ASTRO C207 Radiative Processes in Astrophysics
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Problem Set 8

1. Saha and the Redshift of Recombination

- electron de Broglie wavelength: $\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$
- electron degeneracy: $\xi \equiv n_e \lambda^3 \approx 4.1(10^{-16}) n_e T^{-\frac{3}{2}}$
- $\gamma \equiv \ln\left(\frac{1}{\xi}\right) \approx$ 10-30
- Assume $U_x(T) \propto T^0, \approx 1$

(1)

From Saha

$$\begin{aligned}
\frac{n_{j+1} n_e}{n_j} &= \left(\frac{2\pi m_e k_B T}{h^2}\right)^{\frac{3}{2}} \left(\frac{2U_{j+1}(T)}{U_j(T)}\right) e^{-\frac{\chi}{k_B T}} \\
\frac{n_{j+1}}{n_j} &= \frac{1}{\xi} \frac{2U_{j+1}(T)}{U_j(T)} e^{-\frac{\chi}{k_B T}} \\
0 &\approx \gamma - \frac{\chi}{k_B T} \longleftarrow n_{j+1} \approx n_j \\
k_B T &\approx \frac{\chi}{\gamma}
\end{aligned}$$

(2) I don't understand how $\frac{\Delta T}{T} \approx \left(\frac{d \log\left(\frac{n_{j+1}}{n_j}\right)}{d \log(T)}\right)^{-1}$

$$
\begin{aligned}
R \equiv \frac{n_{j+1}}{n_j} &\approx \frac{1}{\xi} e^{-\frac{\chi}{k_B T}} \\
\frac{d\ln(R)}{d\ln(T)} &= \frac{d\ln(R)}{dT}\left(\frac{d\ln(T)}{dT}\right)^{-1} \\
&\approx \frac{d}{dT}\left[\gamma - \frac{\chi}{k_B T}\right]\cdot T \\
&= \left\{\frac{d}{dT}\left[\ln\left(\frac{T^{\frac{3}{2}}}{4.1(10^{-16} n_e)}\right)\right] + \frac{\chi}{k_B T^2}\right\}\cdot T \\
&= \frac{3}{2} + \frac{\chi}{k_B T} \\
&\approx \frac{3}{2} + \frac{\chi}{\chi/\gamma} \\
&= \frac{3}{2} + \gamma \longleftarrow \gamma \approx \text{10-30} \\
&\approx \gamma \\
\frac{\Delta T}{T} \approx \left(\frac{d\ln(R)}{d\ln(T)}\right)^{-1} &\approx \frac{1}{\gamma}
\end{aligned}
$$

(3)

$$
\begin{aligned}
\frac{n_{j+1}}{n_j} &= \frac{g_{j+1}}{g_j} e^{-\frac{E_{j+1}-E_j}{k_B T}} \\
&\approx e^{-\gamma \frac{E_{j+1}-E_j}{\chi}} \\
&\approx e^{-\gamma} \\
&\ll 1 \longleftarrow \gamma \gg 1
\end{aligned}
$$

2. Pulsar Dispersion Measure

(1)

(2)

(3)

(4)