Least-Squares Technical Document

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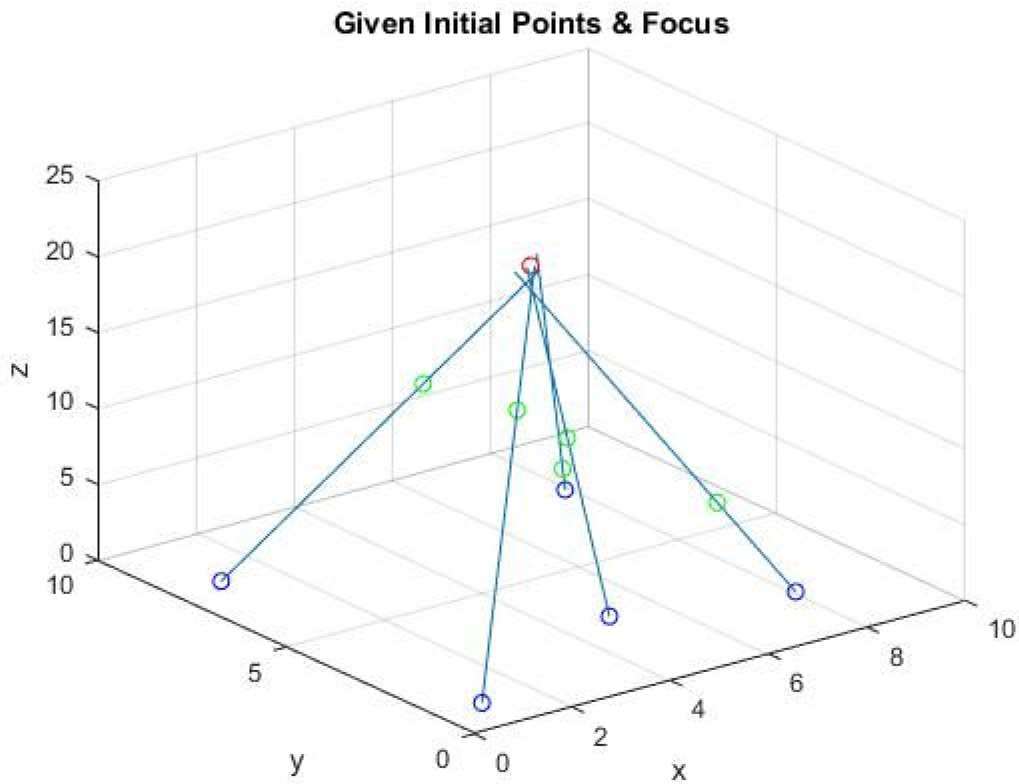
Problem #1: Least-Squares Crossing Lines

Problem

We are given a set of data that are on the XY plane and a matching set of data in space. The spatial data also happen to lie on a plane, which is tilted and offset from the XY plane. The two data sets represent a common configuration: the output of image-processing where the spatial data have been sensed as the planar data using an X-ray imaging device. For simplicity, model the X-rays as coming from a point source. The data given do not perfectly cross. Estimate the focus as the point that minimizes line error.

Algorithm

To estimate the focus that minimizes the line error, I had to do some calculations based on the notes provided by Dr. Randy E. Ellis. First of all, I had to find the unit direction vector by normalizing the difference between SpacialPoints and PlanarPoints. The left and right sides of the equation were found by subtracting the direction vector multiplied by its transpose from the identity matrix, and adding it to the sums. The focus was found by using the left hand side and right hand side of the equation and solving. The focus that was generated is [5.0844; 5.1362; 20.5131]. The error was measured as the root mean square distance from the focus point to each intersecting line. The RMS error obtained was 0.2970. To find the residual for each line I solved for zj, which is Dj times the focus minus Dj times rj. Here, rj is the jth data point in the planar data matrix, P, and the focus is the least squares focal point. To find the RMS error I found the dot product of the residual with itself, which produced a scalar. The total sum was divided by m to find the average and finally square rooted to produce the RMS error. The data was plotted (below), the blue dots are the data in space, green dots is the data on the XY plane, red dot is the focus point, and the blue lines are the lines of each pair of points in relation to the focus. I observed that the lines are slightly off so they do not intersect the focus point. However, since the RMS error was relatively small, the lines are close to intersecting the focus.


Given Initial Points & Focus
z
25
20
15
10
5
0
y
10
5
0
x
0
2
4
6
8
10

Problem #2: Least-Squares Planar Registration

Problem

We are given corresponding sets of XY points. These points are rotated, translated, and may have “noise” added to simulate a real-world situation. Recover the least-squares rotation matrix and translation vector that takes each vector $\vec{p_j}$ to its corresponding vector $\vec{q_j}$by the planar displacement $\vec{q_j} = \hat{R}\vec{p_j} + \hat{t}$ Where $\hat{R}$ is the estimated rotation matrix and $\hat{t}$ is the estimated translation vector

Algorithm

I used the algorithm provided by Dr. Randy E. Ellis in the notes to estimate the rotation matrix and translation vector that transformed the the data in P into the the data in Q. First, I found the rotation matrix by finding the means (Pmean and Qmean) and zero means (A and B) of P and Q. The zero means, A and B, were converted into complex numbers, rowA and rowB, using the x as the real part and y times i as the imaginary part. The next step was to find r phi by using rowB and multiplying it by rowA transpose. I used all of this information to calculate the final rotation matrix. I found the translation vector by subtracting the rotation matrix multiplied by Pmean from Qmean. This process produced the following results:

1. aP.dat and aQ.dat
   Estimated rotation Matrix: R = [0.5183   -0.8552;   0.8552   0.5183]
   Estimated translation vector: t =[-1.6111; -5.4188]
   RMS error: RMSerror = 0.1621

2. bP.dat and bQ.dat
   Estimated rotation Matrix: R = [-0.7105   0.7037; -0.7037   -0.7105]
   Estimated translation vector: t = [-3.1415; 1.6180]
   RMS error: RMSerror = 0.1513

3. cP.dat and cQ.dat
   Estimated rotation Matrix: R = [-0.7034   0.7108; -0.7108   -0.7034]
   Estimated translation vector: t = [-3.3106; 1.5086]
   RMS error: RMSerror = 0.6307

4. dP.dat and dQ.dat
   Estimated rotation Matrix: R = [0   NaN; 0   NaN]
   Estimated translation vector: t = [NaN; NaN]
   RMS error: RMSerror = NaN

The translation vector for bP and bQ contained the familiar constants -pi and phi. cP and cQ data represented an ellipse, which consequently had a higher RMS error. This is due to the outlier data that may have significantly impacted the mean, which is used to calculate the RMS error. dP and dQ was much harder to solve because r phi is 0, and so the norm function will produce NaN when it tries to compute 0/0. As expected, this resulted in the translation vector to contain only 0s. This means that there is no planar displacement, but we know that there is! These strange results are because the dP contains two points that are the same, but these are not plotted to the same values in dQ.