MA 341 w/ Dmitry Zenkov

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3 midterms, 1 final
"webworks" ≈ webassign for HW

expect an email. If not received by ~~Fri~~ Thurs morning, email!

Differential Equations

1) Prerequisites: MA 242 (partials, continuity), MA 241 (more important: integration techniques) (symbolic manipulation with formulae)

Equation ("to be solved")

- algebraic eqns: $5x + 10 = 0$ (linear)
  - quadratic: $x^2 - 2x + 4 = 0$

- difficult to solve: $x + 2\sin x = 0$

identifying and classifying eqns by type to know how to solve them

	Solt'n	
$x^3 - 1$	real:	$x = 1$
(simple cubic)	complex:	2 more

Functional Equations: unknowns are functions

ex)

$$y^2(x) = 1 - \sin^2(x)$$

$y(x)$ is an unknown function

$\hookrightarrow y(x) = \pm \sin(x)$

Differential Equations are functional equations that contain derivatives of the unknown quantity

ex) $y(x) = ?$

$$\frac{dy}{dx} = y(x) \quad \text{or} \quad \frac{dy}{dx} = y$$

Origins

- Classical Mechanics
- Physics
- Engineering

1. Ordinary Differential Equations (ODE's)
2. Partial Differential Equations (PDE's)

1. ODE's: unknown functions are functions of a single variable

2. PDE's: unknown functions depend on two or more variables

ex| traveling waves on strings:

in this case, speed of sound

u-displacement

$u(x, t)$: unknown!

wave eqn., right? $\longrightarrow$ $\underbrace{\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}}_{\text{PDE}}$

Sorta. Satisfies it.

- yup: wave eqn.

Order of an ODE

1) $\dfrac{dy}{dx} = y(x)$

2) $\dfrac{d^2y}{dx^2} = y(x)$

The order of an ODE is the highest order of the derivative of the unknown function.

1) is first-order. 2) is second-order

$y' = \frac{dy}{dx}$ $\quad y''(x) = \frac{d^2y}{dx^2}$ $\quad y^{(n)}(x)$

nth deriv wrt x

$$\frac{dy}{dx} - y + \left(\frac{d^2y}{dx^2}\right)^2 = 0$$

Second-order ordinary differential eqn

$$y(x) + \left(\frac{dy}{dx}\right)^2 = \sin\left(\frac{d^4y}{dx^4}\right) \quad (3)$$

4th-order ODE

F - a function of n+2 vars

$$F(\underset{\text{input for unknown functions}}{x}, \underbrace{y(x)}_{\text{unknown function}}, y'(x), \ldots, y^n(x)) = 0$$

this is a differential eqn.
A relationship between all of those quantities.

ex| For (3),

$F(u_1, u_2, u_3)$

$= u_1 + u_2^2 - \sin u_3$

$F\left(y(x), \frac{dy}{dx}, \frac{d^4y}{dx^4}\right) = 0$

Combine

$$y(x) + \left(\frac{dy}{dx}\right)^2 - \sin\left(\frac{d^4y}{dx^4}\right) = 0$$