ASTRO C207 Radiative Processes in Astrophysics
Fall 2021 Problem Set 3

1. Rotating Magnetic Dipole

(1) For electric dipoles:

$$P = \frac{2}{3}\frac{|\ddot{\vec{p}}|^2}{c^3}$$

For magnetic dipoles:

$$P = \frac{2}{3}\frac{|\ddot{\vec{m}}|^2}{c^3}$$

$\vec{m}$ is rotating about the axis of rotation

$$\begin{aligned}
\vec{m} &= \begin{bmatrix} m\sin(\alpha)\cos(\omega t) \\ m\sin(\alpha)\sin(\omega t) \\ m\cos(\alpha) \end{bmatrix} \\
\ddot{\vec{m}} &= \begin{bmatrix} -\omega^2 m\sin(\alpha)\cos(\omega t) \\ -\omega^2 m\sin(\alpha)\sin(\omega t) \\ 0 \end{bmatrix} \\
|\ddot{\vec{m}}|^2 &= \omega^4 m^2 \sin^2(\alpha)
\end{aligned}$$

Definitely had to look up how to derive the magnetic field from the magnetic dipole

$$\begin{aligned}
\vec{B} &= \frac{3\hat{r}(\vec{m}\cdot\hat{r}) - \vec{m}}{r^3} \longleftarrow \text{because CGS, that's why} \\
&= m\frac{2cos(\theta)\hat{r} + sin(\theta)\hat{\theta}}{r^3} \\
B_0 &= m\frac{2}{R^3} \longleftarrow \theta = 0 \\
m &= \frac{B_0 R^3}{2}
\end{aligned}$$

$$\begin{aligned}
P &= \frac{2}{3}\frac{\omega^4\sin^2(\alpha)B_0^2R^6}{4c^3} \\
&= \frac{\omega^4\sin^2(\alpha)B_0^2R^6}{6c^3}
\end{aligned}$$

$$\boxed{P = \frac{\omega^4\sin^2(\alpha)B_0^2R^6}{6c^3}}$$

(2)

$$
\begin{aligned}
E_{\text{rot}} &= \frac{1}{2} I\omega^2 \\
&= \frac{1}{5} MR^2\omega^2 \\
P &= \dot{E}_{\text{rot}} \\
\frac{\omega^4 \sin^2(\alpha) B_0^2 R^6}{6c^3} &= \frac{1}{5} MR^2 \cdot 2\omega\dot{\omega} = \\
\dot{\omega} &= \frac{\omega}{\tau} \\
\tau &= \frac{6c^3 \cdot \frac{2}{5}}{\omega^2 \sin^2(\alpha) B_0^2 R^4}
\end{aligned}
$$

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\boxed{\tau = \frac{6c^3 \cdot \frac{2}{5}}{\omega^2 \sin^2(\alpha) B_0^2 R^4}}
$$

(3)