Buffon's Needle

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December 29, 2018

The problem

In the 18th century, the following problem was posed by a French aristocrat, Georges-Louis Leclerc, Comte de Buffon.

Suppose we have a floor made of parallel strips of wood, each the same width, and we drop a needle onto the floor. What is the probability that the needle will lie across a line between two strips?

The Problem

This is a problem in geometric probability and can be solved exactly using calculus, to give an answer in terms of the length of the needle l and the gap between the strips of wood, t. For $l < t$ then this probability is

$$P = \frac{2l}{t\pi}$$

Getting the answer

- The distance from the center of the needle to the nearest line is a random number between 0 and $t/2$. The angle between the needle and this nearest line is random between 0 and 90 degrees (or, in radians, between 0 and $\pi/2$).
- The needle crosses the line if

$$x \leq \frac{l}{2} \cos \theta$$

- This probability is given by

$$\int_{\theta=0}^{\pi/2} \int_{x=0}^{(l/2)\cos\theta} \frac{4}{t\pi} dx d\theta = \frac{2l}{t\pi},$$

when the needle is shorter than the gap.

Estimating Pi

For centuries, people have spent time constructing ways to give more and more accurate estimates for π, an irrational, transcedental number which has infinite digits in it's decimal expansion.

Estimating Pi

- Several methods have been devised to estimate π numerically, the simplest being to attempt to accurately measure the circumference of a circle of known radius.
- Laplace realised in 1812 that π could be estimated by using the needle experiment.
- If n needles are dropped and h of them cross the line, then we would estimate P with $\frac{h}{n}$ and rearrange the previous equation to give
$$\pi \approx \frac{2ln}{th}.$$

Doing this simulation

- Pick our $l < t$ and the number of needles to drop n
- For each needle generate a random x and θ and if $x \leq l/2 \cos\theta$ add than needle to h, the number which cross the lines.
- We can do this easily in R.

Some code to show this

```
l = 1
t = 2
n = 10000
x = runif(n, min = 0, max =  t/2)
theta = runif(n, min=0, max = pi/2)
length_line = l/2*cos(theta)
h =length(which(x<length_line))
estimate = 2*l*n/(t*h)
estimate
```

```
## [1] 3.10752
```

- Increasing *n* will give a more accurate answer.

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- How accurate is *pi*, the estimate stored in R?
- Would we do better with a better estimate of π?
- Other methods which don't require using π exist (see website).

Lazzarini

- In 1901, an Italian mathematician Lazzarini claimed to have done this experiment over 3000 times by hand, getting a correct answer to 6 decimal places.
- Given the intermediate numbers he recorded, the probability of getting the answer he got it extremely small, indicating he may have continued the experiment until he got a number he wanted. [1][2]

References

1. https://en.wikipedia.org/wiki/Buffon%27s_needle
2. https://www.maa.org/sites/default/files/pdf/upload_library/22/Allendoerfer/1995/Badger.pdf