PY 202

- Do intro to webassign <u>tonight</u>!

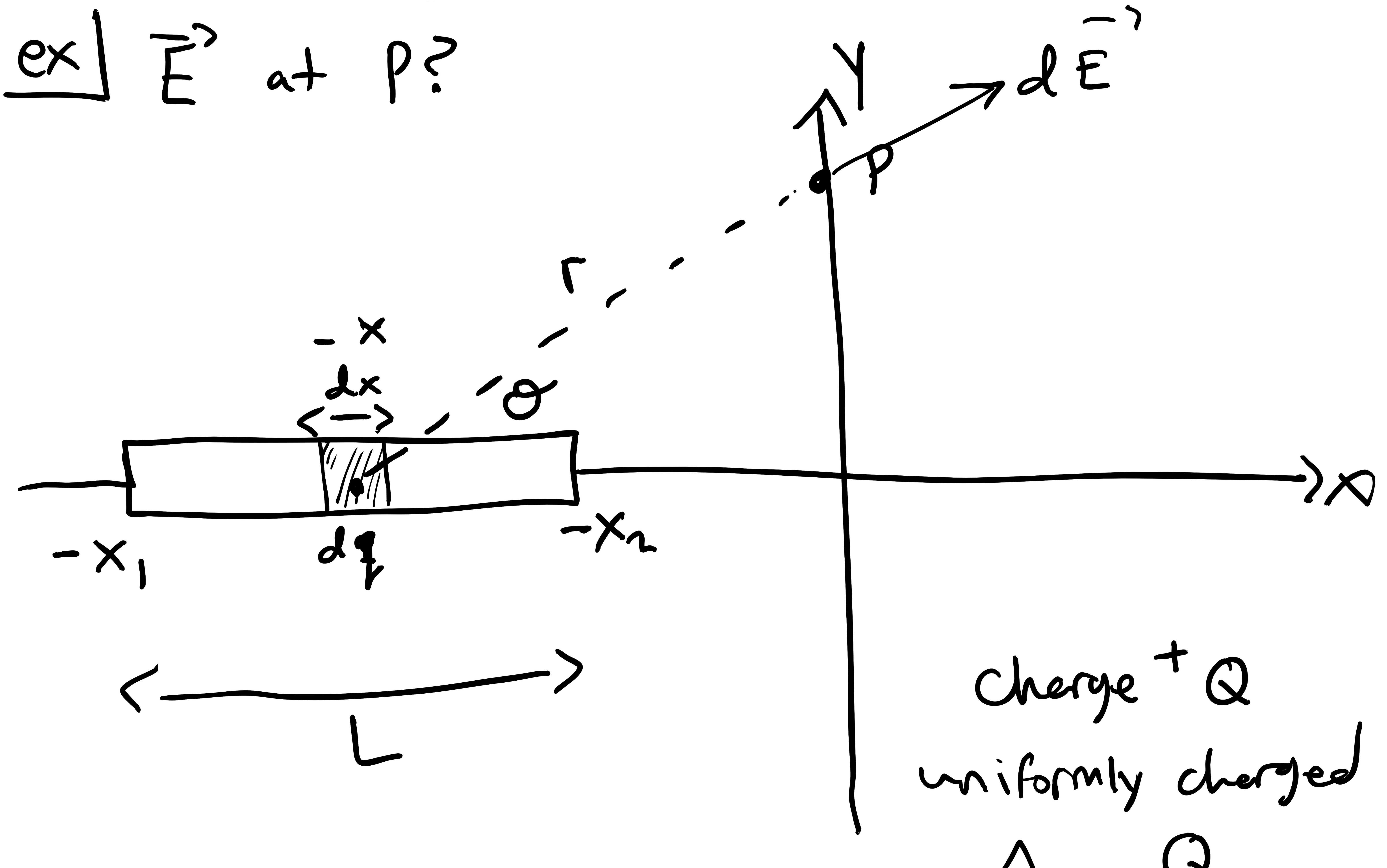
Continuous Q Distributions

$$dE = k\frac{dq}{r^2}$$

$$\vec{E} = \int d\vec{E}$$

$$E_x = \int dE_x \qquad E_y = \int dE_y$$

<u>ex</u> $\vec{E}$ at P?



Charge $+Q$
uniformly charged

$$\lambda = \frac{Q}{L}$$

$$d\vec{E} = dE_r \hat{r} = dE_x \hat{i} + dE_y \hat{j}$$

$$dE_r = k \frac{dq}{r^2}$$

$$dE_x = k \frac{dq}{r^2} \cos\theta$$

$$dE_y = k \frac{dq}{r^2} \sin\theta$$

dq ?

$$Q = \lambda \cdot L$$

$$dq = \lambda \cdot dx$$

using geometry:

$$\sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{-x}$$

$$\Rightarrow x = -y \cot\theta$$

$$\partial x = -y \frac{\partial}{\partial\theta} \cot\theta$$

Now we have dx!

$$dx = y \csc^2\theta \, d\theta$$

$$dE_x = k\lambda \frac{y \csc^2\theta \, d\theta}{y^2/\sin^2\theta} \cos\theta \, d\theta$$

$$dE_x = \frac{k\lambda}{y} \cos\theta \, d\theta$$

$$E_x = \int dE_x = \int_{\theta_1}^{\theta_2} \frac{k\lambda}{y} \cos\theta \, d\theta$$

$$= \frac{k\lambda}{y} \left(\sin\theta_2 - \sin\theta_1 \right)$$

$$dE_y = \frac{k\lambda}{y} \sin\theta \, d\theta$$

$$E_y = \int dE_y = \int_{\theta_1}^{\theta_2} \frac{k\lambda}{y} \sin\theta \, d\theta$$

$$= \frac{k\lambda}{y} \left(\cos(\theta_2) - \cos\theta_1 \right)$$

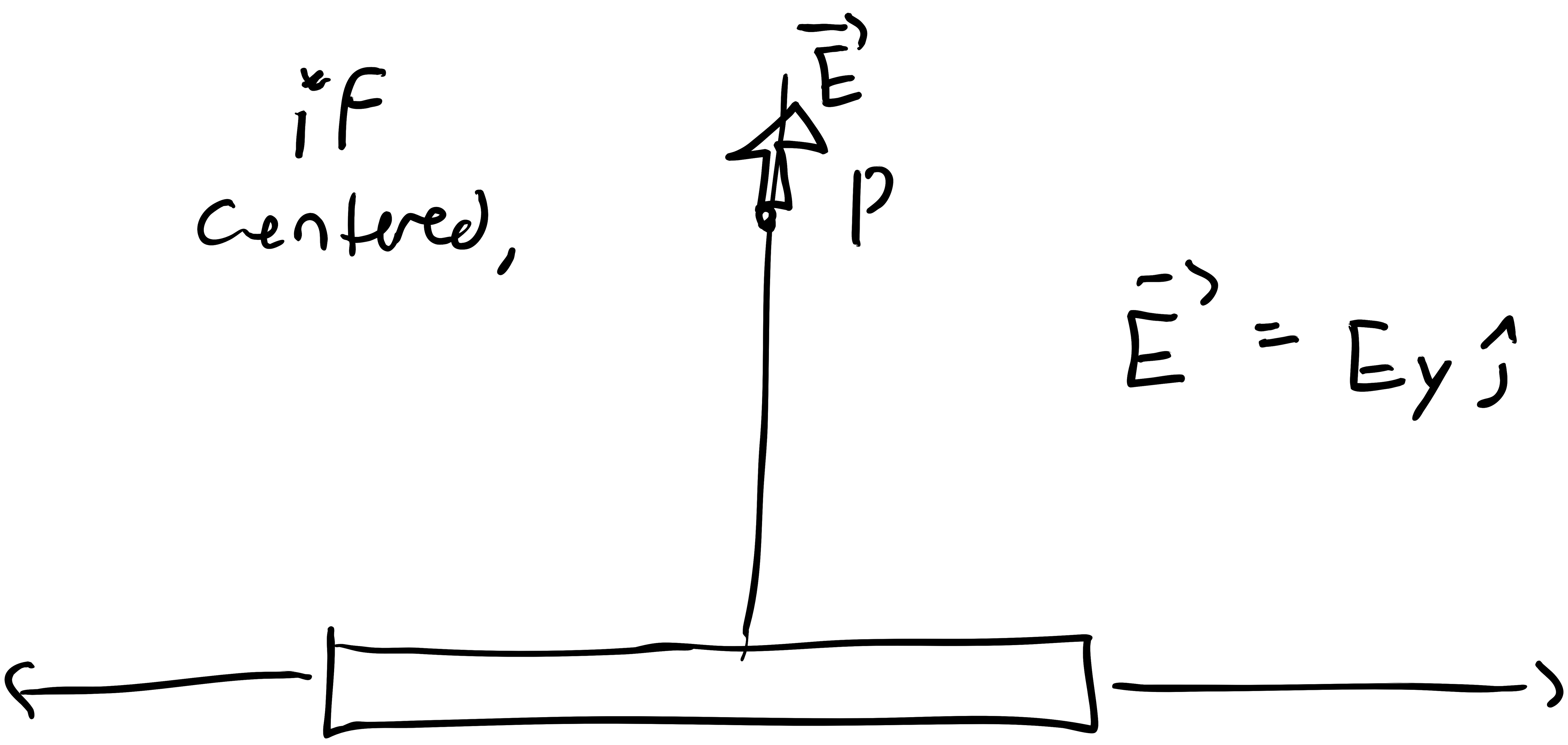
$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

Since $\frac{\sin\theta}{y} = \frac{1}{r}$,

$$E_x = k\lambda\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

(distance instead of θ)

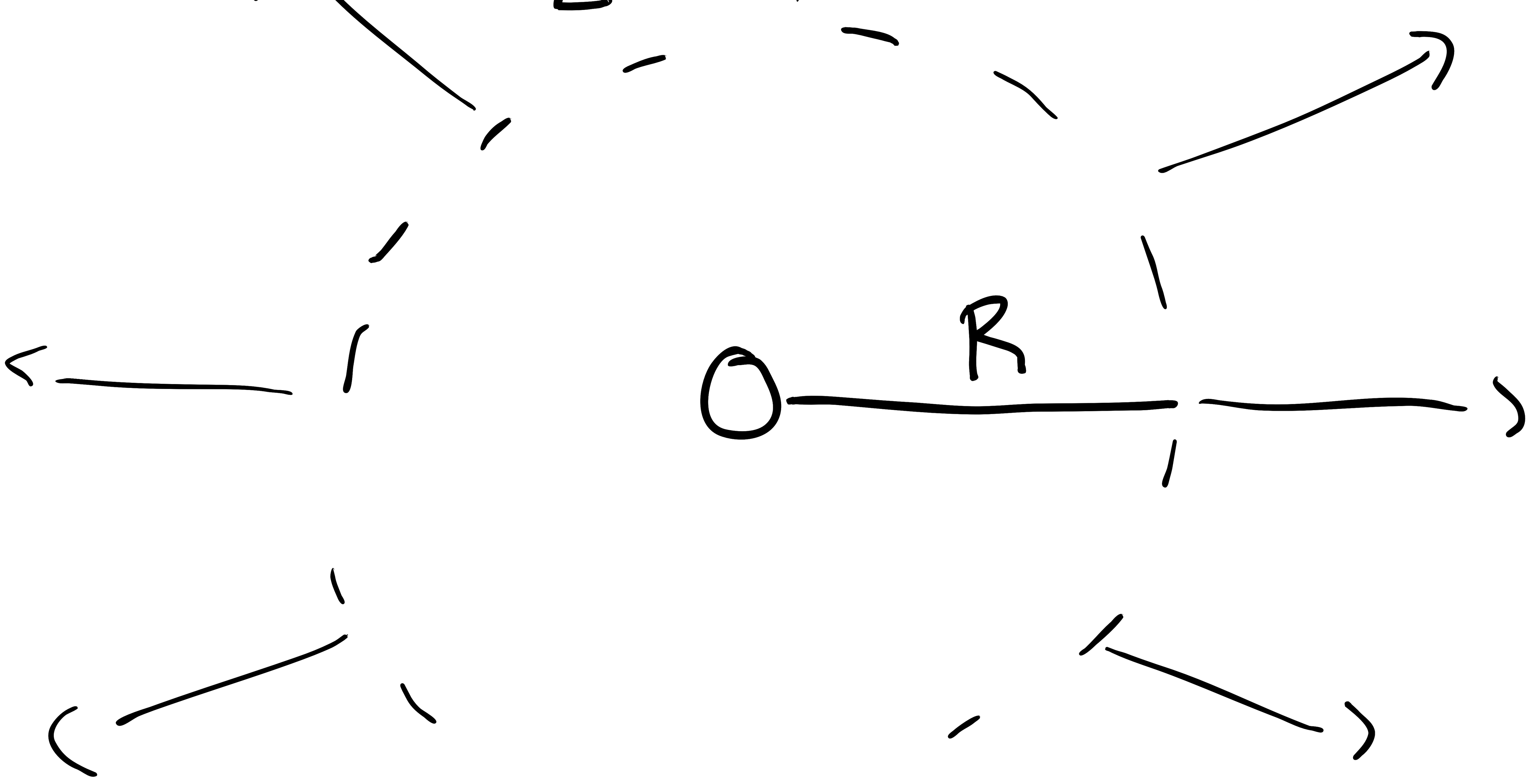
if centered,

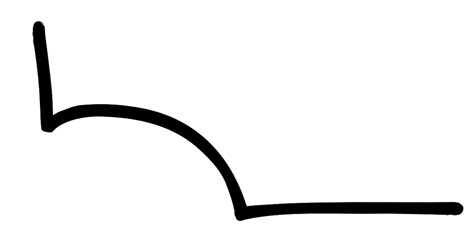


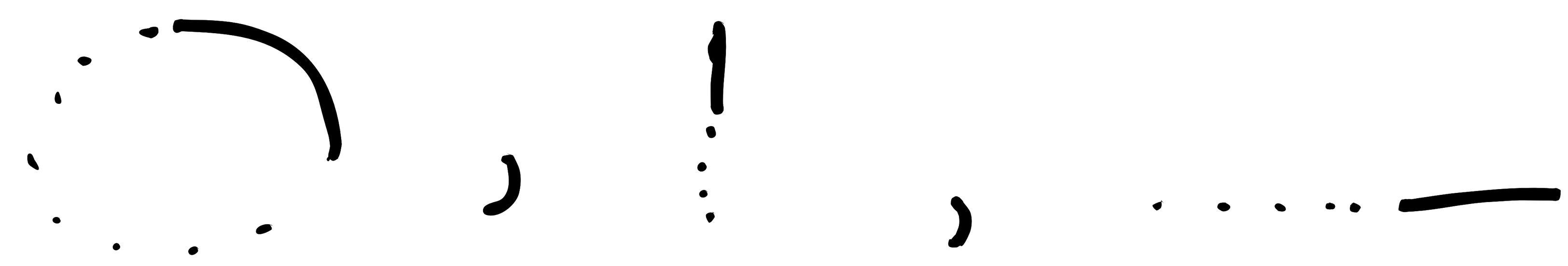
$\vec{E} = E_y\,\hat{j}$

(bc symmetry!)

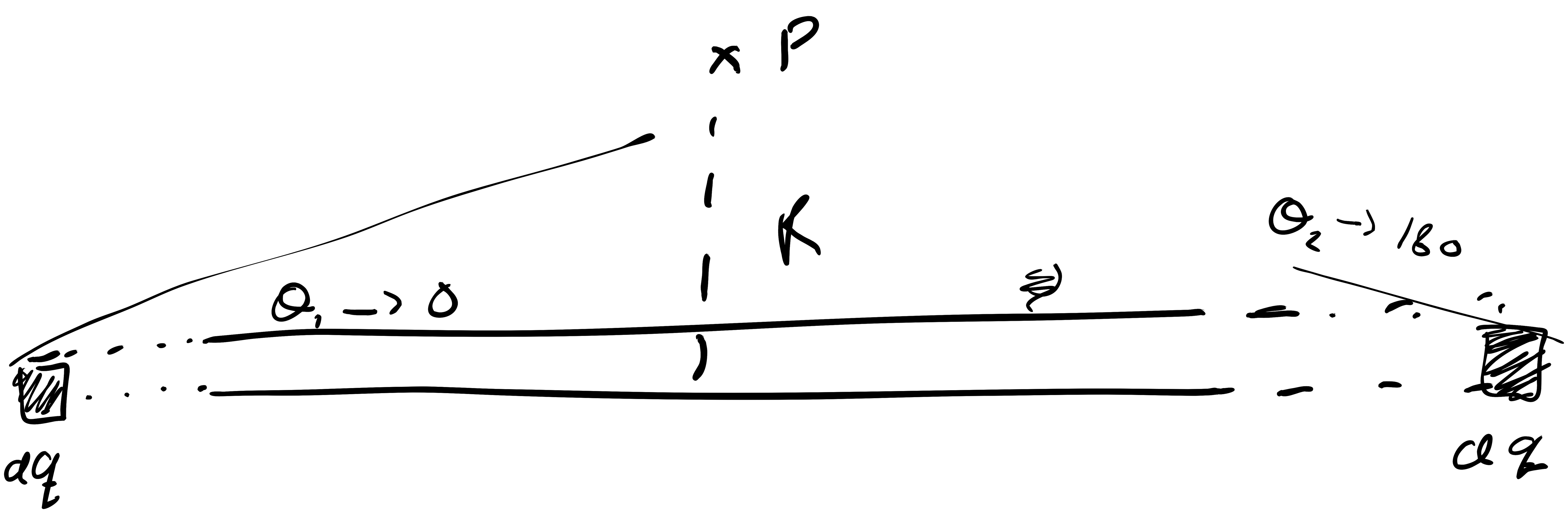
This $\vec{E}$-field is radial.



Could do $\vec{E}$ for 

be 

Infinite Line of Charge



$$E_x = 0$$

$$E_x = \frac{k\lambda}{R}\left(\sin \overset{180^\circ}{\theta_2} - \sin \overset{0^\circ}{\theta_1}\right) = 0$$

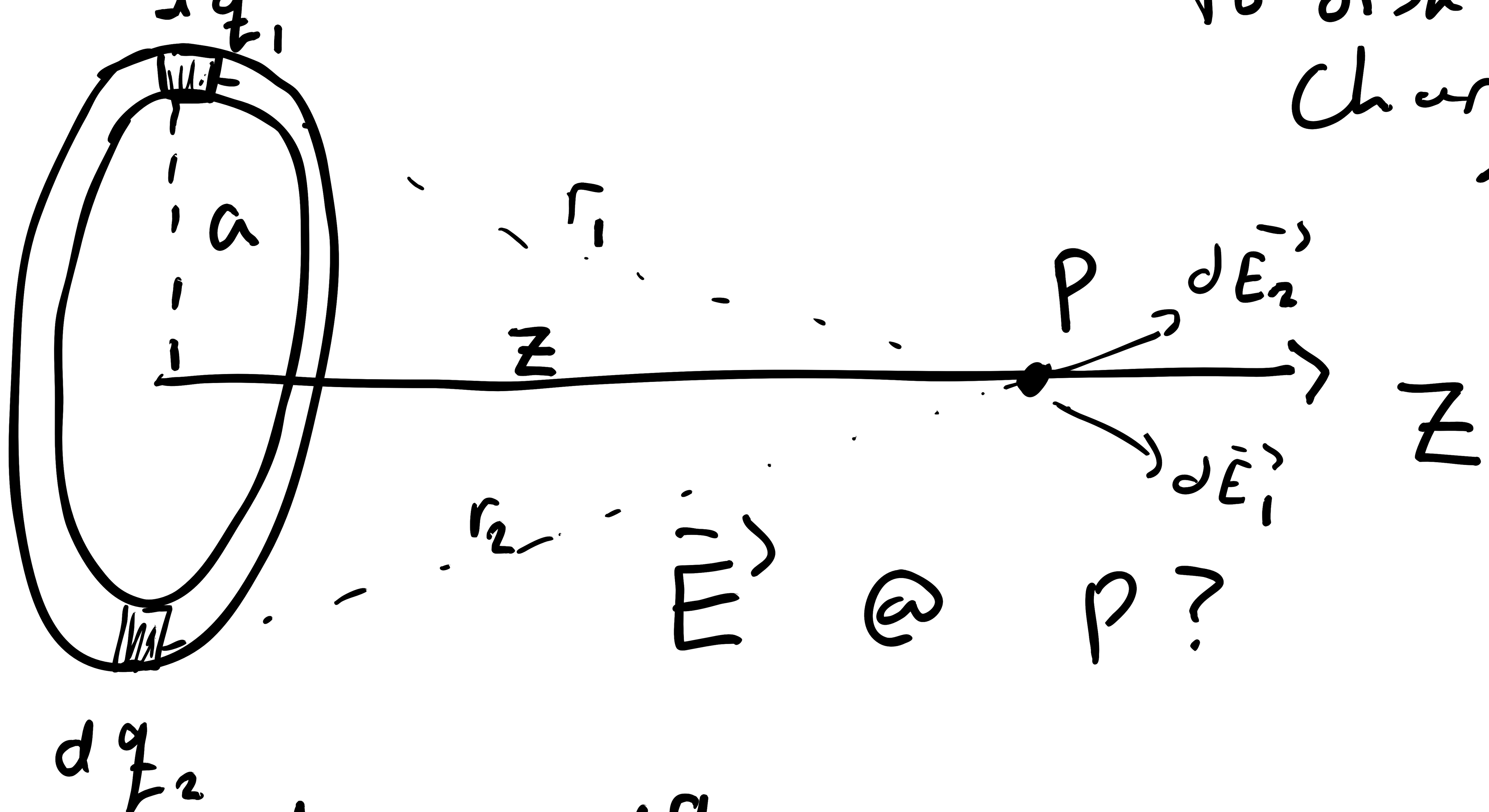
$$E_R = -\frac{k\lambda}{R}\left(\overset{-1}{\cos\theta_2} - \overset{1}{\cos\theta_1}\right) =$$

forgot this minus earlier

$$= \frac{2k\lambda}{R}$$

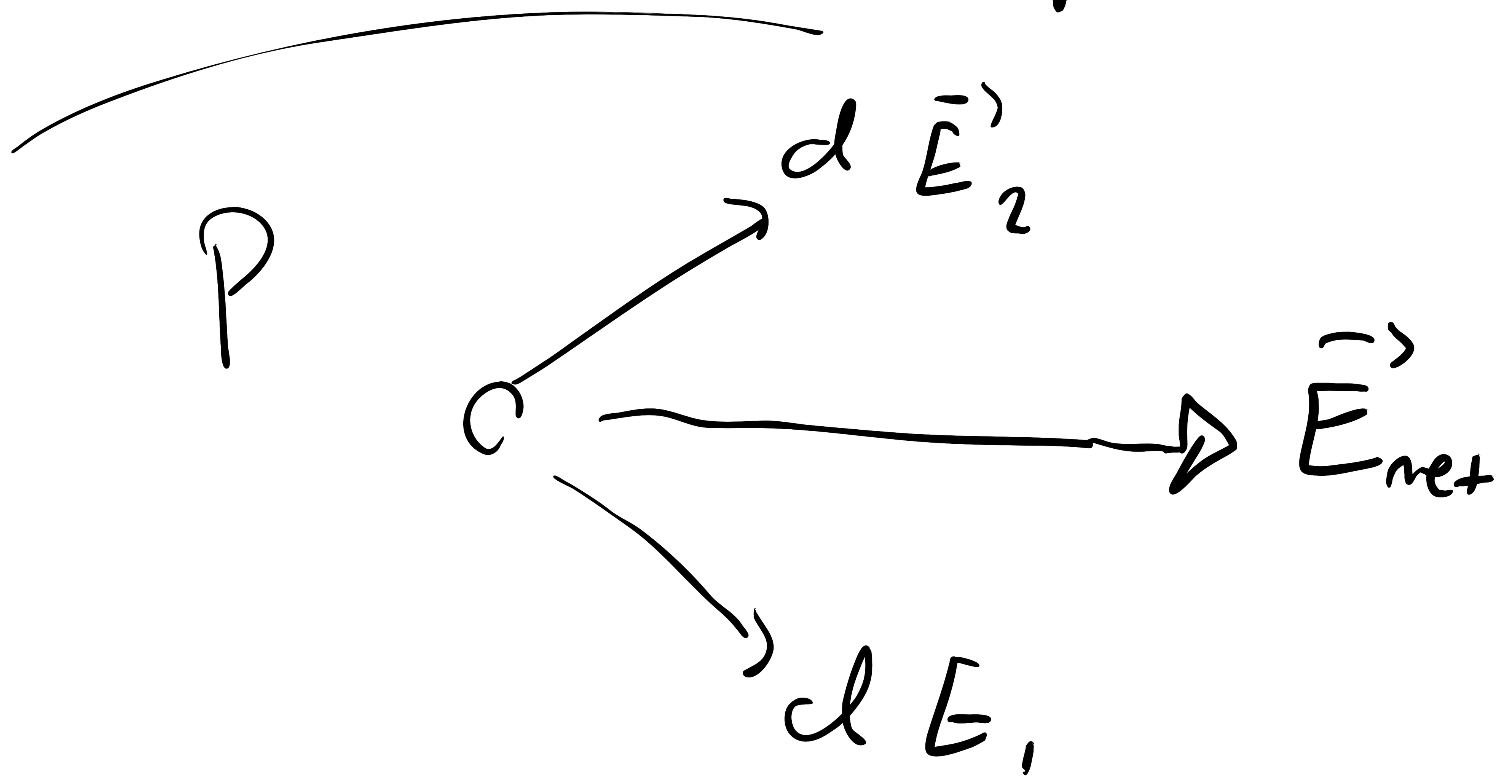
$$\vec{E} = E_R = \frac{2k\lambda}{R}\hat{r}$$ (radial direction w.r.t wire)

Ring of Charge Q (on our way to disk of charge)

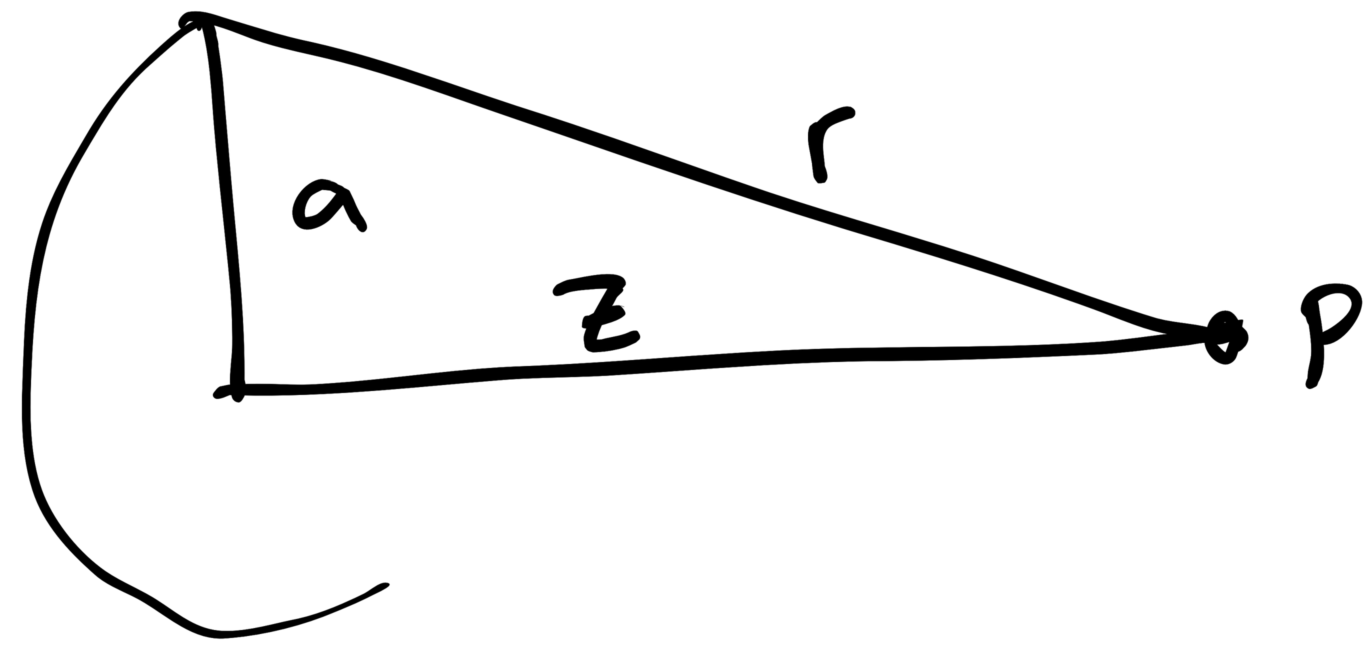


$\vec{E}$ @ P?

$$dE = \frac{k\,dq}{r^2}$$



consider △:



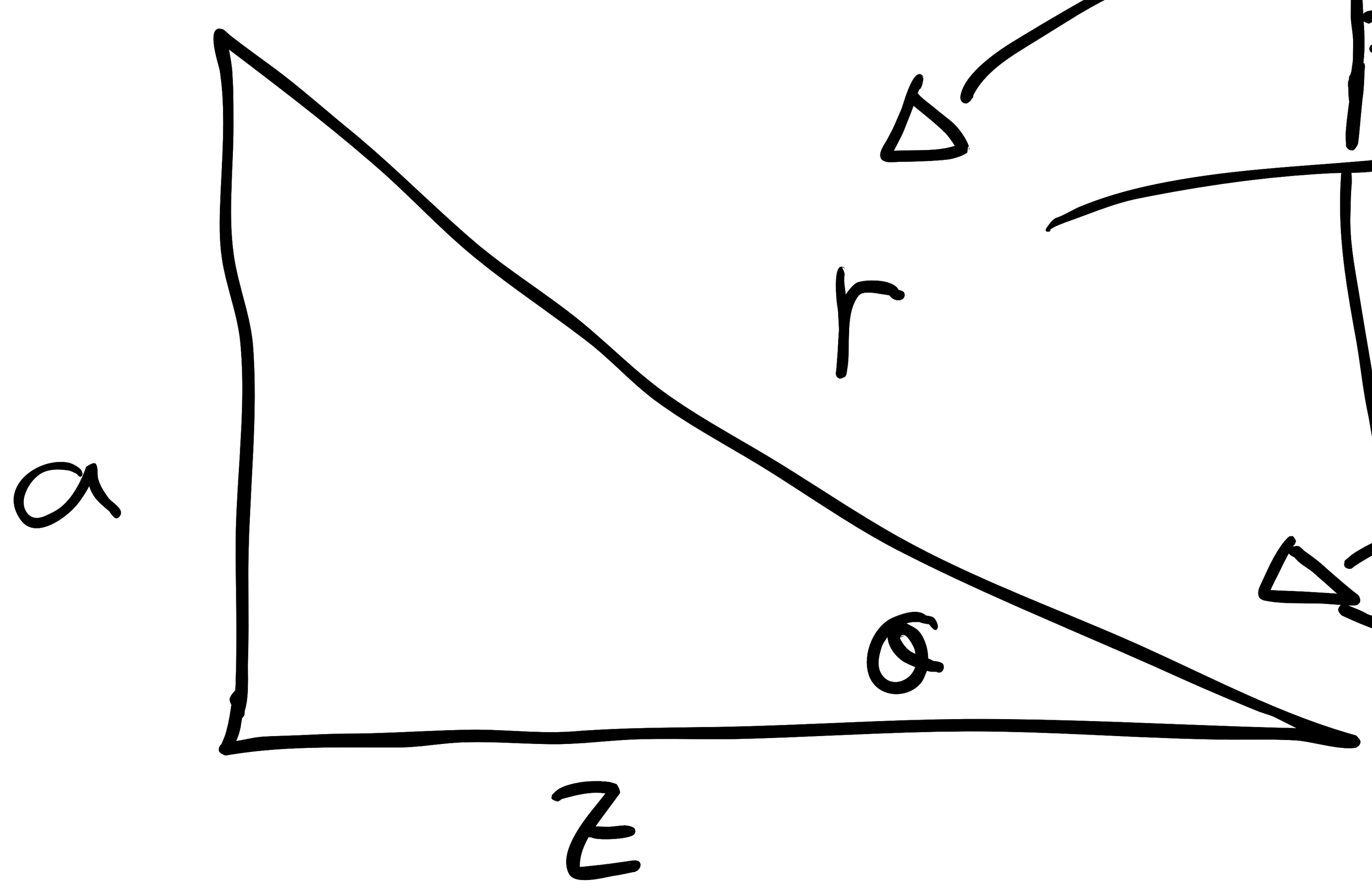
$$r^2 = a^2 + z^2$$

$$E = \int dE = \int k\,\frac{dq}{r^2}\cos\theta$$

$$E_z = \int dE_z = \int k\,\frac{dq}{r^2}\cos\theta$$

$$= \frac{k}{a^2+z^2}\cos\theta\underbrace{\int dq}_{Q}$$

$$= \frac{k\,z\,Q}{(a^2+z^2)^{3/2}}$$



$$\cos\theta = \frac{z}{r}$$

Disk $(r = R)$ of Surface Charge Density σ

$$E_{disk} = \int dE_{ring}$$

Came in handy! → $dE_{ring} = \dfrac{kz\,dq}{(a^2+z^2)^{3/2}}$

$2\pi a\,da$ ↓ $2\pi a\,da$

$$dq_{ring} = \sigma\, dA_{ring} = 2\pi a\,da$$

$$E_{ring} = \int dE_r = 2\sigma k z \int_0^R \frac{a\,da}{(a^2+z^2)^{3/2}}$$

$a\,da$

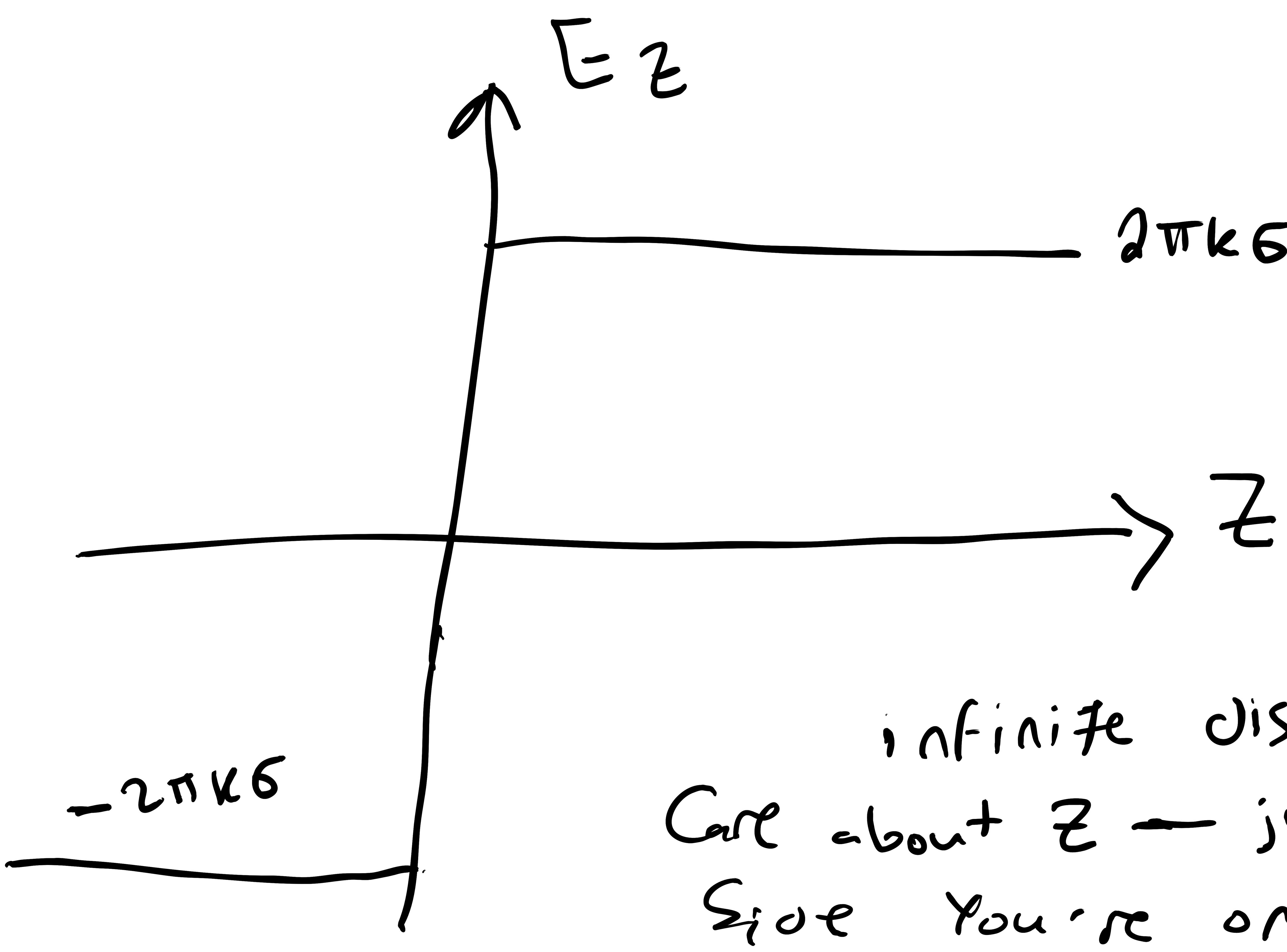
$$E_z = \frac{2\pi k \sigma z}{\sqrt{z^2}}\left(1 - \frac{1}{\sqrt{1+\frac{R^2}{z^2}}}\right)$$

if $z > 0$, $\sqrt{z^2} = +|z|$

if $z < 0$, $\sqrt{z^2} = -|z|$

$R \to \infty$: $\left(1 - \frac{1}{\sqrt{1+\cdots}}\right) \to 1$

$$E_z = \pm 2\pi k \sigma$$

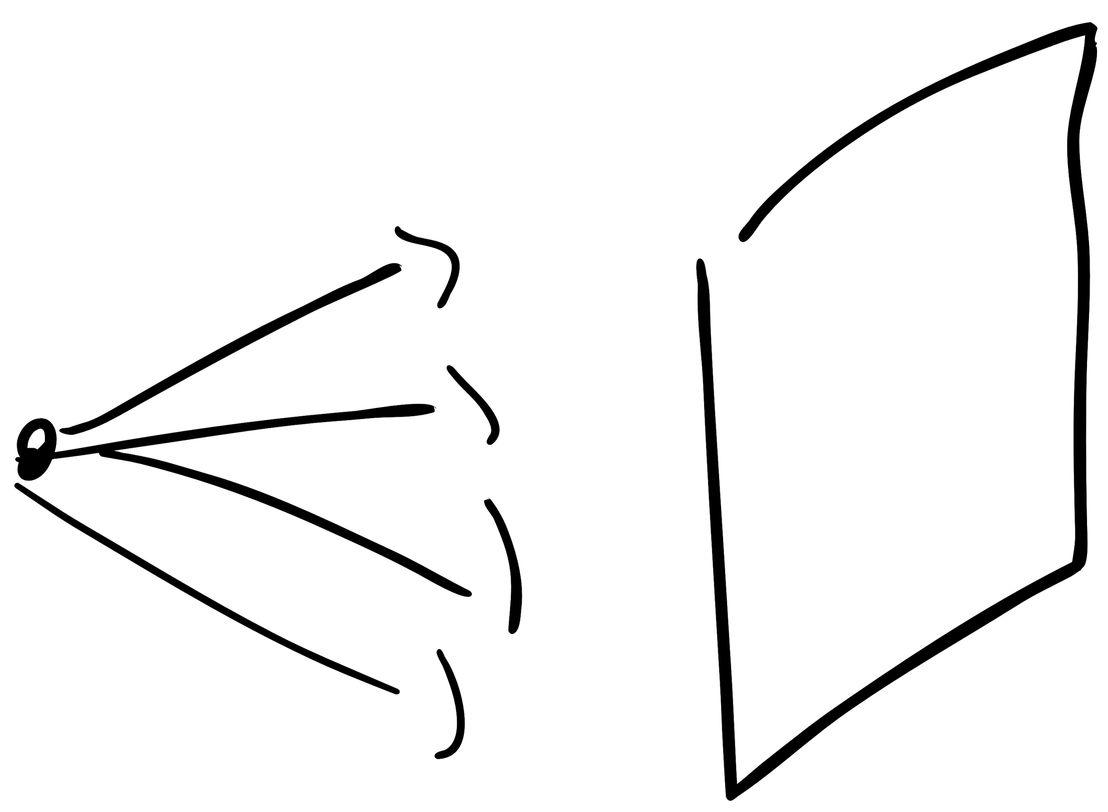


infinite disk don't care about z — just which side you're on.

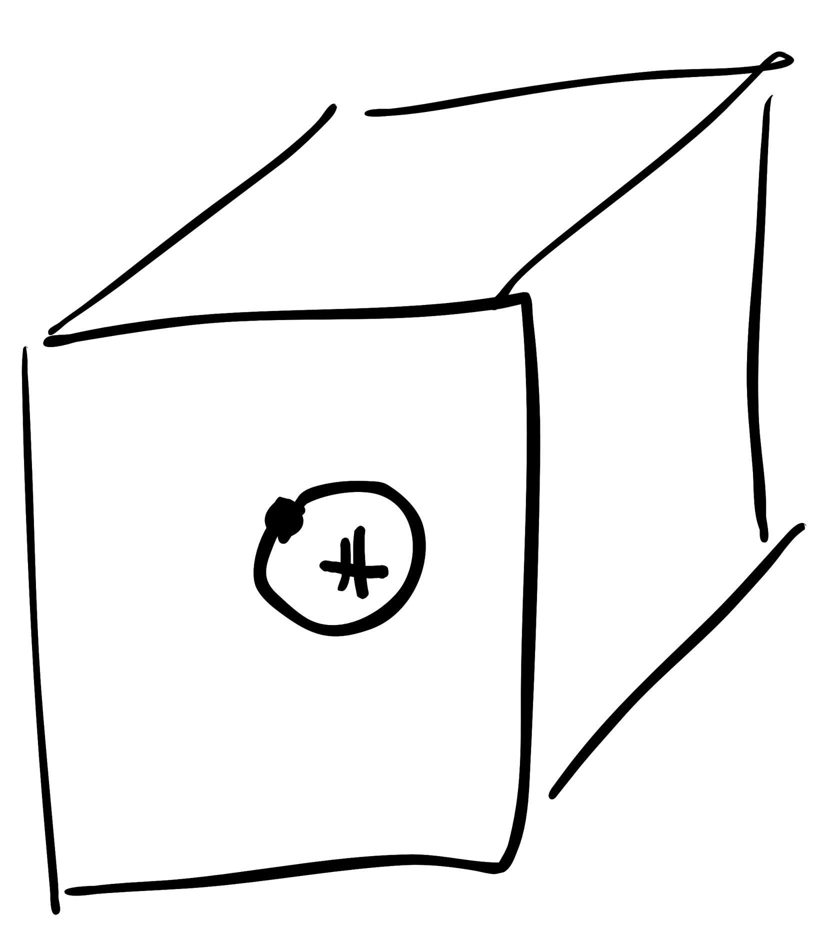
Electric Flux and Gauss's Law

electric flux: $\phi_{el} = \vec{E} \cdot \vec{A}$ (fish through a net)

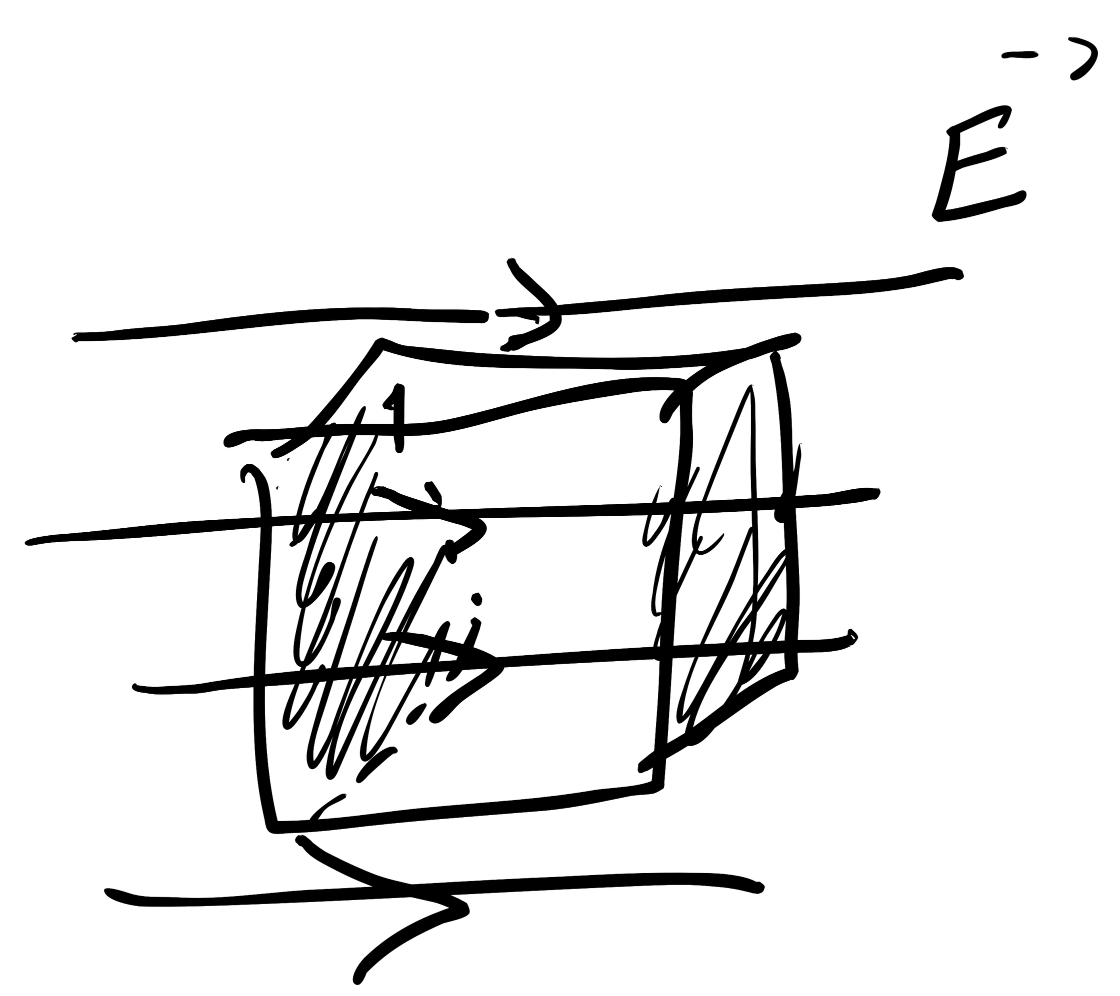
$= EA\cos\theta$



or:



net flux $\neq 0$



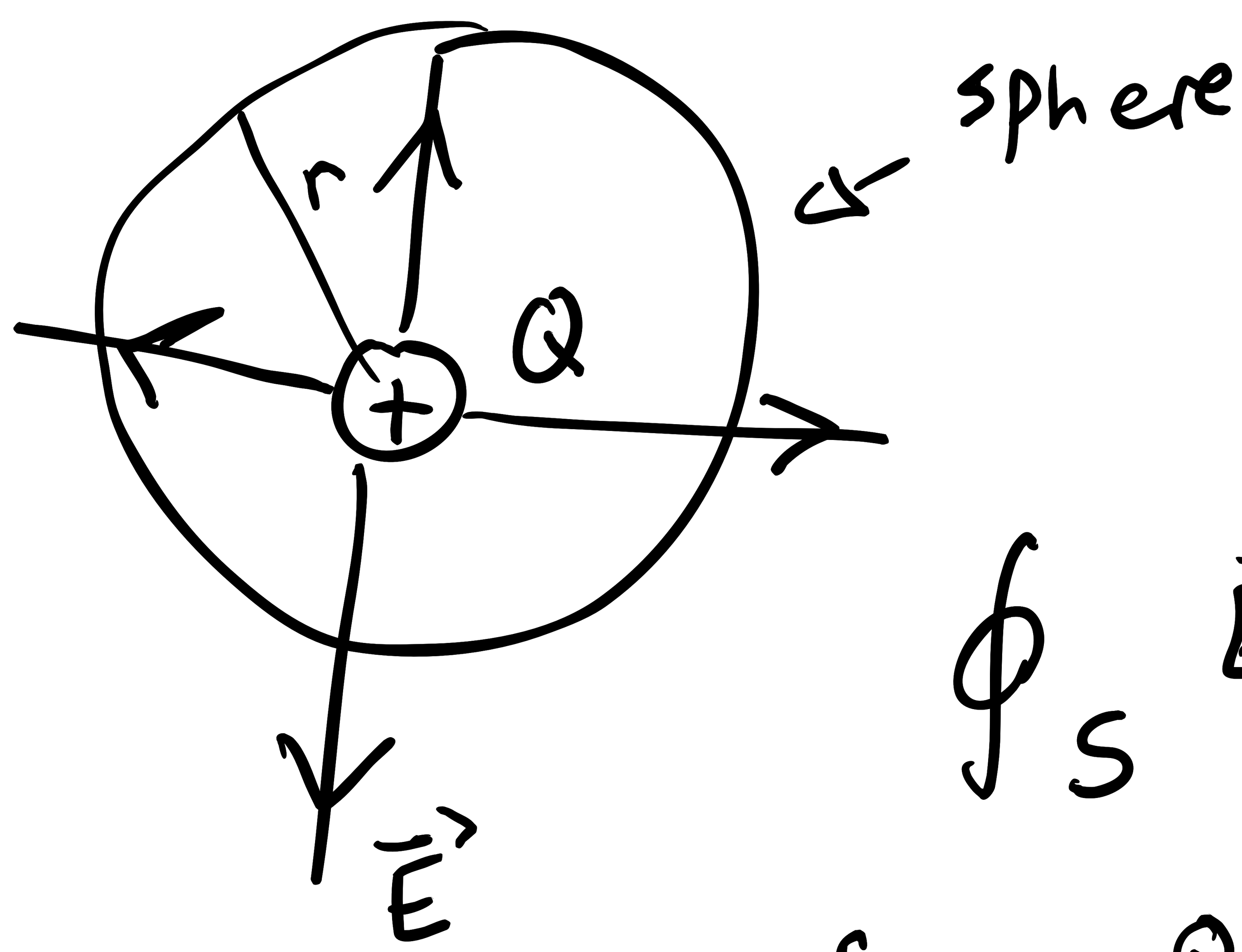
net flux $= 0$

box w/ net ϕ_e has enclosed charge. In fact, if no enclosed charge, net $\phi_e = 0$. (If a line goes in, it must go out.)

Gauss's Law:

$$\phi_S = \underbrace{\oint \vec{E} \cdot \hat{n}\, dA}_{\text{electric flux through closed surface } S} = \frac{Q_{enc}}{\varepsilon_0}$$

ex)



$$\oint_S \vec{E} \cdot \hat{n}\, dA =$$

$$= \oint_S k \frac{Q}{r^2} \hat{r} \cdot \hat{r}\, dA_{sphere}$$

$$= \frac{kQ}{r^2} 4\pi r^2$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$k\,\frac{Q}{r^2}\;4\pi r^2$$

$$= \frac{1}{\cancel{4\pi}\varepsilon_0}\;\frac{Q}{\cancel{r^2}}\;\cancel{4\pi r^2}$$

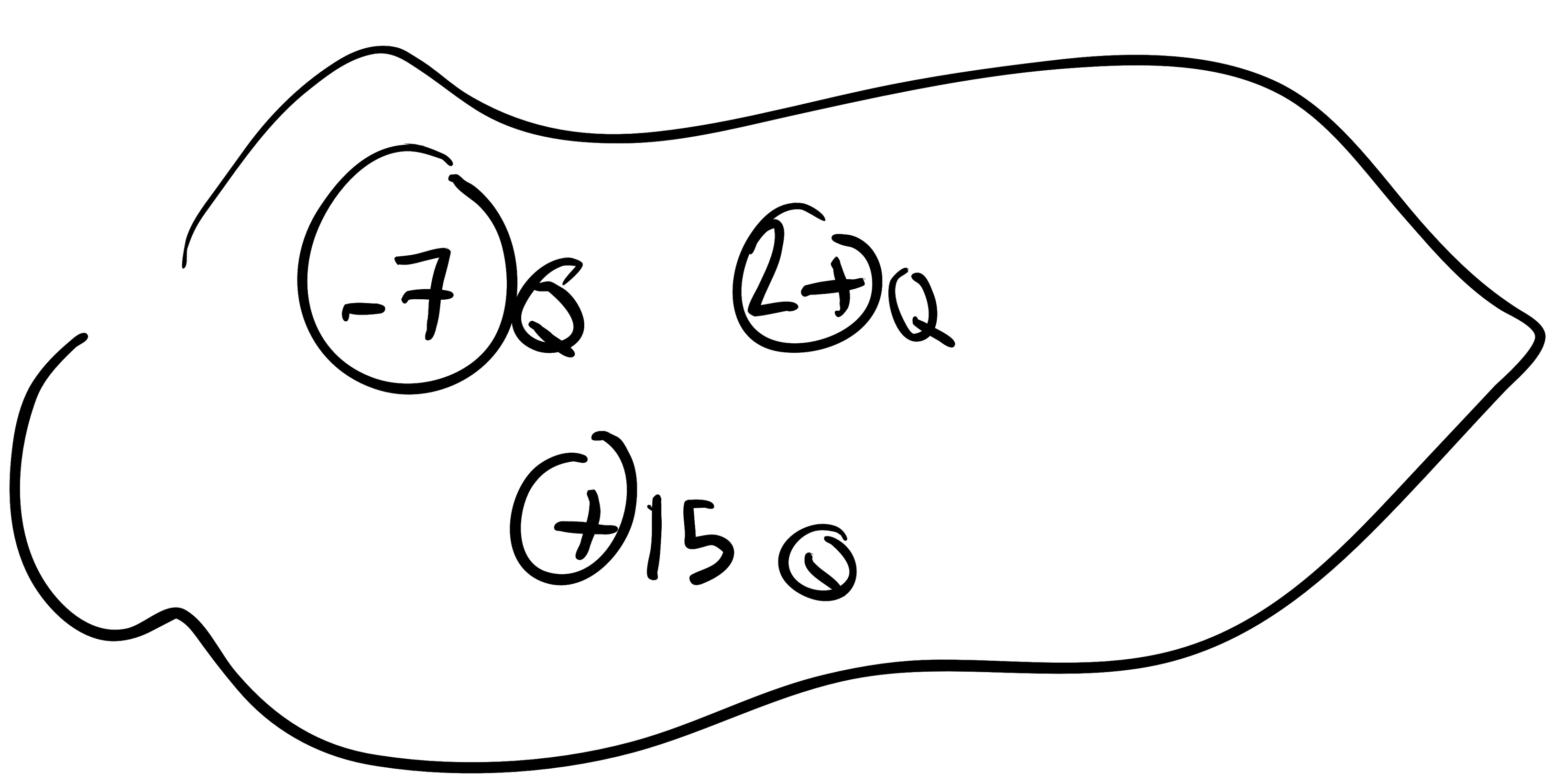
$$= \frac{Q}{\varepsilon_0}$$

proof!

Gauss's Law says

$$\phi_s = \frac{Q_{enc}}{\varepsilon_0}$$

ex)



$$\phi = \frac{Q_{enc}}{\varepsilon_0}$$

$$= \frac{10\,Q}{\varepsilon_0}$$