# ASTRO C207 Radiative Processes in Astrophysics
Lydia Lee
Problem Set 10

## 1. Powering Radio Lobes

(1) Using points $(10\text{MHz}, 10^5\text{Jy})$ and $(2 \cdot 10^4\text{MHz}, 500\text{Jy})$ on the curve and

$$F_\nu \propto \nu^{\frac{1+p}{2}}$$
$$\frac{\Delta(\log F_\nu)}{\Delta(\log \nu)} = \frac{1+p}{2}$$

(where $p < 0$)

$$p \approx -2.4$$

which jives reasonably well with the fitted line (which gives $p \approx -2.6$)

(2)

$$n \cdot d\gamma = C\gamma^p \cdot d\gamma$$
$$U_e = m_e c^2 C \frac{1}{p+2} \gamma^{p+2}|_{\gamma_{\min}}^{\gamma_{\max}} \longleftarrow E = \gamma m_e c^2$$
$$= \frac{m_e c^2}{p+2} C \left(\gamma_{\max}^{p+2} - \gamma_{\min}^{p+2}\right)$$
$$\approx -\frac{m_e c^2}{p+2} C \gamma_{\min}^{p+2}$$

$$U_e = \frac{m_e c^2}{p+2} C \left(\gamma_{\max}^{p+2} - \gamma_{\min}^{p+2}\right) \approx -\frac{m_e c^2}{p+2} C \gamma_{\min}^{p+2}$$

(3)

$$\omega_{\text{cyc}} = \frac{qB}{m_e c}$$
$$U_B = \frac{B^2}{8\pi}$$

Finding $\gamma_{\min}$ in terms of $\omega_m$ and $B$

$$\begin{aligned}
\omega_m &= \frac{3}{2}\gamma_{\min}^2 \omega_{\text{cyc}} \sin\alpha \\
\gamma_{\min} &= \sqrt{\frac{\omega_m}{\omega_{\text{cyc}}}\frac{2}{3}\frac{1}{\sin\alpha}} \\
&= \sqrt{\frac{2\omega_m m_e c}{3\sin\alpha e B}} \\
&= \sqrt{\frac{2m_e c}{3\sin\alpha e}}\,\omega_m^{\frac{1}{2}} B^{-\frac{1}{2}} \\
&= A_1 \omega_m^{\frac{1}{2}} B^{-\frac{1}{2}} \longleftarrow A_1 \equiv \sqrt{\frac{2m_e c}{3\sin\alpha e}}
\end{aligned}$$

Note that $e$ is the charge of an electron because no one can figure out what $q$ is across contexts.

Finding $C$ in terms of $\omega_m$, $B$, $L_m$, and the volume $V$

$$\begin{aligned}
L_\nu &\approx \frac{2}{3} C \frac{U_B \sigma_T c}{\nu_{\text{cyc}}} \left(\frac{\nu}{\nu_{\text{cyc}}}\right)^{\frac{1+p}{2}} \times V \\
&= \frac{2(2\pi)^{-\frac{3+p}{2}}}{3} C U_B \sigma_T c \omega^{\frac{1+p}{2}} \omega_{\text{cyc}}^{-\frac{3+p}{2}} \times V \\
C &\approx L_m \frac{3}{2(2\pi)^{-\frac{3+p}{2}}} \frac{\omega_{\text{cyc}}^{\frac{3+p}{2}}}{U_B \sigma_T c \omega_m^{\frac{1+p}{2}} V} \\
&= L_m \frac{3}{2(2\pi)^{-\frac{3+p}{2}}} \frac{8\pi \left(\frac{eB}{m_e c}\right)^{\frac{3+p}{2}}}{B^2 \sigma_T c \omega_m^{\frac{1+p}{2}} V} \longleftarrow U_B = \frac{B^2}{8\pi} \\
&= \frac{12\pi}{(2\pi)^{-\frac{3+p}{2}}} L_m \frac{\left(\frac{e}{m_e c}\right)^{\frac{3+p}{2}}}{\sigma_T c V} \omega_m^{-\frac{1+p}{2}} B^{\frac{p-1}{2}} \\
&= A_2 \frac{L_m}{V} \omega_m^{-\frac{1+p}{2}} B^{\frac{p-1}{2}} \longleftarrow A_2 \equiv \frac{12\pi}{(2\pi)^{-\frac{3+p}{2}}} \frac{\left(\frac{e}{m_e c}\right)^{\frac{3+p}{2}}}{\sigma_T c}
\end{aligned}$$

and I'm not going to simplify the expression for $A_2$ because what's a computational cycle or two in this day and age?

Plugging things into the expression for $U_e$

$$\begin{aligned}
U_e &= \frac{m_e c^2}{p+2} C \left(\gamma_{\max}^{p+2} - \gamma_{\min}^{p+2}\right) \\
&\approx -\frac{m_e c^2}{p+2} C \gamma_{\min}^{p+2} \\
&= -\frac{m_e c^2}{p+2} \cdot A \frac{L_m}{V} \omega_m^{-\frac{1+p}{2}} B^{\frac{p-1}{2}} \cdot \left(A_1 \omega_m^{\frac{1}{2}} B^{-\frac{1}{2}}\right)^{2+p} \\
&= A \frac{L_m}{V} \nu_m^{\frac{1}{2}} B^{-\frac{3}{2}}
\end{aligned}$$

(4)

$$E = 2V(U_e + U_B)$$
$$\frac{dE}{dB} = 2V\left(\frac{dU_e}{dB} + \frac{dU_B}{dB}\right)$$
$$= 2V\left(-\frac{3}{2}\frac{U_e}{B} + 2\frac{U_B}{B}\right)$$
$$= \frac{2V}{B}\left(-\frac{3}{2}U_e + 2U_B\right)$$

Discounting nonphysical limits like $B = \infty$ and $B = 0$,

$$U_B = \frac{3}{4}U_e$$

(5)

$$U_B = \frac{3}{4}U_e$$
$$\frac{1}{8\pi}B^2 = \frac{3}{4}A\frac{L_m \nu_m^{\frac{1}{2}}}{V}B^{-\frac{3}{2}}$$
$$AL_m\frac{\nu_m^{\frac{1}{2}}}{V}6\pi = B^{\frac{7}{2}}$$
$$B = \left(\frac{AL_m \nu_m^{\frac{1}{2}}}{V}\right)^{\frac{2}{7}}$$

and then

$$\gamma_{\min} \approx \sqrt{\frac{2\nu_m}{3\nu_{\rm cyc}}}$$

$$B \approx 4 \cdot 10^{-5}\mathrm{G}$$
$$\gamma_{\min} \approx 100$$

(6)

$$E = 2V(U_e + U_B)$$

$$\boxed{E \approx 10^{61}\text{erg}}$$

(7) • $E_{\text{created}} = \frac{1}{10} m_{\text{food}} c^2$

$$m_{\text{food}} \approx 10^{41}\text{gram}$$

If SMBHs are $10^5$ to $10^6$ times larger than the sun, that's roughly $100\times$ more masssive than a SMBH, suggesting the SMBH hypothesis doesn't hold up.