MA 405

office SAS 3208

HW assigned every Fri and due following Fri

Tests sound tough; check Koofers

Final is 5/10/17, 8 – 11 AM

A – likely [85, 89]
A likely ~~[89, 93]~~ [90, 94]
A + likely [95, 100]

highly doable

Intro to Course

Linear Algebra

Linear → having to do with lines or planes (geometric)

Algebra → Solving equations involve variables, numbers, etc. ("solve for x")

Two Main Kinds of Probs

1) $\underbrace{A}_{\text{matrix}} x = \underbrace{b}_{\text{vector}}$ known.

Solve for x.

2) $A x = \lambda x$

number (eigenvalue)

"Eigenvalue Problem"

TONS of applications can be solved in this way.

- Chemistry: solve rxn equations
- Physics: compute simulated model of: propagating waves, air flow, heat diffusion.

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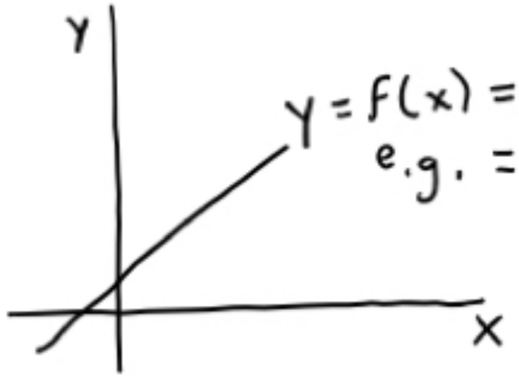
- economics: Markets Production + demand

Linear Systems of Eqns

(1.1–1.3)



$y = f(x) = mx + b$

e.g. $= \frac{x}{2} + 1$

$\to \quad y - \frac{1}{2}x = 1$

any line can be $a_1 y + a_2 x = b$

Two variables, x, y

Two coeff, $a_1 = 1$, $a_2 = \frac{1}{2}$

$\boxed{\text{def}}$ A linear equation in variables

$(x_1, x_2, \dots, x_n)$ is an

equation that can be written in form

$$a_1 x_1 + a_2 x_2 + \dots + a_n x_n = b,$$

$(a_1, a_2, \dots, a_n)$ coeff and b are numbers.

ex| $3x_1 - 5x_2 + 2 = x_1 - x_2$

Linear?

Yes:

$2x_1 - 4x_2 = -2$

with two variables: eqn. of a line

ex| $x_2 = 2(\sqrt{3} - x_1) + x_3$

Linear?

yes: 3 variables :
eqn. of a plane

ex| $x_3 + x_2 + 3x_1^2 = \sqrt{x_3}$

Linear?

no: quadratic factor;
also, power of 1/2.

Powers of vars <u>must be 1</u>.

$\boxed{\text{def}}$ A <u>system</u> of linear eqns is a collection of two or more linear eqns. <u>with the same variables.</u>

<u>ex</u>

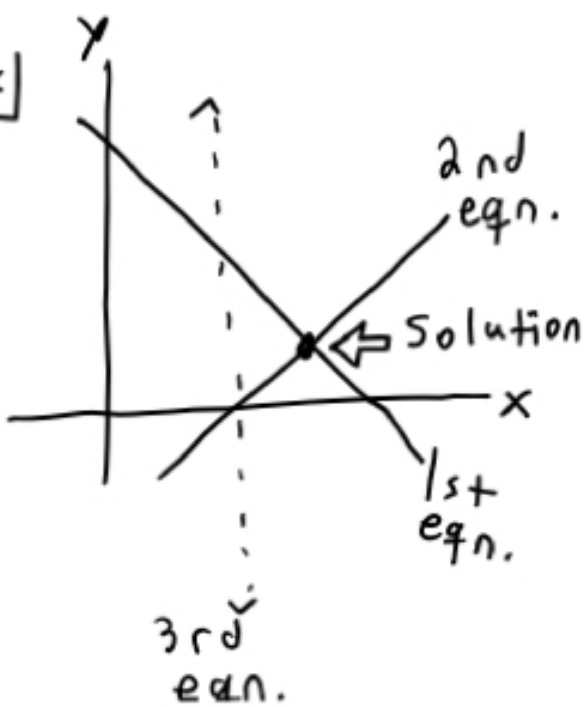
$$\begin{cases} 2x_1 + x_2 - \frac{3}{2}x_3 = 8 \\ x_1 + 4x_3 \qquad = 6 \end{cases}$$

x_2 isn't missing ($a_2 = 0$) $+ 0x_2$

Two eqns w/ 3 vars x_1, x_2, x_3

$\boxed{\text{def}}$ A <u>solution</u> to a system of linear eqns. is $(s_1, s_2, s_3, \ldots, s_n)$ a set of #'s which can be plugged into <u>all</u> equations from the system and have each be true.

<u>ex</u>



In 2 vars, a solution is a point of intersection of all the lines.

→ Possible to have <u>no</u> soltn!

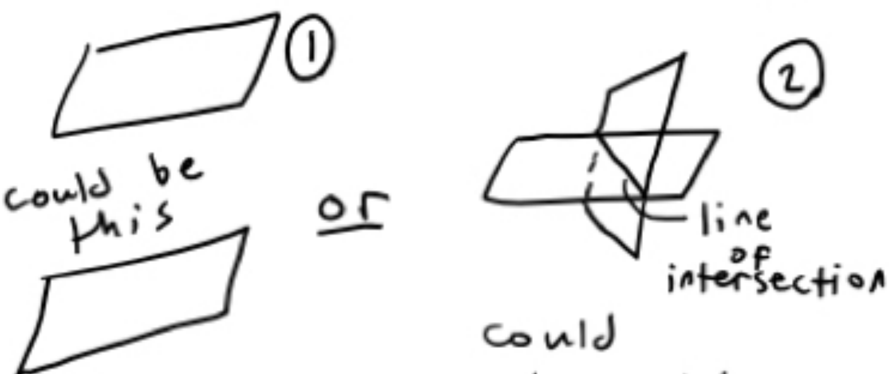
Add 3rd line: no solutions for this 3-eqn. system ex]

$$2x_1 + x_2 - \tfrac{3}{2}x_3 = 8 \rightarrow \text{Plane}$$

$$x_1 + 0x_2 + 4x_3 = 6 \rightarrow \text{Plane}$$

3 variables, 2 equations:

Solution = wherever these two planes intersect (probably a line)



could be this.

(no intersection)

$$2x_1 + x_2 - \tfrac{3}{2}x_3 = 8$$

$$x_1 + 0x_2 + 4x_3 = 6$$

a possible solution: $(2, 11/2, 1)$

Pick $x_1 = 2$. Then $x_3 = 1$.
And then $x_2 = \frac{11}{2}$.

This means it's case 2: they intersect in a <u>line</u>.

i.e. if the planes intersect in <u>one</u> point, there are <u>infinitely</u> <u>many</u> solutions.

$\boxed{\text{def}}$ For any linear system, the solution set (of all poss. soltns) is either an <u>infinite set</u> (line, plane), a <u>single point</u>* (two lines intersecting), or there are <u>no</u> <u>points</u> in solution set.

ex]
$$\left.\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 2x_1 - 3x_2 + 2x_3 &= 14 \\ 3x_1 + x_2 - x_3 &= -2 \end{aligned}\right] \text{Linear system}$$

3 eqns., 3 vars (3 diff. planes)

⟶ How can 3 planes intersect?

▸ 2 planes can intersect as a line, or not at all.

⇒ How to add 3rd plane?

⟶ How can you manipulate these eqns?

Substitution: $X_1 = 6 - 2x_2 - 3x_3$

sub into others, eliminate x_1

$$(12 - 4x_2 - 6x_3) - 3x_2 + 2x_3 = 14$$

$$(18 - 6x_2 - 9x_3) + x_2 - x_3 = -2$$

subst. and elim. performed.

Repeat! Well, first, simplify:

$$-7x_2 - 4x_3 = 2$$

$$-5x_2 - 10x_3 = -20$$

Now you can substitute again! It doesn't matter <u>which</u> <u>way</u> you substitute.

$$-7x_2 - 4x_3 = 2$$

$$x_2 + 2x_3 = 4$$

scaling eqn. by $-1/5$; linear system unchanged

Substitute, solve for x_3: plug into two prior eqns. for x_2: plug x_2, x_3 into original 3-eqn. system.

→ Last step: substitute x_3 to get x_2, then x_1. (Back-substitution)

These operations:

1) Replacement
2) Interchange ("doesn't matter)
3) Scaling ($-1/5$)