ASTRO C207 Radiative Processes in Astrophysics
Fall 2021 Problem Set 1

1. Practice with $j_\nu, \alpha_\nu, S_\nu, I_\nu$

(1)
- $n_{\text{gas}} \sim 10\text{cm}^{-3}$
- $\rho_{\text{dust}}/\rho_{\text{gas}} = 0.01$
- $r_{\text{grain}} = 0.1\mu\text{m} = 10^{-5}\text{cm}$
- $\rho_{\text{grain}} \sim 3\frac{\text{g}}{\text{cm}^3}$

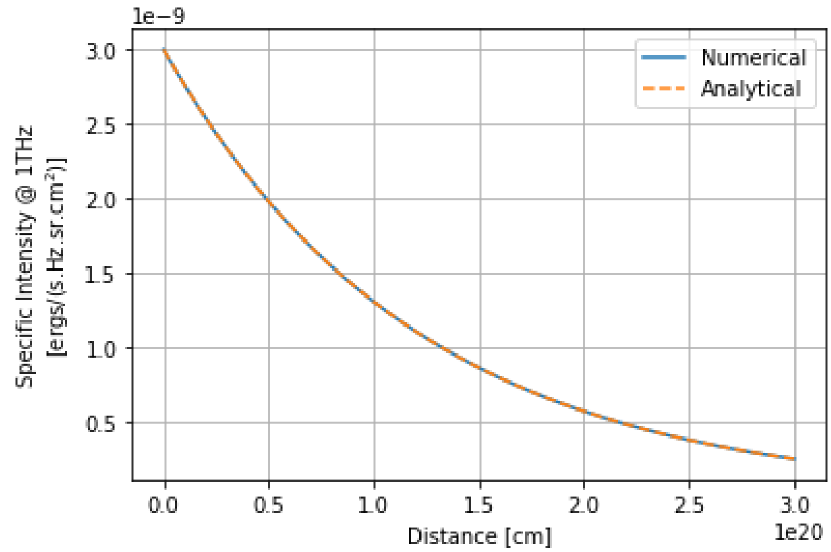
$$
\begin{aligned}
\rho_{\text{gas}} &= \frac{2}{N_0} n_{\text{gas}} \\
&\approx \frac{1}{3}\left(10^{-22}\right)\frac{\text{g}}{\text{cm}^3} \\
\rho_{\text{dust}} &= \frac{\rho_{\text{gas}}}{100} \\
&\approx \frac{1}{3}(10^{-24})\frac{\text{g}}{\text{cm}^3}
\end{aligned}
$$

$$
\begin{aligned}
n_{\text{dust}} &= \frac{\rho_{\text{dust}}}{m_{\text{grain}}} \\
&= \frac{\rho_{\text{dust}}}{V_{\text{grain}}\rho_{\text{grain}}} \\
&= \frac{\rho_{\text{dist}}}{\frac{4}{3}\pi r_{\text{grain}}^3 \rho_{\text{grain}}} \\
&\approx \frac{1}{12\pi}(10^{-9})\frac{\text{particles}}{\text{cm}^3}
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
n_{\text{dust}} &\approx \frac{1}{12\pi}(10^{-9})\frac{\text{particles}}{\text{cm}^3} \\
&\approx 2.65(10^{-11})\frac{\text{particles}}{\text{cm}^3}
\end{aligned}
}
$$

(2)
- $I_{\nu 0} = 3(10^{-3})\frac{\text{erg}}{\text{s}\cdot\text{Hz}\cdot\text{sr}\cdot\text{cm}^2}$
- $\nu = 1\text{THz}$

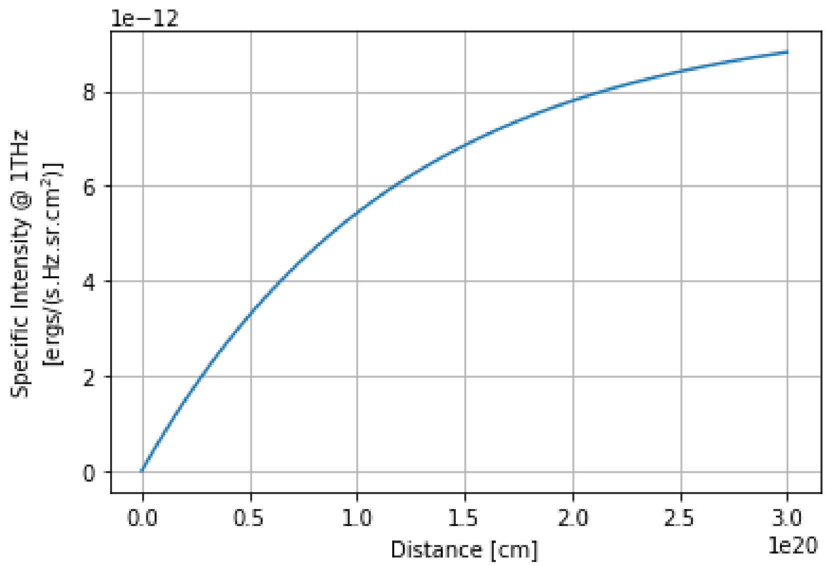
$$\begin{aligned}\alpha_\nu &= n_{\text{dust}}\sigma_{\text{grain}} \\ &= n_{\text{dust}}\cdot\pi r_{\text{grain}}^2 \\ &\approx \frac{1}{12}(10^{-19})\frac{1}{\text{cm}}\end{aligned}$$



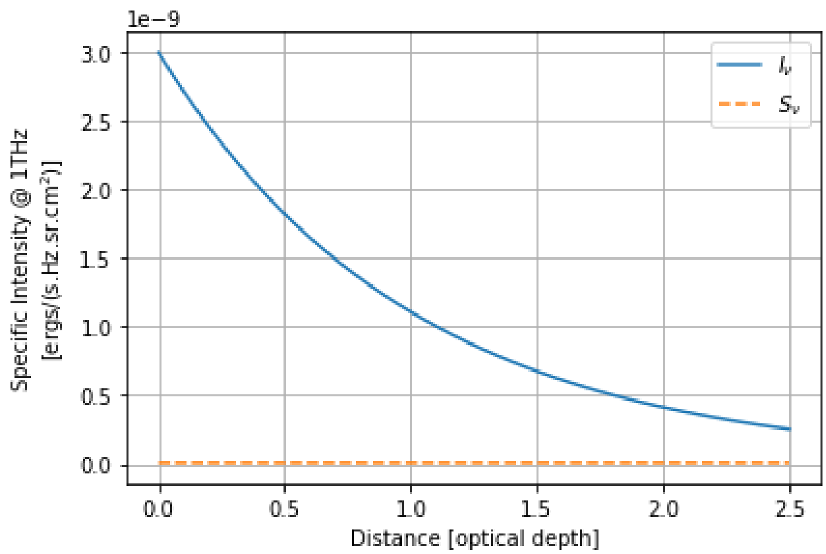
(3) • $T = 50\text{K}$

$$\begin{aligned} B_\nu(\nu, T) &= \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{k_B T}} - 1} \\ &\approx 9.6(10^{-12})\frac{\text{erg}}{\text{s}\cdot\text{cm}^2\cdot\text{sr}\cdot\text{Hz}} \\ j_\nu &= B_\nu \sigma_{\text{grain}} n_{\text{dust}} \\ &\approx 8(10^{-32})\frac{\text{erg}}{\text{s}\cdot\text{cm}^3\cdot\text{sr}\cdot\text{Hz}} \end{aligned}$$

$$I_{\nu,\text{medium}}(s) = \frac{j_\nu}{\alpha_\nu}\left(1 - e^{-\alpha_\nu s}\right)$$



(4) Combining the answers from before



2. Brightness, Magnitudes, and Photons

(1) • $D_{\text{Keck}} = 10\text{m}$

$$F_i = \int_{\lambda_{\text{start}}}^{\lambda_{\text{stop}}} F_\lambda(\lambda)\phi(\lambda)d\lambda$$

Matching x-axes for the integral involved interpolating and resampling F_λ and ϕ over wavelengths $\{\lambda[0], \lambda[1], \ldots, \lambda[N-1]\}$, i.e.

$$F_{\lambda,\text{resample}}[i] = F_\lambda(\lambda[i])$$
$$\phi_{\text{resample}}[i] = \phi(\lambda[i])$$

I ended up just using the wavelengths provided with the Vega data, so computationally, resampling F_λ didn't do anything.

The total photon flux accounted for changing wavelength and was taken as

$$\sum_{i=0}^{N-2} \frac{F_{\lambda,\text{resample}}[i]\phi_{\text{resample}}[i]}{\frac{hc}{\lambda[i]}} \cdot (\lambda[i+1] - \lambda[i])$$

.

Multiplying the total photon flux by the area of the receiver $A_{\text{Keck}} = \pi\left(\frac{D_{\text{Keck}}}{2}\right)^2$ gives the photon count rate.

$$\boxed{\text{count rate} \approx 7.3(10^{11})\,\frac{\text{photons}}{\text{s}}}$$

(2)
- $x_{\text{Vega}} = 8\text{pc}$
- $D_{\text{Vega}} = 2.5R_{\odot}$

First, checking if Vega's size in the sky exceeds the telescope's receiving beam...

$$\theta_{\text{Vega}} = \arctan\left(\frac{D_{\text{Vega}}}{x_{\text{Vega}}}\right) \qquad \theta_{\text{beamwidth}}(\lambda) \approx \frac{1.22\lambda}{D_{\text{Keck}}}$$

...and it doesn't.

Using the resampling from earlier,

$$F_i = \sum_{i=0}^{N-2} F_{\lambda,\text{resample}}[i]\phi_{\text{resample}}[i] \cdot (\lambda[i+1] - \lambda[i])$$

$$I_\lambda = \frac{F_i}{\Delta\lambda\,\theta_{\text{Vega}}^2}$$

where $\Delta\lambda$ is the passlength.

$$\boxed{I_\lambda \approx 8(10^{13})\frac{\text{erg}}{\text{s}\cdot\text{cm}^2\cdot\text{sr}\cdot\text{cm}}}$$

(3) Halving the distance to Vega still doesn't make it larger than the telescope's receiving beam.
$P \propto \frac{1}{r^2}$.
The solid angle that Vega occupies in the sky also scales up by $4\times$.

Halving the distance to Vega would increase the number of photons by 4x.
The specific intensity wouldn't change; this is because the solid angle that Vega occupies in the sky (θ^2_{Vega}) would scale up by $4\times$ as well.

3. Dust Bowl

(1)
- $D_{\text{grain}} = 100\mu\text{m}$
- $x_{\text{vis}} \approx 1.5\text{m}$
- $\tau_{\text{hard-to-see}} \approx 3$

$$\begin{aligned} n_{\text{dust,air}} &= \frac{\alpha}{\sigma_{\text{grain}}} \\ &= \frac{1}{\lambda_{\text{mfp}}\sigma_{\text{grain}}} \\ &= \frac{1}{\frac{x_{\text{vis}}}{\tau_{\text{hard-to-see}}} \cdot \pi D_{\text{grain}}^2/4} \end{aligned}$$

$$\boxed{n_{\text{dust,air}} \approx 2.6(10^8)\,\frac{\text{particles}}{\text{m}^3}}$$

(2) • $z_{\text{air}} \approx 8(10^4)\text{cm}$

$$n_{\text{dust,ground}} \approx \frac{1}{D_{\text{grain}}^3}$$
$$z_{\text{ground}} n_{\text{dust,ground}} = z_{\text{air}} n_{\text{dust,air}}$$

$$\boxed{z_{\text{ground}} \approx 20\text{cm}}$$