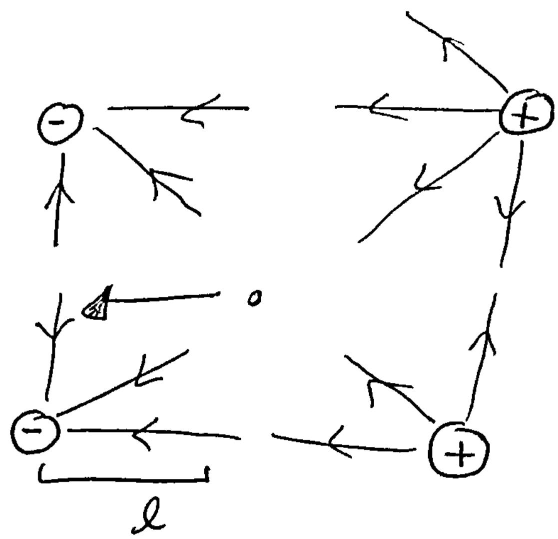
PY 202 V of Continuous q Distributions

- SPS pitch
- Quiz!

$$V = -\int E\, dl$$



$E \neq 0$

(net left)

$$V = \frac{kQ}{r} = -\int_a^b E \cdot l \cos(0) = -El$$

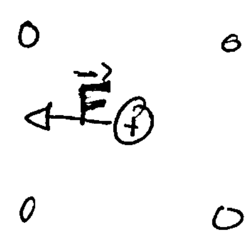
$= ?$

C, $E \neq 0$, $V \neq 0$

I thought wrong, but answered correctly (I typed D)

$\boxed{E \neq 0, \quad V = 0}$

but



$$V_{\substack{each \\ corner}} = \frac{kq}{(d/2)} \Rightarrow \frac{k(+Q)}{(d/2)} + \frac{k(+Q)}{(d/2)} + \frac{k(-Q)}{(d/2)} + \frac{k(-Q)}{(d/2)}$$

$= 0$ potential

Takeaway: Study $\underline{V}$!

Point charges: $$V = \frac{kQ}{r}$$

$$\text{“} V_p = \sum_i V_i \text{”}$$

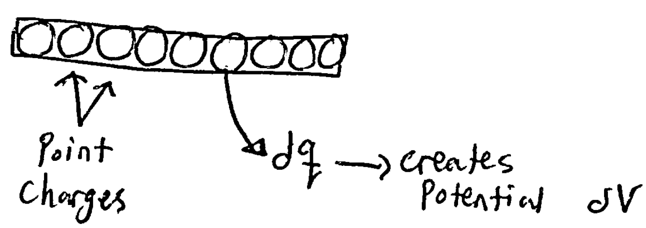
$$\left. V_p = \frac{kQ_1}{r_{1,p}} + \frac{kQ_2}{r_{2,p}} + \dots \right]$$ Potential at point $P(x, y)$

location of charge

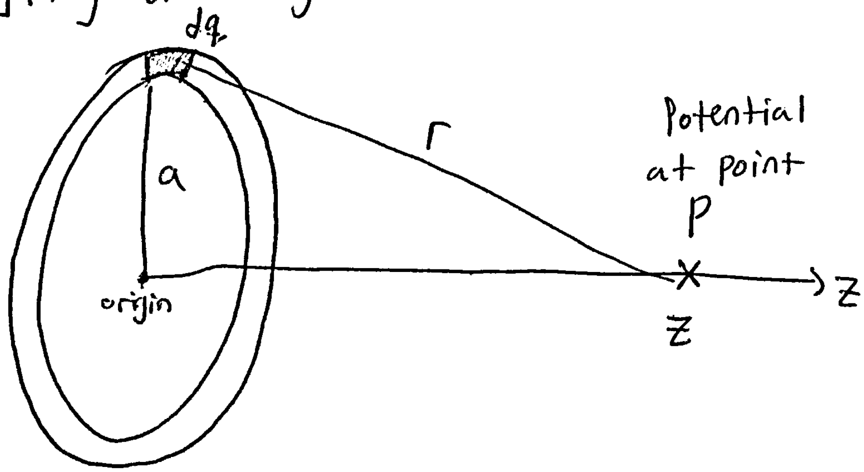
point of interest

Continuous Charge Distributions: $$\text{“} V = \sum_i V_i \text{”} \longrightarrow V = \int dV = \int \frac{k\,dq}{r}$$

rod:



$\underline{\text{ex}}$| ring of charge Q (total)



$$V_p = \int \frac{k\,dq}{r} = \int \frac{k\,dq}{\sqrt{a^2 + z^2}}$$

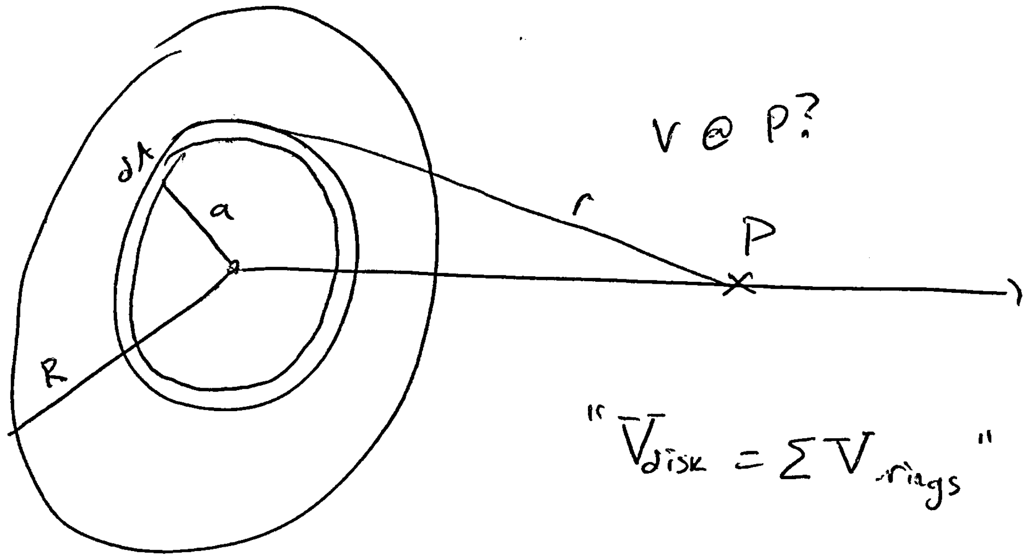
$$= \frac{K}{\sqrt{a^2+z^2}} \int dq = \boxed{\frac{kQ}{\sqrt{a^2+z^2}}} + \text{constant}$$

V due to ring of charge Q

constant $\downarrow$ 0

V at ∞: $V=0$

ex) disk of charge



V @ P?

Sum up rings

" $V_{disk} = \sum V_{rings}$ "

$$V = \int \frac{k\,dq}{r} \quad \leftarrow r^2 = a^2+z^2$$

dq = charge on one ring

$$\frac{dq}{dA} = \frac{Q}{A_{ring}}$$

$$dq = \frac{Q}{A_{ring}} \underset{\uparrow}{dA} = \sigma(2\pi a)\,da$$

area of a ring

$$V = \int \frac{k\,dq}{r} = K\int_0^R \frac{\sigma\, 2\pi a\, da}{\sqrt{a^2+z^2}} = \ldots = 2\pi K\sigma\left(\sqrt{z^2+R^2} - \sqrt{z^2}\right)$$

$u = a^2+z^2$

$du = 2a\,da$

V and $\vec{E}$

(finding one from the other)

$$\boxed{V = -\int \vec{E}\cdot d\vec{l}}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$-dV = \vec{E}\cdot d\vec{l} = E_x dx + E_y dy + E_z dz$$

$$\frac{-dV}{dx} = E_x + E_y\underbrace{\frac{dy}{dx}}_{0} + E_z\underbrace{\frac{dz}{dx}}_{0}$$

(cartesian coordinates pieces are independent)

$$\boxed{-\frac{\partial V}{\partial x} = E_x}$$

↳ Can find E-field if given $V(x,y,z)$

$$\boxed{\begin{aligned}-\frac{\partial V}{\partial y} &= E_y \\ -\frac{\partial V}{\partial z} &= E_z\end{aligned}}$$

$$\boxed{-\vec{\nabla} V = \vec{E}}$$

$$V = \frac{KQ_1}{r_{1p}} + \frac{KQ_2}{r_{2p}} + \frac{KQ_3}{r_{3p}} \quad \text{(adding scalers)}$$

$$\vec{E} = \vec{E}(Q_1) + \vec{E}(Q_2) + \vec{E}(Q_3) \quad \text{(adding vectors)}$$

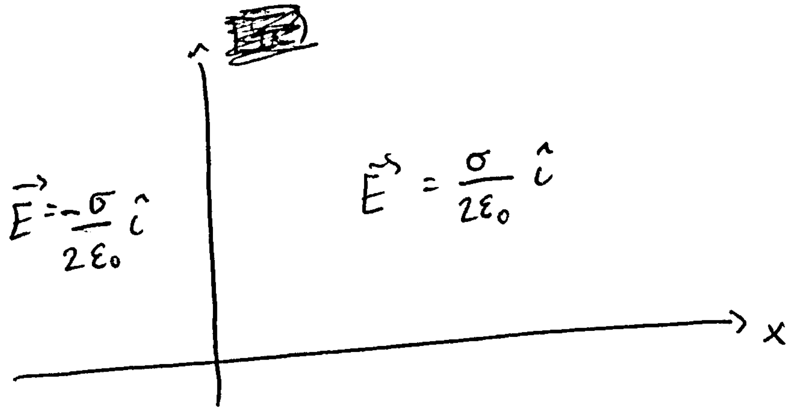
- Sometimes, one or the other is easier.

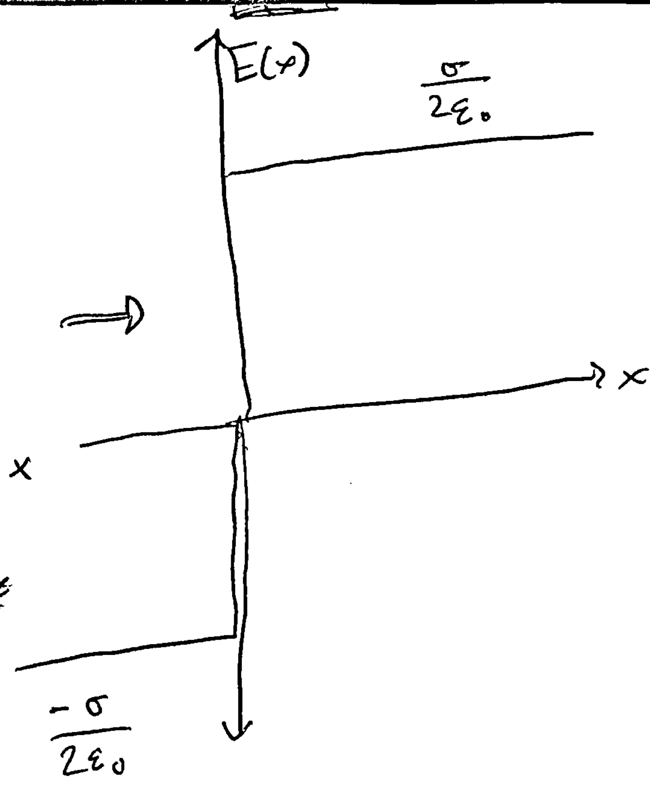
To summate:

$$\boxed{-\vec{\nabla} V = \vec{E}} \quad \text{and} \quad \boxed{V = -\int \vec{E} \cdot d\vec{l}}$$

(Relationships Between $\vec{E}$ and V)

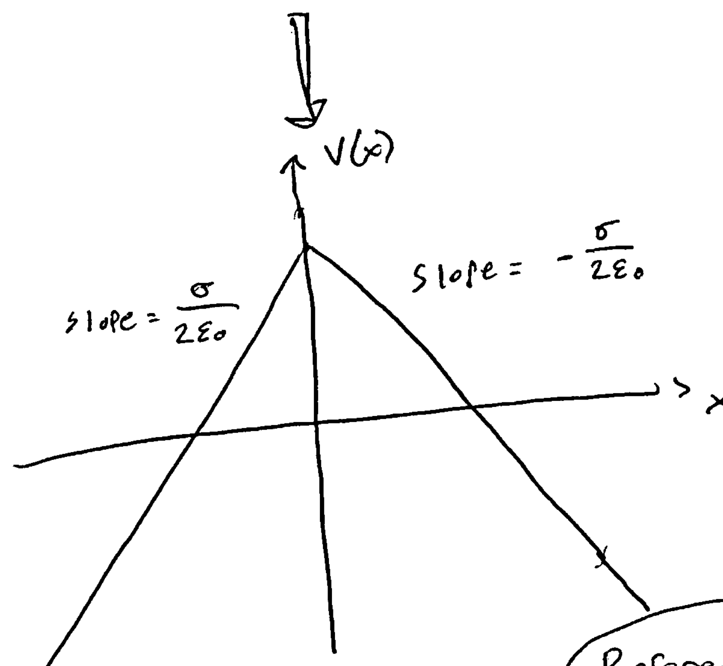
ex] infinite sheet of charge





if E is const, and

$\vec{E} = -\nabla \vec{V}$, $\vec{V}$ is linear.



$\underline{x>0}$: $V = -\int \vec{E}\cdot d\vec{l} = -\int \frac{\sigma}{2\varepsilon_0}\,dx = -\frac{\sigma}{2\varepsilon_0}x + V_0$

Reference Point determines V_0

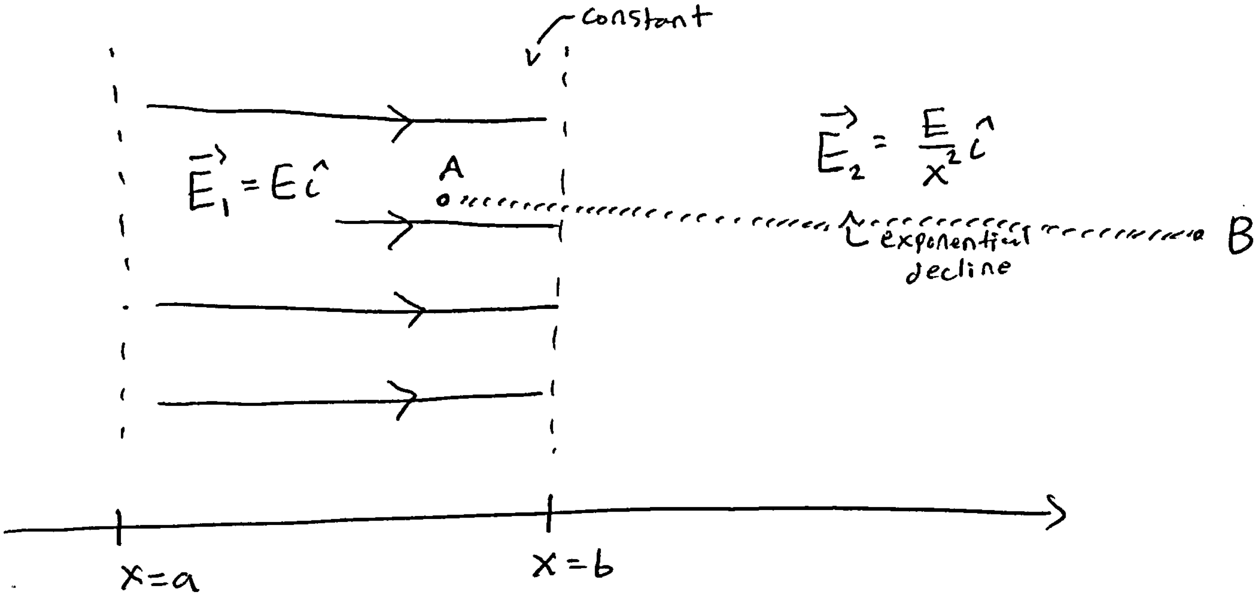
$V = -\infty$ at $x = \infty$

$V = 0$, $E \neq 0$

function slope

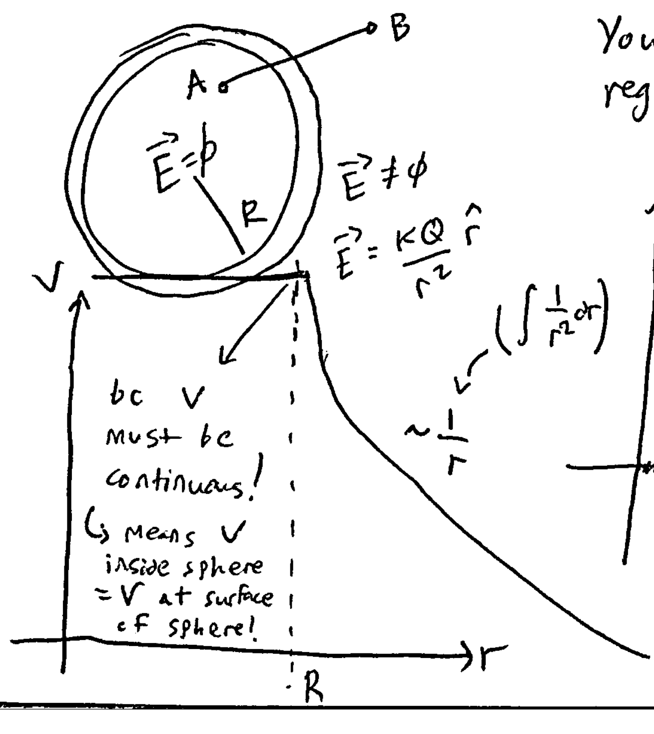
$$V = -\int \vec{E} \cdot d\vec{l}$$

But what if,

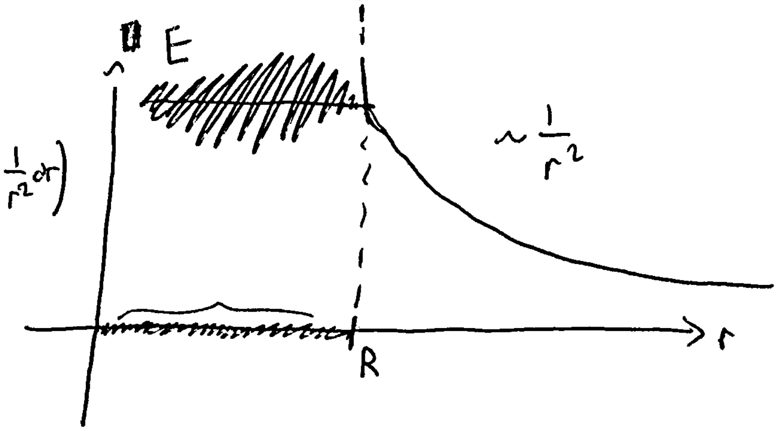


in this case, $V = -\int_A^B \vec{E} \cdot d\vec{l} = \left(-\int_B^{x=b} \vec{E}_2 \cdot d\vec{l}\right) + \left(-\int_{x=b}^{A} \vec{E} \cdot d\vec{l}\right)$

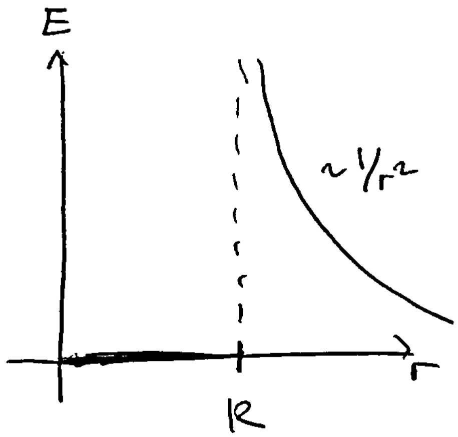
ex] spherical shell

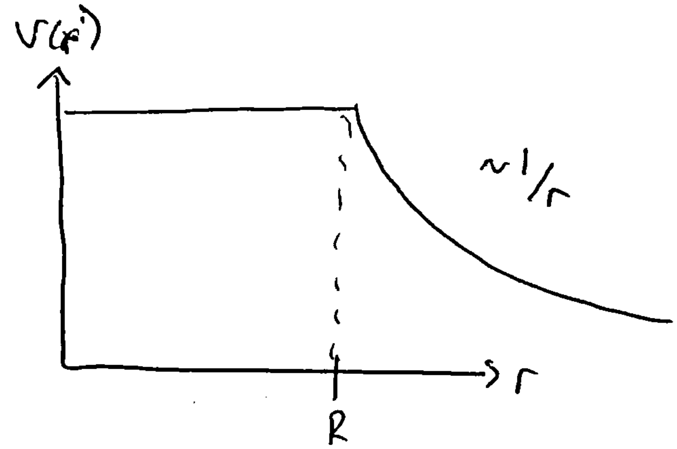


You must split integral into the regions w/ different electric fields



(spherical shell)



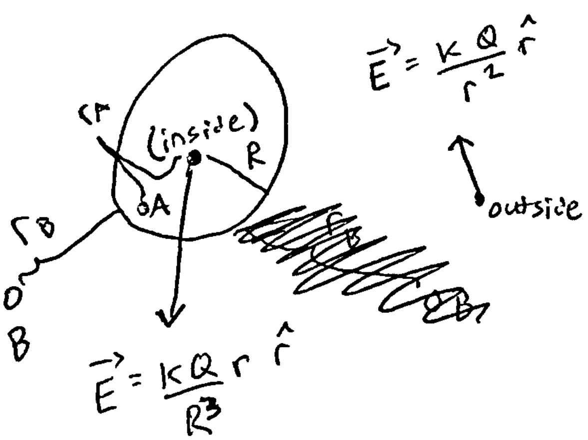


ex| Shell $r = 20$ cm

$$V(r = 10 \text{ cm from center}) = V(r = 20 \text{ cm}) = \frac{kQ}{20 \text{ cm}}$$

for spherical shell, V on inside is the same as V on surface!

ex| uniformly charged sphere (obv. non-conducting)



$$V = -\int_B^A \vec{E}\cdot d\vec{l} = \left. -\int_B^R \frac{kQ}{r_B^2}\, dr \right| \text{outside}$$

$$\left. -\int_R^A \frac{kQ}{R^3} r_A\, dr \right| \text{inside}$$