. Write a complete mathematical proof of the following statements.

- (a) If S is a subspace of another subspace $V \subset \mathbb{R}^n$, i.e. $S \subset V$, then the orthogonal complement V^\perp of V is a subspace of S^\perp , i.e. $V^\perp \subset S^\perp$.
- (b) If $Q_1 \in \mathbb{R}^{n \times m}$ and $Q_2 \in \mathbb{R}^{m \times p}$ have orthonormal columns, then the product matrix $Q_1 Q_2$ does also. $\hookrightarrow C_1(Q_2) \in \text{Null}(Q_1^T)$
- (c) [Challenge] If $Q \in \mathbb{R}^{n \times n}$ has orthonormal columns and is also upper triangular, then Q must be diagonal. (Bonus: Does $Q = I$? Why or why not?) \hookrightarrow

Laplace Transform

→ diff eq

$$Y = Y(t) \xrightarrow{\mathcal{L}} Y(s) = \mathcal{L}(Y)$$

$\underline{w}^T \underline{v} = 0 \quad \hat{s} + \hat{g} = \hat{f} + \hat{g}$
 $\underline{w}^T \underline{s} = 0 \quad c\hat{f} = c\hat{f}$

$$\begin{aligned} \text{ex)} \quad f(x) &= 1 \\ F &= Z(1) \\ F(s) &= \int_0^\infty \end{aligned}$$

differentiation \xrightarrow{L} Polynomial Multiplication
Polynomial multiplication \xrightarrow{L} differentiation

diff eq $\xrightarrow{\gamma}$ algebraic equation

solve

solve

~~13 need~~

$$f = f(t)$$

$$F = \mathcal{L}f = \hat{f}$$

$$F(s) = (\mathcal{L}(f))(s)$$

$$= \mathcal{L}(f(t))(s)$$

the hard
part

Def

$$f = f(t) \quad [0, \infty) \rightarrow \mathbb{R}$$

2

only \leq left.

$$\mathcal{L}F = F : \mathcal{L} \rightarrow \mathbb{R}$$

new domain

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$R^T = R, \quad R^{-1} \text{ exists}$$

$$R = \begin{bmatrix} q_1^T a_1 & q_1^T a_2 & 0 \\ q_2^T a_1 & q_2^T a_2 & 0 \\ q_3^T a_1 & q_3^T a_2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) Find the QR decomposition $A = QR$, where Q has orthonormal columns and R is upper triangular and invertible.
 [50pts] 1. QR Decomposition and Least-Squares. [Similar to Ch. 3 Review #14, 33]

Directions: This exam has THREE (3) problems. Show your work for each problem and justify your answers unless otherwise directed. Solve the challenge parts at the end!

4/12/17

MA 105 Section 001 - Vallian

trans	cos t	trans	Exam 3	e^{at}
$\frac{1}{s-a}$	$\frac{s}{s^2+a^2}$	$\frac{a}{s^2+a^2}$	$\frac{t^{n-1}}{s^n}$	$\frac{1}{s}$
$(-\infty, \infty)$	$(0, \infty)$	$(0, \infty)$	$(0, \infty)$	$(0, \infty)$
$F(s)$	(t)	(t)	(t)	(t)

e^{t^2} ! have 2
 \hat{e}^{t^2}
 \neq