HW04

Lydia Lee

1

In [1]:
```
## Lydia Lee

%matplotlib inline
import numpy as np
from scipy import signal

from astropy.visualization import quantity_support
import matplotlib.pyplot as plt
from pprint import pprint

import matplotlib as mpl
mpl.rcParams['agg.path.chunksize'] = 50000000
```

In [2]:
```
c = 3e10 # cm/s, speed of light
kB = 1.4e-16 # cgs, Boltzmann constant
m = 7.335e-23 # g, mass of CO2 molecule
A21 = 10 # s^-1, Einstein A of the transition
L21 = 15e-4 # cm, wavelength of the transition
F21 = c / L21 # Hz, frequency of the transition

def lorentz(nu, A=A21, nu0=F21):
    """Return a Lorentzian line profile function evaluated at the specified frequencies.
    Arguments:
        nu: spectral frequency [Hz]
        A: Einstein A coefficient [s^-1]
        nu0: center frequency [Hz]"""
    ans = A/(4*np.pi)**2 / ((nu - nu0)**2 + (A/(4*np.pi))**2)
    return ans / ans.sum()

def doppler(nu, T, m, nu0=F21):
    """Return a Gaussian line profile function evaluated at the specified frequencies.
    Arguments:
        nu: spectral frequency [Hz]
        T: Temperature [K]
        m: Mass of the particle [g]
        nu0: center frequency [Hz]"""
    dnu = np.sqrt(2 * kB * T / m) * nu0/c
    ans = 1./(dnu * np.sqrt(np.pi)) * np.exp(-(nu-nu0)**2/dnu**2)
    return ans / ans.sum()
```

```
def voigt(nu, T, m, A=A21, nu0=F21, mode='fft'):
    """Return a Voigt line profile function evaluated at the specified frequencies.
    Arguments:
        nu: spectral frequency [Hz]
        T: Temperature [K]
        m: Mass of the particle [g]
        A: Einstein A coefficient [s^-1]
        nu0: center frequency [Hz]
        mode: fft uses FFT for convolution, anything else uses
            polynomial multiplication.
    """
    phi_lorentz = lorentz(nu=nu, A=A, nu0=nu0)
    phi_doppler = doppler(nu=nu, m=m, T=T, nu0=nu0)
    f_conv = signal.fftconvolve if mode == 'fft' else np.convolve
    phi_voigt = f_conv(phi_lorentz, phi_doppler, mode='same')
    phi_voigt_norm = phi_voigt/np.trapz(phi_voigt, nu)
    return phi_voigt_norm

def sigma(nu, A, T, m, nu0=F21, mode='fft'):
    """Return the cross-section for photon absorption."""
    lamb0 = c/nu0
    return lamb0**2/(8*np.pi) * A * voigt(nu=nu, T=T, m=m, A=A, nu0=nu0, mode=mode)
```

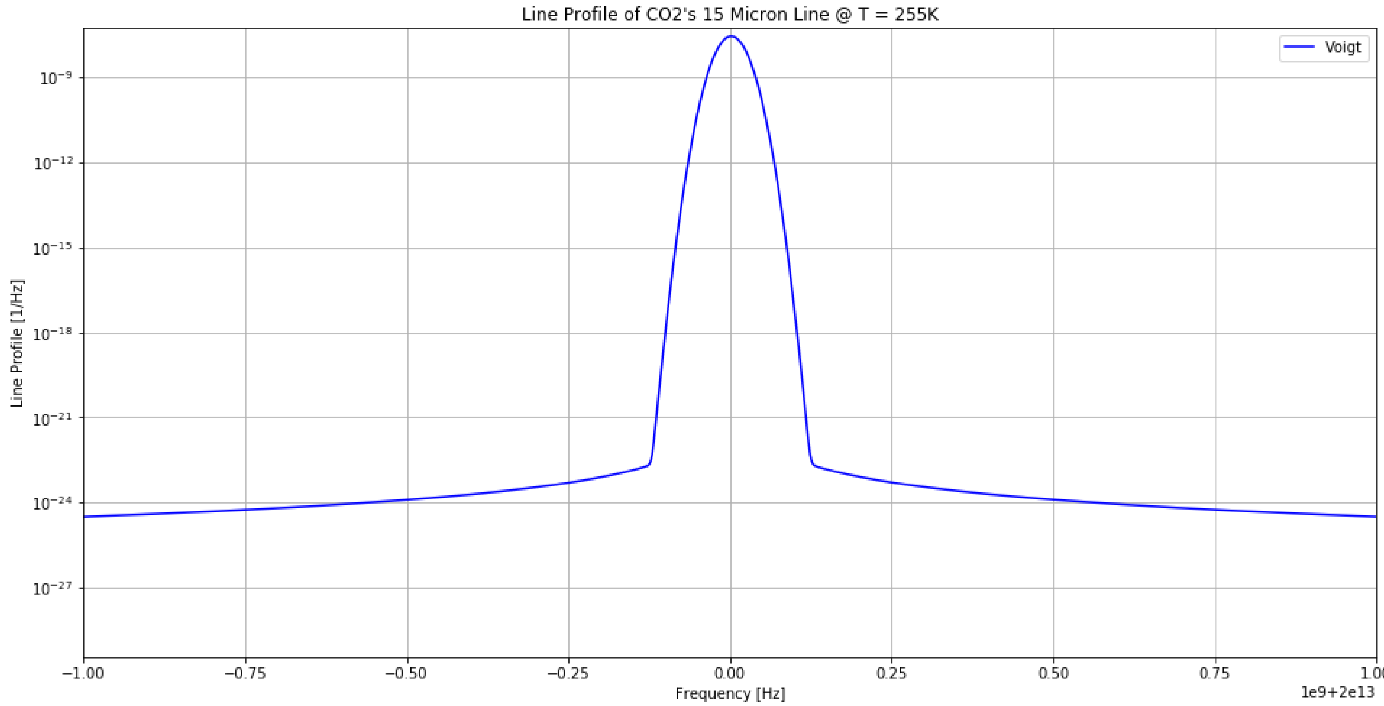
1.1

In [3]:
```
nu_vec_narrow = np.arange(0.99*F21, 1.01 * F21, F21*1e-7)
T = 255 # K
```

In [4]:
```
prof_voigt = voigt(nu=nu_vec_narrow, T=T, m=m, A=A21, nu0=F21, mode=None)
```

In [5]:
```
plt.rcParams['figure.figsize'] = (16, 8)
plt.semilogy(nu_vec_narrow, prof_voigt, 'b', label='Voigt')

plt.xlabel('Frequency [Hz]')
plt.ylabel('Line Profile [1/Hz]')
plt.title(f'Line Profile of CO2\'s 15 Micron Line @ T = {T}K')
plt.ylim((min(prof_voigt), 2*max(prof_voigt)))
plt.xlim((.99995*F21, 1.00005*F21))

plt.grid(True)
plt.legend()
```

Out[5]:
```
<matplotlib.legend.Legend at 0x29ecf683710>
```



1.2

```
In [6]: hbar = 1e-27 # erg.s
        h = hbar*2*np.pi # erg/Hz
        tau0 = 3 # optical depth at line center
        dnu = nu_vec_narrow[1]-nu_vec_narrow[0]

        def planck_fun(nu_vec, T):
            '''
            Inputs:
                nu_vec: Numpy vector of floats. Frequencies in Hz.
                T: Float. Temperature in K.
            Outputs:
            '''
            return 2*h*nu_vec**3/c**2 * 1/(np.exp(h*nu_vec/(kB*T)) - 1)
```

```
In [7]: # # nu_vec_broad = np.arange(min(nu_vec_narrow)-.1*F21, max(nu_vec_narrow)+.1*F21, dnu/2)
        # nu_vec_broad = np.arange(F21/10, F21*10, F21/50000)

        # nu_min = min(nu_vec_narrow)
        # nu_max = max(nu_vec_narrow)
        # nu_overlap = [float(nu) for nu in nu_vec_broad if (nu < nu_max) and (nu > nu_min)]

        # # Combining non-uniform steps into a single axis and removing the screwy overlap
        # idx_overlap_broad = [i for i,nu in enumerate(nu_vec_broad) if float(nu) in nu_overlap]
        # nu_vec_broad_clipped = np.array([nu_vec_broad[i] for i,_ in enumerate(nu_vec_broad) if i not in
        # nu_vec_unsorted = np.concatenate([nu_vec_broad_clipped, nu_vec_narrow])
        # idx_sort = np.argsort(nu_vec_unsorted)
        # nu_vec = np.array([nu_vec_unsorted[i] for i in idx_sort])
```

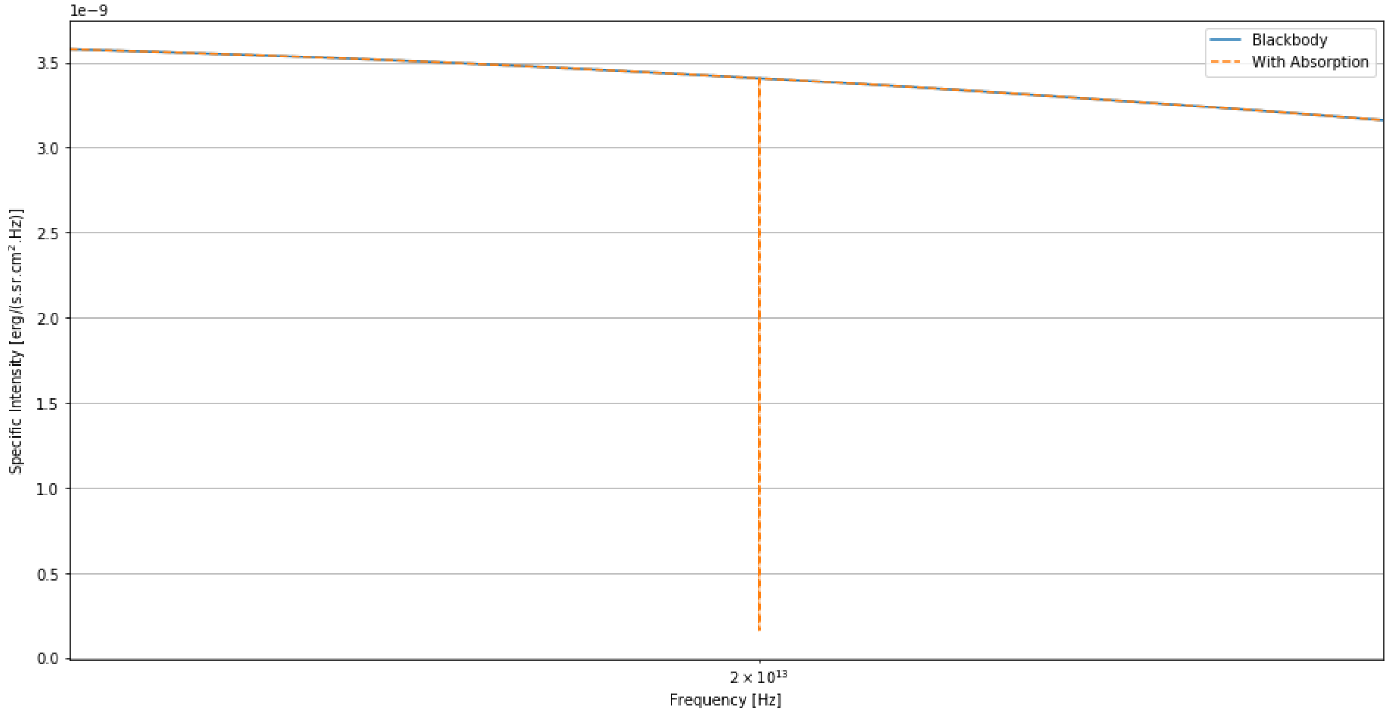
In [8]:
```
# # Incorporating 15um line absorption
# ## x-section (as a function of frequency), separated because of convolution
# sigma_vec_broad = sigma(nu_vec_broad, A=A21, m=m, T=T, nu0=F21)
# sigma_vec_narrow = sigma(nu_vec_narrow, A=A21, m=m, T=T, nu0=F21)
# sigma_vec_broad_clipped = np.array([sigma_vec_broad[i] for i,_ in enumerate(sigma_vec_broad) if
# sigma_vec_unsorted = np.concatenate([sigma_vec_broad_clipped, sigma_vec_narrow])
# sigma_vec = np.array([sigma_vec_unsorted[i] for i in idx_sort])

# ## getting column density of CO2 given tau=3 @ line center
# idx = np.argwhere(np.diff(np.sign(nu_vec-F21))).flatten()[0] # idx of nearest nu to nu0
# N = tau0/sigma_vec[idx]

# ## getting optical depth across different frequencies
# tau_vec = N * sigma_vec
# tau_vec_narrow = N * sigma_vec_narrow
```

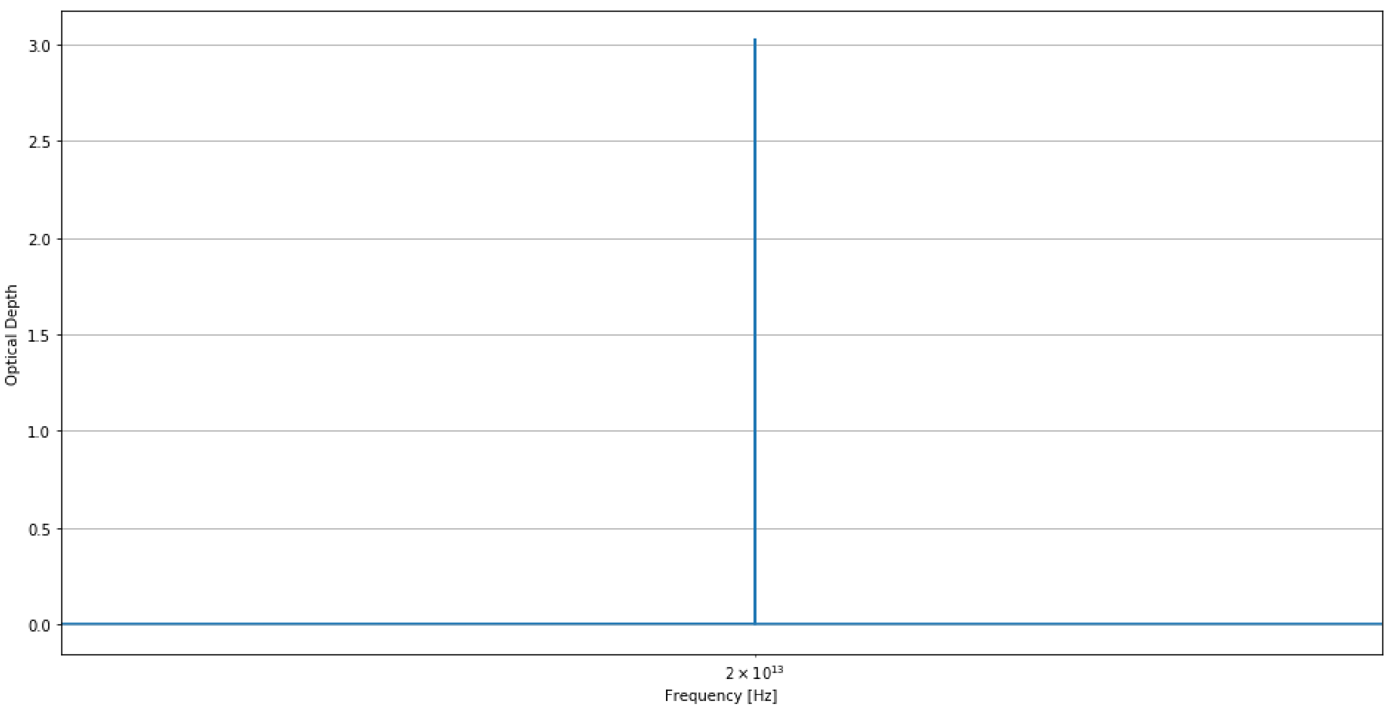
In [9]:
```
nu_vec = np.arange(F21*.9, F21*1.1, dnu/2)
sigma_vec = sigma(nu_vec, A=A21, m=m, T=T, nu0=F21, mode='fft')
sigma_vec_narrow = sigma(nu_vec_narrow, A=A21, m=m, T=T, nu0=F21, mode=None)

## getting column density of CO2 given tau=3 @ line center
idx = np.argwhere(np.diff(np.sign(nu_vec-F21))).flatten()[0] # idx of nearest nu to nu0
N = tau0/sigma_vec[idx]

## getting optical depth across different frequencies
tau_vec = N * sigma_vec
tau_vec_narrow = N*sigma_vec_narrow
```

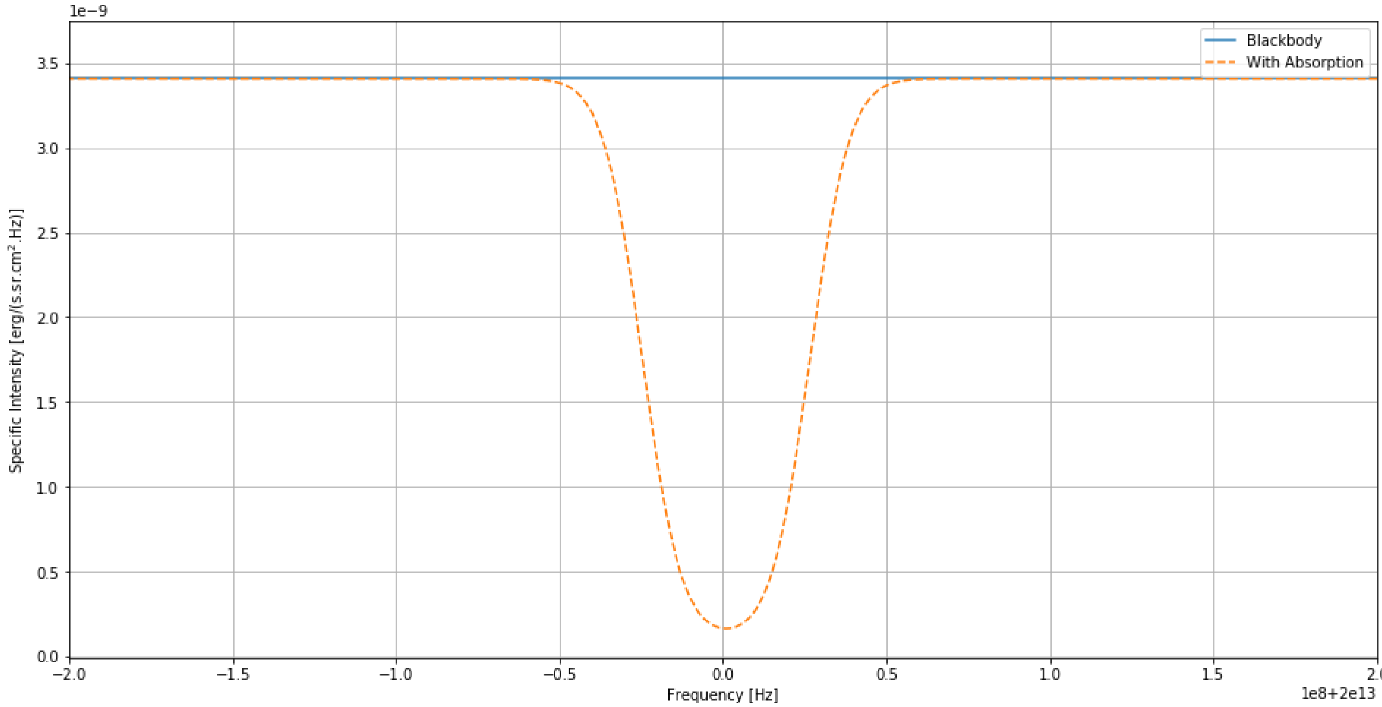
In [10]:
```
# Blackbody radiation
B_blackbody = planck_fun(nu_vec=nu_vec, T=T)
B_blackbody_narrow = planck_fun(nu_vec=nu_vec_narrow, T=T)
B_wabs = np.multiply(B_blackbody, np.exp(-tau_vec))
B_wabs_narrow = np.multiply(B_blackbody_narrow, np.exp(-tau_vec_narrow))
B_diff = B_blackbody - B_wabs
B_diff_narrow = B_blackbody_narrow - B_wabs_narrow
```

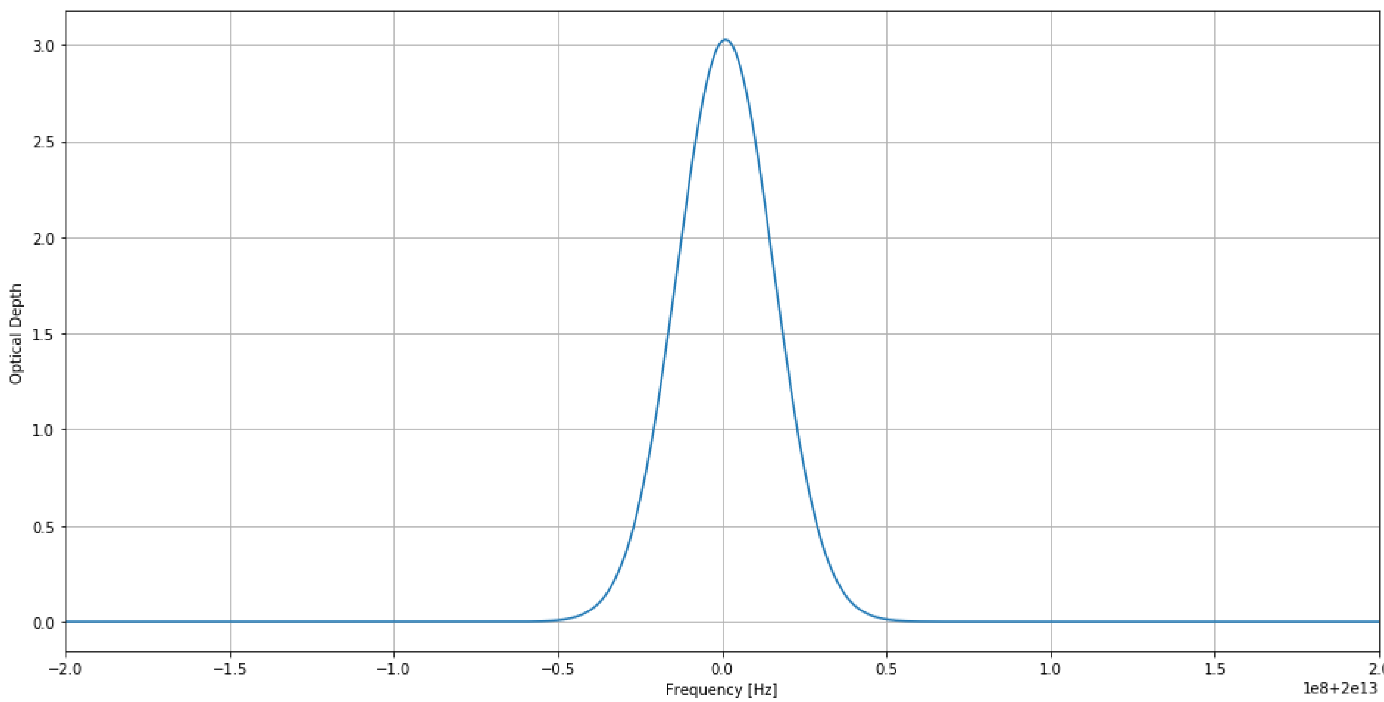
In [11]:
```
## Plotting over the full frequency range & a narrower range for visibility
for full_range in (True, False):
    xmin = min(nu_vec) if full_range else F21*.99999
    xmax = max(nu_vec) if full_range else F21 * 1.00001

    plt.figure()
    f = plt.semilogx if full_range else plt.plot
    f(nu_vec, B_blackbody, label='Blackbody')
    f(nu_vec, B_wabs, '--', label='With Absorption')
    plt.xlabel('Frequency [Hz]')
    plt.ylabel('Specific Intensity [erg/(s.sr.cm$^2$.Hz)]')
    plt.legend()
    plt.grid(True)
    plt.xlim((xmin, xmax))

    plt.figure()
```

```
f(nu_vec, tau_vec)
plt.xlabel('Frequency [Hz]')
plt.ylabel('Optical Depth')
plt.xlim((xmin, xmax))
plt.grid(True)
```









1.3

```
In [12]: full_range = True # gets rid of screwy edge condition
         B_diff_used = B_diff if full_range else B_diff_narrow
         nu_vec_used = nu_vec if full_range else nu_vec_narrow
```

```
    tau_vec_used = tau_vec if full_range else tau_vec_narrow

    # Trapezoidal integration for total absorption by CO2
    F_diff = np.trapz(B_diff_used, nu_vec_used) * 4*np.pi

    # Absorption in optically thick region by CO2
    idx_thick = [i for i, tau in enumerate(tau_vec_used) if tau > 1]
    nu_vec_thick = [nu_vec_used[i] for i in idx_thick]
    B_diff_thick = [B_diff_used[i] for i in idx_thick]
    F_diff_thick = 4*np.pi * np.trapz(B_diff_thick, nu_vec_thick)

    print(f'Blackbody:\t\t\t{4*np.pi*np.trapz(B_blackbody, nu_vec)} erg/(s.cm^2)')
    print(f'With Absorption:\t\t{4*np.pi*np.trapz(B_wabs, nu_vec)} erg/(s.cm^2)')
    print(f'Total Absorption:\t\t{F_diff} erg/(s.cm^2)')
    print(f'->  Optically Thick:\t\t{F_diff_thick} erg/(s.cm^2)')
    print(f'Optically Thick Fraction:\t{F_diff_thick/F_diff}')
```

```
Blackbody:                        170628.74319788706 erg/(s.cm^2)
With Absorption:                170626.61807835393 erg/(s.cm^2)
Total Absorption:                2.125119533145682 erg/(s.cm^2)
->  Optically Thick:                1.5679168612901773 erg/(s.cm^2)
Optically Thick Fraction:        0.7378017268371194
```

1.4

```
In [13]: sigma_blackbody = 2 * np.pi**5 * kB**4 / (15 * c**2 * h**3) # blackbody constant = Fblackbody/T^4

         F_Earth_woabs = sigma_blackbody * T**4
         F_Earth_compensate = (F_Earth_woabs + F_diff)
         T_new = (F_Earth_compensate / sigma_blackbody)**.25
         print(f'OG Temperature:\t\t{T} K')
         print(f'New Temperature:\t{T_new} K')
         print(f'Temp. Difference:\t{T_new - T} K')
```

```
OG Temperature:                255 K
New Temperature:        255.00045633474454 K
Temp. Difference:        0.00045633474454120915 K
```

```
In [14]: # voigt_scratch = voigt(nu=nu_vec, T=T, m=m, A=A21, nu0=F21)
         # sigma_scratch = sigma(nu=nu_vec, A=A21, T=T, m=m, nu0=F21)
         # plt.semilogy(nu_vec, sigma_scratch, 'b', label='$\sigma$')
         # plt.semilogy(nu_vec, voigt_scratch, 'r', label='Voigt')
         # plt.xlim((min(nu_vec_fine), max(nu_vec_fine)))
         # plt.legend()
```