1.2

$$F(x, y, y', \dots, y^{(n)}) = 0$$

single ODE

x: indepentent variable
y: dependent variable

A solution is a function $y = \phi(x)$:

such that $F(x, \phi(x), \phi'(x), \dots, \phi^{(n)}(x))$ [redacted]
equals 0 identically.

ex $y' + y = 0$ [redacted] $\iff \dfrac{dy}{dx} + y(x) = 0$

Q: is $\phi(x)$ [redacted] $= 0$ a solution?

$\phi'(x) = 0$

$$y' + y = 0$$
$$\phi'(x) + \phi(x) = 0$$
$$0 + 0 = 0$$
$$0 = 0$$

Yes.

Q: is $\phi(x) = e^{-x}$ a solution?

$\phi'(x) = -e^{-x}$

$y' + y = 0$
$\phi'(x) + \phi(x) = 0 \longrightarrow e^{-x} - e^{-x} = 0$

$0 = 0$ Yes.

a: $\phi(x) = e^x$?

$$\phi'(x) + \phi(x) = 0$$

$$e^x + e^x = 2e^x \neq 0$$ <u>No.</u>

(Not a solution)

in fact, any

$$\phi(x) = Ce^{-x}$$

is solution (for constant C)

<u>ex</u> $y'' + y = 0$ (2nd-order ODE)

[me: trig] 1) $\phi(x) = 0$, $\phi'(x) = 0$, $\phi''(x) = 0$

$0 = 0$

$\phi(x) = 0$ is a solution

2) $\phi(x) = \sin x$

$\phi'(x) = \cos x$

$\phi''(x) = -\sin x$

[redacted]

$\sin x - \sin x = 0 = 0$

$\phi(x) = \sin x$ is a solution.

y' [redacted] $= 3y^{2/3}$ (1st-order ODE)

1) $\phi(x) = 0$: $\phi'(x) = 0$: $0 = 0 \cdot 0^{2/3} = 0$

solution

2) $\phi(x) = x^3$

$\phi'(x) = 3x^2$ [redacted]

$3x^2$ [redacted] $= 3(x^3)^{2/3} = 3x^2$

$\phi(x) = x^3$ is a solution.

How ~~should~~ could these solutions be found?

1st-Order ODE's

$$F(x, y, y') = 0 \qquad \text{ex} \quad x - y + (y')^3 = 0$$

Assume: this eqn. can be simplified: rewritten as $y' = f(x, y)$

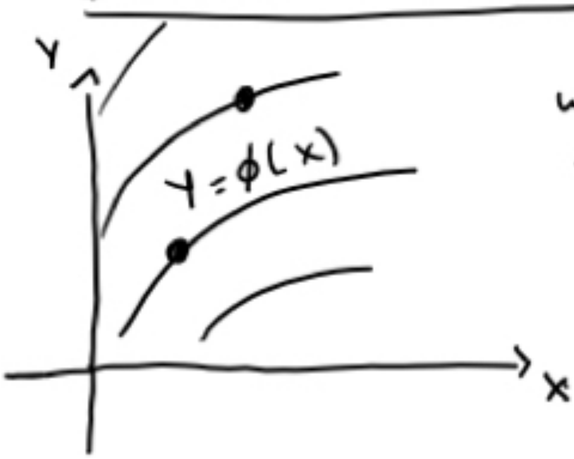
$$(y')^3 = -(x - y)$$

ex $y' = f(x, y) = \sqrt[3]{\text{[redacted]}\ y - x}$

▷ When do solutions of $y' = f(x, y)$ exist?

related to that:

Initial Value Problem



what mechanism singles out solutions from possible set? (Infinitely many available.)

$Y = \phi(x)$

$y' = y$

Given two pieces of information:

- an eqn. $y' = f(x, y)$
- a condition $y(x_0) = y_0$

is there a solution $y = \phi(x)$?

want $y' = f(x)$ that satisfies condition $y(x_0) = y_0$

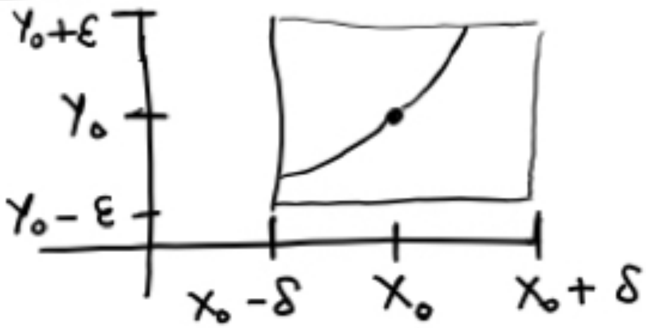
<u>ex)</u> $y' + y = 0$ Solve this...
$y(0) = 1$ ... Subject to this.

$Y = 0$ does not solve the IVP

$Y = e^{-x}$ <u>does</u> solve the IVP

Initial Value Problem:

– $y' = f(x,y)$ subject to $y(x_0) = y_0$



If $f(x,y)$ and $\dfrac{\partial f}{\partial y}(x,y)$ are continuous in a (sufficiently small) rectangle around (x_0, y_0), the IVP has a unique solution $y = \phi(x)$ for $x - a < x < x + a$, where a is a sufficiently small positive real number.

<u>A simpler condition</u>: $f(x,y)$ is continuously differentiable in the rectangle (often easier to check)

ex $\quad y' + y = 0, \quad y(0) = 1$

$$y' = -y$$

$$f(x,y) = -y$$

continuously differentiable everywhere (Calc III)

ex| $y' = 3y^{2/3}$, $y(0) = 0$

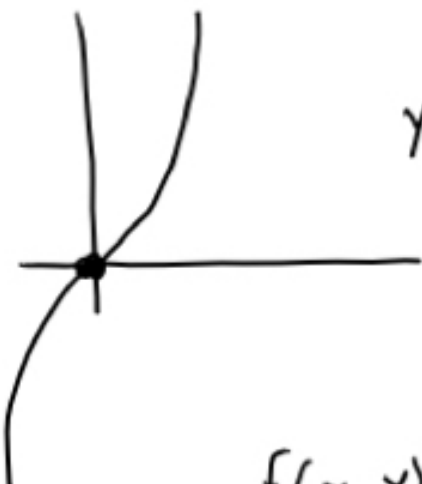
1) $y = 0$ (solution)

2) $y = x^3$ (solution)

(uniqueness is about this: the solution curves do not intersect.)

1 sol'n 2 sol'ns

2) $y = x^3$



$y=0$ and $y=x^3$ intersect at the origin (break the uniqueness property). Can't do IVP!

$f(x,y) = 3y^{2/3}$ okay

$\frac{\partial f}{\partial y} = 2y^{-1/3} = \frac{2}{\sqrt[3]{y}}$ ← DNE @ 0 — not continuous

neither version of IVP prerequisite is satisfied; can't do it. Thus, no <u>unique</u> solution (as per IVP).