

# 4T2: The Short-Time Fourier Transform (2 of 2)

***Xavier Serra***

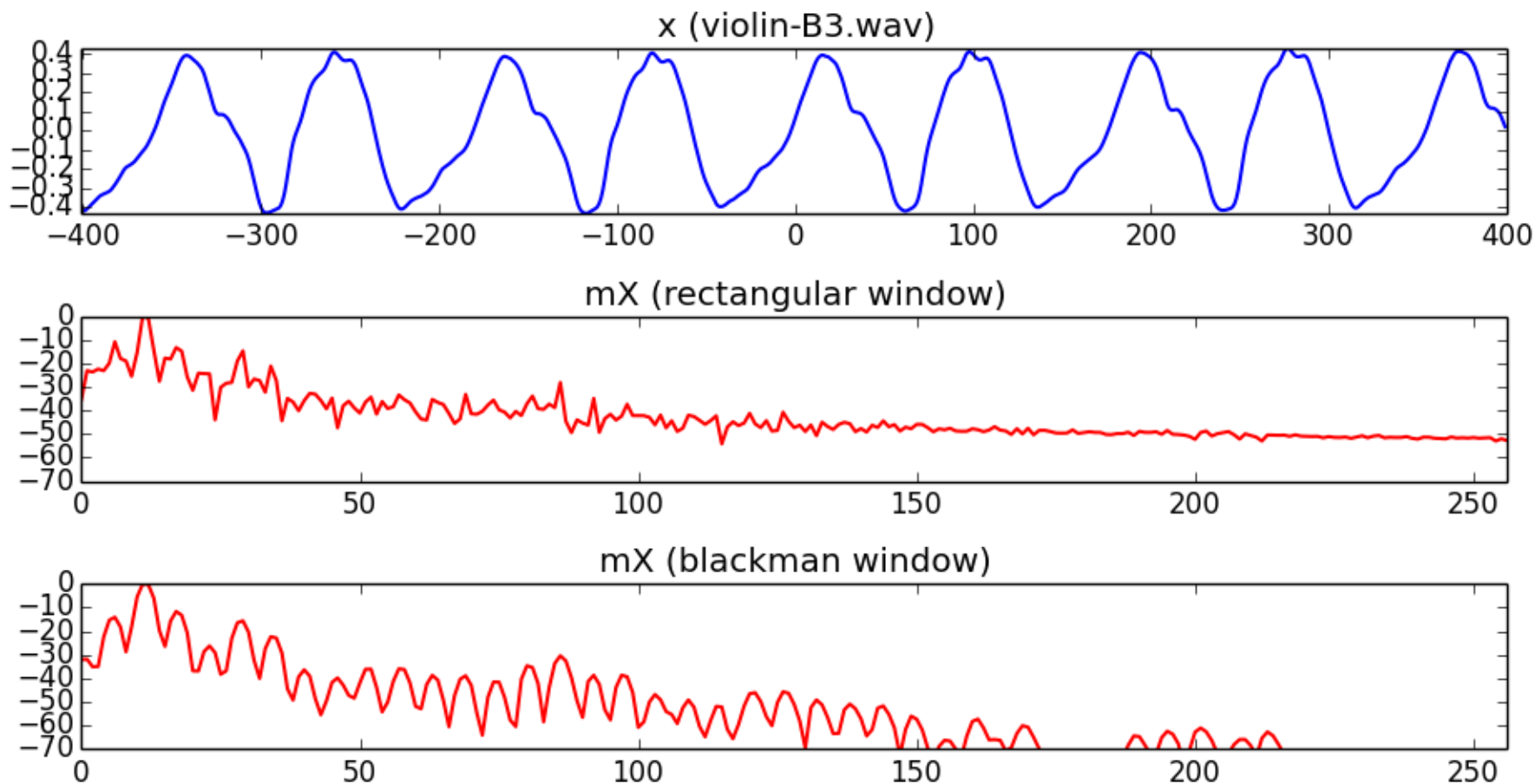
Universitat Pompeu Fabra, Barcelona

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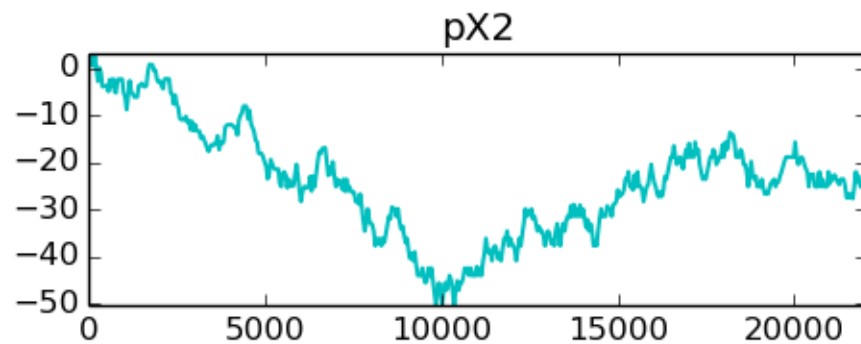
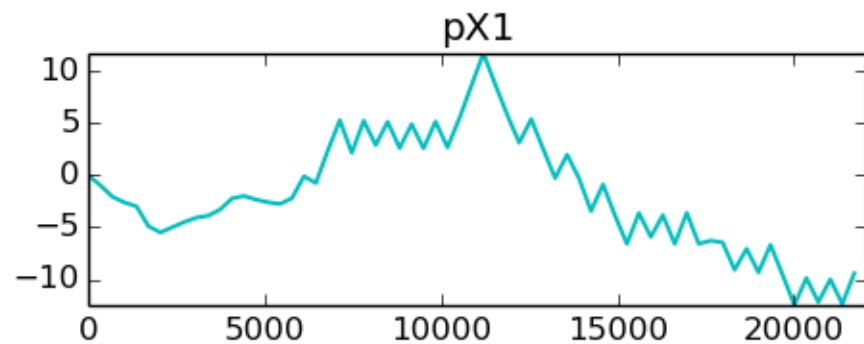
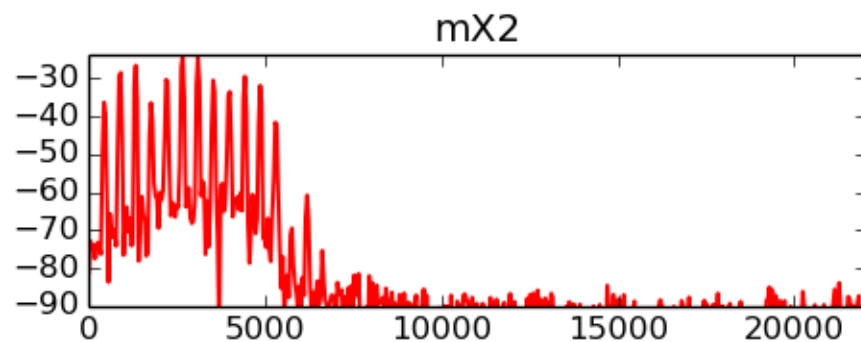
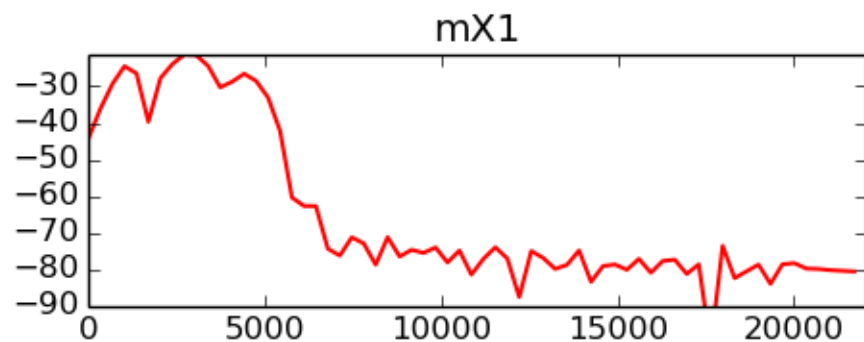
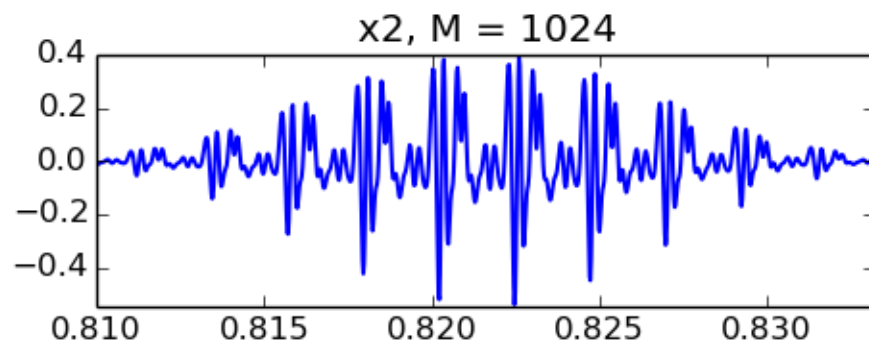
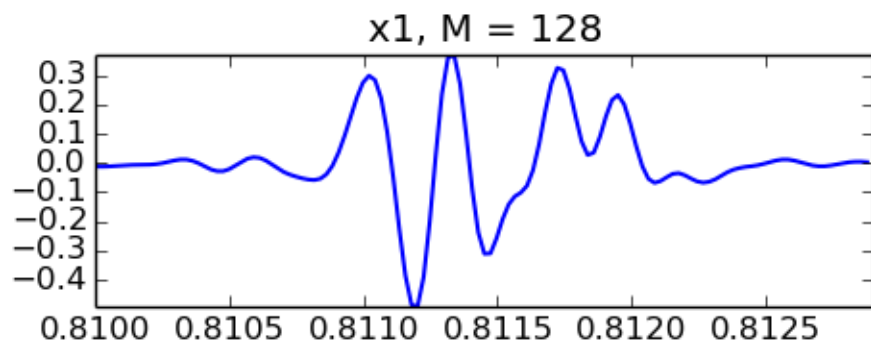
- STFT and analysis window
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# STFT and analysis window

$$X_l[k] = \sum_{n=-N/2}^{N/2-1} w[n] x[n+lH] e^{-j2\pi kn/N} \quad l=0,1,\dots,$$

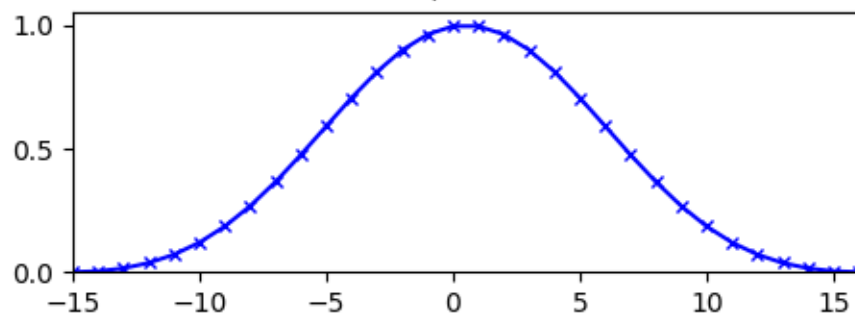


# Window size

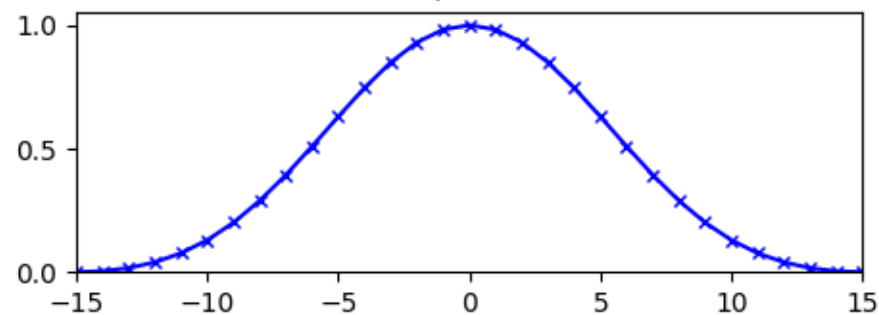


# Even-odd size window

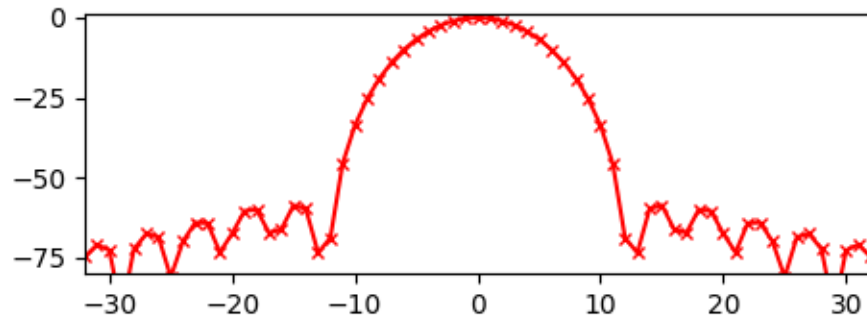
w1, M=32



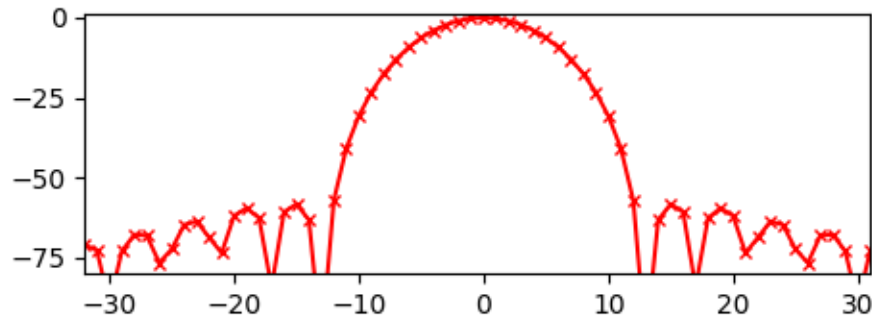
w2, M=31



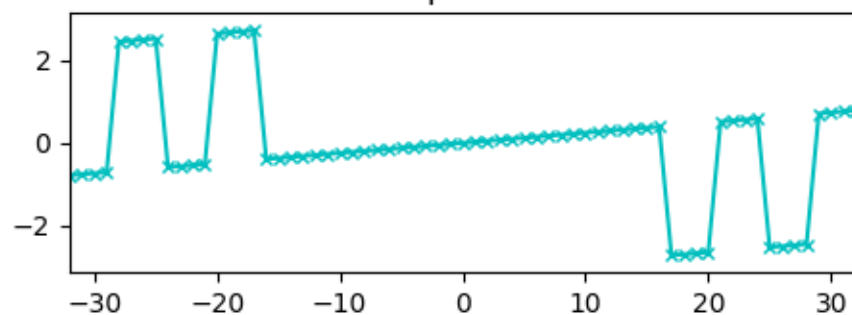
mW1



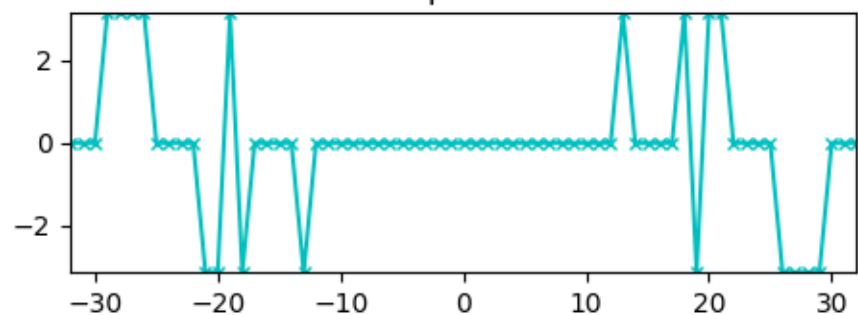
mW2



pW1

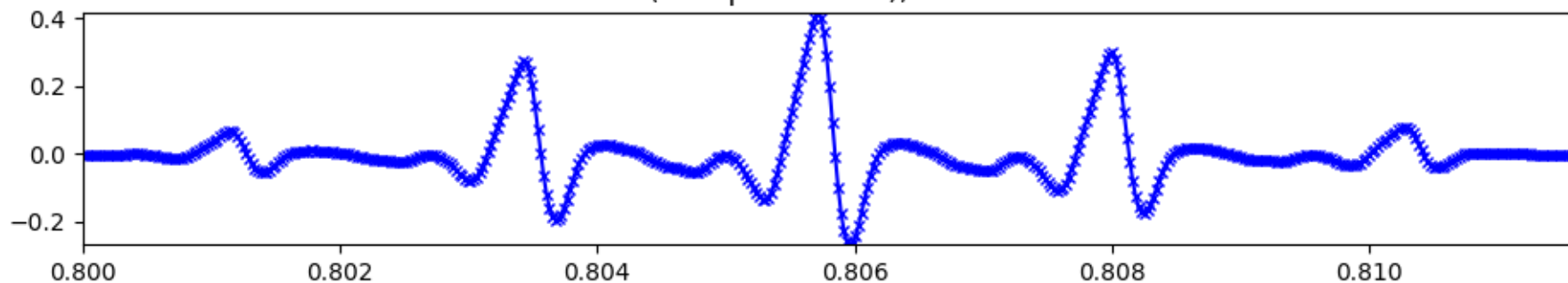


pW2

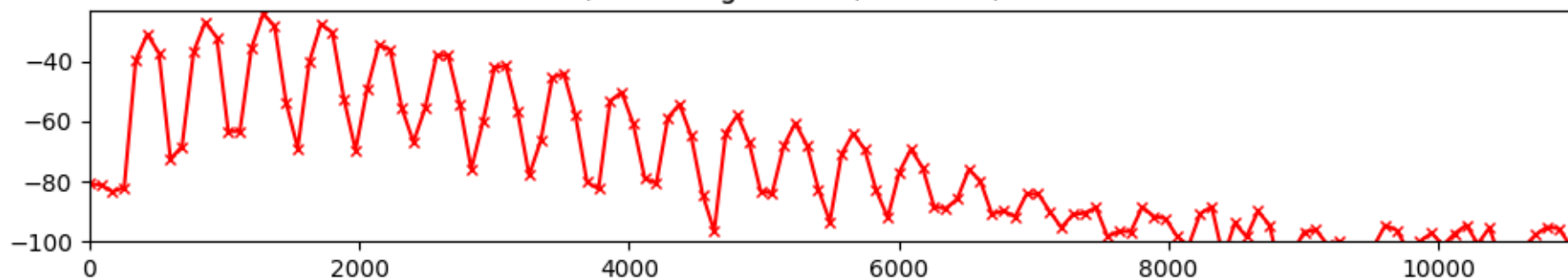


# FFT size

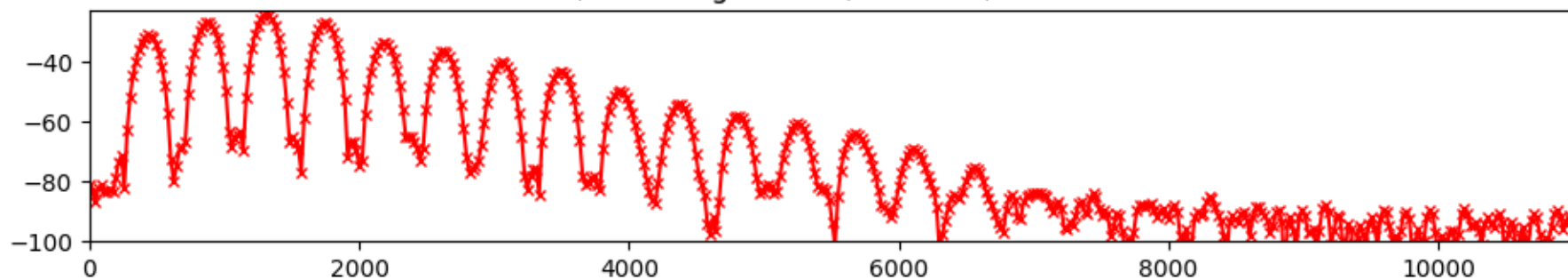
x (trumpet-A4.wav), M = 512



mX, hamming window, M = 512, N = 512



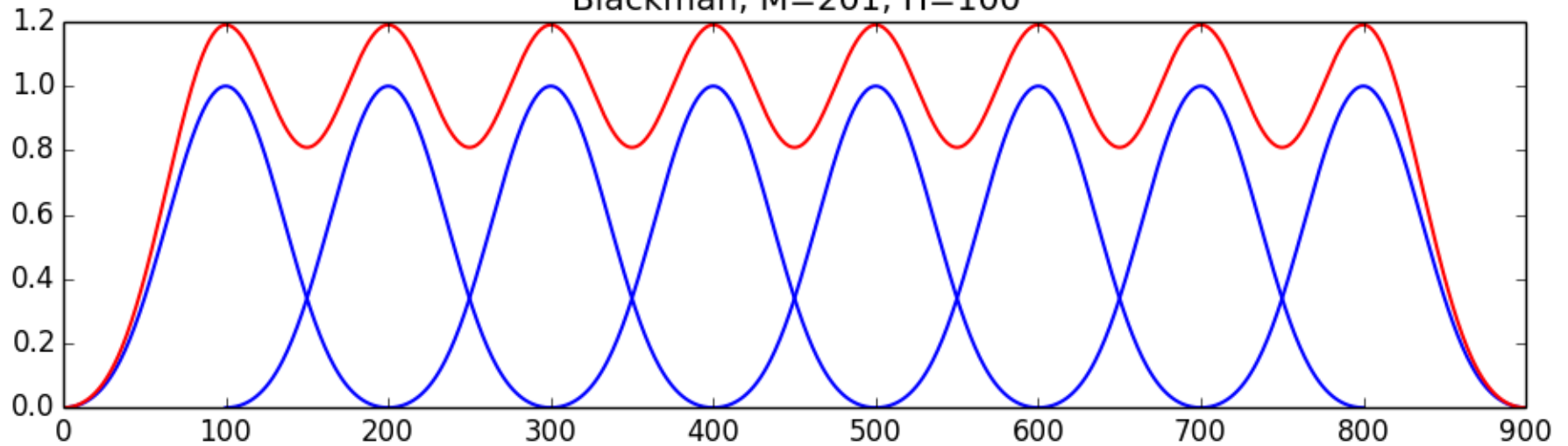
mX, hamming window, M = 512, N = 2048



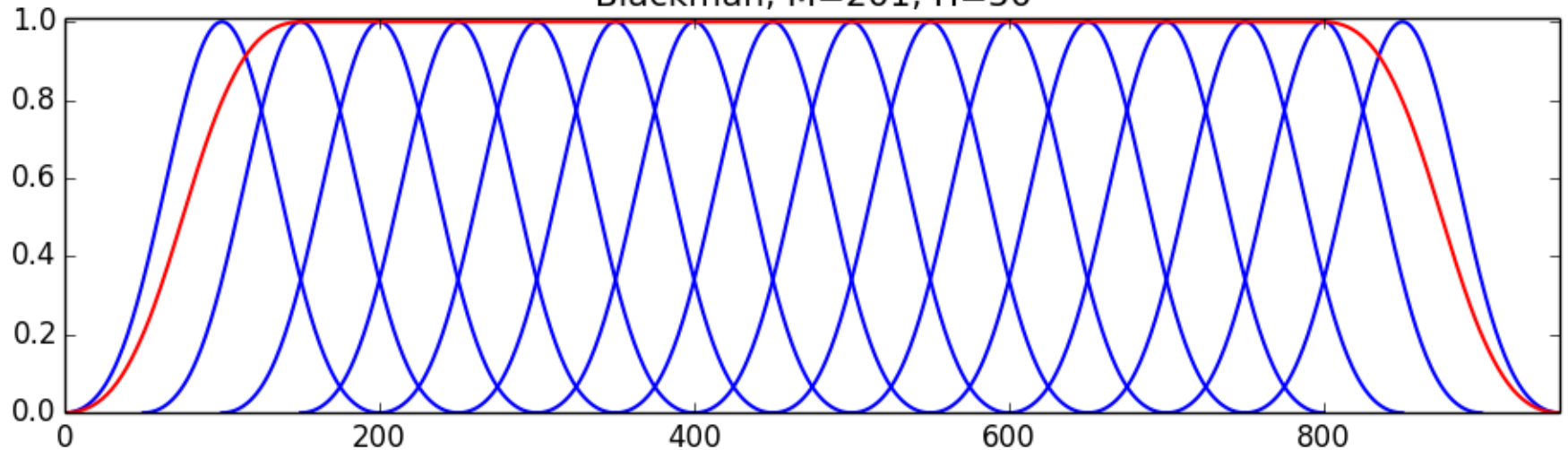
# Hop size

$$A_w[n] = \sum_{l=0}^{L-1} w[n-lH] = c$$

Blackman, M=201, H=100

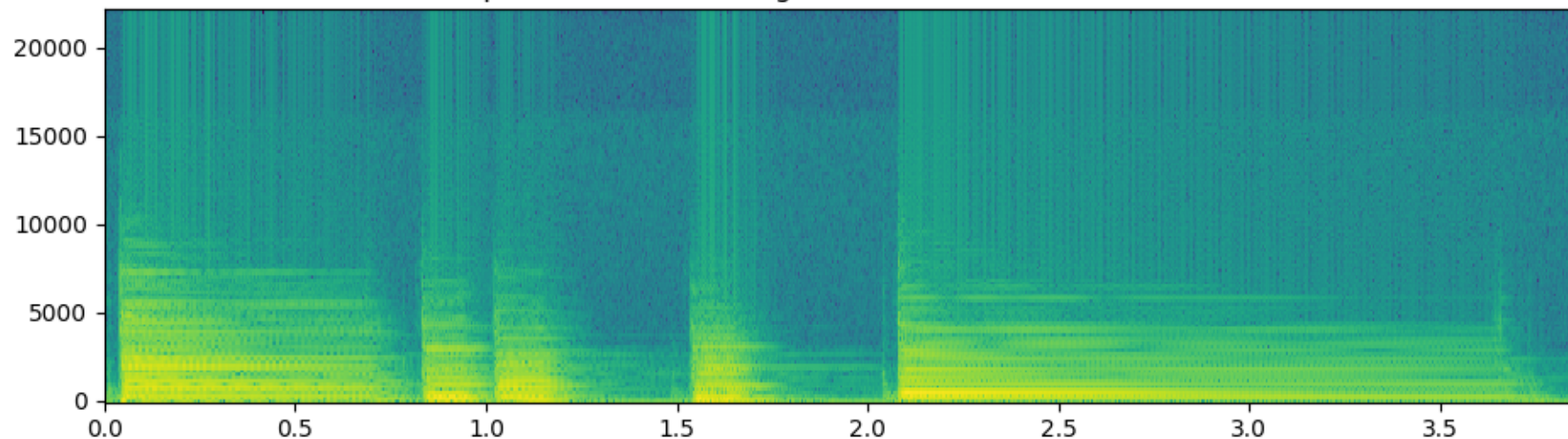


Blackman, M=201, H=50

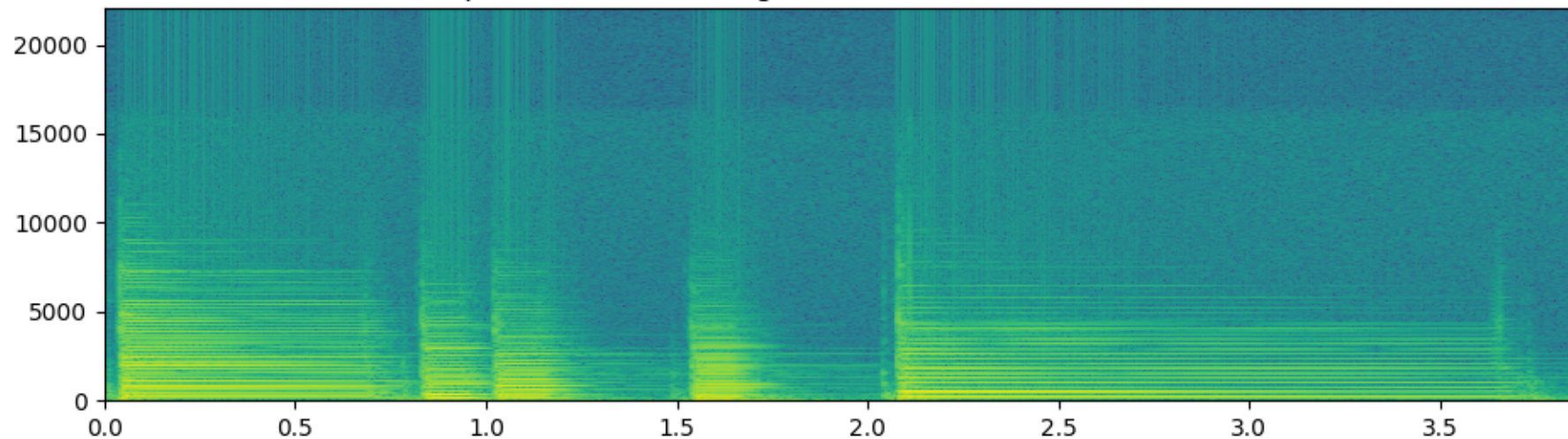


# Time-frequency compromise

mX (piano.wav), Hamming window, M=256, N=256, H=128



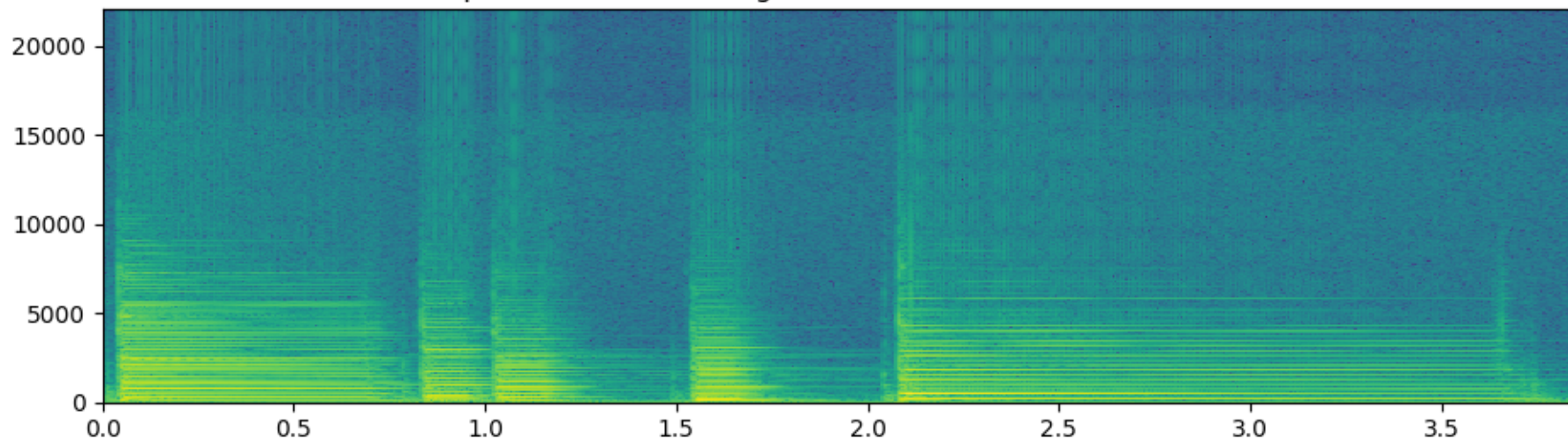
mX (piano.wav), Hamming window, M=1024, N=1024, H=128



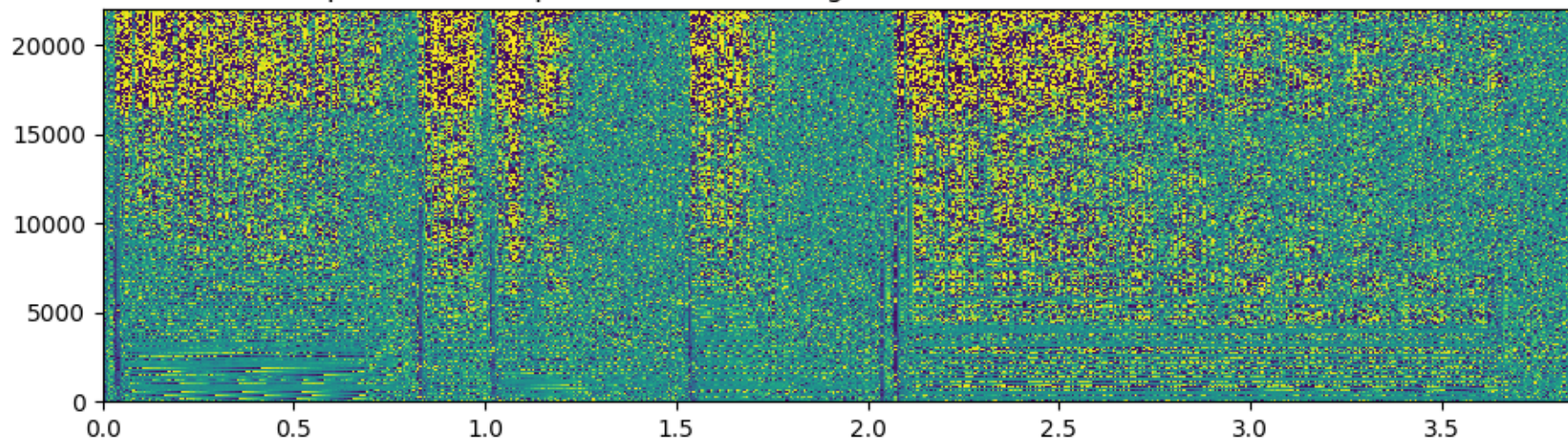


# Amplitude and phase spectrogram

mX (piano.wav), Hamming window, M=1001, N=1024, H=256



pX derivative (piano.wav), Hamming window, M=1001, N=1024, H=256



# Inverse STFT

$$y[n] = \sum_{l=0}^{L-1} \text{Shift}_{lH, n} \left[ \frac{1}{N} \sum_{k=-N/2}^{N/2-1} X_l[k] e^{j2\pi kn/N} \right]$$

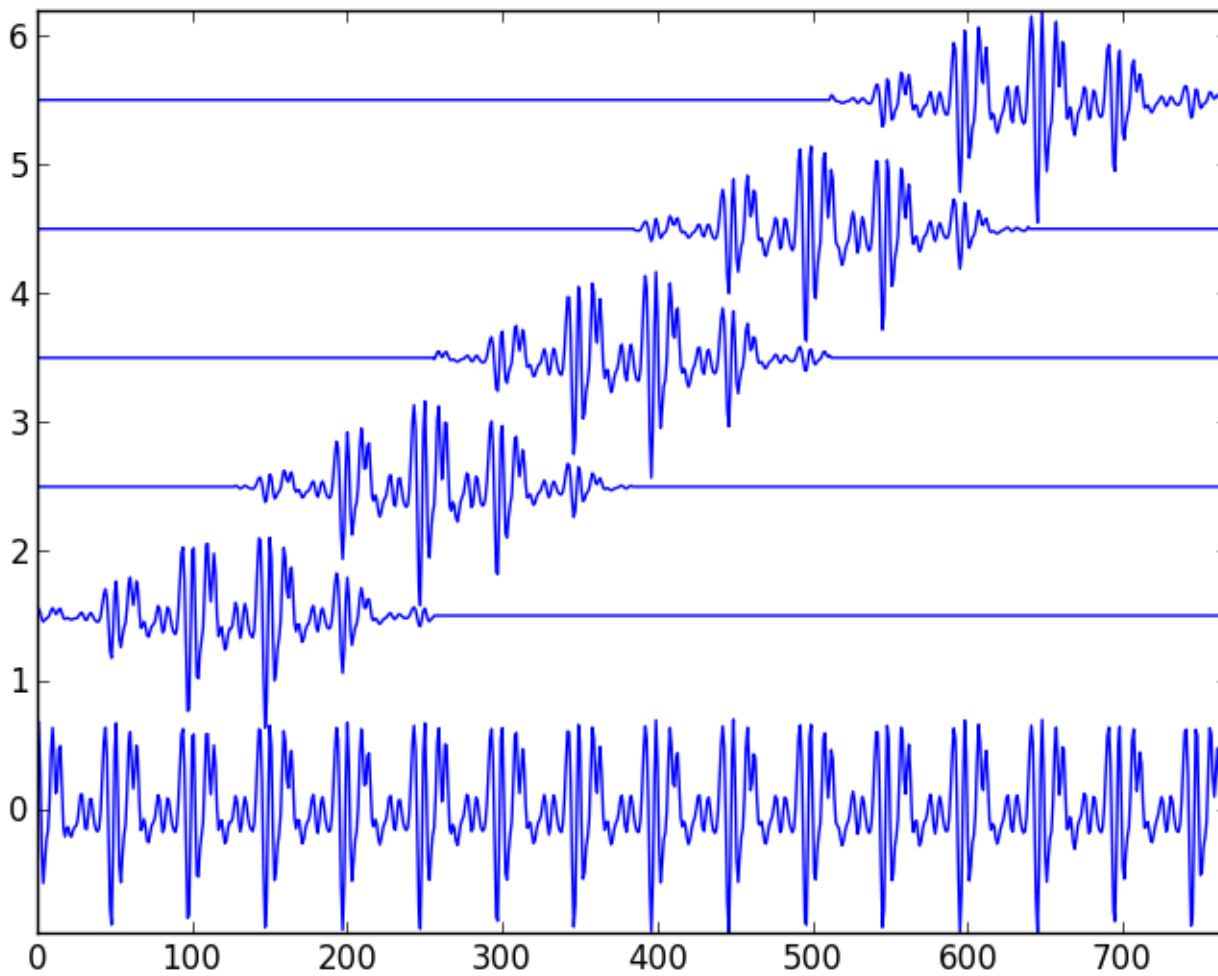
each output frame is:

$$yw_l[n] = x(n + lH) w[n]$$

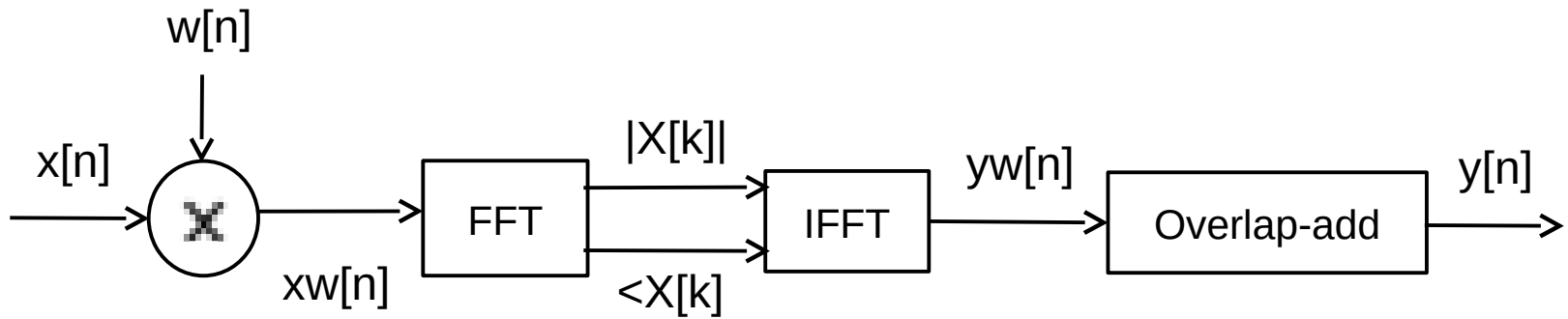
and the output sound is:

$$y[n] = \sum_{l=0}^{L-1} yw_l[n] = x[n] \sum_{l=0}^{L-1} w[n - lH]$$

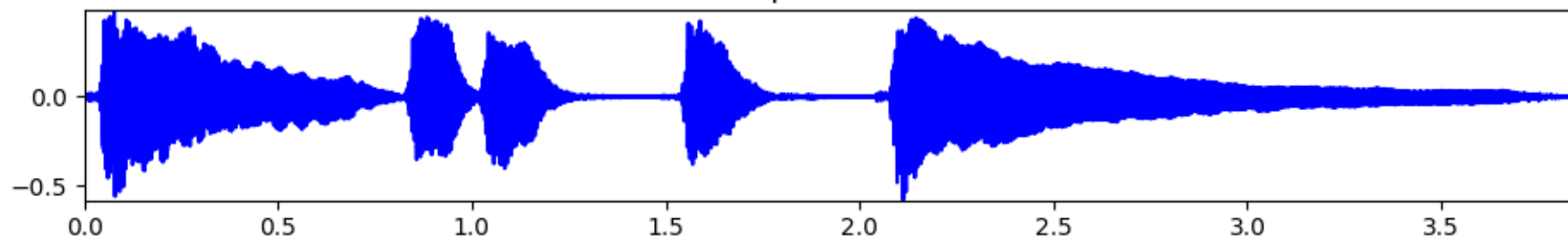
$$yw_l[n] = w[n]x[n+lH] \quad l=0,1,\dots,$$



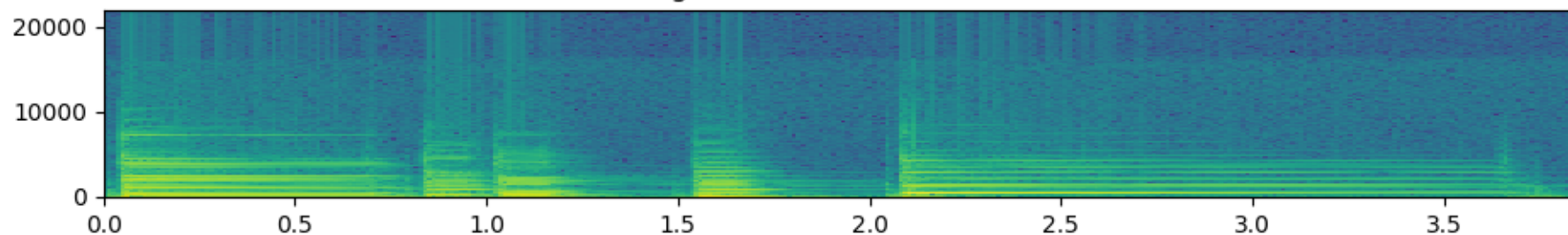
# STFT system



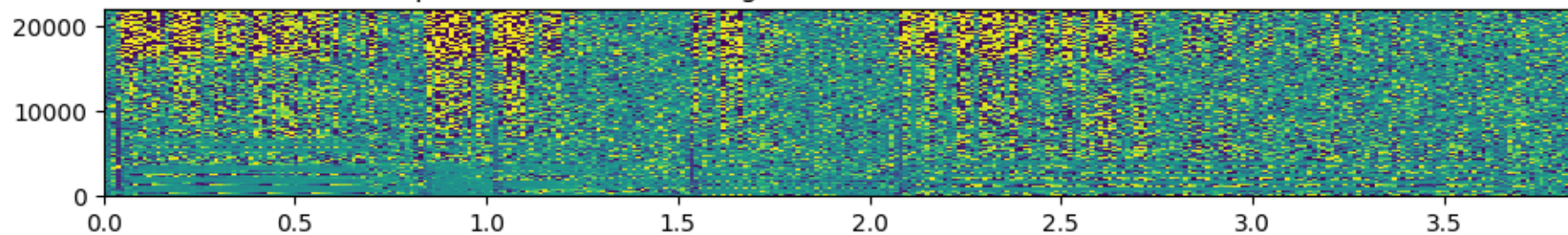
x (piano.wav)



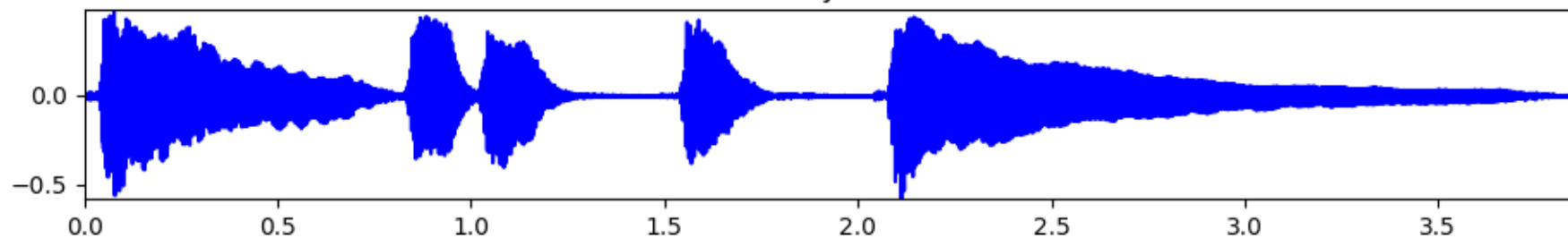
mX, Hamming window, M=1024, N=1024, H=512



pX derivative, Hamming window, M=1024, N=1024, H=512



y



# References and credits

- More information in:  
<https://en.wikipedia.org/wiki/STFT>  
[https://en.wikipedia.org/wiki/Window\\_function](https://en.wikipedia.org/wiki/Window_function)  
<http://en.wikipedia.org/wiki/Spectrogram>
- Reference on the STFT by Julius O. Smith: <https://ccrma.stanford.edu/~jos/sasp/>
- Sounds from: <http://www.freesound.org/people/xserra/packs/13038/>
- Slides released under CC Attribution-Noncommercial-Share Alike license and code under Affero GPL license. All available from <https://github.com/MTG/sms-tools>



# 4T2: The Short-Time Fourier Transform (2 of 2)

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