Question 6

1. $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$

$= \frac{1}{N}\left(\sum_{i=1}^{N} a + b\sum_{i=1}^{N} x_i\right)$

$= \frac{1}{N}(Na) + b\frac{1}{N}\sum_{i=1}^{N} x_i$

$= a + bm(x)$

2. $\operatorname{cov}(x,x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(x))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$

$= s^2$

3. Let $Z = a+bY$, $z_i = a+by_i$

$m(z) = m(a+bY) = a + bm(Y)$

$\operatorname{cov}(X,Z) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(z_i - m(Z))$

$z_i - m(z) = (a+by_i) - (a+bm(Y))$

$= b(y_i - m(Y))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\cdot b(y_i - m(Y))$

$= b\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(y))\right)$

$= b\operatorname{cov}(x,y)$

4. Let $U = a+bX$ and $Z = a+bY$

$m(U) = a+bm(x)$ $\quad m(Z) = a+bm(Y)$

$u_i - m(U) = b(x_i - m(x))$ $\quad z_i - m(Z) = b(y_i - m(Y))$

$\operatorname{cov}(U,Z) = \frac{1}{N}\sum_{i=1}^{N}(u_i - m(U))(z_i - m(Z))$

$= \frac{1}{N}\sum_{i=1}^{N}[b(x_i - m(x))][b(y_i - m(y))]$

$= b^2\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(y))\right]$

$= b^2\operatorname{cov}(x,y)$

5. yes $\operatorname{med}(a+bX) = a + b(\operatorname{med}(x))$

when $b>0$, the transformation $a+bx$ keeps values in the same order, so the data point that was in the middle before the transformation is still in the middle afterwards, which is why the median transforms as $a + b\operatorname{med}(x)$

$IQR(x) = Q_3(x) - Q_1(x)$

For $b>0$ $\quad Q_1(a+bX) = a + bQ_1(x)$ $\quad Q_3(a+bX) = a+bQ_3(x)$

$IQR(a+bX) = Q_3(a+bX) - Q_1(a+bX)$

$= (a + bQ_3) - (a + bQ_1)$

$= b(Q_3 - Q_1)$

$= b\, IQR(x)$

6. Let $X = \{0,2\}$

$m(x) = \frac{0+2}{2} = 1 \Rightarrow (m(x))^2 = 1$

$X^2 = \{0^2, 2^2\} = \{0,4\} \Rightarrow m(x^2) = \frac{0+4}{2} = 2$

$m(x^2) = 2 \neq 1 = (m(x))^2$

Let $X = \{0,4\}$

$m(x) = \frac{0+4}{2} = 2 \Rightarrow \sqrt{m(x)} = \sqrt{2}$

$\sqrt{x} = \{\sqrt{0}, \sqrt{4}\} = \{0,2\} \Rightarrow m(\sqrt{x}) = \frac{0+2}{2} = 1$

$m(\sqrt{x}) = 1 \neq \sqrt{2} = \sqrt{m(x)}$

nonlinear transformation like X^2 or $\sqrt{x}$ do not translate in the same way.