ex| Make augmented matrix & reduce to echelon form for:

$$\begin{aligned} 2x_1 + x_2 &= 0 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 + x_4 &= 0 \\ x_3 + 2x_4 &= 5 \end{aligned}$$

▷ pivots? Solution?

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$R_1 \leftarrow R_1 - R_2$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$$R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 3R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & -4 & -3 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{array}\right]$$

$$R_3 \leftrightarrow R_4$$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -3 & 0 \end{array}\right]$$

$R_4 \leftarrow R_4 + 4R_3$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 5 & 5 \end{array}\right]$$

$R_4 \leftarrow R_4/5$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$R_3 \leftarrow R_3 - 2R_4$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_3 - R_4$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1 + R_2 + R_3 - R_4$

$x_1 = 1$

$x_2 = 4$

$x_3 = -2$

$x_4 = 1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

pivots

<u>Teach</u> (+ class)

wanna eliminate the lower staircase of 1's

$$R_2 \leftarrow R_2 - \tfrac{1}{2} R_1$$

<u>or</u>

$$R_2 \leftrightarrow R_1$$

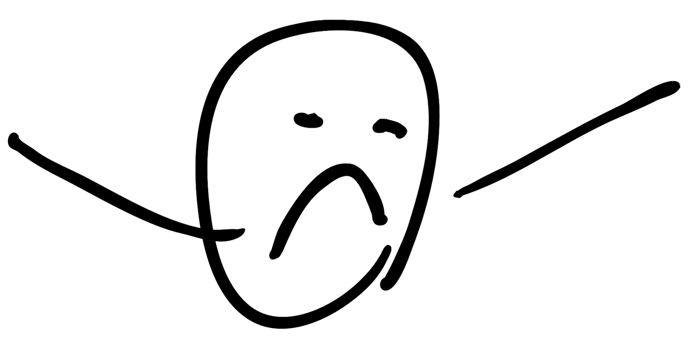
$$R_2 \leftarrow R_2 - R_1$$

$$\boxed{\cancel{R_3 \leftarrow R_3 -}}$$

$$R_3 \leftarrow 3R_3 + R_2$$

pivots in all 4 cols:
Consistent ✓

$x_4 = 4$ $\quad x_2 = 2$
$x_3 = -3$ $\quad x_1 = -1$



I swapped the 5 and 0 when I shouldn't have, still wrong!

▷ This was a band matrix, anyway.

## Vectors + Geometry

In augmented matrices,
- → rows are eqns. for system
- → columns are coeffs. for diff. variables

★ row ops on aug. matrix correspond to familiar algebraic manipulations

You can think of linear systems as rows. But you can also think in columns.

Thinking abt Linear Systems in Terms of Columns: use <u>Vectors</u>!

$\boxed{\text{def}}$ A (column) vector is a matrix with only one column.
A (row) vector is a matrix with only one row.

ex

$$\begin{bmatrix} -3 \\ 1 \end{bmatrix} \quad \underline{1 \text{ col}}: \quad \vec{v} = \langle -3, 1 \rangle$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \langle 1, 2, 3 \rangle$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \end{bmatrix} \langle 5, 4, 3, 2, 1 \rangle$$

<u>1 row</u>

<u>Notation:</u> set of all vectors w/ $n$ rows and 1 column:

$\mathbb{R}^n$ ($n$-dimensional space, sorta)

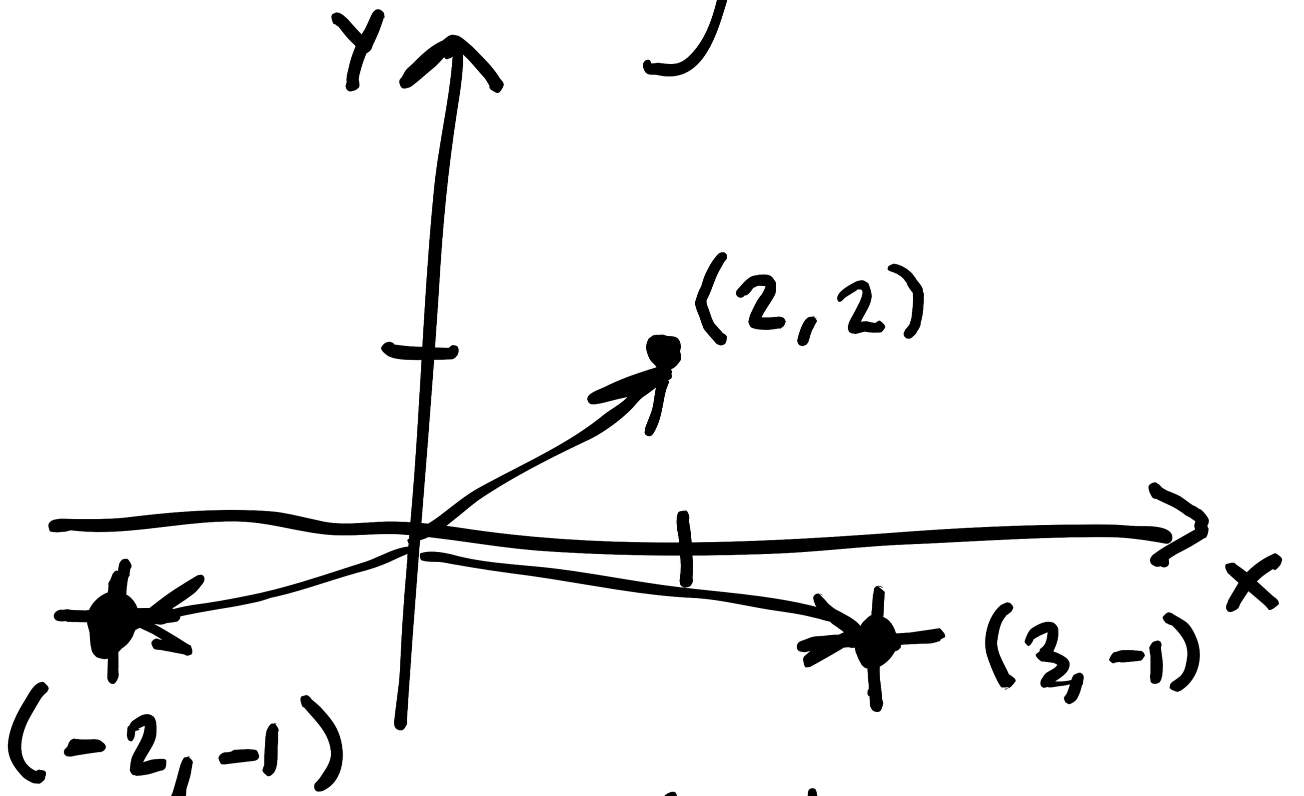
★ <u>Order matters!</u>

$$\begin{bmatrix} 5 \\ -1 \end{bmatrix} \neq \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

★ A vector is basically an <u>ordered</u> list of numbers.

<u>ex</u> Visualizing vecs in $\mathbb{R}^2$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$




vectors can be represented as points in plane or as arrows (useful for addition)

<u>ex</u>

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= -1 \\ x_1 \qquad\quad + 5x_3 &= 6 \end{aligned}$$

<u>Vector equation</u>

$$x_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

▷ Find a <u>linear combination</u> of vectors to solve the linear system.

aug matrix

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & -1 \\ 1 & 0 & 5 & 6 \end{array}\right]$$

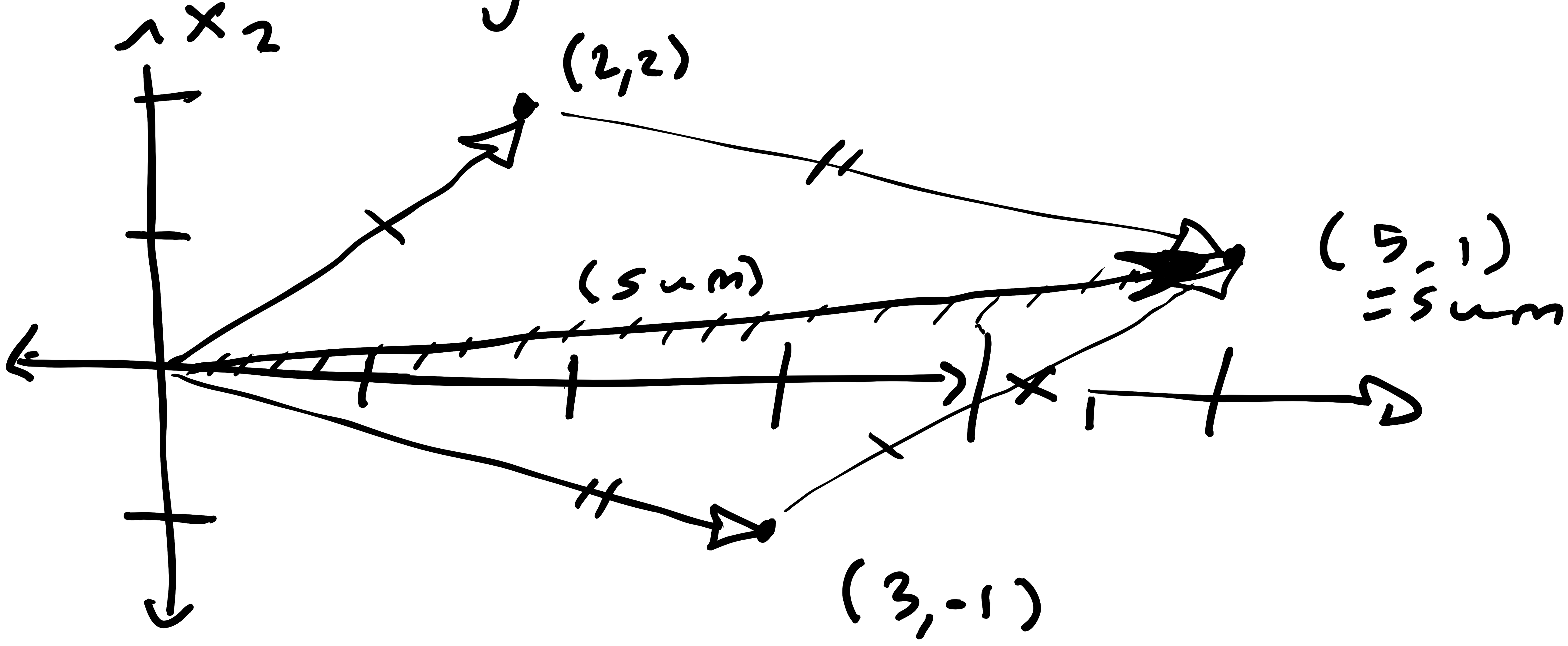
Row reduction

# Scaling and Adding Vectors

(★) Both of these happen <u>entrywise</u>

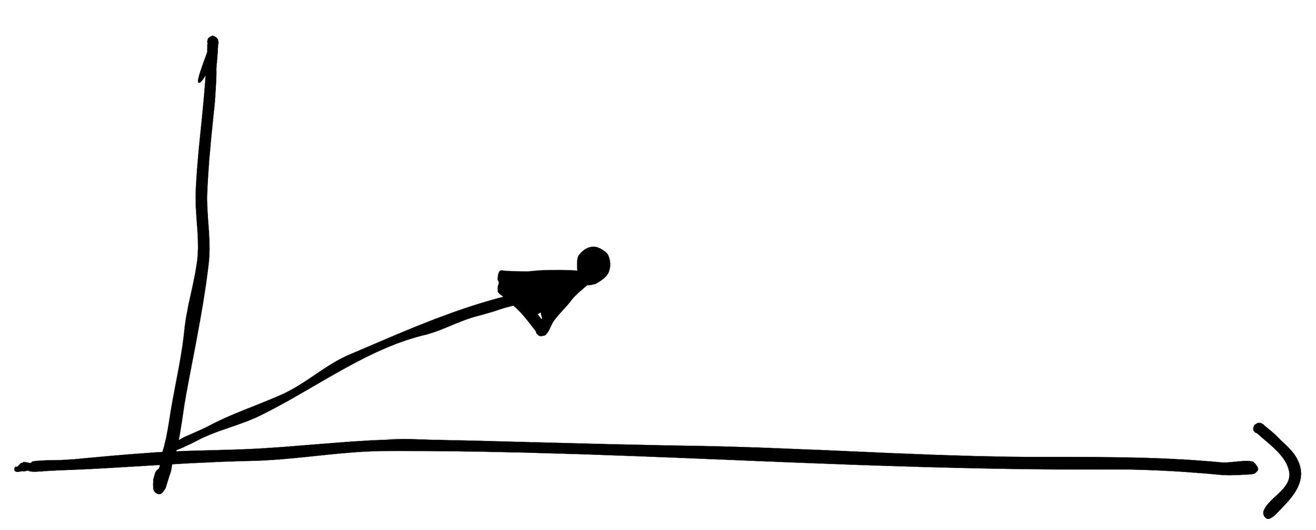
ex) $\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

"Parallelogram Rule"




The sum of two vectors is like the fourth vertex of a parallelogram whose other vertices are 0, u, v (sum of u + v)

ex) $2 \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$



Q]

$$w = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad v = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

$$w + v = ?$$

Nope

Vectors not same size can't be added! (These objects live in different spaces: $\mathbb{R}^4$ vs $\mathbb{R}^3$.)

$\boxed{\text{def}}$ A <u>linear combination</u> of vectors

$v_1, v_2, v_3, v_4, \ldots, v_n \Rightarrow$ All in $\mathbb{R}^\ell$

is a sum

$c_1 v_1 + c_2 v_2 + \ldots + c_n v_n$

where $c_1, c_2, \ldots, c_n$ are all numbers.

$\rightarrow$ it's okay for some or all of c's to be 0.