MA 341

Laplace, II

Recap

$$f(t) \xrightarrow{\mathcal{L}} F(s) = (\mathcal{L}f)(s) = \mathcal{L}(f(t))(s) = \hat{f}(s)$$

$$\int_0^\infty f(t) \cdot e^{-st} \, dt$$

| $f$ | $1$ | $t^n$ | $e^{at}$ | $\cos at$ | $\sin at$ |
|---|---|---|---|---|---|
| $F$ | $1/s$ | $\frac{n!}{s^{n+1}}$ | $\frac{1}{s-a}$ | $\frac{s}{s^2+a^2}$ | $\frac{a}{s^2+a^2}$ |
| $\mathcal{D}$ | $(0,\infty)$ | $(0,\infty)$ | $(0,\infty)$ | $(0,\infty)$ | $(0,\infty)$ |

$\mathcal{L}$

ex] $y = y(t)$

$y'' - y = -2t \qquad y(0) = 0 \qquad y'(0) = 2$

$y(t) \xrightarrow{\mathcal{L}} Y = Y(s)$

solve

diff eqn $\longrightarrow$ alg eqn

easy

$\mathcal{L}$ Properties $\left(f(t) \Longrightarrow F(s)\right)$ $\mathcal{D} = (\alpha, \infty)$

① exp. multiplication $\xRightarrow{\mathcal{L}}$ translation

$$\mathcal{L}(e^{at} f(t)) \Longrightarrow F(s-a) \longrightarrow \mathcal{D} = (\alpha + a, \infty)$$

$s - a > \alpha$

$\Downarrow$

$s > \alpha + a$

② nth order "differentiation" $\xRightarrow{\mathcal{L}}$ nth order "Polynomial multiplication"

$$\mathcal{L}(f^{(n)}(t)) \Longrightarrow s^n \cdot F(s)$$

correction terms (from integration by parts)

$$\left\{ \begin{array}{l} - s^{n-1} \cdot f(0) \\ - s^{n-2} \cdot f^{(1)}(0) - \dots - s^0 f^{(n-1)}(0) \end{array} \right.$$

ex

$$\left.\begin{array}{l} y'' - y = -2t \\ \hline y(0) = 0 \\ y'(0) = 2 \end{array}\right\} \xRightarrow{\mathcal{L}} \mathcal{L}(y'' - y) = \mathcal{L}(-2t)$$

$$\mathcal{L}(y'') - \mathcal{L}(y) = -2\,\mathcal{L}(t^1)(s)$$

due to linearity

$$\mathcal{L}(y'') - Y(s) = -2\left(\frac{1}{s^2}\right)$$

$$s^2 \cdot Y \underbrace{- sY(0) - s^0 y'(0)}_{\text{correction terms}} - Y$$

from the table

$$s^2 Y - 0 - 2 - Y = \frac{-2}{s^2}$$

$$Y(s^2 - 1) = 2 - \frac{2}{s^2}$$

$$\boxed{Y = \left(\frac{2}{s^2 - 1}\right)\left(1 - \frac{1}{s^2}\right)}$$

use table!

$$\boxed{Y = \frac{2}{s^2}} = 2 \cdot \frac{1}{s^2} = 2 \cdot \frac{1!}{s^{1+1}}$$

So, $Y(s) = 2 \cdot \mathcal{L}(t^1) = \mathcal{L}(2t)$, so $\xrightarrow{\mathcal{L}^{-1}} \boxed{y(t) = 2t}$