Homogeneous Systems

$$\frac{\partial x}{\partial t} = Ax$$

① Find $A - \lambda I$

② $\det|A - \lambda I|$

③ solve for λ_i

Real λ

① if multiplicity = 1,

② Plug each λ_i into $A - \lambda I$

③ solve $\begin{bmatrix} A - \lambda I \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

For members n_i of $\vec{n}$

④ $\vec{n} = \begin{bmatrix} n_1 \\ f(n_1) \end{bmatrix}$, e.g. (Plug in the best n_1)

⑤ gen solution

$$\boxed{\vec{x}(t) = c_1 e^{\lambda_1 t} \vec{n}_1 + c_2 e^{\lambda_2 t} \vec{n}_2}$$

① if Multiplicity > 1,

④ Find $\vec{n}_i$'s as above

⑤ solve $\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} p_1 \\ \vdots \\ p_2 \end{bmatrix} = \begin{bmatrix} n_1 \\ \vdots \\ n_n \end{bmatrix}$

for elements of $\vec{p}$

$$\vec{p} = \begin{bmatrix} p_1 \\ \vdots \\ f(p_1) \end{bmatrix} \Rightarrow$$

⑥ Pick a p_1 that best solves matrix

⑦ general solution

$$\boxed{\vec{x}(t) = c_1 e^{\lambda_1} \vec{n}_1 + c_2\left(e^{\lambda_1 t} \cdot t\, \vec{n}_1 + e^{\lambda_1 t} \vec{p}_1\right)}$$

Complex λ

⓪ Solve for $\vec{n}$'s (or just $\vec{n}_1$)

① write out $\vec{x}_1(t) = c_1 e^{\lambda_1 t} \vec{n}_1$

② $e^{\lambda_1 t} = \cos(\lambda_1) + i\sin(\lambda_1)$

③ split $(\quad)\vec{n}_1$ into Re, Im

④ $\vec{x}_1(t) = u(t) + iv(t)$

⑤ general solution

$$\boxed{\vec{x}(t) = C_1 \vec{u}(t) + C_2 \vec{v}(t)}$$

determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = a\begin{vmatrix} e & f \\ h & k \end{vmatrix} - b\begin{vmatrix} d & f \\ g & k \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

diff eq

Non-Homogeneous Systems

Undetermin'd Coefficients

$$\vec{x}' = A x + \begin{bmatrix} QP_1 \\ QP_2 \end{bmatrix}$$

① solve homogeneous version:

$$\vec{x}_c = c_1 e^{\lambda_1 t} \vec{n}_1 + c_2 e^{\lambda_2 t} \vec{n}_2$$

② $$\vec{x}_p = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_2 \end{bmatrix} + t \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} + \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \cos(3t) + \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} e^{5t} + \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} \sin(3t)$$

Construct general (matrix) quasipolynomial incorporating all elements of given QP

③ $x(t) = \vec{x}_c + \vec{x}_p$

④ Plug into $\vec{x}' = A x$

VoP

① find $\vec{x}_c = c_1 (e^{\lambda_1 t} \vec{n}_1) + c_2 (e^{\lambda_2 t} \vec{n}_2)$

Varia
Par

$\vec{x} = A$

① solve
for li

e.g. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \rightarrow$

② drop

R

e.g.

③ expres
in t

e g

④ split

e.g.

⑤ general

4. split $\vec{u}$ into [redacted]: Re, Im

e.g. $\vec{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + i\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

5. general solution

$x(t) = c_1\Big(e^{\alpha t}$

$\boxed{\text{VoP}}$

1. find $\vec{x}_c = c_1\left(e^{\lambda_1 t}\vec{n}_1\right) + c_2\left(e^{\lambda_2 t}\vec{n}_2\right)$ ...

2. squish columns $\left(e^{\lambda_1 t}\vec{n}_1\right), \left(e^{\lambda_2 t}\vec{n}_2\right)$, etc. into $X(t)$

3. Find $X^{-1}(t)$

4. $x(t) = X(t)c + X(t)\int X^{-1}(t)\, f(t)\,dt$

5. IVP: $x(t_0) = x_0$

$$x(t) = X(t)X^{-1}(t_0)x_0 + X(t)\int_{t_0}^{t} X^{-1}(s)f(s)\,ds$$