CTFS

$$a_k = \frac{1}{T}\int_{<T>} x(t)e^{-jk\omega_o t}dt$$
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_o t}$$

FINISH

DTFS

$$a_m = \frac{1}{N}\sum_{n\in<N>} x[n]e^{jm\omega_o n}$$
$$x[n] = \sum_{m\in<N>} a_m e^{jm\omega_o n}$$

FINISH

CTFT

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$
$$X(j\omega_1, j\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t_1,t_2)e^{-j\omega_1 t_1}e^{-j\omega_2 t_2}dt_1 dt_2$$
$$x(t_1,t_2) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(j\omega_1, j\omega_2)e^{j\omega_1 t_1}e^{j\omega_2 t_2}d\omega_1 d\omega_2$$

. . . .

1. Linear
   $\alpha x(t) + \beta y(t) \longleftrightarrow \alpha X(j\omega) + \beta Y(j\omega)$
2. Time Shift
   $x(t-\tau) \longleftrightarrow X(j\omega)e^{-j\omega\tau}$
3. Conjugation
   $x^*(t) \longleftrightarrow X^*(-j\omega)$
4. Conjugate Symmetry
   $x(t) \in \mathbb{R} \Longrightarrow x(t) = x^*(t)$
   $\cdots \Longrightarrow X(j\omega) = X^*(-j\omega)$
5. Differentiation
   $\frac{dx}{dt} \longleftrightarrow j\omega X(j\omega)$
6. Integration
   $\int_{-\infty}^{t} x(\tau)d\tau \longleftrightarrow \frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
7. Time/Frequency Scaling
   $x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|}X(\frac{j\omega}{\alpha})$
8. Convolution
   $x(t) * y(t) \longleftrightarrow X(j\omega)Y(j\omega)$
9. Differentiation 2
   $-jtx(t) \longleftrightarrow \frac{dX}{d\omega}$
10. Frequency Shift
    $e^{j\omega_o t}x(t) \longleftrightarrow X(j(\omega - \omega_o))$
11. Multiplication
    $x(t)y(t) \longleftrightarrow \frac{1}{2\pi}X(j\omega) * Y(j\omega)$
12. Separability (Fubini's Theorem?)
    $X(j\omega_1, j\omega_2) = X(j\omega_1)X(j\omega_2)$

FINISH

DTFT (many repeats from CTFT)

$$X(e^{j\omega}) = \sum_{n=\infty}^{\infty} x[n]e^{-j\omega n}$$
$$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(e^{j\omega})e^{j\omega n}d\omega$$

. . . .

$$\text{Given} \sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$
$$H(e^{j\omega}) = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} a_k e^{-j\omega k}}$$

. . . .

1. Time Reversal
   $x[-n] \longleftrightarrow X(e^{-j\omega})$
2. Conjugation/Even Symmetry
   $x[n] \in \mathbb{R} \Longrightarrow X(e^{j\omega} = X^*(e^{-j\omega}))$
   $\cdots \wedge x[n] = x[-n] \Longrightarrow X(e^{j\omega}) \in \mathbb{R}$
3. Differencing
   $x[n] - x[n-1] \longleftrightarrow X(e^{j\omega}) \cdot (1 - e^{-j\omega})$
4. Accumulation
   $\sum_{m=-\infty}^{n} x[m] \longleftrightarrow \frac{1}{1-e^{-j\omega}}X(e^{j\omega}) + \pi X(e^0)\sum_l \delta(\omega - kl)$
5. Differentiation
   $nx[n] \longleftrightarrow j\frac{dX}{d\omega}$
6. Parseval's
   $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{<2\pi>} |X(e^{j\omega})|^2 d\omega$
7. Multiplication
   $x[n]y[n] \longleftrightarrow X(e^{j\omega}) \circledast Y(e^{j\omega})$
   $\cdots \longleftrightarrow \frac{1}{2\pi}\int_{<2\pi>} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

FINISH

$\boxed{\textbf{DFT}}$ (finite length sequences)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(\frac{2\pi}{N})kn} k \in [0, N-1] \longrightarrow X[k_1, k_2] = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x[n_1, n_2] e^{-j(\frac{2\pi}{N_1})k_1 n_1} e^{-j(\frac{2\pi}{N_2})k_2 n_2}$$
$$= N a_k$$
$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j(\frac{2\pi}{N})kn} \longrightarrow x[n_1, n_2] = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum k_2 = 0^{N_2-1} X[k_1, k_2] e^{j(\frac{2\pi}{N_1})k_1 n_1} e^{j(\frac{2\pi}{N_2})k_2 n_2}$$

. . . .

1. Conjugate Symmetry
   $x[n] \in \mathbb{R} \Longrightarrow X[k] = X^*[N-k] \Longrightarrow X[\frac{N}{2}] \in \mathbb{R}$

$\boxed{\textbf{Laplace}}$ asdf

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$asdf$