[xcolor=dvipsnames]beamer
tikz,xcolor,soul media9 []beamerthemeMadrid
[shadow]beamerinnerthemerounded []beamercolorthemerose
Theorem [theorem]Proposition [theorem]Corollary
enumerate graphicx amsmath,amssymb hyperref
[english]babel
[T1]fontenc ifthen latexsym,times,graphics,amssymb,fancybox,fancyhdr,bm
natbib
color
Sweave

Working with Data in R [1]

Steps

1. *Import* data from various sources: the web, a database, a stored file.
2. *Clean* and format the data. Usually this means rows are observations and columns are variables.
3. *Analyze* the data using visualizations, modelling, or other methods.
4. *Communicate* your results.

[1] Slide developed from G. Grolemund and H. Wickham.

Working with Data in R [1]

Steps

1. *Import* data from various sources: the web, a database, a stored file.
2. *Clean* and format the data. Usually this means rows are observations and columns are variables.
3. *Analyze* the data using visualizations, modelling, or other methods.
4. *Communicate* your results.

In this class, we learn tools and strategies for completing each step.

[1] Slide developed from G. Grolemund and H. Wickham.

Functional Programming [2]

Function Definition

A function is a machine which turns input objects (*arguments*) into an output object (*return value*), according to a definite rule.

- Programming is writing functions to transform inputs into outputs easily and correctly.
- Good programming takes big transformations and breaks them down into smaller and smaller ones until you come to tasks which the built-in functions can do.

[2] Slide developed from C.R. Shalizi and A.C. Thomas (2014).

What is R?

What is R?

R is an open-source statistical programming software used by industry professionals and academics alike.

This means that R is supported by a community of users.

Will use R extensively in this class

- Download R at: `https://www.r-project.org`
- Download RStudio at: `https://www.rstudio.com`

What is R?

R is an open-source statistical programming software used by industry professionals and academics alike.

This means that R is supported by a community of users.

Will use R extensively in this class

- Download R at: `https://www.r-project.org`
- Download RStudio at: `https://www.rstudio.com`

Using R and RStudio

- The *editor* allows you to type and save code that you may want to reuse later.
- Basic interaction with R happens in the *console*. The is where you type R code.

Figure 1: Image of RStudio from G. Grolemund and H. Wickham

A Quick Example...

Type the following into your console:

```
> # Create a vector in R names "x"
> x <- c(5, 29, 13, 87)
> x
```

```
[1]  5 29 13 87
```

Two important ideas:

1. Commenting
2. Assignment

A Quick Example...

Type the following into your console:

```
> # Create a vector in R
> x <- c(5, 29, 13, 87)
> x
```

```
[1]  5 29 13 87
```

Two important ideas:

1. Commenting
    - Anything after the # isn't evaluated by R.
    - Used to leave notes for humans reading your code.
    - Very important in our class. Comment your code!
2. Assignment

A Quick Example...

Type the following into your console:

```
> # Create a vector in R
> x <- c(5, 29, 13, 87)
> x
```

```
[1]  5 29 13 87
```

Two important ideas:

1. Commenting
2. Assignment
   - The `<-` symbol means assign `x` the value `c(5, 29, 13, 87)`.
   - Could use = instead of `<-` but this is discouraged.
   - All assignments take the same form: `object_name <- value`.
   - `c()` means "concatenate".
   - Type `x` into the console to print its assignment.

A Quick Example...

Type the following into your console:

```
> # Create a vector in R names "x"
> x <- c(5, 29, 13, 87)
> x
```

```
[1]  5 29 13 87
```

Note

The [1] tells us that 5 is the first element of the vector.

```
> # Create a vector in R names "x"
> x <- 1:50
> x
```

```
 [1]  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18
[19] 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
[37] 37 38 39 40 41 42 43 44 45 46 47 48 49 50
```

Variable Types, Vectors, & Matrices

Variable Types

R has a variety of variable types (or *modes*).

Modes

1. Numeric (3.7, 15907, 80.333)
2. Complex (1 + 2i)
3. Character ("Columbia", "Statistics is fun!", "HELLO WORLD")
4. Logical (TRUE, FALSE, 1, 0)

In this class, we are primarily concerned with numeric, character, and logical.

Let's check this out in R

'Numeric' variable type

```
> x <- 2
> mode(x)
```

```
[1] "numeric"
```

```
> typeof(x)
```

```
[1] "double"
```

```
> y <- as.integer(3)
> typeof(y)
```

```
[1] "integer"
```

Let's check this out in R

'Complex' variable type

```
> z <- 1 - 2i
> z
```

```
[1] 1-2i
```

```
> typeof(z)
```

```
[1] "complex"
```

Let's check this out in R

'Character' variable type

```
> name <- "Columbia University"
> name
```

```
[1] "Columbia University"
```

```
> typeof(name)
```

```
[1] "character"
```

Let's check this out in R

'Logical' variable type

```
> a <- TRUE
> b <- F
> a
```

```
[1] TRUE
```

```
> b
```

```
[1] FALSE
```

```
> typeof(a)
```

```
[1] "logical"
```

Data Types

There are many data types in R.

Data Types

- Vectors
- Scalars
- Matrices
- Arrays
- Lists
- Dataframes

Data Types

There are many data types in R.

Data Types

Vectors

- All elements must be the same type (mode).
- More to come in this lecture!

- Scalars
- Matrices
- Arrays
- Lists
- Dataframes

Data Types

There are many data types in R.

Data Types

- Vectors

Scalars

- Treated as one-element vectors in R.

- Matrices
- Arrays
- Lists
- Dataframes

Data Types

There are many data types in R.

Data Types

- Vectors
- Scalars

Matrices

- An array (rows and columns) of values.
- All values must be the same type (mode).
- More to come this lecture!

- Arrays
- Lists
- Dataframes

Data Types

There are many data types in R.

Data Types

- Vectors
- Scalars
- Matrices

Arrays

- Similar to matrices, but with more than two dimensions.

- Lists
- Dataframes

Data Types

There are many data types in R.

Data Types

- Vectors
- Scalars
- Matrices
- Arrays

Lists

- Like a vector, but elements can be of different modes.
- Won't study lists explicitly, but encounter them all the time.

- Dataframes

Data Types

There are many data types in R.

Data Types

- Vectors
- Scalars
- Matrices
- Arrays
- Lists

Dataframes

- Like a matrix, but elements can be of different modes.
- More to come next week!

Check Your Understanding

What mode are the following variables?

1. `3*TRUE`?
2. `"147"`?

Check Your Understanding

What mode are the following variables?

1. `3*TRUE`?
2. `"147"`?

Solutions

```
> 3*TRUE # Logicals in arithmetic
```

```
[1] 3
```

```
> mode(3*TRUE)
```

```
[1] "numeric"
```

```
> mode("147")
```

```
[1] "character"
```

Vectors and Matrices in R

Recall: Vectors

- Variable types are called modes.
- All elements of a vector are the same mode.
- Scalars are just single-element vectors.

Recall: Matrices

- All elements of a matrix are the same mode.
- A matrix is treated like a vector in R with two additional attributes: number of rows and number of columns.

Building Vectors in R

- Use the *concatenate* function `c()` to define a vector.

Some Examples

Defining a numeric vector:

```
> x <- c(2, pi, 1/2, 3^2)
> x
```

```
[1] 2.000000 3.141593 0.500000 9.000000
```

A character vector:

```
> y <- c("NYC", "Boston", "Philadelphia")
> y
```

```
[1] "NYC"          "Boston"       "Philadelphia"
```

Building Vectors in R

- The syntax `a:b` produces a *sequence* of integers ranging from a to b.
- The *repetition* function `rep(val, num)` repeats the value `val` a total of `num` times.

Some Examples

A sequential list of integers:

```
> z <- 5:10
> z
```

```
[1]  5  6  7  8  9 10
```

Using `rep()` to create a 1's vector:

```
> u <- rep(1, 18)
> u
```

```
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Building Vectors in R

- Alternately, could allocate space and then fill in element-wise.

Some Examples

```
> v <- c()
> v[1] <- TRUE
> v[2] <- TRUE
> v[3] <- FALSE
> v
```

```
[1]  TRUE  TRUE FALSE
```

Building Vectors in R

- The *concatenate* function c() can be nested.

Some Examples

```
> vec1 <- rep(-27, 3)
> vec1
```

```
[1] -27 -27 -27
```

```
> vec2 <- c(vec1, c(-26, -25, -24))
> vec2
```

```
[1] -27 -27 -27 -26 -25 -24
```

Building Matrices in R

- Use the function `matrix(values, nrow, ncol)` to define your matrix.
- In R, matrices are stored in *column-major order* (determines where the number go as in the following example).

Some Examples

Building a matrix that fills in by column:

```
> mat <- matrix(1:9, nrow = 3, ncol = 3)
> mat
```

```
     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
```

Building Matrices in R

- Use the function `matrix(values, nrow, ncol)` to define your matrix.
- In R, matrices are stored in *column-major order* (determines where the number go as in the following example).

Some Examples

Building a matrix that fills in by row:

```
> new_mat <- matrix(1:9, nrow = 3, ncol = 3, byrow = TRUE)
> new_mat
```

```
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
```

Building Matrices in R

- Alternately, could allocate space and fill in element-wise.
- Tell R the size of the matrix beforehand.

Some Examples

Allocating the space for a matrix then filling it in:

```
> this_mat <- matrix(nrow = 2, ncol = 2)
> this_mat[1,1] <- sqrt(27)
> this_mat[1,2] <- round(sqrt(27), 3)
> this_mat[2,1] <- exp(1)
> this_mat[2,2] <- log(1)
> this_mat
```

```
         [,1]  [,2]
[1,] 5.196152 5.196
[2,] 2.718282 0.000
```

Building Matrices in R

- The *row bind* function `rbind()` also works, though it can be costly computationally. Similarly for *column bind* function `cbind()`.

Some Examples

```
> vec1 <- rep(0, 4)
> vec2 <- c("We're", "making", "matrices", "!")
> final_mat <- rbind(vec1, vec2)
> final_mat
```

```
     [,1]    [,2]     [,3]       [,4]
vec1 "0"     "0"      "0"        "0"
vec2 "We're" "making" "matrices" "!"
```

Recall, matrix entries must all be the same type.

Building Matrices in R

- Name columns (rows) of a matrix using `colnames()` (`rownames()`).

Some Examples

```
> this_mat # Defined previously
```

```
         [,1]  [,2]
[1,] 5.196152 5.196
[2,] 2.718282 0.000
```

```
> colnames(this_mat) # Nothing there yet
```

```
NULL
```

Building Matrices in R

- Name columns (rows) of a matrix using `colnames()` (`rownames()`).

Some Examples

```
> colnames(this_mat) <- c("Column1", "Column2")
> this_mat
```

```
      Column1 Column2
[1,] 5.196152   5.196
[2,] 2.718282   0.000
```

Mixing Variable Modes

- When variable modes are mixed in vectors or matrices, R picks the 'least common denominator'.
- Use the *structure* function str() to display the internal structure of an R object.

Example

```
> vec <- c(1.75, TRUE, "abc")
> vec
```

```
[1] "1.75" "TRUE" "abc"
```

```
> str(vec)
```

```
 chr [1:3] "1.75" "TRUE" "abc"
```

Help in R

- Use a single question mark `?` to get help about a specific function using form `?function name`.
  - Provides a description, lists the arguments (to the function), gives an example, etc.
- Use the double question mark to get help with a topic using form `??topic`.

How to get help in R

```
> # What does the str() function do?
>
> # Function help:
> ?str
> # Fuzzy matching:
> ??"structure"
```

Help in R

Code example.

Subsetting Vectors

- Use square brackets [] to extract elements or subsets of elements.

Example

```r
> y <- c(27, -34, 19, 7, 61)
> y[2]
```

```
[1] -34
```

```r
> y[3:5]
```

```
[1] 19  7 61
```

```r
> y[c(1, 4)]
```

```
[1] 27  7
```

Subsetting Vectors

- Use the same strategy to reassign elements of a vector.

Example

```
> y <- c(27, -34, 19, 7, 61)
> y
```

```
[1]  27 -34  19   7  61
```

```
> y[c(1, 4)] <- 0
> y
```

```
[1]   0 -34  19   0  61
```

Subsetting Vectors

- Negative values can be used to exclude elements.

Example

```
> y <- c(27, -34, 19, 7, 61)
> y
```

```
[1]  27 -34  19   7  61
```

```
> y[-c(1, 4)]
```

```
[1] -34  19  61
```

```
> y <- y[-1]
> y
```

```
[1] -34  19   7  61
```

Subsetting Matrices

- `mat[i,j]` returns the $(i,j)th$ element of `mat`.
- `mat[i, ]` returns the i^{th} row of `mat`.
- `mat[ ,j]` returns the j^{th} column of `mat`.

```
> mat <- matrix(1:8, ncol = 4)
> mat
```

```
     [,1] [,2] [,3] [,4]
[1,]    1    3    5    7
[2,]    2    4    6    8
```

```
> mat[, 2:3]
```

```
     [,1] [,2]
[1,]    3    5
[2,]    4    6
```

Subsetting Matrices

- Can use column names or row names to subset as well.
- Negative values are used to exclude elements.

```
> this_mat
```

```
       Column1 Column2
[1,] 5.196152   5.196
[2,] 2.718282   0.000
```

```
> this_mat[, "Column2"]
```

```
[1] 5.196 0.000
```

```
> this_mat[, -1]
```

```
[1] 5.196 0.000
```

An Extended Example: Image Data

But First...Packages!

What are packages?

- Packages are collections of functions, data, or code that extend the capabilities of base R.
- Some packages come pre-loaded but others must be downloaded and installed using function `install.packages("package name")`.
- An installed R package must be loaded in each session it is to be used using function `library("package name")`.

```
> # Installing the "pixmap" package.
> install.packages("pixmap")
> library("pixmap")
```

Image Data Example [3]

- Images are made up of pixels which are arranged in rows and columns (like a matrix).
- Image data are matrices where each element is a number representing the intensity or brightness of the corresponding pixel.
- We will work with a greyscale image with numbers ranging from 0 (black) to 1 (white).

[3] Example developed from N. Matloff, "The Art of R Programming: A Tour of Statistical Software Design".

Image Data Example (cont.)

```
> library(pixmap)
> casablanca_pic <- read.pnm("casablanca.pgm")
> casablanca_pic
```

```
Pixmap image
  Type          : pixmapGrey
  Size          : 360x460
  Resolution    : 1x1
  Bounding box  : 0 0 460 360
```

```
> plot(casablanca_pic)
```

Image Data Example (cont.)

Image Data Example (cont.)

```
> dim(casablanca_pic@grey)
```

```
[1] 360 460
```

```
> casablanca_pic@grey[360, 100]
```

```
[1] 0.4431373
```

```
> casablanca_pic@grey[180, 10]
```

```
[1] 0.9882353
```

Image Data Example (cont.)

Let's erase Rick from the image.

```
> casablanca_pic@grey[15:70, 220:265] <- 1
```

```
> plot(casablanca_pic)
```

Image Data Example (cont.)

Image Data Example (cont.)

- Use R's `locator()` function to find the rows and columns corresponding to Rick's face.
- A call to the function allows the user to click on a point in a plot and then the function returns the coordinates of the click.

Check Your Understanding

Using matrix z, what is the output of the following?

```
> z
```

```
     First Second Third
[1,]     1      4     7
[2,]     2      5     8
[3,]     3      6     9
```

1. `z[2:3, "Third"]`?
2. `c(z[,-(2:3)], "abc")`?
3. `rbind(z[1,], 1:3)`?

Check Your Understanding

```
> z
```

```
     First Second Third
[1,]     1      4     7
[2,]     2      5     8
[3,]     3      6     9
```

Solutions

```
> z[2:3, "Third"]
```

```
[1] 8 9
```

```
> c(z[,-(2:3)], "abc")
```

```
[1] "1"   "2"   "3"   "abc"
```

Check Your Understanding

```
> z
```

```
     First Second Third
[1,]     1      4     7
[2,]     2      5     8
[3,]     3      6     9
```

Solutions

```
> rbind(z[1,], 1:3)
```

```
     First Second Third
[1,]     1      4     7
[2,]     1      2     3
```

More with Vectors & Matrices and Linear Algebra Review

Reminder: Vector Algebra

Define vectors:

$$A = (a_1, a_2, \ldots, a_N), \quad \text{and} \quad B = (b_1, b_2, \ldots, b_N).$$

Then for c a scalar,

- $A + B = (a_1 + b_1, a_2 + b_2, \ldots, a_N + b_n)$.
- $cA = (ca_1, ca_2, \ldots, ca_N)$.
- Dot product: $A \cdot B = a_1 b_1 + a_2 b_2 + \ldots + a_N b_N$.
- Norm: $\|A\|^2 = A \cdot A = {a_1}^2 + {a_2}^2 + \ldots + {a_N}^2$.

Reminder: Matrix Algebra

Define matrices:

$$A = \begin{pmatrix} a_1 & a_3 \\ a_2 & a_4 \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} b_1 & b_3 \\ b_2 & b_4 \end{pmatrix}.$$

Then for c a scalar,

- $A + B = \begin{pmatrix} a_1 + b_1 & a_3 + b_3 \\ a_2 + b_2 & a_4 + b_4 \end{pmatrix}.$
- $cA = \begin{pmatrix} ca_1 & ca_3 \\ ca_2 & ca_4 \end{pmatrix}.$
- Matrix Multiplication: $AB = \begin{pmatrix} a_1 b_1 + a_3 b_2 & a_1 b_3 + a_3 b_4 \\ a_2 b_1 + a_4 b_2 & a_2 b_3 + a_4 b_4 \end{pmatrix}.$

  What if the dimensions of A and B are different?

Define matrices:

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{pmatrix}, \quad \text{and } B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}.$$

- The *transpose* of A is a $m \times n$ matrix:

$$t(A) = \begin{pmatrix} a_{1,1} & a_{2,2} & \cdots & a_{n,1} \\ a_{1,2} & a_{2,2} & \cdots & a_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1,m} & a_{2,m} & \cdots & a_{n,m} \end{pmatrix}.$$

- The *trace* of the square matrix B is the sum of the diagonal elements: $tr(B) = b_{1,1} + b_{2,2}$.

Reminder: Matrix Operations

Define matrices:

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{pmatrix}, \quad \text{and } B = \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}.$$

- The *determinant* of square matrix B is $det(B) = b_{1,1}b_{2,2} - b_{1,2}b_{2,1}$.
  How do you find the determinant for an $n \times n$ matrix?
- The *inverse* of square matrix B is denoted B^{-1} and
  $BB^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and

$$B^{-1} = \frac{1}{det(B)} \begin{pmatrix} b_{2,2} & -b_{1,2} \\ -b_{2,1} & b_{1,1} \end{pmatrix}.$$

How do you find the inverse of an $n \times n$ matrix?

Functions on Numeric Vectors

Useful R Functions

R function	Description
`length(x)`	Length of a vector x
`sum(x)`	Sum of a vector x
`mean(x)`	Arithmetic mean of a vector x
`quantiles(x)`	Sample quantiles of a vector x
`max(x)`	Maximum of a vector x
`min(x)`	Minimum of a vector x
`sd(x)`	Sample standard deviation of a vector x
`var(x)`	Sample variance of a vector x
`summary(x)`	Summary statistics of vector x

Reminder...

To access the help documentation of a known R function, use syntax `?function`.

Example: Functions on Numeric Vectors

Example

To investigate the dependence of energy expenditure (y) on body build, researches used underwater weighing techniques to determine the fat-free body mass (x) for each of seven men. They also measured the total 24-hour energy expenditure for each man during conditions of quiet sedentary activity. The results are shown in the table.

Subject	1	2	3	4	5	6	7
x	49.3	59.3	68.3	48.1	57.61	78.1	76.1
y	1,894	2,050	2,353	1,838	1,948	2,528	2,568

```
> # Define covariate and response variable
> x <- c(49.3,59.3,68.3,48.1,57.61,78.1,76.1)
> y <- c(1894,2050,2353,1838,1948,2528,2568)
```

Example: Functions on Numeric Vectors (continued)

Example

Subject	1	2	3	4	5	6	7
x	49.3	59.3	68.3	48.1	57.61	78.1	76.1
y	1,894	2,050	2,353	1,838	1,948	2,528	2,568

```
> n <- length(x) # Sample size
> n
```

```
[1] 7
```

```
> max(x)
```

```
[1] 78.1
```

```
> sd(x)
```

```
[1] 12.09438
```

Example: Functions on Numeric Vectors (continued)

Example

Subject	1	2	3	4	5	6	7
x	49.3	59.3	68.3	48.1	57.61	78.1	76.1
y	1,894	2,050	2,353	1,838	1,948	2,528	2,568

```
> summary(x) # Summary statistics
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  48.10   53.46   59.30   62.40   72.20   78.10
```

```
> summary(y)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   1838    1921    2050    2168    2440    2568
```

Element-Wise Operations for Vectors

Vectors x and y must have the same length. Let a be a scalar.

Element-Wise Operators

Operator	Description
a + x	Element-wise scalar addition
a * x	Element-wise scalar multiplication
x + y	Element-wise addition
x * y	Element-wise multiplication
x ^ a	Element-wise power
a ^ x	Element-wise exponentiation
x ^ y	Element-wise exponentiation

Recycling

Recall that a scalar is just a vector of length 1. When a shorter vector is added to a longer one, the elements in the shorter vectored are repeated. This is *recycling*.

Some Examples

```
> u <- c(1,3,5)
> v <- c(1,3,5)
> v + 4 # Recycling
```

```
[1] 5 7 9
```

```
> v + c(1,3) # Recycling
```

```
[1] 2 6 6
```

```
> v + u
```

```
[1]  2  6 10
```

Some Examples

Note: Operators are functions in R.

```
> u <- c(1,3,5)
> v <- c(1,3,5)
> '+'(v,u)
```

```
[1]  2  6 10
```

```
> '*'(v,u)
```

```
[1]  1  9 25
```

Line of Best Fit Example

Recall the energy expenditure versus fat-free body mass example.

Example

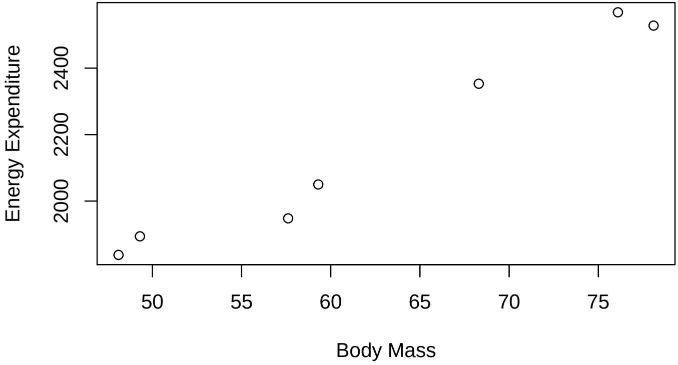
To investigate the dependence of energy expenditure (y) on body build, researches used underwater weighing techniques to determine the fat-free body mass (x) for each of seven men. They also measured the total 24-hour energy expenditure for each man during conditions of quiet sedentary activity. The results are shown in the table.

Subject	1	2	3	4	5	6	7
x	49.3	59.3	68.3	48.1	57.61	78.1	76.1
y	1,894	2,050	2,353	1,838	1,948	2,528	2,568

Line of Best Fit Example (cont.)

Let's find the line of best fit.

```
> plot(x,y, xlab = "Body Mass", ylab = "Energy Expenditure")
```



Line of Best Fit Example (cont.)

Recall:

For the line of best fit, $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ where

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}, \quad \text{and} \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}.$$

Solution:

```
> # First, compute x and y deviations
>     dev_x <- x - mean(x)
>     dev_y <- y - mean(y)
> # Next, compute sum of squares of xy and xx
>     Sxy <- sum(dev_x * dev_y)
>     Sxx <- sum(dev_x * dev_x)
```

Line of Best Fit Example (cont.)

Recall:

For the line of best fit, $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ where

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}, \quad \text{and} \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}.$$

Solution:

```
> # Compute the estimated slope
>     Sxy/Sxx
```

```
[1] 25.01184
```

```
> # Compute the estimated intercept
>     mean(y) - (Sxy/Sxx) * mean(x)
```

```
[1] 607.6539
```

Functions for Numeric Matrices

Useful R Functions

R Function	Description
`A %*% B`	Matrix multiplication for compatible matrices A, B.
`dim(A)`	Dimension of matrix A.
`t(A)`	Transpose of matrix A.
`diag(x)`	Returns a diagonal matrix with elements x along the diagonal.
`diag(A)`	Returns a vector of the diagonal elements of A.
`solve(A,b)`	Returns x in the equation $b = Ax$.
`solve(A)`	Inverse of A where A is a square matrix.
`cbind(A,B)`	Combine matrices horizontally for compatible matrices A, B.
`rbind(A,B)`	Combine matrices vertically for compatible matrices A, B.

Solve the system of equations: $\begin{cases} 3x - 2y + z = -1 \\ x + \frac{1}{2}y - 12z = 2 \\ x + y + z = 3 \end{cases}$

Recall,

We can represent the system using matrices as follows:

$$\begin{pmatrix} 3 & -2 & 1 \\ 1 & \frac{1}{2} & -12 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}.$$

Then we would like to solve for vector (x, y, z).

System of Linear Equations Example (cont.)

Recall,

$$\begin{pmatrix} 3 & -2 & 1 \\ 1 & \frac{1}{2} & -12 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

Solution:

```
> # Define matrix A
>     A <- matrix(c(3,1,1,-2,1/2,1,1,-12,3), nrow = 3)
> # Define vector b
>     b <- c(-1, 2, 3)
> # Use the solve function
>     solve(A, b)
```

```
[1] 1 2 0
```

System of Linear Equations Example (cont.)

Recall,

$$\begin{pmatrix} 3 & -2 & 1 \\ 1 & \frac{1}{2} & -12 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

Let's use matrix multiplication to check that $\mathbf{x} = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix}^T$ is the correct solution to our system of equations.

Solution

```
> x <- c(1, 2, 0) # Define solution vector x
> A %*% x         # Then check with matrix multiplication
```

```
     [,1]
[1,]   -1
[2,]    2
[3,]    3
```

Element-wise Operations for Matrices

Let A and B be matrices of the same dimensions. Let a be a scalar.

Element-wise Operators

Operator	Description
`a + A`	Element-wise scalar addition
`a * A`	Element-wise scalar multiplication
`A + B`	Element-wise addition
`A * B`	Element-wise multiplication
`A ^ a`	Element-wise power
`a ^ A`	Element-wise exponentiation
`A ^ B`	Element-wise exponentiation

Eigenvalue Example

Check if 5 is an eigenvalue of the matrix $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$.

Recall,

If λ is an eigenvalue of a square matrix A, then $Av = \lambda v$ for some non-zero vector v. Equivalently, if λ is a eigenvalue of A then $det(A - 5\mathrm{I}) = 0$ where I is an identity matrix.

Eigenvalue Example

Check if 5 is an eigenvalue of the matrix $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$.

Solution:

```
> # Define matrix A
> A <- matrix(c(1, -2, -2, 4), nrow = 2, byrow = TRUE)
> # Define a 2 by 2 identity matrix
> identity <- diag(2)
> identity
```

```
     [,1] [,2]
[1,]    1    0
[2,]    0    1
```

Eigenvalue Example

Check if 5 is an eigenvalue of the matrix $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$.

Solution:

```
> # Check if 5 is an eigenvalue of A
> det(A - 5*identity)
```

```
[1] 0
```

Filtering

Logical and Relational Operators

<table>
<tr><th>Logical Operator</th><th>Description</th></tr>
<tr><td>!</td><td>Negation (NOT)</td></tr>
<tr><td>&</td><td>AND</td></tr>
<tr><td>|</td><td>OR</td></tr>
</table>

Relational Operator	Description
<, >	Less than, greater than
<=, >=	Less than or equal to, greater than or equal to
==	Equal to
!=	Not equal to

Examples of Logical and Relational Operators

Some Basic Examples

```
> 1 > 3
```

```
[1] FALSE
```

```
> 1 == 3
```

```
[1] FALSE
```

```
> 1 != 3
```

```
[1] TRUE
```

Examples of Logical and Relational Operators

Some Basic Examples

```
> (1 > 3) & (4*5 == 20)
```

```
[1] FALSE
```

```
> (1 > 3) | (4*5 == 20)
```

```
[1] TRUE
```

Examples of Logical and Relational Operators

Some Basic Examples

```
> c(0,1,4) < 3
```

```
[1]  TRUE  TRUE FALSE
```

```
> which(c(0,1,4) < 3)
```

```
[1] 1 2
```

```
> which(c(TRUE, TRUE, FALSE))
```

```
[1] 1 2
```

Examples of Logical and Relational Operators

Some Basic Examples

```
> c(0,1,4) >= c(1,1,3)
```

```
[1] FALSE  TRUE  TRUE
```

```
> c("Cat","Dog") == "Dog"
```

```
[1] FALSE  TRUE
```

Filtering Examples

Sometimes we would like to extract elements form a vector or matrix that satisfy certain criteria.

Extracting Elements from a Vector

```
> w <- c(-3, 20, 9, 2)
> w[w > 3] ### Extract elements of w greater than 3
```

```
[1] 20  9
```

```
> ### What's going on here?
> w > 3
```

```
[1] FALSE  TRUE  TRUE FALSE
```

```
> w[c(FALSE, TRUE, TRUE, FALSE)]
```

```
[1] 20  9
```

Filtering Examples

```
> w <- c(-3, 20, 9, 2)
> ### Extract elements of w with squares between 3 and 10
> w[w*w >= 3 & w*w <= 10]
```

```
[1] -3  2
```

```
> w*w >= 3 ### What's going on here?
```

```
[1] TRUE TRUE TRUE TRUE
```

```
> w*w <= 10
```

```
[1]  TRUE FALSE FALSE  TRUE
```

```
> w*w >= 3 & w*w <= 10
```

```
[1]  TRUE FALSE FALSE  TRUE
```

Filtering Examples

Extracting Elements from a Vector

```
> w <- c(-1, 20, 9, 2)
> v <- c(0, 17, 10, 1)
> ### Extract elements of w greater than elements from v
> w[w > v]
```

```
[1] 20  2
```

```
> ### What's going on here?
> w > v
```

```
[1] FALSE  TRUE FALSE  TRUE
```

```
> w[c(FALSE, TRUE, FALSE, TRUE)]
```

```
[1] 20  2
```

Filtering Examples

Filtering Elements of a Matrix

```
> M <- matrix(c(rep(4,5), 5:8), ncol=3, nrow=3)
> M
```

```
     [,1] [,2] [,3]
[1,]    4    4    6
[2,]    4    4    7
[3,]    4    5    8
```

```
> ### We can do element-wise comparisons with matrices too.
> M > 5
```

```
      [,1]  [,2] [,3]
[1,] FALSE FALSE TRUE
[2,] FALSE FALSE TRUE
[3,] FALSE FALSE TRUE
```

Filtering Examples

```
> M
```

```
     [,1] [,2] [,3]
[1,]    4    4    6
[2,]    4    4    7
[3,]    4    5    8
```

```
> M[,3] < 8
```

```
[1]  TRUE  TRUE FALSE
```

```
> M[M[,3] < 8, ]
```

```
     [,1] [,2] [,3]
[1,]    4    4    6
[2,]    4    4    7
```

Filtering Examples

Reassigning Elements of a Matrix

```
> M
```

```
     [,1] [,2] [,3]
[1,]    4    4    6
[2,]    4    4    7
[3,]    4    5    8
```

```
> ### Assign elements greater than 5 with zero
> M[M > 5] <- 0
> M
```

```
     [,1] [,2] [,3]
[1,]    4    4    0
[2,]    4    4    0
[3,]    4    5    0
```

Check Your Understanding

Using matrix z, what is the output of the following?

```
> z
```

```
     First Second Third
[1,]     1      1     9
[2,]     2      0    16
[3,]     3      1    25
```

1. `z[z[, "Second"], ]`?
2. `z[, 1] != 1`?
3. `z[(z[, 1] != 1), 3]`?

Check Your Understanding

```
> z
```

```
     First Second Third
[1,]     1      1     9
[2,]     2      0    16
[3,]     3      1    25
```

Solutions

```
> z[z[, "Second"], ]
```

```
     First Second Third
[1,]     1      1     9
[2,]     1      1     9
```

Check Your Understanding

```
> z
```

```
     First Second Third
[1,]     1      1     9
[2,]     2      0    16
[3,]     3      1    25
```

Solutions

```
> z[, 1] != 1
```

```
[1] FALSE  TRUE  TRUE
```

```
> z[(z[, 1] != 1), 3]
```

```
[1] 16 25
```

A Quick Note

```
> z
```

```
     First Second Third
[1,]     1      1     9
[2,]     2      0    16
[3,]     3      1    25
```

```
> z[(z[, 1] != 1), 3]
```

```
[1] 16 25
```

```
> z[(z[, 1] != 1), 3, drop = FALSE]
```

```
     Third
[1,]    16
[2,]    25
```

NA and NULL Values

NA and NULL

- `NA` indicates a missing value in a dataset.
- `NULL` is a value that doesn't exist and is often returned by expressions and functions whose value is undefined.

Example

```
> length(c(-1, 0, NA, 5))
```

```
[1] 4
```

```
> length(c(-1, 0, NULL, 5))
```

```
[1] 3
```

NA and NULL

Example

```
> ### Use na.rm = TRUE to remove NA values
> t <- c(-1,0,NA,5)
> mean(t)
```

```
[1] NA
```

```
> mean(t, na.rm = TRUE)
```

```
[1] 1.333333
```

```
> ### NA values are missing, but NULL values don't exist.
> s <- c(-1, 0, NULL, 5)
> mean(s)
```

```
[1] 1.333333
```

NA and NULL

NULL can be used is to build a vector in the following way:

```
> # Define an empty vector
>     x <- NULL
> # Fill in the vector
>     x[1] <- "Blue"
>     x[2] <- "Green"
>     x[3] <- "Red"
>     x
```

```
[1] "Blue"  "Green" "Red"
```

- NULL is commonly used to build vectors in loops with each iteration adding another element.
- Filling in pre-allocated space is less expensive (computationally) than adding an element at each step.
- Loops will be introduced next lecture.

A Note on Lists

Lists

A list structure combines objects of different modes.

Recall, in vectors and matrices all elements must have the same mode.

To define a list:

- Use the function "`list()`":

```
list(name1 = object1, name2 = object2, ...)
```

List component names (called tags) are optional.

Line of Best Fit Example

Recall, the energy expenditure versus fat-free body mass example one more time.

Example

To investigate the dependence of energy expenditure (y) on body build, researches used underwater weighing techniques to determine the fat-free body mass (x) for each of seven men. They also measured the total 24-hour energy expenditure for each man during conditions of quiet sedentary activity. The results are shown in the table.

Subject	1	2	3	4	5	6	7
x	49.3	59.3	68.3	48.1	57.61	78.1	76.1
y	1,894	2,050	2,353	1,838	1,948	2,528	2,568

Lists

Let's make a list of the values we've calculated for this example.

```
> # Combine data into single matrix
>   data <- cbind(x, y)
> # Summary values for x and y
>   sum_x <- summary(x)
>   sum_y <- summary(y)
> # We computed Sxy and Sxx previously
>   est_vals <- c(Sxy/Sxx, mean(y) - Sxy/Sxx*mean(x))
```

Lists

```
> # Define a list with different objects for each element
>   body_fat <- list(variable_data = data,
+                    summary_x = sum_x, summary_y = sum_y,
+                    LOBF_est = est_vals)
```

Extracting Components of Lists

Extract an individual component "c" from a list called `lst` in the following ways:

- `lst$c`
- `lst[[i]]` where "c" is the i^{th} component.
- `lst[["c"]]`

Extracting Components of Lists

Energy expenditure versus fat-free body mass example

```
> # Extract the first list element
> body_fat[[1]]
```

```
         x    y
[1,] 49.30 1894
[2,] 59.30 2050
[3,] 68.30 2353
[4,] 48.10 1838
[5,] 57.61 1948
[6,] 78.10 2528
[7,] 76.10 2568
```

Lists

Energy expenditure versus fat-free body mass example

```
> # Extract the Line of Best Fit estimates
> body_fat$LOBF_est
```

```
[1]  25.01184 607.65386
```

```
> # Extract the summary of x
> body_fat[["summary_x"]]
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  48.10   53.46   59.30   62.40   72.20   78.10
```