MA 405 (Day 3)

before: - Matrices

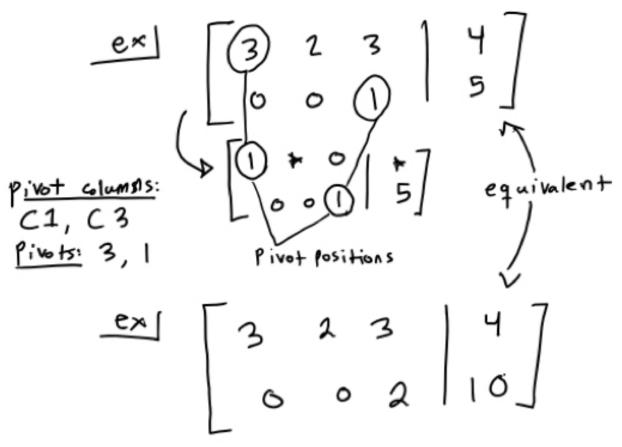
=> reduced echelon

rous

def A pivot position is the location of a leading 1 in the reduced echelon form of a matrix,

def A Pivot Glump is any column containing a pivot position

[def] A pivot is a (nonzero) # in a pivot position



Pivot columns: just a scale on Rz: Same C1, C3 reduced echelon form as above. Pivots: 3,2 (Same p. Cols, p. pos,3, Jiff pivs.)

equivalent matrices will have some piv. cols and pos's, but maybe diff piv's.

Pivot Positions and columns
do not change under
row operations.

e×]

$$3 \times_{1} + 2 \times_{2} + 3 \times_{3} = 4$$

 $\times_{3} = 5$

Soltans: Solution set is a line; infinitely many solutions:

 $3\times_1+2\times_2=-11$ We can see it's the eqn. for a line.

×3=5

In echelon form, non-pivot columns correspond to free variables in your solution (if system is consistent).

ex In above, $x_1 = -2x_2 - 11$ $x_2 = Free$ If you have multiple non-pivot Columns, you. II have infinite solutions, but each extra free war adds on extra dimension to your solution set.

[1 0 5 | 0]

Dalready in reduced exhelon form

> Pivot columns: C1, C2

Pivots: 1, 1

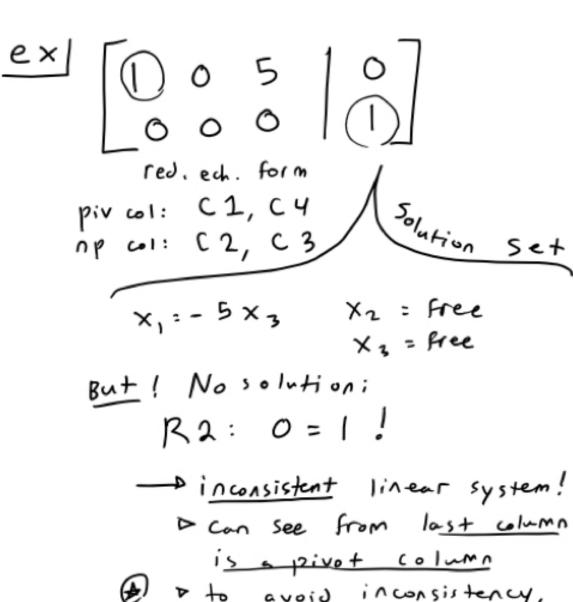
Solution?

- C3: non-pivot column

- X3 is a free variable

Solution Set $\begin{cases}
X_1 = -5 \times 3 \\
X_2 = -2 \times 3 + 1 \\
X_3 \text{ free}
\end{cases}$

What kind of geometric object is this? D 3 vers, but only 1 gree vor: a line in 3D-space. P Matrix looks like two Planes intersecting; of course it would be a line! $\begin{bmatrix}
1 & 0 & 5 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}$ $\begin{cases}
X_{2} & \text{free} \\
X_{3} & \text{free}
\end{cases}$ $\begin{cases}
\text{reduced exhelon form} \\
\text{Pivot col}: C1, C4 \\
\text{(Pivots: 1, 1)}
\end{cases}$ $\begin{cases}
\text{Non-Pivot col}: C2, C3
\end{cases}$ $\begin{cases}
X_{1} = -5 \times 3 \\
\times 4 = 0
\end{cases}$ Geometrically, this is a plane (two free variables)



is a pivot column

To avoid inconsistency,

pivot columns con't be

the last column of

augmented matrix.

Thm: If an (row) echelon form of an augmented matrix contains a row of the form

[0 0 0 0 0 | b] b \(\psi \)

then the associated linear system is inconsistent.

① For which H's h is system consistent?

[| h | 4]
2 | lo | 4]

1 A linear system w/ 4 variables, 3 equations can have what possible solution sets?

a) nothing b) single point c) line d) 20-plane e) 30-plane f) 40-plane ① $h \neq 5$ (if h = 5, $R_2 = [001 - 4]$ if $R_1 \leftarrow 2R_1$,

[2 2h | 8]

then $h \neq 6$. For $R_1 \leftarrow 3R_1$,

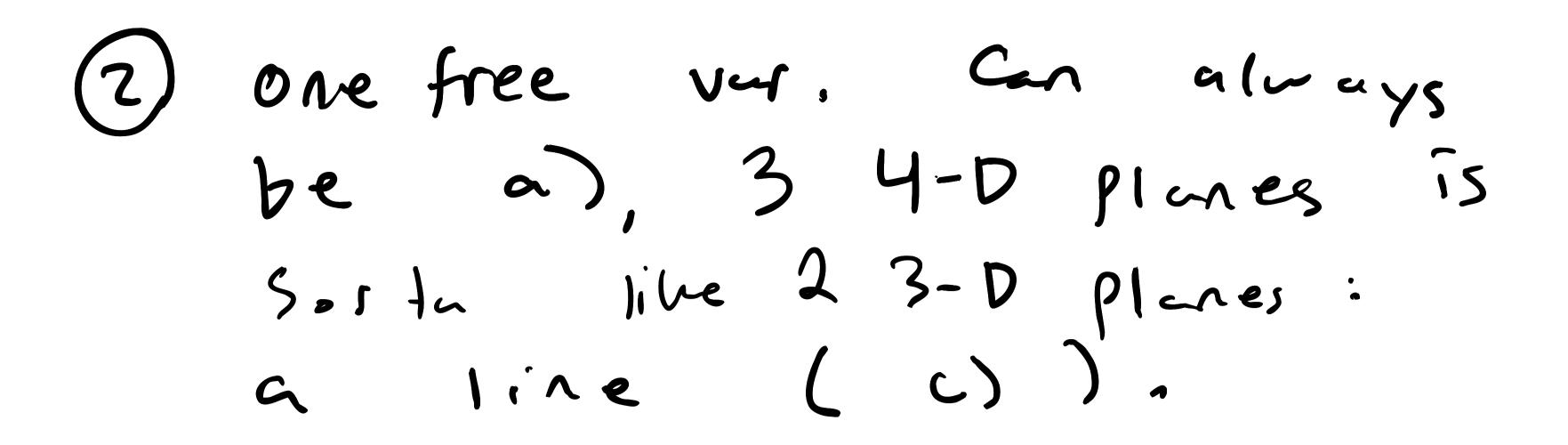
[3 h | 12]

meh. $h \neq 5$!

[4 4h | 116] $-2R_2$ -20

I'm Pretty sure

it's $h \in \mathbb{Z}^{3h \in \mathbb{N}}$ $h \neq 5$

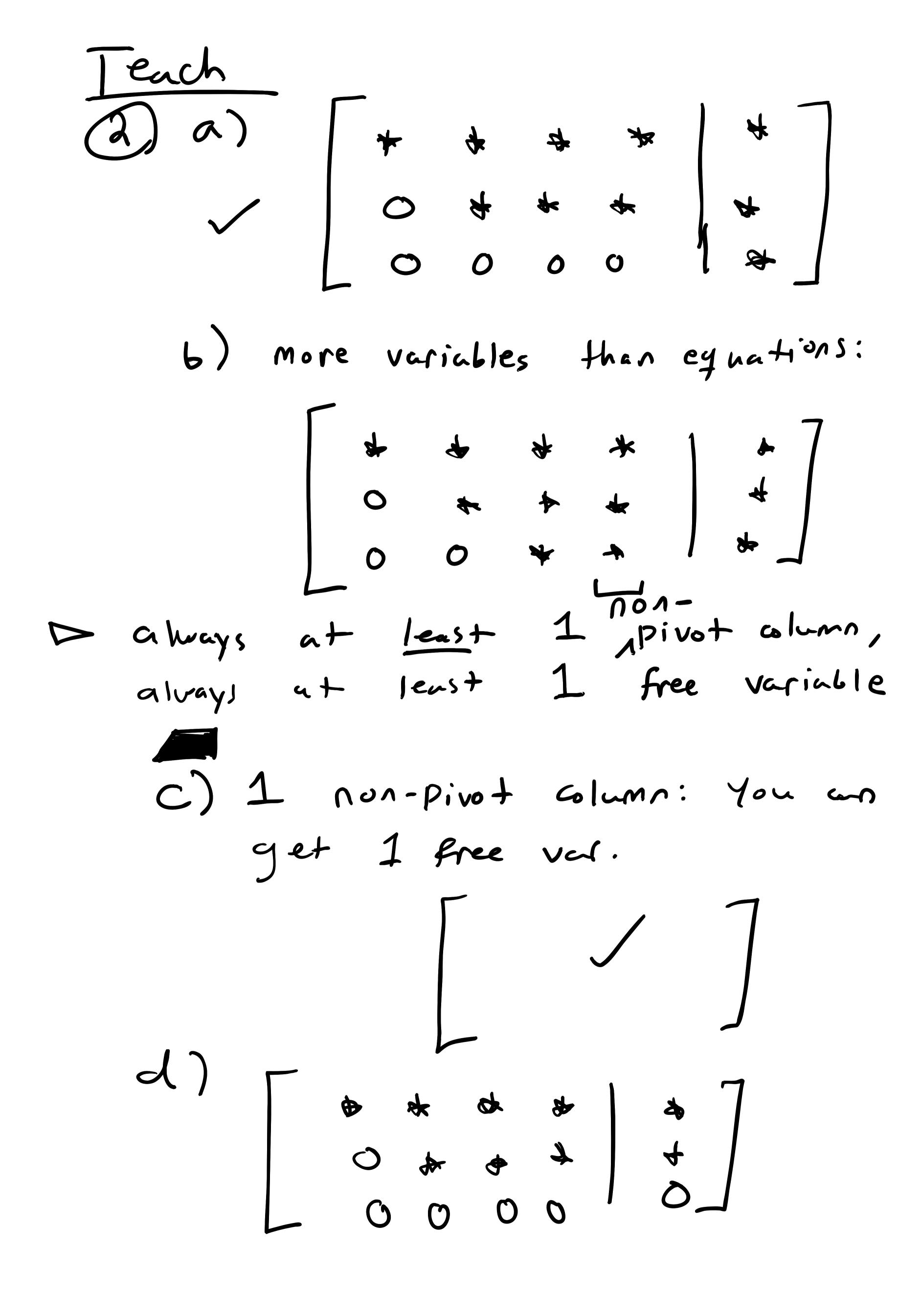


I'11 say a), c)

makes it a bit more Obvious.

Teach DReduce. R2-2R, 5 1 h 4 J 5 10-2h 1-4 J 10-21 70 h 7 5 Con+ divide by 0: 10-21 70

h 75



- d) sust use two non-pivot
- e) just use three non-pivot columns:

(trivial linear system)