

ASTRO C207 Radiative Processes in Astrophysics

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Problem Set 10

1. Powering Radio Lobes

- (1) Using points (40MHz, $2.3 \cdot 10^4 \text{Jy}$) and ($2 \cdot 10^4 \text{MHz}$, 500Jy) on the curve and using

$$F_\nu \propto \nu^{\frac{1+p}{2}}$$

$$\frac{\Delta(\log F_\nu)}{\Delta(\log \nu)} = \frac{1+p}{2}$$

(where $p < 0$)

TODO numerical

- (2)

$$n \cdot d\gamma = C\gamma^p \cdot d\gamma$$

$$U_e = \int_{\gamma_{\min}}^{\infty} C\gamma^p \cdot d\gamma$$

$$= C \left(\frac{1}{1+p} \gamma^{1+p} \right) \Big|_{\gamma=\gamma_{\min}}^{\gamma=\infty}$$

$$= -C \frac{\gamma_{\min}^{1+p}}{1+p} \leftarrow 1+p < 0$$

TODO

- (3)

$$v_{\text{cyc}} = \frac{qB}{m_e c}$$

$$U_B = \frac{B^2}{8\pi}$$

$$v_m = \frac{3}{2} \gamma_{\min}^2 v_{\text{cyc}} \sin \alpha$$

$$\gamma_{\min} = \sqrt{\frac{2v_m}{3v_{\text{cyc}} \sin \alpha}}$$

$$= A_1 v_m^{\frac{1}{2}} B^{-\frac{1}{2}} \text{ where } A_1 \equiv \sqrt{\frac{2m_e c}{3q \sin \alpha}}$$

$$L_\nu \approx \frac{2}{3} C \frac{U_B \sigma_{TC}}{v_{\text{cyc}}} \left(\frac{\nu}{v_{\text{cyc}}} \right)^{\frac{1+p}{2}} \times V$$

$$C \approx \frac{3L_m v_{\text{cyc}}^{\frac{3+p}{2}}}{2U_B \sigma_{TC} v_m^{\frac{1+p}{2}} V}$$

$$= A_2 \frac{L_m B^{-\frac{1+p}{2}}}{v_m^{\frac{1+p}{2}} V} \text{ where } A_2 \equiv \frac{12\pi \left(\frac{q}{m_e c} \right)^{\frac{3+p}{2}}}{\sigma_{TC}}$$

$$\begin{aligned}
 U_e &= \frac{-1}{1+p} C \gamma_{\min} \\
 &= A \frac{L_m B^{-\frac{2+p}{2}} v_m^{-\frac{p}{2}}}{V}
 \end{aligned}$$

check algebra

(4)

$$\begin{aligned}
 E &= 2V(U_e + U_B) \\
 \frac{dE}{dB} &= 2V \left(\frac{dU_e}{dB} + \frac{dU_B}{dB} \right) \\
 &= 2V \left(-\frac{3}{2} \frac{U_e}{B} + 2 \frac{U_B}{B} \right) \\
 &= \frac{2V}{B} \left(-\frac{3}{2} U_e + 2U_B \right)
 \end{aligned}$$

Discounting nonphysical limits like $B = \infty$ and $B = 0$,

$$U_B = \frac{3}{4} U_e$$

(5)

$$\begin{aligned}
 U_B &= \frac{3}{4} U_e \\
 \frac{1}{8\pi} B^2 &= \frac{3}{4} A \frac{L_m v_m^{\frac{1}{2}}}{V} B^{-\frac{3}{2}} \\
 AL_m \frac{v_m^{\frac{1}{2}}}{V} 6\pi &= B^{\frac{7}{2}} \\
 B &= \left(\frac{AL_m v_m^{\frac{1}{2}}}{V} \right)^{\frac{2}{7}}
 \end{aligned}$$

and then

$$\gamma_{\min} \approx \sqrt{\frac{2v_m}{3v_{\text{cyc}}}}$$

numerical

(6)

$$E = 2V(U_e + U_B)$$

numerical

(7) • $E_{\text{created}} = \frac{1}{10} m_{\text{food}} c^2$

Supermassive black holes are 10^5 to 10^6 times larger than the mass of the sun ($2 \cdot 10^{33}$ g)