ASTRO C207 Radiative Processes in Astrophysics
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Problem Set 10

1. Powering Radio Lobes

(1) Using points $(40\text{MHz}, 2.3 \cdot 10^4\text{Jy})$ and $(2 \cdot 10^4\text{MHz}, 500\text{Jy})$ on the curve and using

$$
\begin{aligned}
D &\propto \nu^{\frac{1+p}{2}} \\
\frac{\Delta(\log D)}{\Delta(\log \nu)} &= \frac{1+p}{2}
\end{aligned}
$$

(where $p < 0$)

TODO numerical

(2)

$$
\begin{aligned}
n \cdot d\gamma &= C\gamma^p \cdot d\gamma \\
U_e &= \int_{\gamma_{\min}}^{\infty} C\gamma^p \cdot d\gamma \\
&= C\left(\frac{1}{1+p}\gamma^{1_p}\right)\Big|_{\gamma=\gamma_{\min}}^{\gamma=\infty} \\
&= -C\frac{\gamma_{\min}^{1+p}}{1+p} \longleftarrow 1+p<0
\end{aligned}
$$

TODO

(3)

$$
\begin{aligned}
\nu_{\text{cyc}} &= \frac{qB}{m_e c} \\
U_B &= \frac{B^2}{8\pi}
\end{aligned}
$$

$$
\begin{aligned}
\nu_m &= \frac{3}{2}\gamma_{\min}^2 \nu_{\text{cyc}} \sin\alpha \\
\gamma_{\min} &= \sqrt{\frac{2\nu_m}{3\nu_{\text{cyc}}\sin\alpha}} \\
&= A_1 \nu_m^{\frac{1}{2}} B^{-\frac{1}{2}} \text{ where } A_1 \equiv \sqrt{\frac{2m_e c}{3q\sin\alpha}}
\end{aligned}
$$

$$
\begin{aligned}
L_\nu &\approx \frac{2}{3}C\frac{U_B\sigma_T c}{\nu_{\text{cyc}}}\left(\frac{\nu}{\nu_{\text{cyc}}}\right)^{\frac{1+p}{2}} \times V \\
C &\approx \frac{3L_m \nu_{\text{cyc}}^{\frac{3+p}{2}}}{2U_B\sigma_T c\nu_m^{\frac{1+p}{2}} V} \\
&= A_2\frac{L_m B^{\frac{-1+p}{2}}}{\nu_m^{\frac{1+p}{2}} V} \text{ where } A_2 \equiv \frac{12\pi\left(\frac{q}{m_e c}\right)^{\frac{3+p}{2}}}{\sigma_T c}
\end{aligned}
$$

$$U_e = \frac{-1}{1+p} C \gamma_{\min}$$
$$= A \frac{L_m B^{\frac{-2+p}{2}} \nu_m^{-\frac{p}{2}}}{V}$$

check algebra

$$E = 2V(U_e + U_B)$$
$$\frac{dE}{dB} = 2V\left(\frac{dU_e}{dB} + \frac{dU_B}{dB}\right)$$
$$= 2V\left(-\frac{3}{2}\frac{U_e}{B} + 2\frac{U_B}{B}\right)$$
$$= \frac{2V}{B}\left(-\frac{3}{2}U_e + 2U_B\right)$$

Discounting nonphysical limits like $B = \infty$ and $B = 0$,

$$U_B = \frac{3}{4}U_e$$

(4)

(5)

(6)

(7)