5T2: Sinusoidal model (2 of 3)

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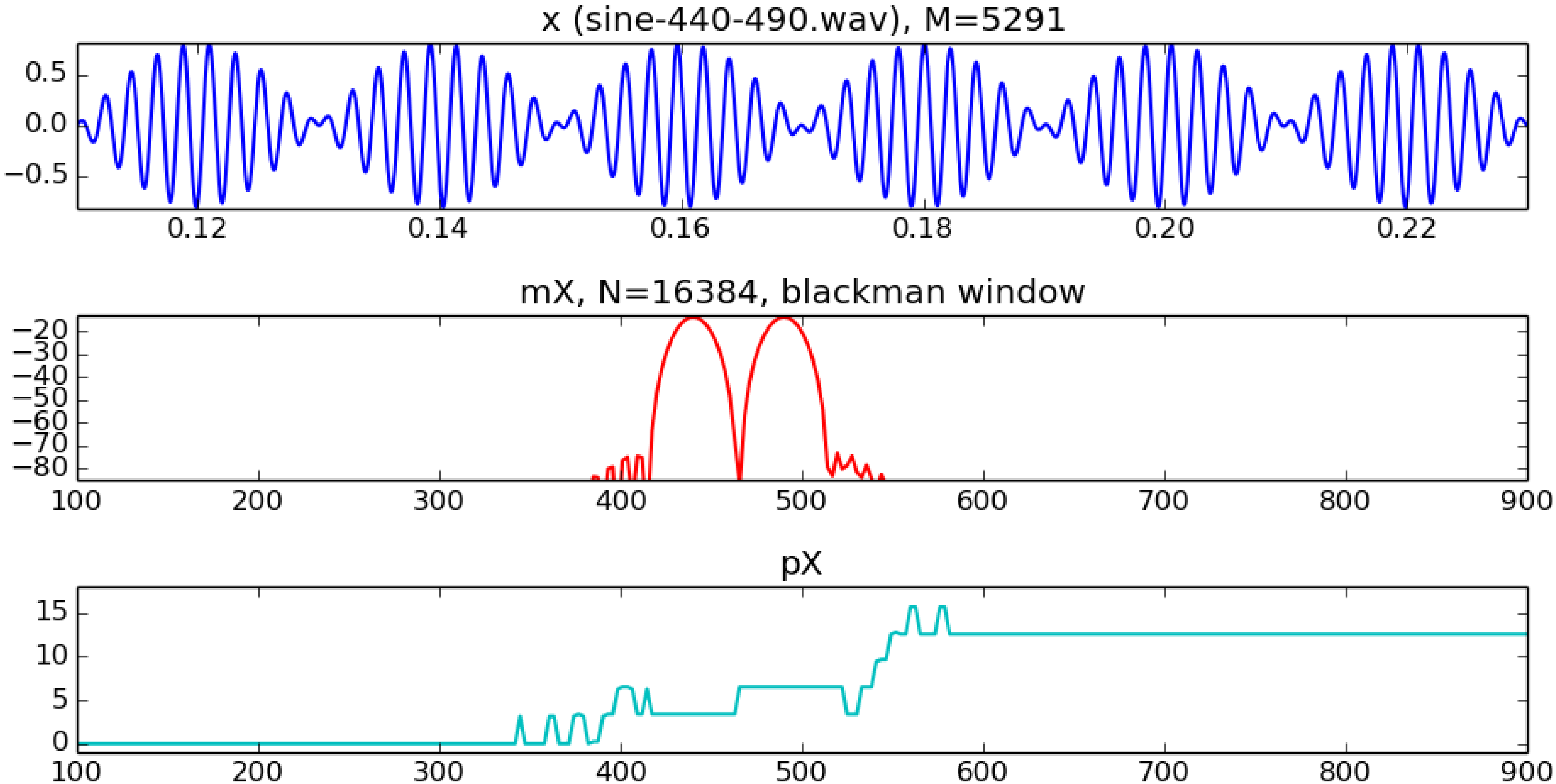
Sinusoidal model

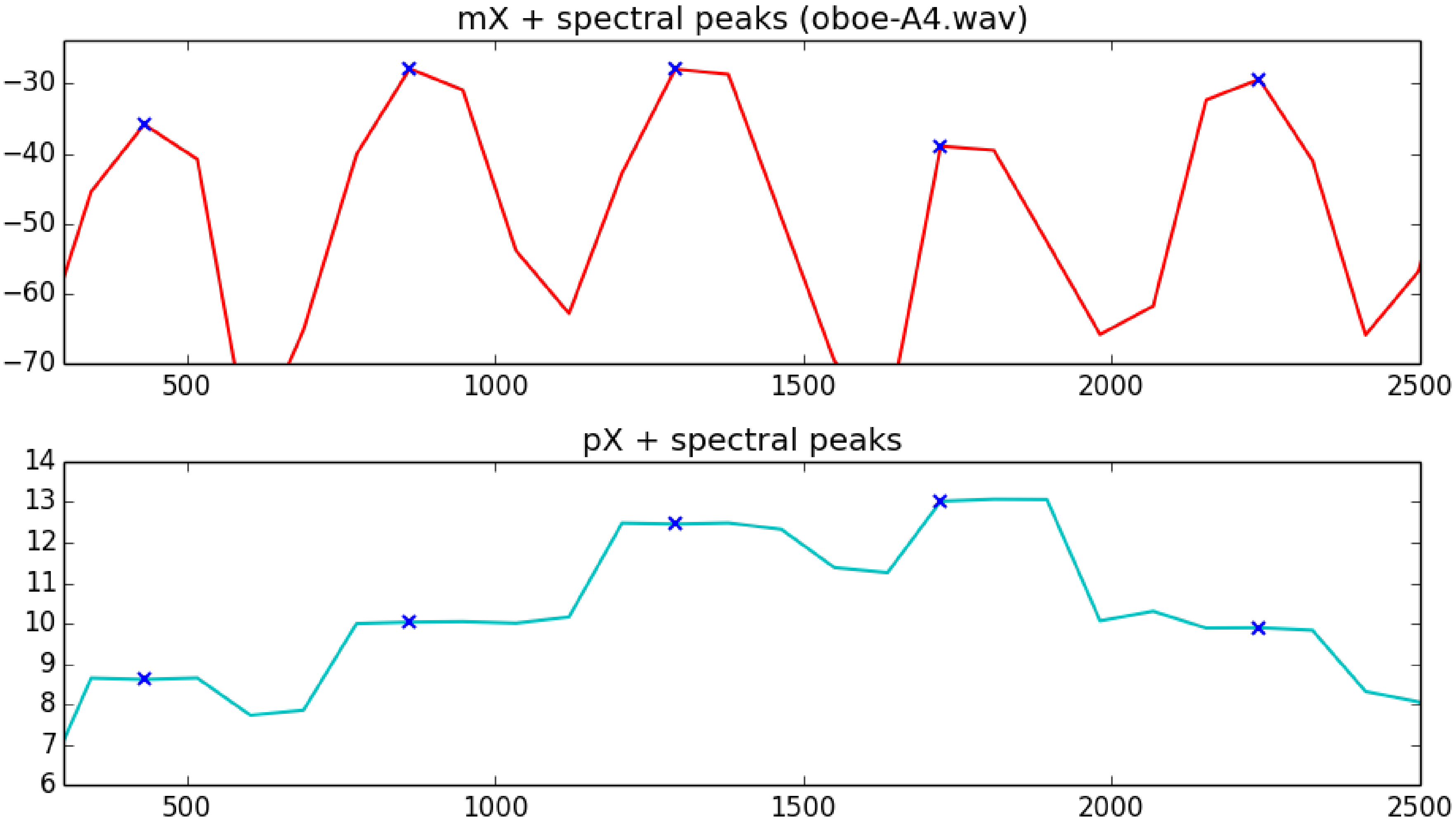
$$y[n]=\sum_{r=1}^{R} A_r[n]\cos\left(2\pi f_r[n]n\right)$$



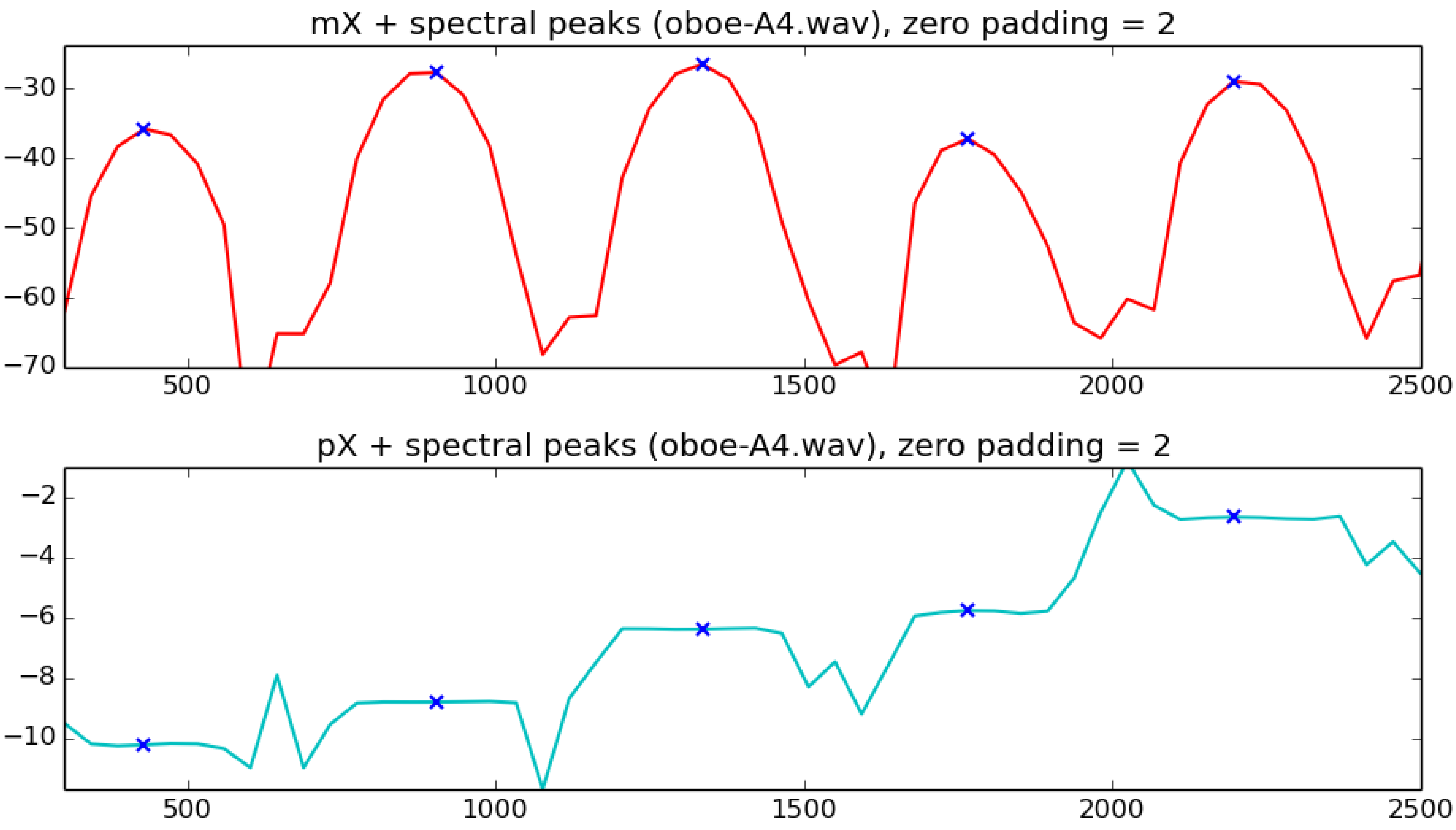
Peak detection

$$p_r = mX[k_0] \quad \text{when} \quad mX[k_0-1] < mX[k_0] > mX[k_0+1]$$

mX = magnitude spectrum ; r = peak number ; k_0 = peak location



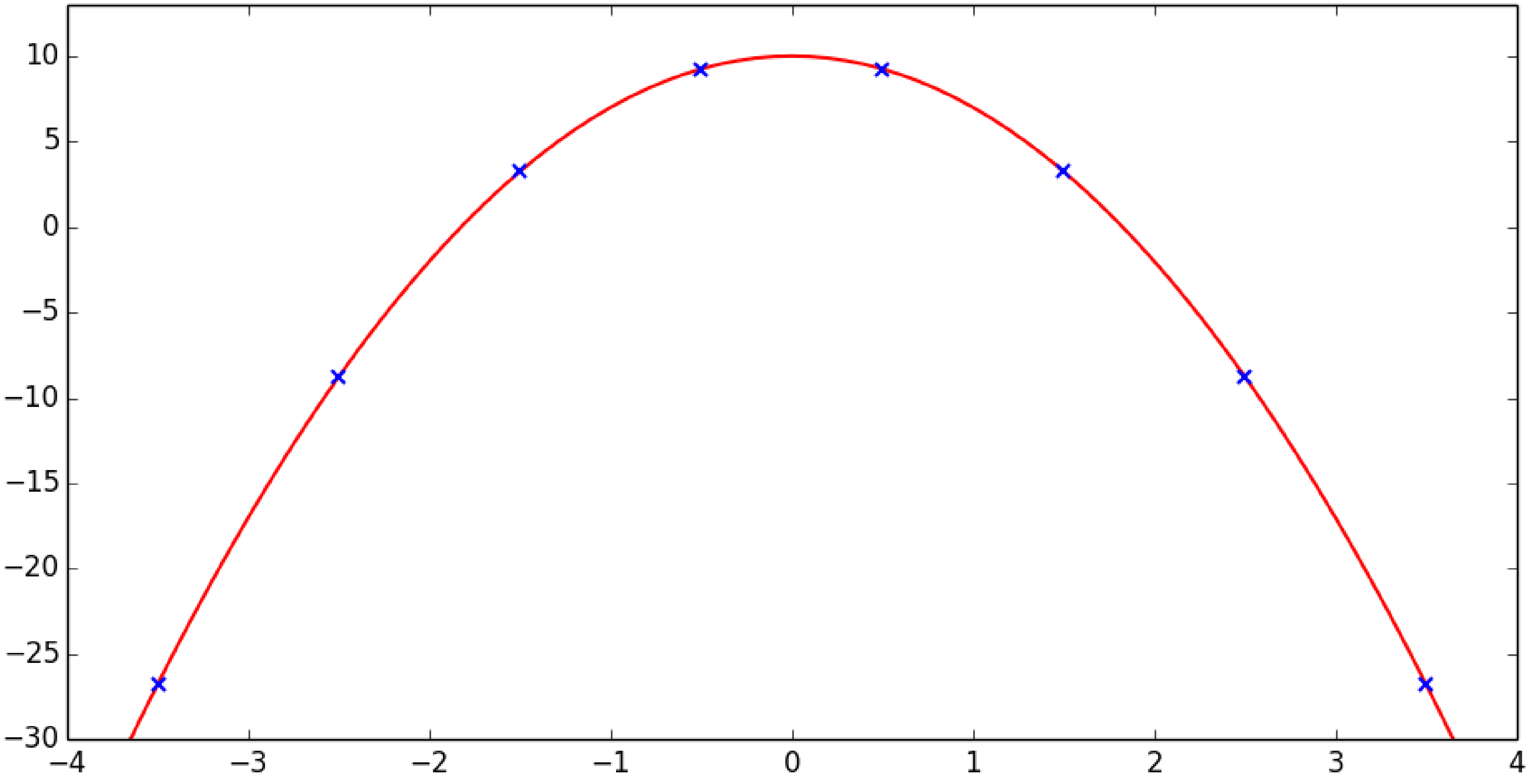
Peak detection with zero-padding



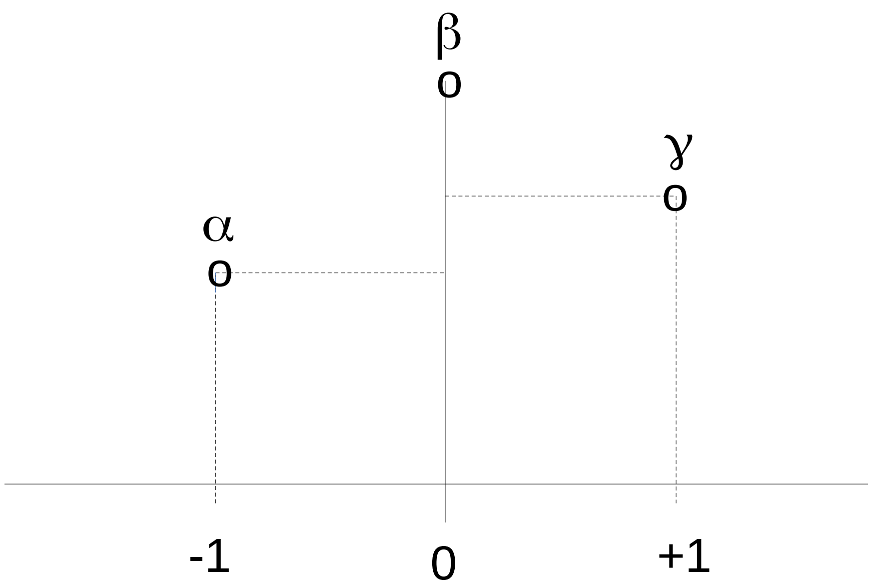
Parabola

$$x[n]=a(n-p)^2+b$$

p: center of parabola
a: concavity measure
b: offset



Peak interpolation

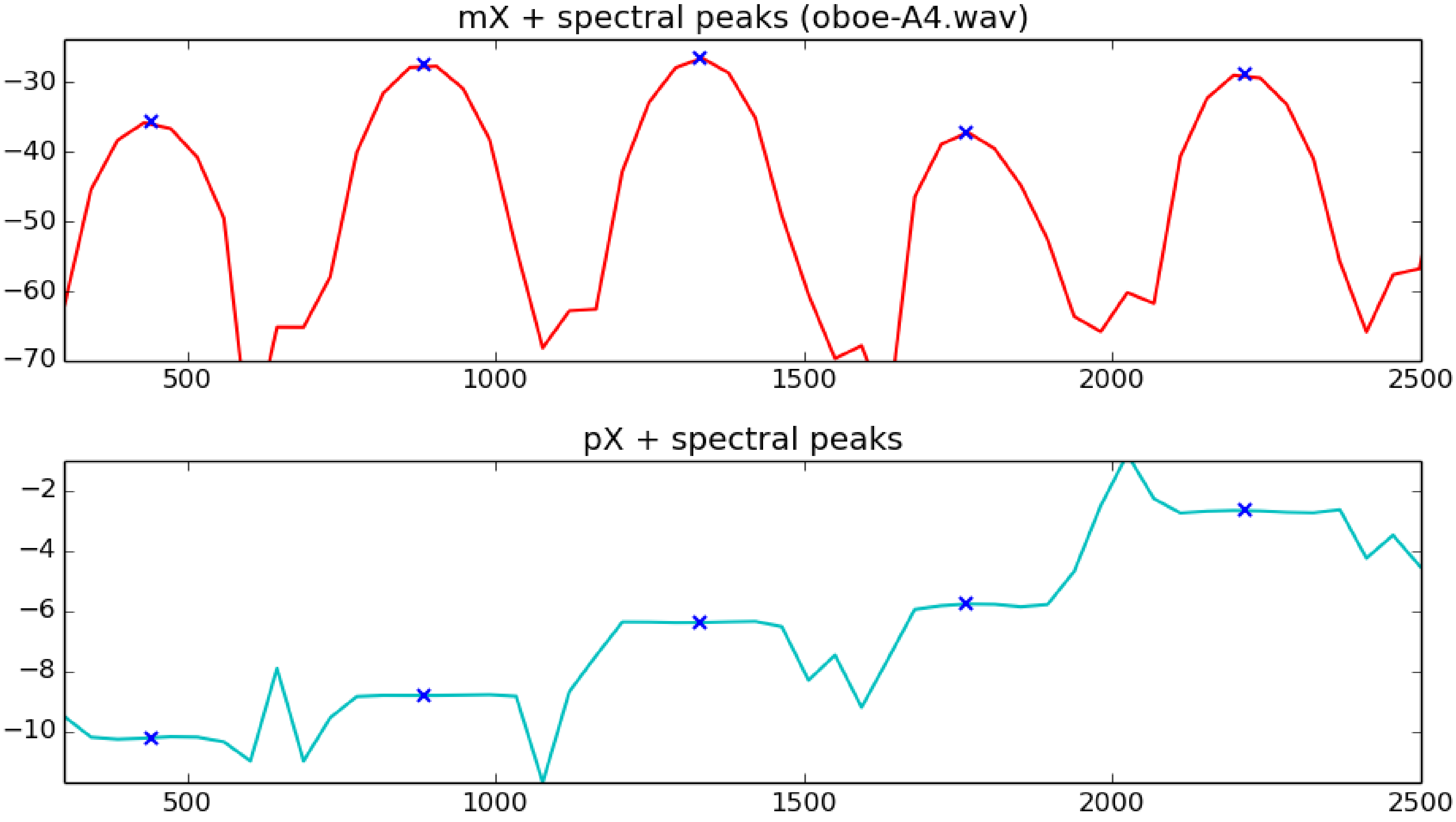


$$x[-1] \;=\; \alpha \;=\; mX[k_\beta - 1]$$
$$x[0] \;=\; \beta \;=\; mX[k_\beta]$$
$$x[1] \;=\; \gamma \;=\; mX[k_\beta + 1]$$

center of the parabola: $\hat{k}_p = k_p + \frac{1}{2}\frac{\alpha - \gamma}{(\alpha - 2\beta + \gamma)}$

amplitude: $\hat{a} = \beta - \frac{1}{4}(\hat{k}_p - k_p)(\alpha - \gamma)$

Peak detection with interpolation



Sinusoidal parameters from peaks

$$\hat{k}_p = k_p + \frac{0.5*(|X[k_p-1]| - |X[k_p+1]|)}{|X[k_p-1]| - 2*|X[k_p]| + |X[k_p+1]|}$$

$$f_p = \frac{f_s * \hat{k}_p}{N}$$

$$A_p = |X[k_p]| - 0.25*(|X[k_p-1]| - |X[k_p+1]|)*(\hat{k}_p - k_p)$$

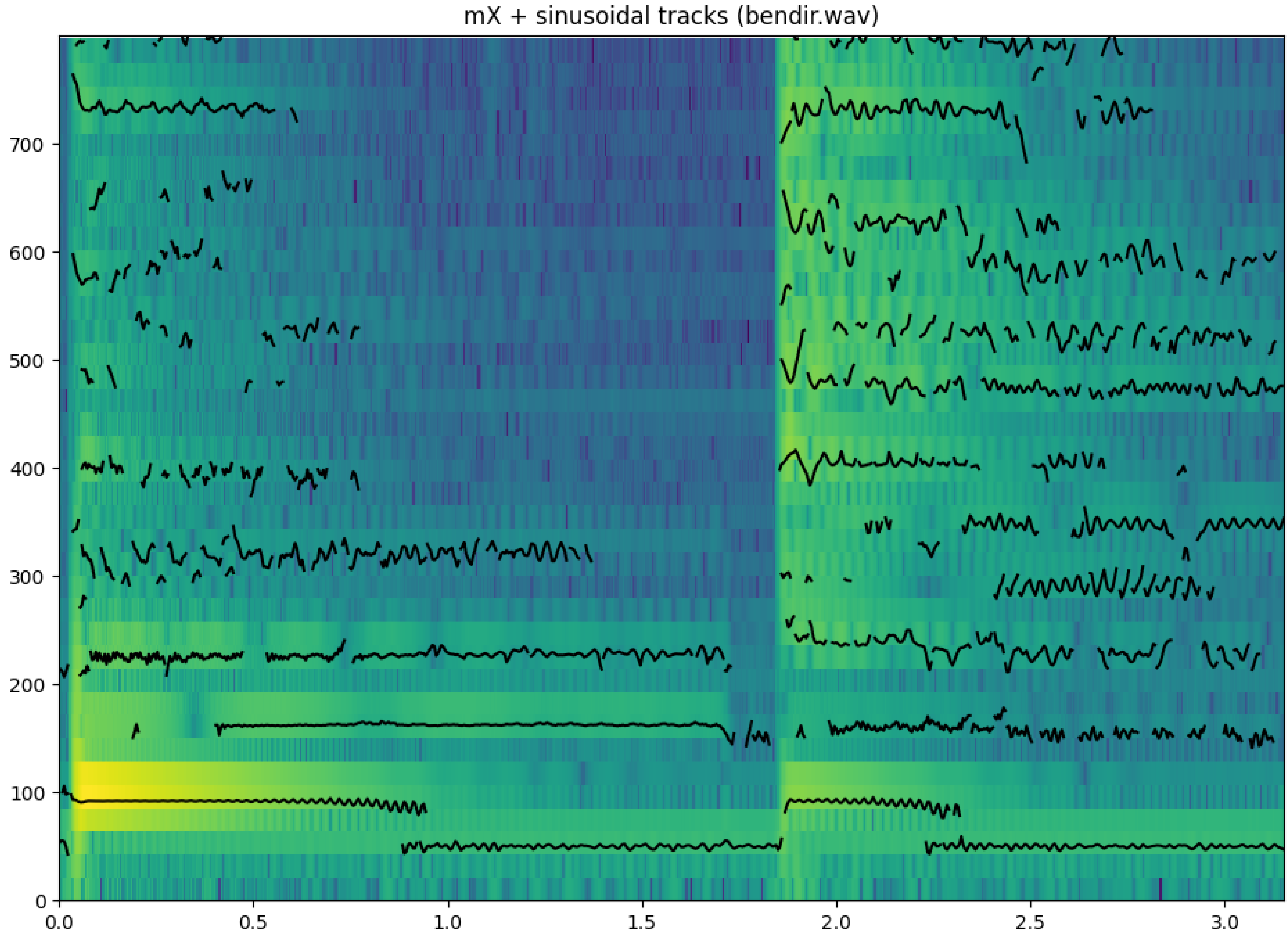
$$ph_p = \measuredangle X[\hat{k}_p]$$

Sinewaves in spectrogram

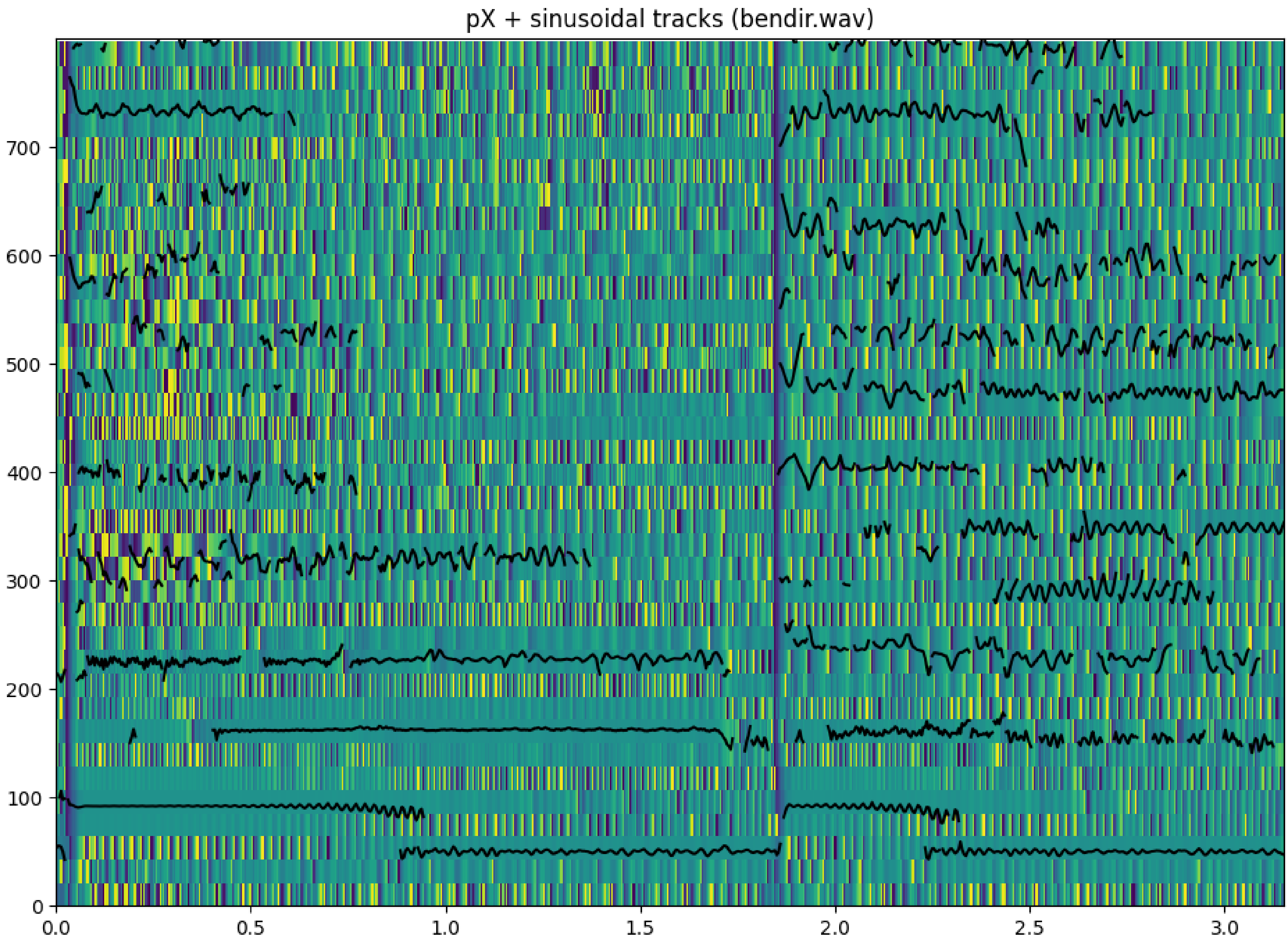
- Sinusoid → **stable** peak track in spectrogram
- Stability identified by
  - frequency and amplitude in succesive frames
  - phase derivative in time/freq

Condition for a peak f_p of frame l to be part of a track t:

$$f_p[l] = f_t[l] \quad \text{if} \quad \left(|f_p[l] - f_t[l-1]| < \textit{threshold} \right)$$
$$\text{and if exists} \quad f_t[l-2], f_t[l-3], \ldots, f_t[l-L]$$



mX + sinusoidal tracks (bendir.wav)
0
100
200
300
400
500
600
700
0.0
0.5
1.0
1.5
2.0
2.5
3.0



pX + sinusoidal tracks (bendir.wav)
700
600
500
400
300
200
100
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

References and credits

- More information in:http://en.wikipedia.org/wiki/Sinusoidal_model
- Reference on sinusoidal modeling by Julius O. Smith: https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_Sinusoids.html
- Sounds from: http://www.freesound.org/people/xserra/packs/13038/
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