ASTRO C207 Radiative Processes in Astrophysics
Lydia Lee

Problem Set 9

1. A Compton Monte Carlo

(1)

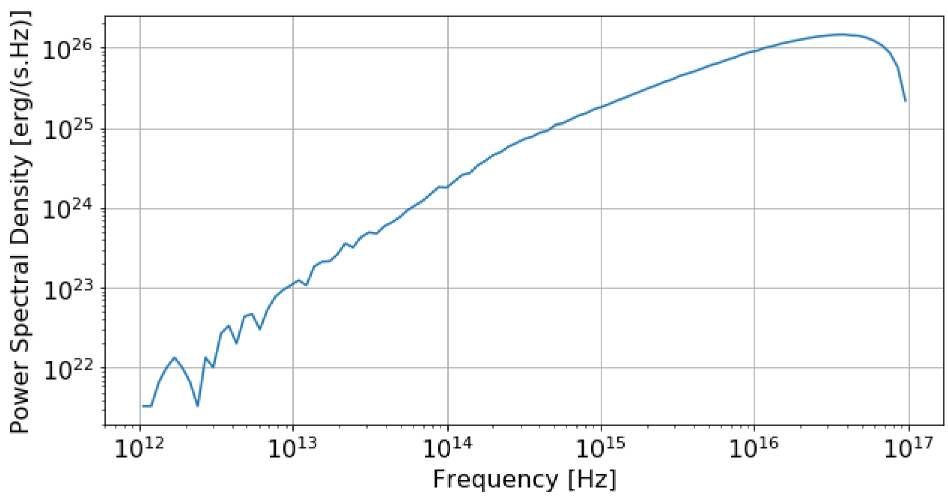


Figure 1: The simulated power spectral density of a single electron with $\gamma = 10$, generated via Monte Carlo sampling of uniformly distributed $\cos\theta_{\text{in}}$ and $\cos\theta'_{\text{out}}$. The cutoff (around x-ray frequencies) doesn't seem terribly unreasonable and corresponds to a few hundred eV.

(2)

$$\frac{dP}{d\nu} = \frac{dP}{d\gamma} \cdot \frac{d\gamma}{d\nu}$$

$$\begin{aligned}
\nu &= \nu_{\text{in}}\gamma^2(1-\beta\cos\theta_{\text{in}})(1+\beta\cos\theta'_{\text{out}}) \\
&\approx \nu_{\text{in}}\gamma^2 \\
\frac{d\gamma}{d\nu} &\approx \frac{1}{2\sqrt{\nu_{\text{in}}}}\nu^{-\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\frac{dP}{d\gamma} &= \frac{dN_\gamma}{d\gamma} \cdot P|_{\text{single photon}} \\
&= N_p A\gamma^{-p} \cdot L_p\gamma^2(1-\beta\cos\theta_{\text{in}})(1+\beta\cos\theta'_{\text{out}})(1-e^{-\tau}) \\
&\approx N_p A\gamma^{-p} \cdot L_p\gamma^2(1-e^{-\tau}) \\
&= N_p L_p A(1-e^{-\tau})\gamma^{2-p} \\
&\approx N_p L_p A(1-e^{-\tau})\left(\frac{\nu}{\nu_{\text{in}}}\right)^{1-\frac{p}{2}}
\end{aligned}$$

where

$$A = \frac{1-p}{(\gamma_{\max})^{1-p} - (\gamma_{\min})^{1-p}}$$

to enforce

$$\int_{\gamma_{\min}}^{\gamma_{\max}} Ax^{-p}dx = 1$$

When push comes to shove,

$$\frac{dP}{d\nu} \propto \nu^{\frac{1-p}{2}}$$

over values of $\nu \in [\nu_{\rm in}(\gamma_{\min})^2, \nu_{\rm in}(\gamma_{\max})^2]$ (roughly). The plot below includes the various constants.

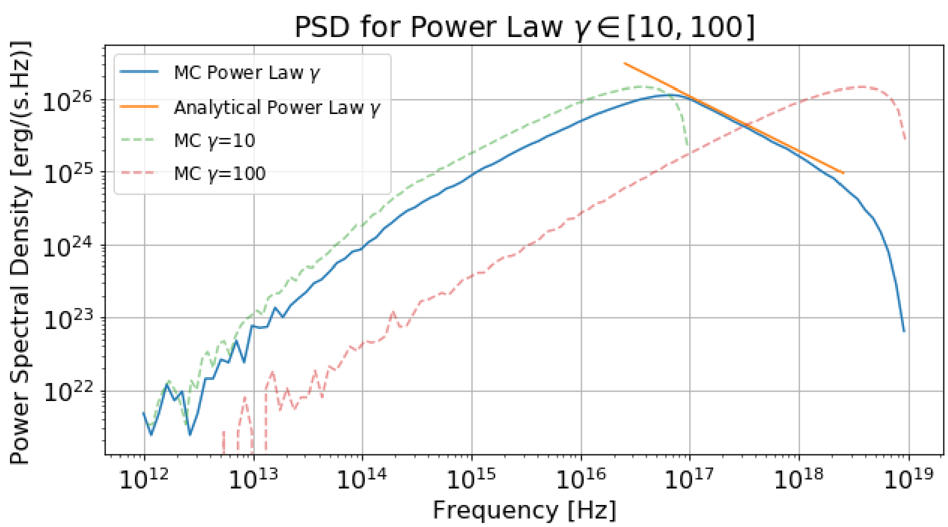


Figure 2: The simulated power spectral density of an ensemble of electrons with a power law distribution of γ, i.e. $dN_\gamma = A\gamma^{-p}d\gamma$ when $\gamma_{\min} \leq \gamma \leq \gamma_{\max}$. The PSDs for constant, single-valued γ at the bounds $\gamma_{\min}$ and $\gamma_{\max}$ are included to sanity check the behavior of the Monte Carlo simulated result past the bounds of their respective cutoff frequencies.

q_a_compton_monte_carlo

November 15, 2021

In [1]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from pprint import pprint
```

In [2]:
```
q = 5e-10 # esu, electron charge
hbar = 1e-27 # erg.s
h_Planck = hbar * 2*np.pi # erg.s
m_e = 1e-27 # g, electron mass
c = 3e10 # cm/s, speed of light
eV2erg = 1.602e-12
```

In [3]:
```
L_s = 1e43 # erg/s, supernova luminosity
E_in = 1 * eV2erg # ergs, initial photon energy
nu_in = E_in / h_Planck # Hz, photon frequency
tau = 0.01 # optical depth for shell of relativistic e-
N_p = 1000000 # number of MC packets

L_p = L_s/N_p # erg/s, MC packet luminosity
```

In [4]:
```
def calc_PSD(gamma, cos_theta_in, cos_theta_out_p, E_in):
    '''
    Inputs:
        gamma: Scalar or NumPy array of gamma.
        cos_theta_in: Scalar or NumPy array of cos(theta_in).
        cos_theta_out_p: Scalar or NumPy array of cos(theta_out').
        E_in: Ergs, scalar or NumPy array of pre-scattering photon energy.
    Returns:
        nu_out_bin_vec: NumPy array of floats. A collection of frequencies of
            photons after Compton scattering against a relativistic electron.
            These are uniformly spaced in the logarithmic scale.
        PSD: NumPy array of floats. Power spectral density, index matched
            ot nu_out_bin_vec, in erg/(s.Hz).
    Raises:
        A stink if (more than one of gamma, cos_theta_in, cos_theta_out_p, and E_in
        are NumPy arrays) and (of those which are arrays, the dimensions don't match).
    '''
    beta = np.sqrt(1 - 1/gamma**2)
    E_out_dat = E_in * gamma**2 \
        * (1 - beta*cos_theta_in_dat) \
        * (1 + beta*cos_theta_out_p_dat)
    L_out_dat = L_p * E_out_dat/E_in * (1-np.exp(-tau))
```

```
        # Binning data and getting counts
#       hist, L_out_bins = np.histogram(L_out_dat, bins=100)
        L_out_bins = np.logspace(np.log10(min(L_out_dat)-.1),
                                 np.log10(max(L_out_dat)+.1),
                                 num=100)
        hist,_ = np.histogram(L_out_dat, bins=L_out_bins)

        # Getting x-axis for for-realsies plotting
        L_out_bin_vec = 0.5 * (L_out_bins[1:]+L_out_bins[:-1])
        E_out_bin_vec = L_out_bin_vec/L_p * E_in / (1-np.exp(-tau))
        nu_out_bin_vec = E_out_bin_vec / h_Planck

        # Normalizing by bin width
        L_out_bin_width_vec = L_out_bins[1:] - L_out_bins[:-1]
        E_out_bin_width_vec = L_out_bin_width_vec/L_p * E_in / (1-np.exp(-tau))
        nu_out_bin_width_vec = E_out_bin_width_vec / h_Planck

        PSD = hist * L_out_bin_vec / nu_out_bin_width_vec

        return nu_out_bin_vec, PSD
```

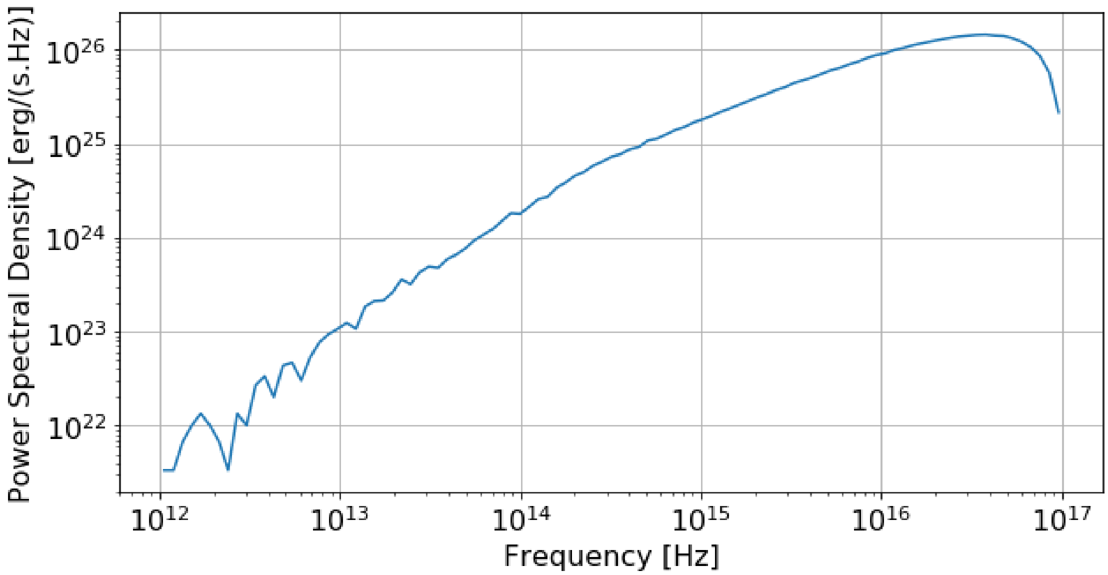
1

```
In [5]: gamma = 10

        # MC setup
        np.random.seed(0) # DEBUG constant seed
        cos_theta_in_dat = np.random.uniform(-1, 1, N_p)
        np.random.seed(1) # DEBUG constant seed
        cos_theta_out_p_dat = np.random.uniform(-1, 1, N_p)
```

```
In [6]: nu_out_single, PSD_single = calc_PSD(gamma,
                                             cos_theta_in_dat,
                                             cos_theta_out_p_dat,
                                             E_in)
```

```
In [7]: plt.rcParams['figure.figsize'] = (10, 5)
        plt.rcParams.update({'font.size': 16})
        # Plotting
        plt.loglog(nu_out_single, PSD_single)
        plt.xlabel("Frequency [Hz]");
        plt.grid(True)
        plt.ylabel("Power Spectral Density [erg/(s.Hz)]");
```

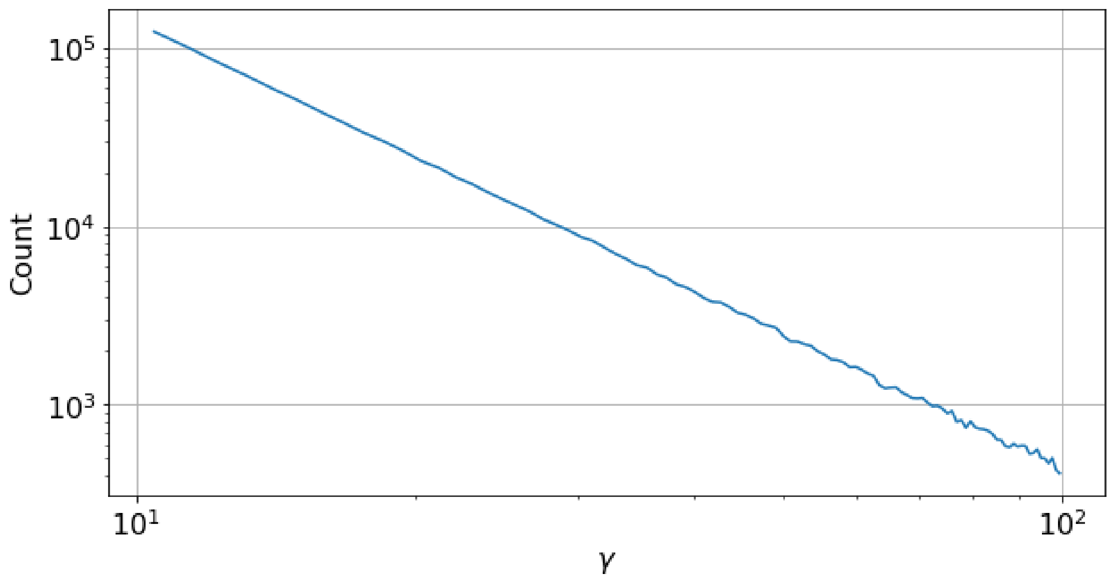


2

```
In [8]: gamma_min = 10
        gamma_max = 100
        p = 2.5
        power_law = lambda xmin, xmax, n, size: \
            ((xmax**(n+1) - xmin**(n+1))*np.random.uniform(size=size) \
             + xmin**(n+1)) ** (1/(n+1))

        # MC setup
        np.random.seed(3) # DEBUG constant seed
        gamma_vec = power_law(gamma_min, gamma_max, -p, N_p)
```

```
In [9]: # Sanity checking gamma
        gamma_hist, gamma_bins = np.histogram(gamma_vec, bins=100);
        plt.loglog(0.5*(gamma_bins[1:]+gamma_bins[:-1]), gamma_hist)
        plt.xlabel('$\gamma$')
        plt.ylabel('Count')
        plt.grid(True)
```




```
In [10]: nu_out_ensemble, PSD_ensemble = calc_PSD(gamma_vec,
                                                  cos_theta_in_dat,
                                                  cos_theta_out_p_dat,
                                                  E_in)
         nu_out_max, PSD_single_max = calc_PSD(gamma_max,
                                               cos_theta_in_dat,
                                               cos_theta_out_p_dat,
                                               E_in)
         nu_out_min, PSD_single_min = calc_PSD(gamma_min,
                                               cos_theta_in_dat,
                                               cos_theta_out_p_dat,
                                               E_in)
```

```
In [11]: # Theory
         nu_min = nu_in * gamma_min**2
         nu_max = nu_in * gamma_max**2
         nu_ensemble_theory = np.logspace(np.log10(nu_min), np.log10(nu_max), 100)

         const_A = (1-p) / (gamma_max**(1-p) - gamma_min**(1-p))
         dgamma_dnu = 1/(2*np.sqrt(nu_in)) * 1/np.sqrt(nu_ensemble_theory)
         dpower_dgamma = N_p * L_p * const_A * (1-np.exp(-tau)) * (nu_ensemble_theory/nu_in)**(1-p/2)
         PSD_ensemble_theory = dpower_dgamma * dgamma_dnu
```

```
In [12]: # Plotting
         plt.loglog(nu_out_ensemble, PSD_ensemble, label='MC Power Law $\gamma$')
         plt.loglog(nu_ensemble_theory, PSD_ensemble_theory, label='Analytical Power Law $\gamma$')
         plt.loglog(nu_out_min, PSD_single_min, '--', label=f'MC $\gamma$={gamma_min}', alpha=0.5)
         plt.loglog(nu_out_max, PSD_single_max, '--', label=f'MC $\gamma$={gamma_max}', alpha=0.5)
         plt.xlabel("Frequency [Hz]");
         plt.ylabel("Power Spectral Density [erg/(s.Hz)]");
         plt.title('PSD for Power Law $\gamma \in [10,100]$');
```

```
plt.grid(True)
plt.legend(prop={'size': 12})
```

Out[12]: <matplotlib.legend.Legend at 0x1a97c95e198>