MA 341

(1) check if LHS/RHS roots =

$$y'' - 2y' + 5y = 2\sin(x)$$

RHS $\rightarrow$ $P(x)\, e^{ax} \begin{array}{c} \cos bx \\ \hline \sin bx \end{array}$

Polynomial not relevant here.

yields $a \pm i b$

a: e arg

b: trig argument

Char eqn

$$r^2 - 2r + 5 = 0$$

$$(r-1)^2 + 4 = 0$$

$$(r-1)^2 = -4$$

$$r - 1 = \pm 2i$$

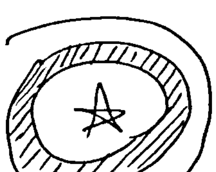
roots $\rightarrow$ $r = 1 \pm 2i$

$\pm i$

don't match (good news!)

(2) Find Y_p

- Y_p should keep structure of the RHS!
- Sine and Cosine come together!



generate Y_p $\rightarrow$

$$\boxed{Y_p = A\cos x + B\sin x}$$

– Substitute this in for y

find

$$Y_p' = B\cos x - A\sin x$$

$$Y_p'' = -A\cos x - B\sin x$$

Plug in: $5y_p - 2y_p' + y_p''$

$5y_p = 5A\cos x + 5B\sin x$

$-2y_p' = 2B\cos x + 2A\sin x$

$= (4A - 2B)\cos x + (2A + 4B)\sin x$

equate to RHS: since $5y_p - 2y_p' + y_p'' = 20\sin x$,

Solve for A, B: $(4A - 2B)\cos x + (2A + 4B)\sin x = 20\sin x$

$\to 4A - 2B = 0$

$\to 2A + 4B = 20$ $\Rightarrow A = 2,\ B = 4$

Plug A, B into y_p: $\boxed{y_p = 2\cos x + 4\sin x}$

ex) $y'' - 2y' + 5y = 20xe^{x}\sin x$ $\quad 1 \pm i$ ①

$20xe^{x}\sin 2x$ $\quad 1 \pm 2i$ ②

$r = 1 \pm 2i$

$20xe^{2x}\sin x$ $\quad 2 \pm i$ ③

① complx pairs $\neq$

$P(x)$ degree 1

exp, sin

$\to \underbrace{(\quad)e^{x}\sin x + (\quad)}_{\text{degree 1 polynomials}} e^{x}\cos x$

$= (A_1x + A_2)e^{x}\sin x$

$+ (B_1x + B_2)e^{x}\cos x$

③

$(A_1x + A_2)e^{2x}\sin x + (B_1x + B_2)e^{2x}\cos x$

② $\left[(A_1x + A_2)e^{x}\sin 2x + (B_1x + B_2)e^{x}\cos 2x\right] x^{k}$, here $k=1$

bc complx pairs match

$$y'' + py' + qy = f(x)$$

Variation of Parameters (á lá Legrange)

1) $y'' + py' - qy = 0 \quad (**)$

Find:

$y_1(x)$, $y_2(x)$ — two independent solutions of (x, y)

2) $y_h(x) = \underbrace{c_1 \, y_1(x) + c_2 y_2(x)}_{\text{constants: } c_1,\ c_2}$

make the constants variables!

Now,

3) For $(*)$

Look for solutions in form

$$y = u_1(x) y_1(x) + u_2(x) y_2(x)$$

$$y' = u_1(x) y_1'(x) + u_2(x) y_2'(x) + \boxed{u_1'(x) y_1(x) + u_2'(x) y_2(x)}$$

set $= 0$ (worry why later)

$$y'' = u_1'(x) y_1'(x) + u_2'(x) y_2'(x) + u_1(x) y_1''(x) + u_2(x) y_2''(x)$$

4) Substitute in y, y', y''

$$q y + p y' + y'' = f(x)$$

$\Downarrow$

$$\underbrace{u_1'(x)\, y_1'(x) + u_2'(x)\, y_2'(x)}_{y''}$$

$$\boxed{\begin{aligned} &+ u_1(x)\, y_1''(x) + u_2(x)\, y_2''(x) \\ &+ p u_1(x) y_1'(x) + p\, u_2(x) y_2'(x) \\ &+ q(u_1(x)\, y_1(x)) + q\, u_2(x)\, y_2(x) \end{aligned}} = 0$$

$$= f(x)$$

$$u_1(x)\left[y_1'' + p(y_1') + q(y_1) \right]$$
$$= 0$$

↳ bc y_1 solves $(**)$

recall ~~other~~ boxed "$=0$" expression

$\triangleright \quad u_1'(x) \cdot y_1'(x) + u_2' y_2'(x) = f(x)$

$\triangleright \quad u_1'(x) y_1(x) + u_2'(x) y_2(x) = 0$