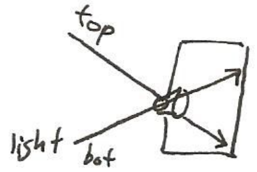
CSC418 A2 David - 1001435501 - lydavid1
Ly

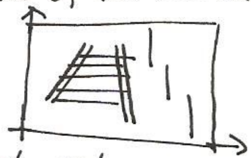
Part A

2. Viewing

a) The image in a pinhole camera is inverted because light travels in a straight line, so light that enters the pinhole from an angle below the hole will appear at the top of the inside and vice-versa.



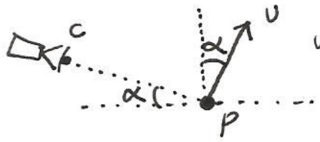
c) a family of parallel lines to $v = (vx, vy, vz)$ will remain parallel after a perspective projection if they are parallel to either the horizontal or vertical axis of the screen



d) Yes, they will converge to a single vanishing pt if this condition is not met.

That pt would be $v = \lim_{\lambda \to \pm\infty} x(\lambda) = f \frac{A + \lambda D}{A_z + \lambda D_z} = f \frac{D}{D_z} = f \begin{vmatrix} D_x / D_z \\ D_y / D_z \\ 1 \end{vmatrix}$

b)



we can get this with an inverse of camera to world transformation matrix (translation/rotation of camera's position/orientation)

- rotate c about p by α degrees
- translate c to some xy-coord in the view plane w/ center p

3. Surfaces

a) surface normal at point $p=(x,y,z)$ using: $f(x,y,z)=(R-\sqrt{(x^2+y^2)})^2+z^2-r^2=0$, $R>r$

- simply find gradient vector ∇f

$$\nabla f(x,y,z) = \langle 2(R-\sqrt{x^2+y^2})(-\tfrac{1}{2}(x^2+y^2)^{-1/2}(2x)), 2(R-\sqrt{x^2+y^2})(-\tfrac{1}{2}(x^2+y^2)^{-1/2}(2y)), 2z \rangle$$

$$= \langle \frac{2R-2\sqrt{x^2+y^2}}{-\sqrt{x^2+y^2}}x, \frac{2R-2\sqrt{x^2+y^2}}{-\sqrt{x^2+y^2}}y, 2z \rangle$$

b) implicit eqn of tangent plane at p

- this is perpendicular to our normal, say our point $p=(x,y,z)$ is actually $p=(x_0,y_0,z_0)$

general eqn of tangent plane:

$$f_x(x_0,y_0,z_0)(x-x_0)+f_y(x_0,y_0,z_0)(y-y_0)+f_z(x_0,y_0,z_0)(z-z_0)=0$$

then we have:

$$-\frac{2R-2\sqrt{x_0^2+y_0^2}}{\sqrt{x_0^2+y_0^2}}x_0(x-x_0) - \frac{2R-2\sqrt{x_0^2+y_0^2}}{\sqrt{x_0^2+y_0^2}}y_0(y-y_0)+2z(z-z_0)=0$$

c) show $q(\lambda)=(R\cos\lambda, R\sin\lambda, r)$ lies on surface

$x=R\cos\lambda \quad y=R\sin\lambda \quad z=r$

$$(R-\sqrt{R^2\cos^2\lambda+R^2\sin^2\lambda})^2+r^2-r^2=0$$

$$\Rightarrow (R-\sqrt{R^2(\cos^2\lambda+\sin^2\lambda)})=0$$

$$\Rightarrow (R-\sqrt{R^2})=0$$

$$\Rightarrow R-R=0$$

d) find tangent vector of $q(\lambda)$

$$q'(\lambda)=\langle -R\sin\lambda, R\cos\lambda, 0\rangle$$

e) show this tangent vetor lies on imp eqn of tangent plane

$x=-R\sin\lambda \quad y=R\cos\lambda \quad z=0$

$$\langle 2(-R\sin\lambda)(1-\frac{R}{\sqrt{R^2(\sin^2\lambda+\cos^2\lambda)}}), 2(R\cos\lambda)(1-\frac{R}{\sqrt{R^2(\sin^2\lambda+\cos^2\lambda)}}), 2(0)\rangle$$

x and y coord:

$$2(-R\sin\lambda)\underbrace{(1-\frac{R}{R})}_{=0} \text{ and } 2(R\cos\lambda)\underbrace{(1-\frac{R}{R})}_{=0}$$

z: 0

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4. Curves

a) tangents B_1' and B_2' at P_4

P_4 @ $t=1$

$B_1(t) = (1-t)^3 P_1 + 3(1-t)^2 t P_2 + 3(1-t)t^2 P_3 + t^3 P_4 \quad , 0 \le t \le 1$

$B_2(t)$ adjust so that $1 \le t \le 2$

$B_2(t) = (1-(t-1))^3 P_4 + 3(1-(t-1))^2(t-1)P_5 + 3(1-(t-1))(t-1)^2 P_6 + (t-1)^3 P_7 \quad , 0 \le t \le 1$

$= (2-t)^3 P_4 + 3(2-t)^2(t-1)P_5 + 3(2-t)(t-1)^2 P_6 + (t-1)^3 P_7$

derivatives

$B_1'(t) = 3(1-t)^2(-1)P_1 + 3(2(1-t)(-1)t)(1(1-t)^2)P_2 + 3(-1(t^2))(2t(1-t))P_3 + 3t^2 P_4$

$B_1'(1) = 3P_4$

$B_2'(t) = 3(2-t)^2 P_4 + \cdots$ * left out as they go to 0

$B_2'(1) = 3(2-1)^2 P_4 = 3P_4$

b) B_1'', B_2'' at P_4

$B_1''(t) = \cdots + 6tP_4$

$B_1''(1) = 6P_4$

$B_2''(t) = 6(2-t)P_4 + \cdots$

$B_2''(1) = 6(2-1)P_4 = 6P_4$

d)
1. easily allows C2 continuity via joining cubic Bezier curves at endpoints
2. minimum-curvature interpolants to a set of points
   (ie w/ n+3 points, if you define a "smoothest" curve that passes through them, it can be represented by n-segments)
3. position and derivatives can be found at start and end of equation
4. good tradeoff between computation and smoothness

c) $B_1(1) = B_2(1) = P_4$ * thus C^0 cts

$P_5 - P_4 = P_4 - P_3$ * by $B_1'(1) = B_2'(1)$

$\boxed{P_5 = 2P_4 - P_3}$

$P_6 - 2P_5 + P_4 = P_4 - 2P_3 + P_2$ * by $B_1''(1) = B_2''(1)$

$\boxed{P_6 = 4P_4 - 4P_3 + P_2}$

P_7 unconstrained if we only require C^2 cts, it will be constrained if we want C^3 cts.