ASTRO C207 Radiative Processes in Astrophysics
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Problem Set 5

1. Good Rovibrations

(1) Assume we end in $n = 0$.

$$\begin{aligned} k_{\text{center}} &= \frac{\nu_{\text{center}}}{c} \\ \nu_{\text{center}} &= ck_{\text{center}} \\ &\approx 3(10^{10}) \cdot 2145 \\ &\approx 6.4(10^{13})\text{Hz} \\ &\approx 1 \times \nu_{0,\text{CO}} \end{aligned}$$

where $\nu_{0,\text{CO}} \approx 6.7(10^{13})$Hz is the natural frequency of CO's vibrational transition.

$$\boxed{|\Delta n| = 1}$$

(2)
- Boltzmann statistics for populations in each J state
- Line intensity $\propto n_{J_{\text{upper}}}$

$$\begin{aligned} \frac{n_{J+1}}{n_J} &= \frac{g_{J+1}}{g_J} e^{-\frac{E_{J+1}-E_J}{k_B T}} \\ \text{where } E_{J+1} - E_J &= \frac{\hbar^2}{2I}\left[(J+1)(J+2) - J(J+1)\right] \\ &= \frac{\hbar^2}{I}(J+1) \longleftarrow \begin{cases} I \approx \mu a_0^2 \\ \mu \equiv m_O \parallel m_C \end{cases} \end{aligned}$$

Sweeping $\frac{n_{J+1}}{n_J}$ with respect to T and choosing $J_{\text{infl}} = 7$ where $\frac{n_{J_{\text{infl}}}}{n_{J_{\text{infl}}-1}} > 1$ and $\frac{n_{J_{\text{infl}}+1}}{n_{J_{\text{infl}}}} < 1$ yields a rough temperature range

INCLUDE

$$\boxed{T \in [416, 526)\text{K}}$$

(3) Looking up the dipole moment $d_{\text{CO}} \approx 0.122$ esu.cm and calculating values relative to the Lyman-α

$$A_{\text{CO}} = A_{\text{Ly}\alpha} \left(\frac{d_{\text{CO}}}{d_{\text{H}}}\right)^2 \left(\frac{\omega_{\text{CO}}}{\omega_{\text{Ly}\alpha}}\right)^3$$

with

$$
\begin{aligned}
\omega_{\mathrm{CO}} &= \frac{(\Delta E)_{\substack{\Delta n=1 \\ \Delta J=\pm 1}}}{\hbar} \\
&= \omega_{0,\mathrm{CO}} \pm \frac{\hbar}{I} J_{\mathrm{upper}}
\end{aligned}
$$

INCLUDE

A varies by more than 10% over the range of J–enough to justify inclusion.

(4)

(5)

A few things weren't accounted for:

- non-constant line profile function vs. frequency
- change in the moment of inertia (internuclear separation) with respect to n and J