Machine Learning Summer School: Day 1

David Carlson

High-Level Introduction to Deep Learning

Part I

What is happening today?

- This class is focused on “Deep Learning:”
  - What it is
  - Some of its many applications
  - How to *build your own* deep networks
- These algorithms *learn* from data to automate tasks and make predictions
- Today, you will learn about what we mean by “deep learning” and *build* your first deep network

Deep Learning is state-of-the-art for *many* applications

- Deep Learning is **not new**—many of the techniques go back decades
- Recent resurgence due to *amazing* performance on benchmark tasks
- One key task was the ImageNet Challenge
  - Want to recognize what is in an image (1 of 1000 categories)
  - Have ~1 million example images
  - Very relevant for things such as image search
- Example images are shown on the right, with predicted categories beneath each image

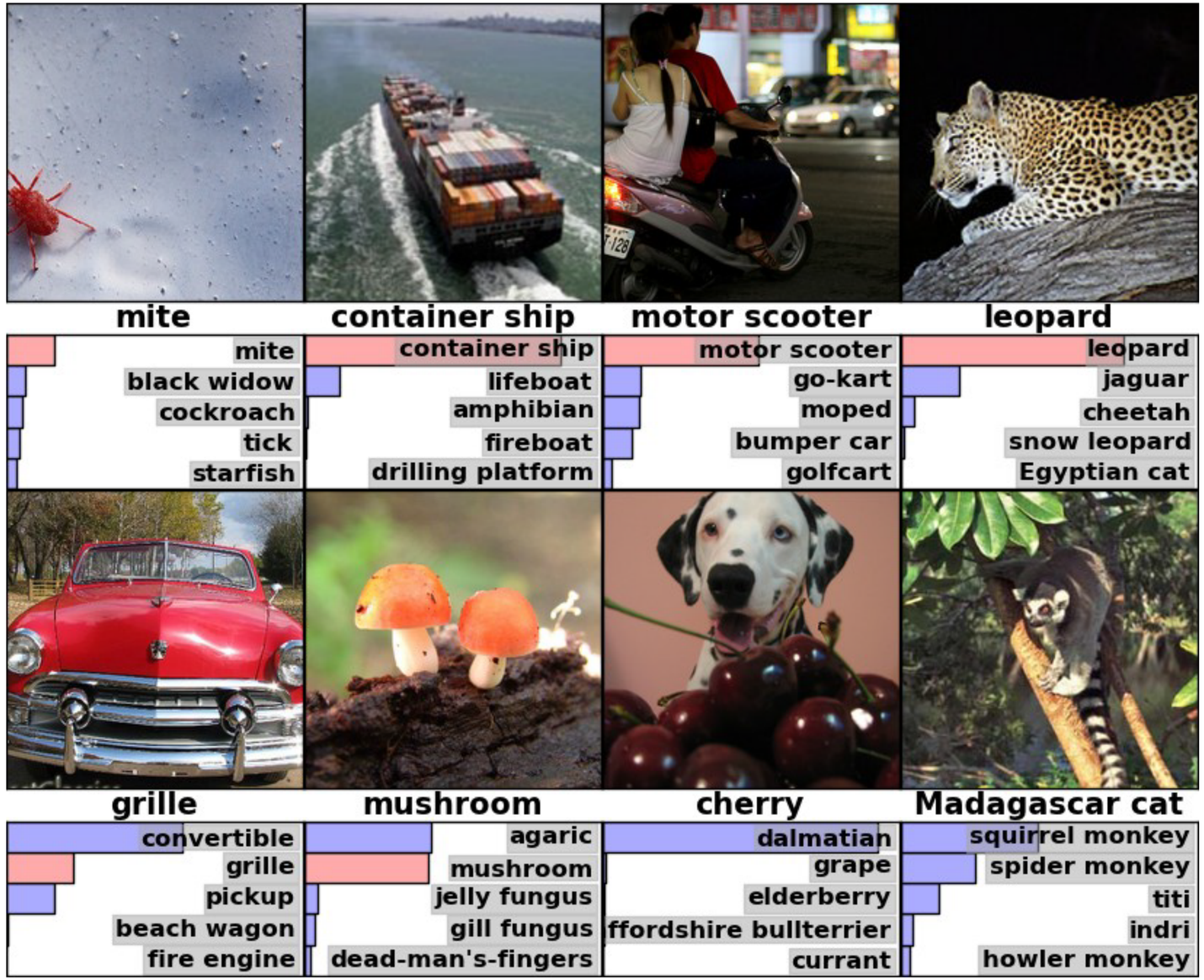
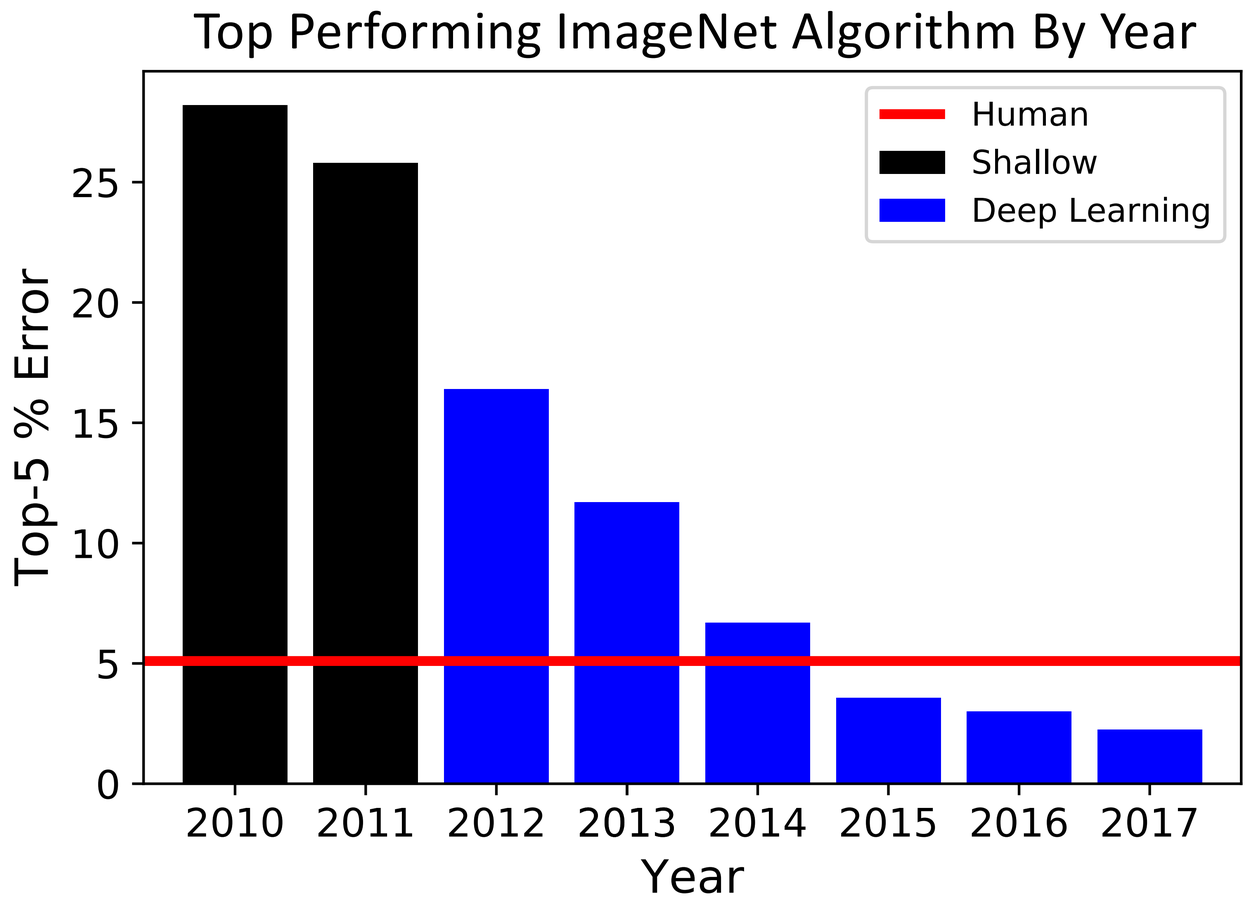


Figure from Krizhevsky et al 2012.

Deep Learning can surpass human performance

- For ImageNet:
  - Deep Learning was a *huge* jump forward
  - State-of-the-art systems **significantly outperform humans** on the same task
- These use "Convolutional Neural Networks," which you will learn about tomorrow



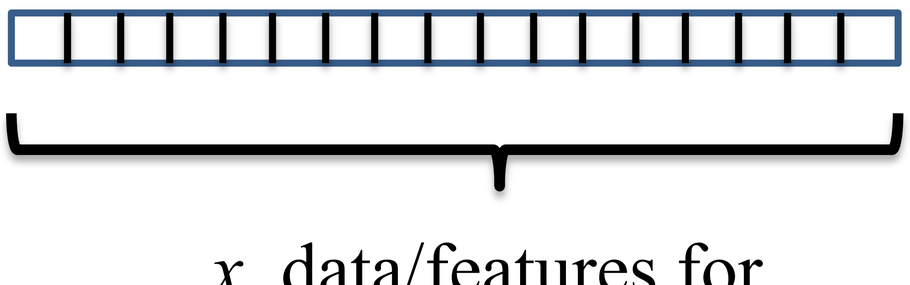
Deep Learning beats human performance in many tasks

- Famously, Google DeepMind trained “AlphaGo” to beat the world champion Go player (a complex game)
- AlphaGo’s largely works by Deep Learning techniques (learned by repeatedly playing the game)
- Many other examples:
  - Voice Recognition
  - Object Detection
  - Text Translation
  - Etc
- So what is Deep Learning?

We need to understand “shallow” learning before “deep” learning

A “SHALLOW” NETWORK

Learning a Predictive Model Based on Labeled Data

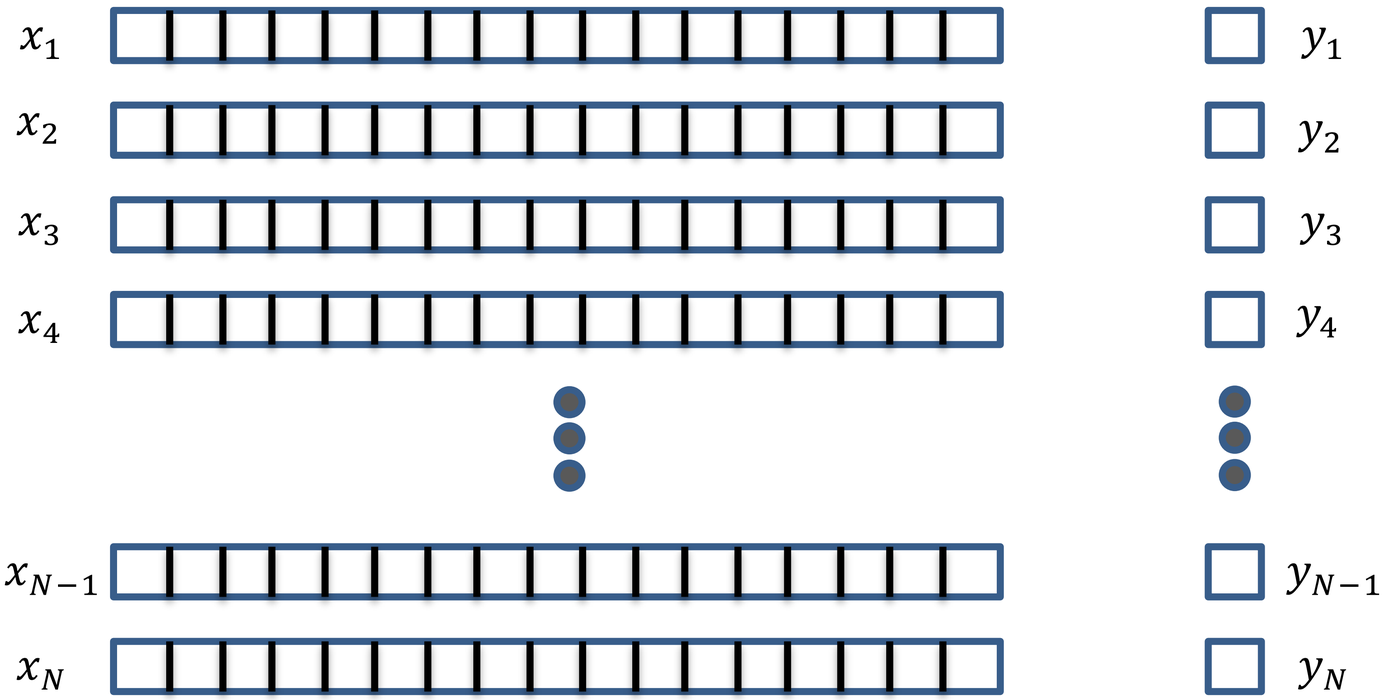


x, data/features for a subject

y, associated label 0/1

End goal: *predict* y from x

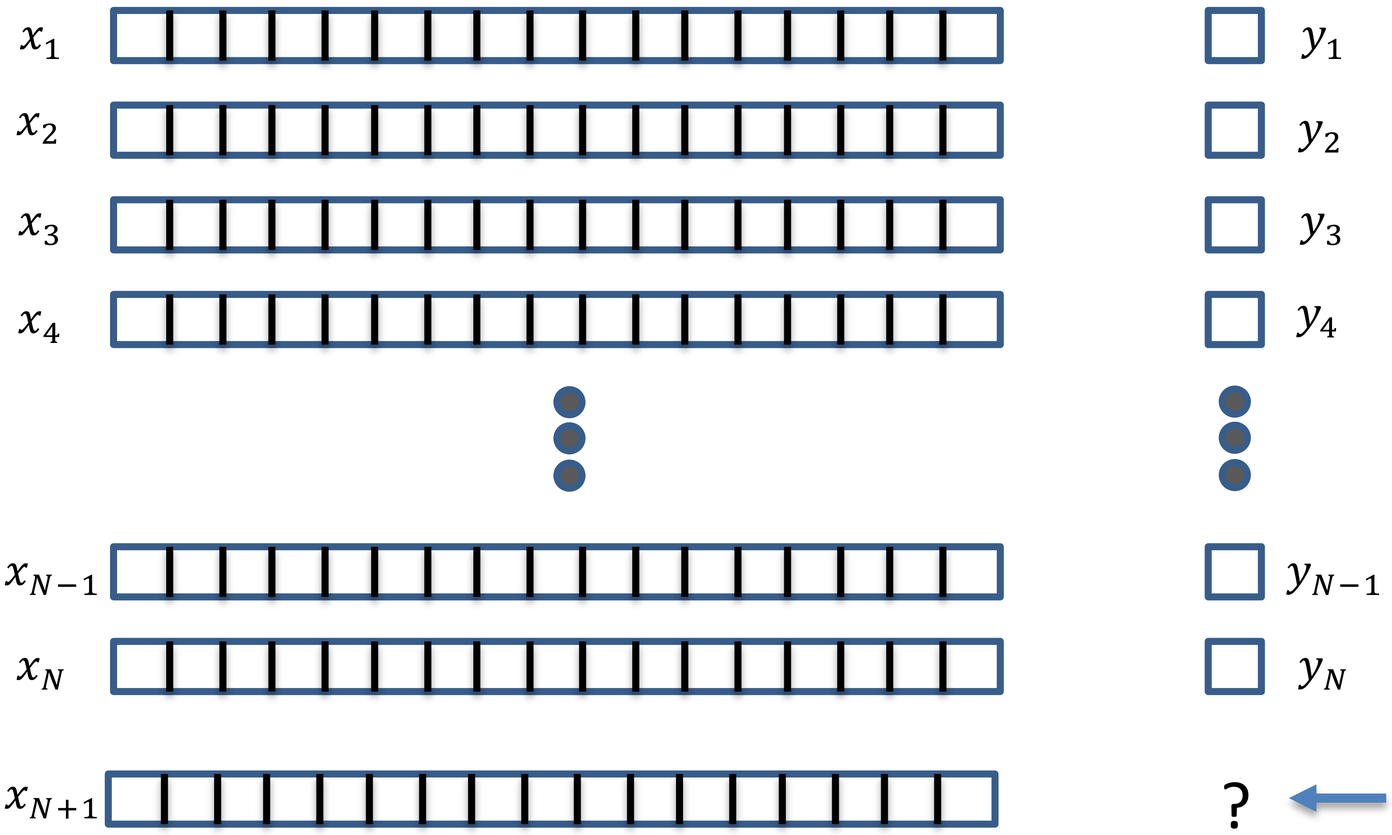
Training Set (Historical Data)



Start by limiting y to a binary outcome:

- False/True
- 0/1

Making Predictions



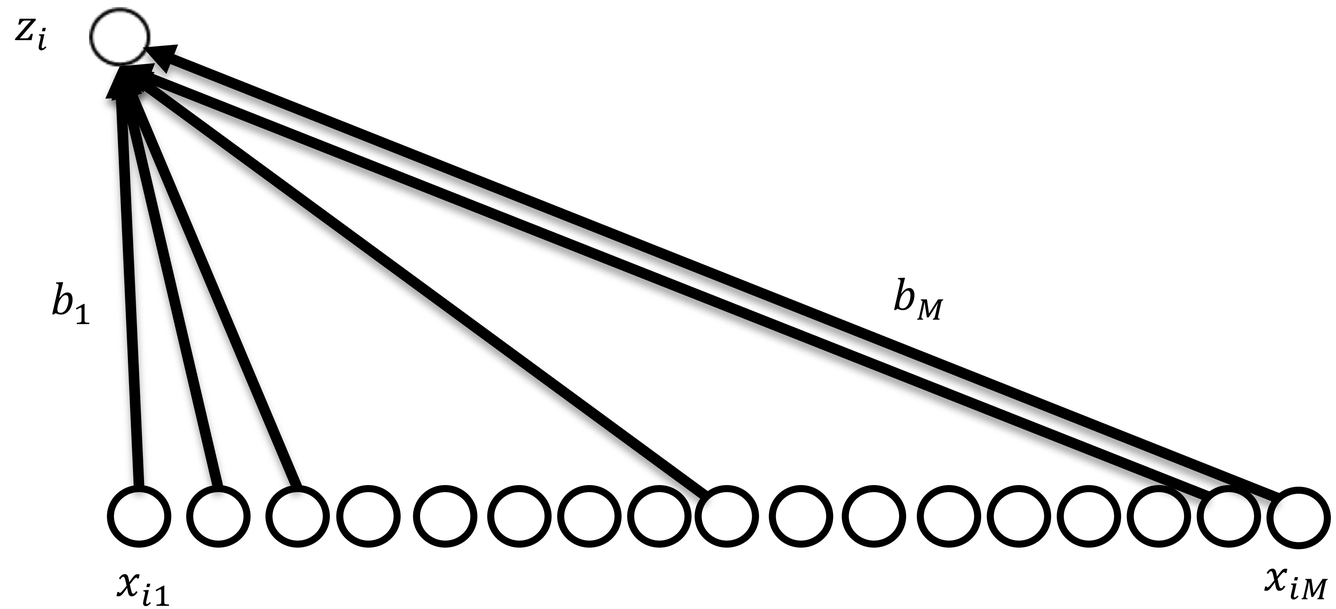
New Data

Start by limiting y to a binary outcome:

- False/True
- 0/1

Want to learn how to *predict* outcome

Linear Predictive Model



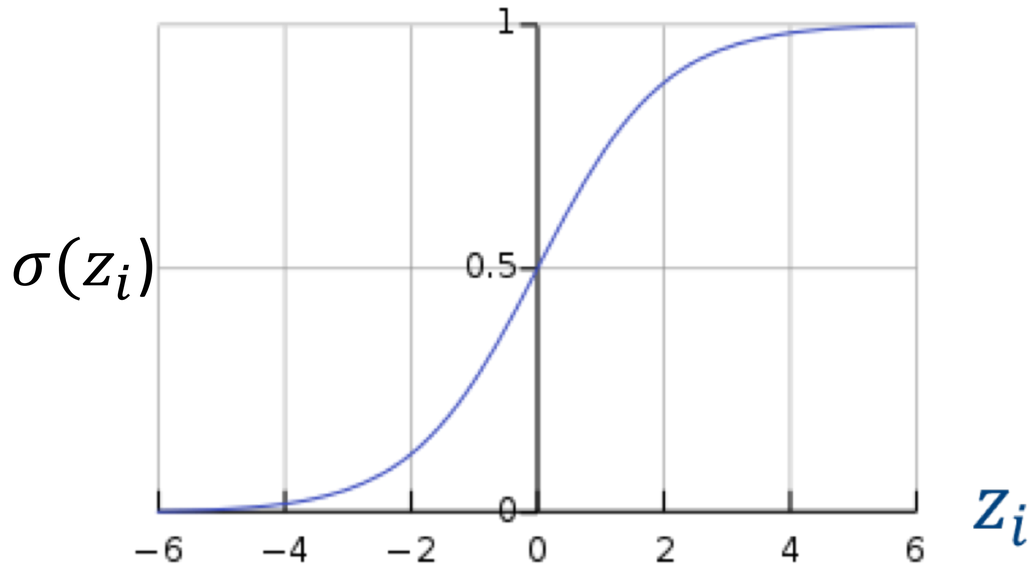
$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM})$$

Convert to a Probability

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

Extra Constant

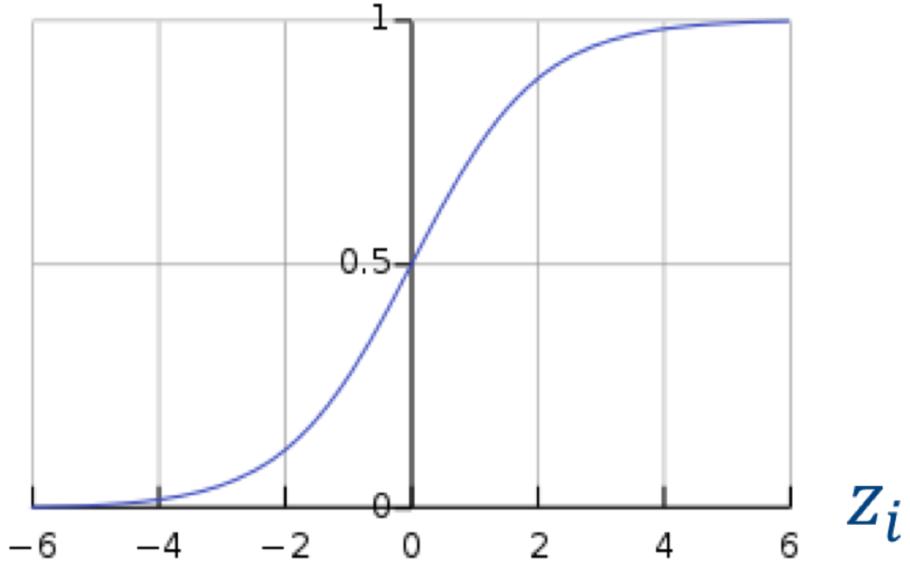
$$p(y_i = 1 | x_i) = \sigma(z_i)$$



Convert to a Probability

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

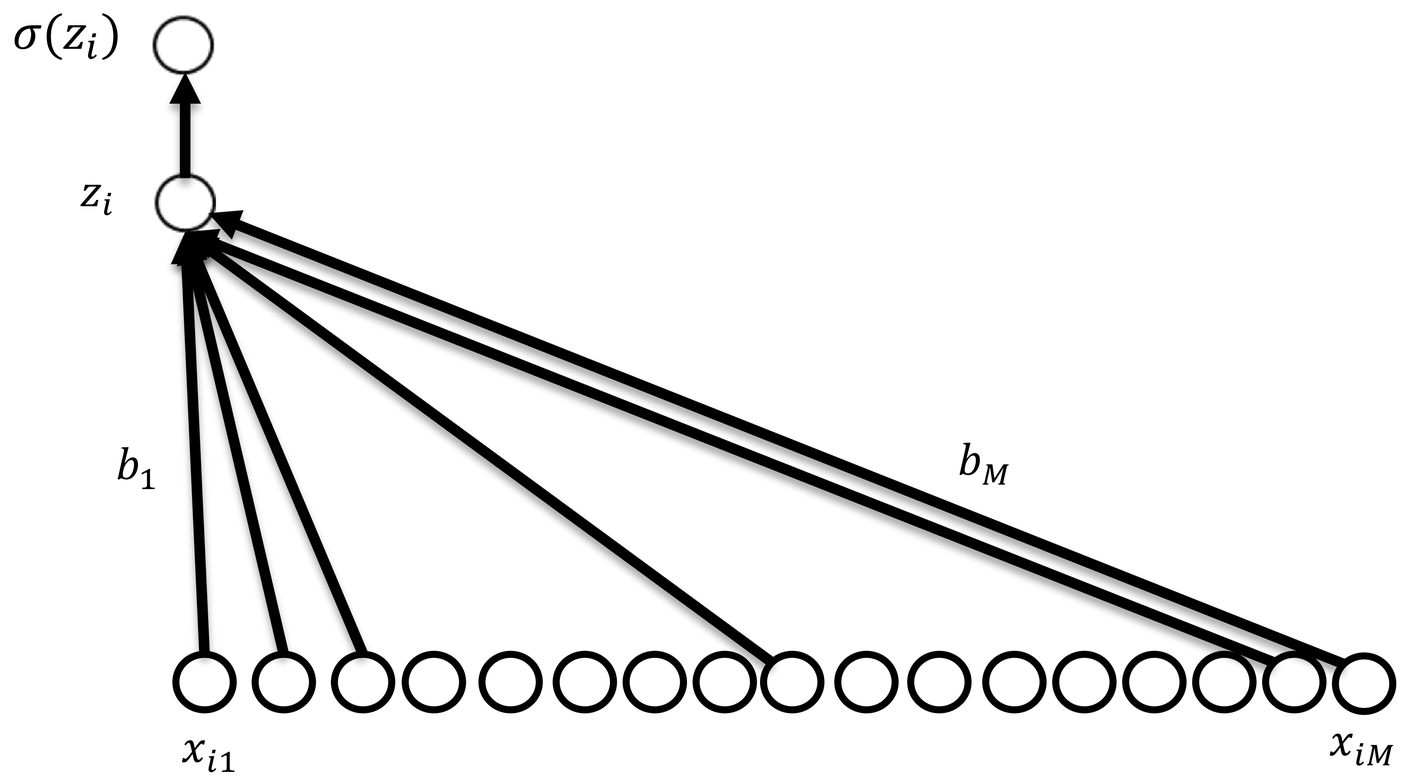
$$p(y_i = 1|x_i) = \sigma(z_i) = \frac{\exp(z_i)}{1+\exp(z_i)}$$



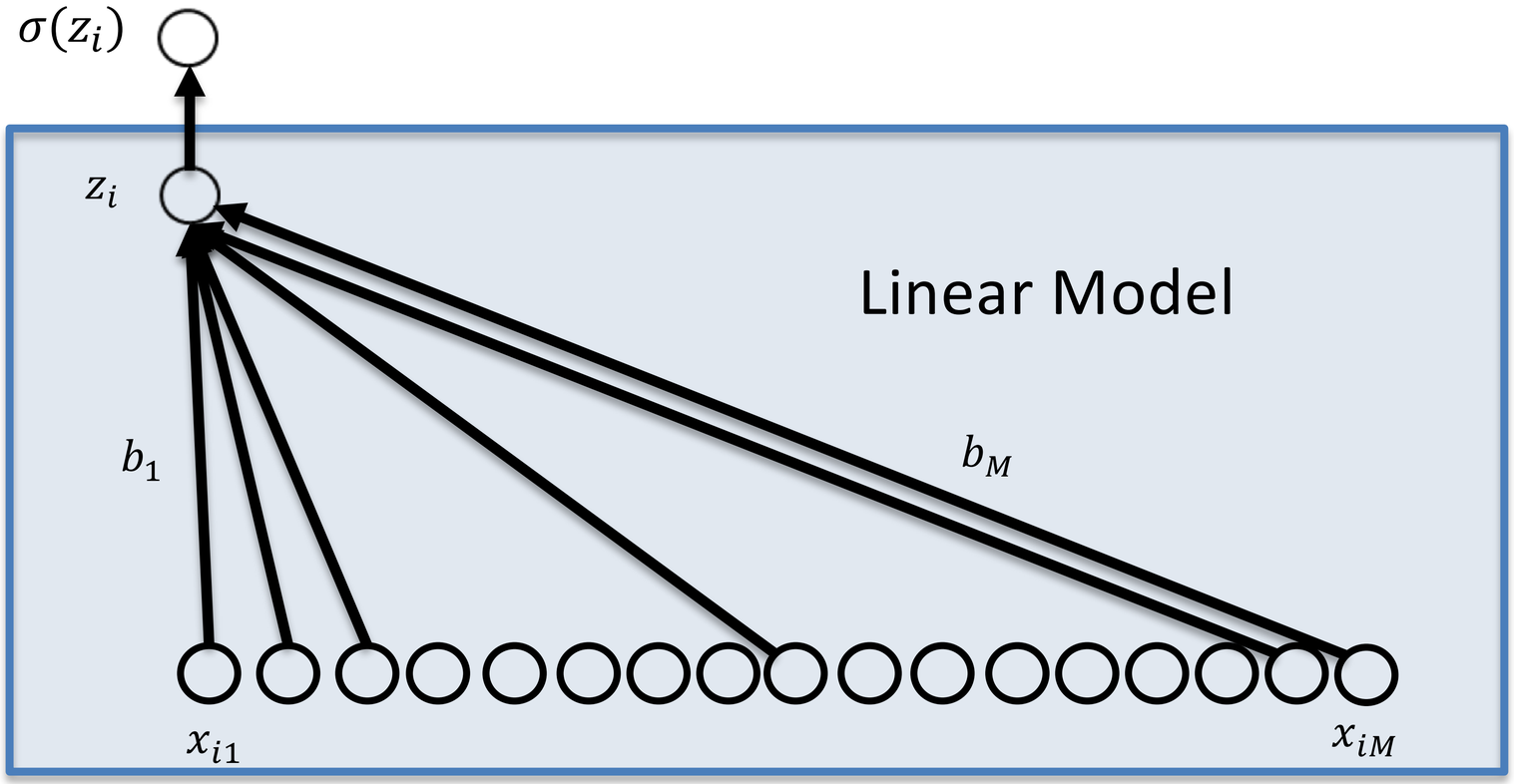
- Large and positive z_i indicates that event $y_i = 1$ is likely

- Large and negative z_i indicates that event $y_i = 0$ is likely

Logistic Regression

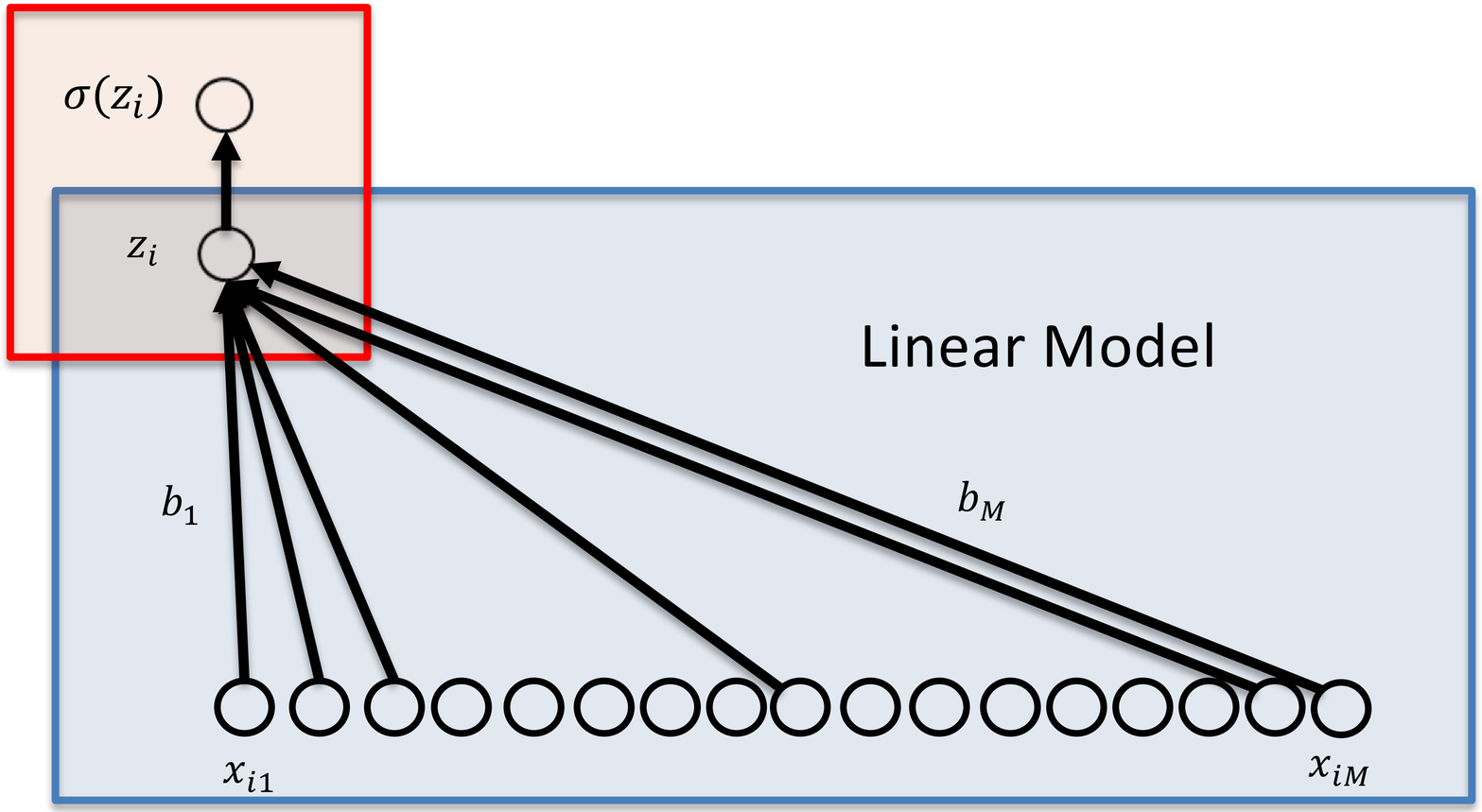


Logistic Regression



Logistic Regression

Convert to Probability

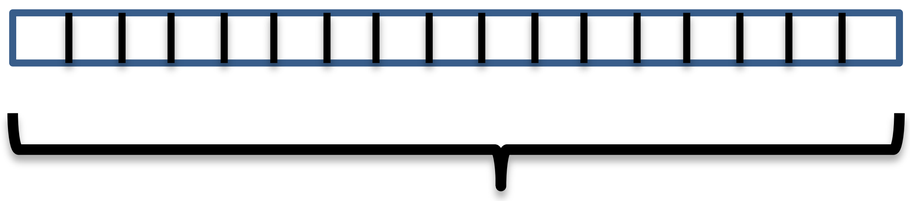


What do the parameters and model mean?

AN EXAMPLE

Example

- Outcome:
  - $y_i = 1$, it rains on day i;
  - $y_i = 0$, it does not rain on day i
- Features:
  - On day i what is the $\{cloud\ cover, humidity, temperature, air\ pressure, \ldots\}$



x_i, features for day i

y_i, did it rain on day i

Example

- Outcome: $y_i = 1$, it rains on day i; $y_i = 0$, it does not rain on day i
- Features: On day i what is the $\{1: cloud\ cover, 2: humidity, 3: temperature, \ldots\}$

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

Cloud Cover Humidity

- If cloud cover is positively related to rainfall, b_1 should be positive

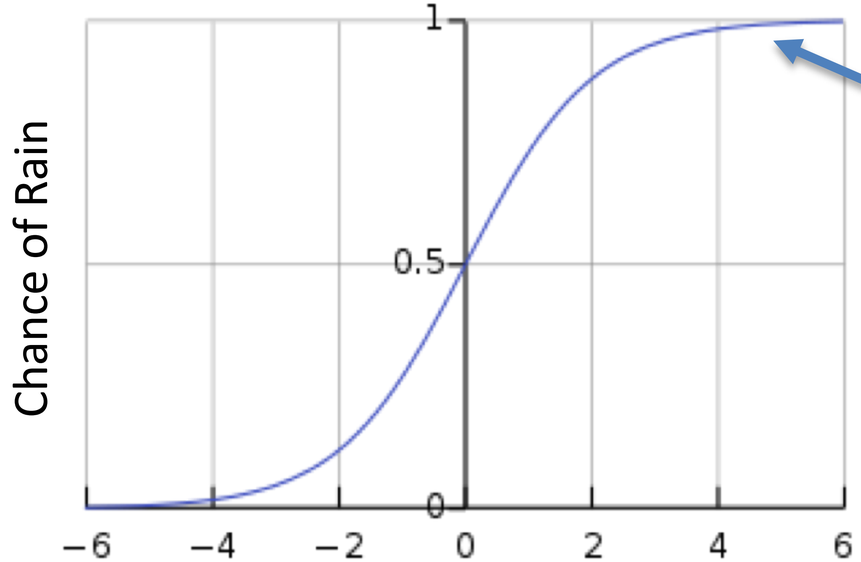
Impact on the Sigmoid Function

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM}) + b_0$$

Cloud Cover

Humidity

$$p(y_i = 1 | x_i) = \sigma(z_i)$$

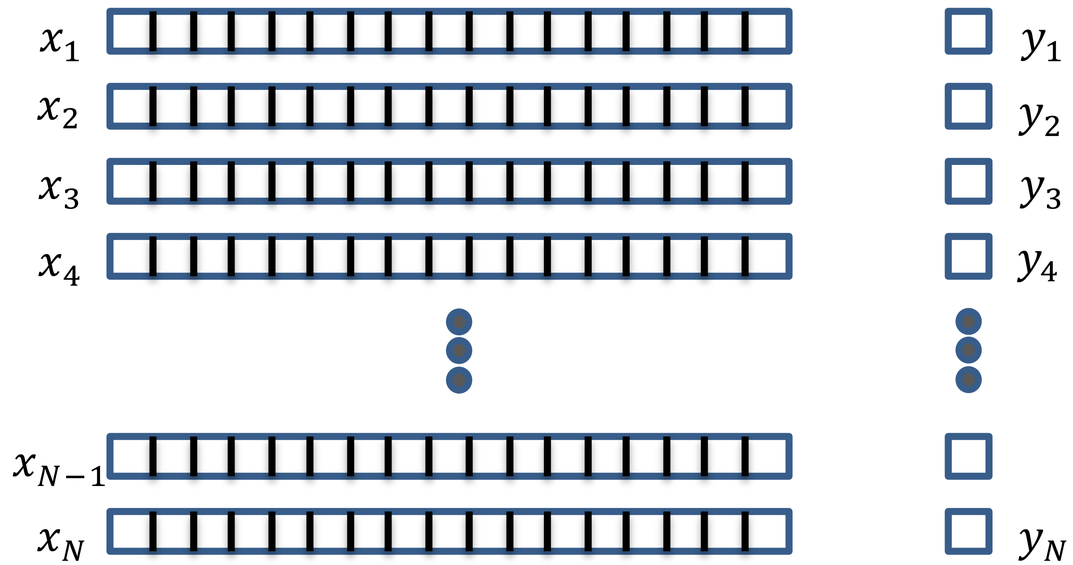


As the value z_i increases, the chance of rain increases

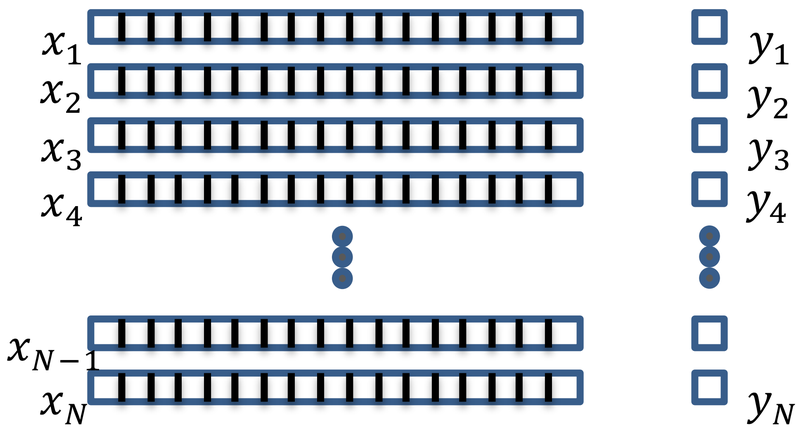
z_i

Building the Training Set

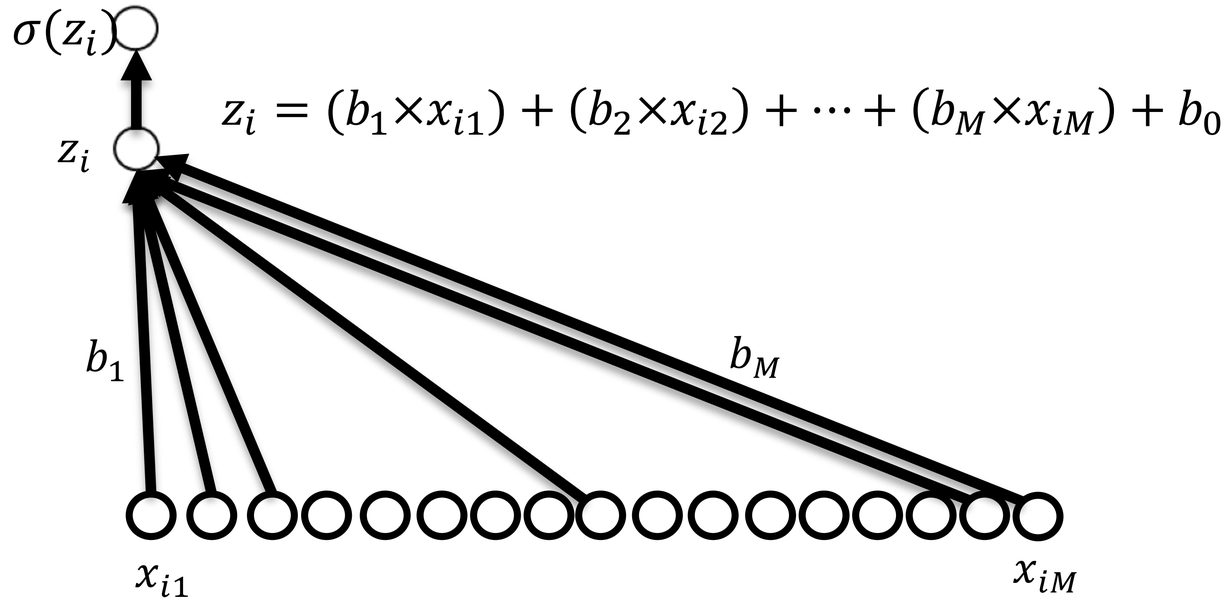
- Need to learn the parameters
- Requires *training data*
- Record data from N days
  - Capture features: $\{cloud\ cover, humidity, temperature, \ldots\}$
  - Did it rain?



Learning Model Parameters



Training Set

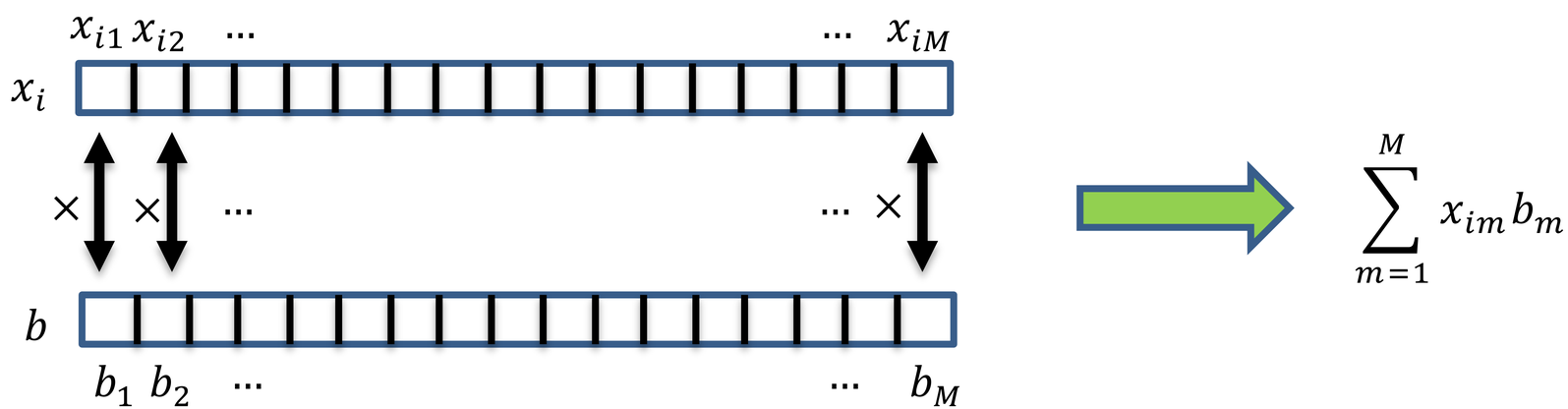


Logistic Regression Model (or "Network")

Learned Parameters $(b_0, b_1, \ldots b_N)$

Interpretation of Logistic Regression

$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM})$$



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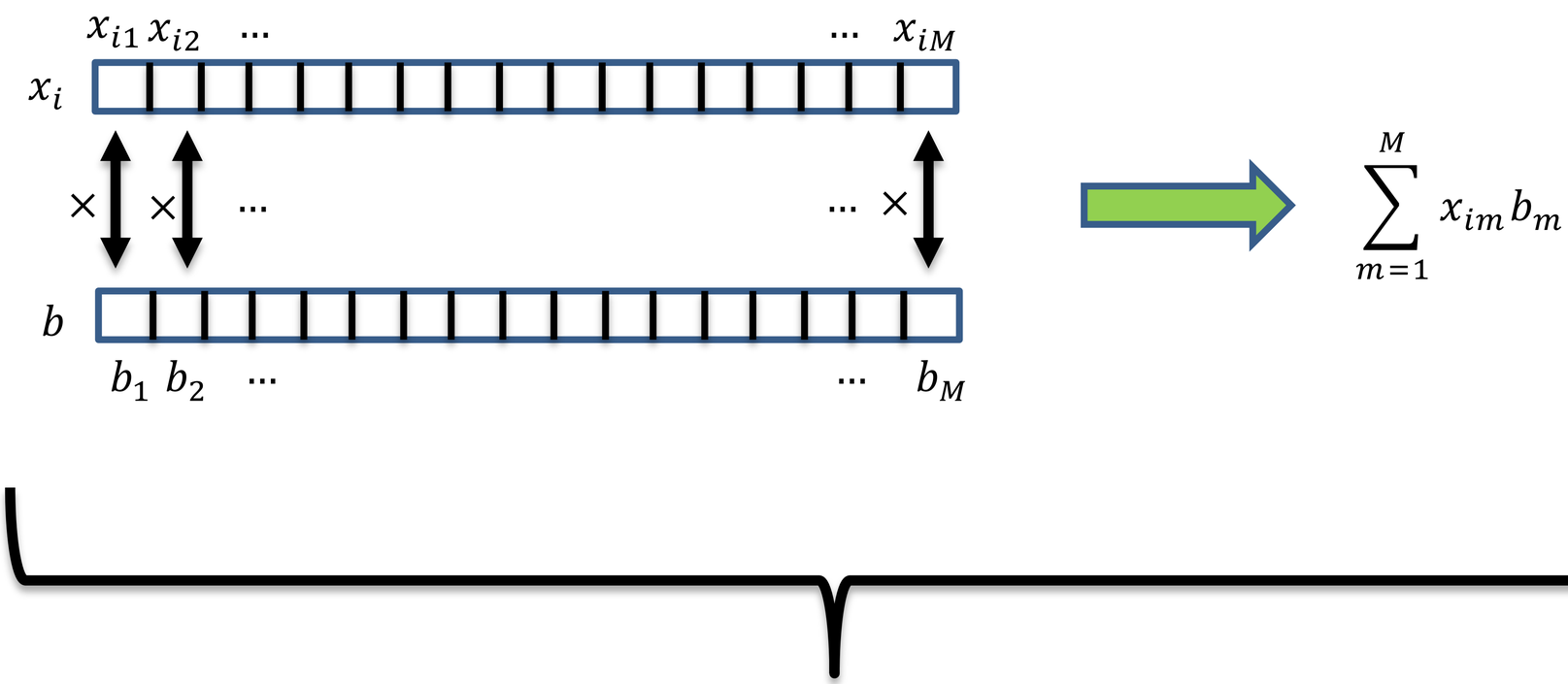


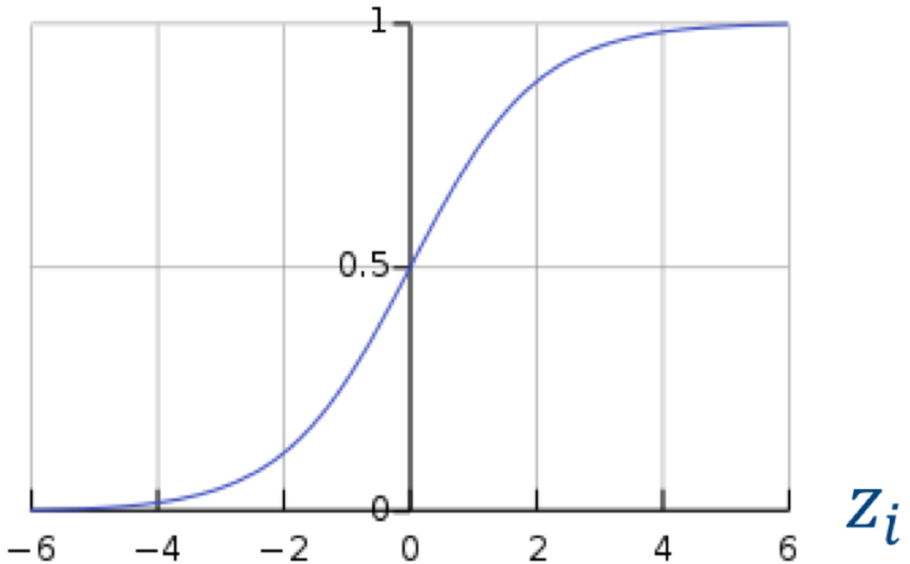
Compact Notation: $x_i \odot b$ (or "inner product")

Interpretation of Logistic Regression

$$z_i = b_0 + (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM})$$

$$= b_0 + x_i \odot b$$

$$p(y_i = 1|x_i) = \sigma(z_i)$$



Interpretation of Logistic Regression

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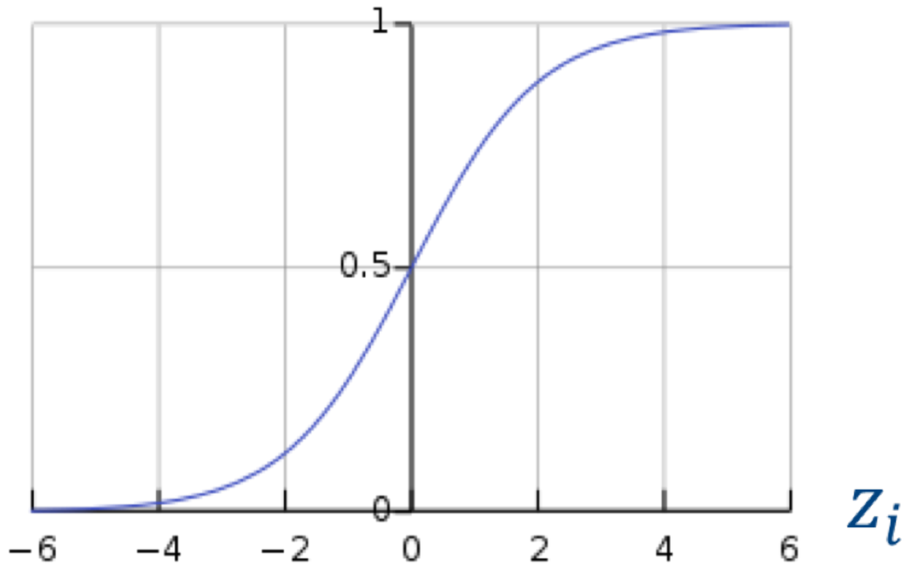


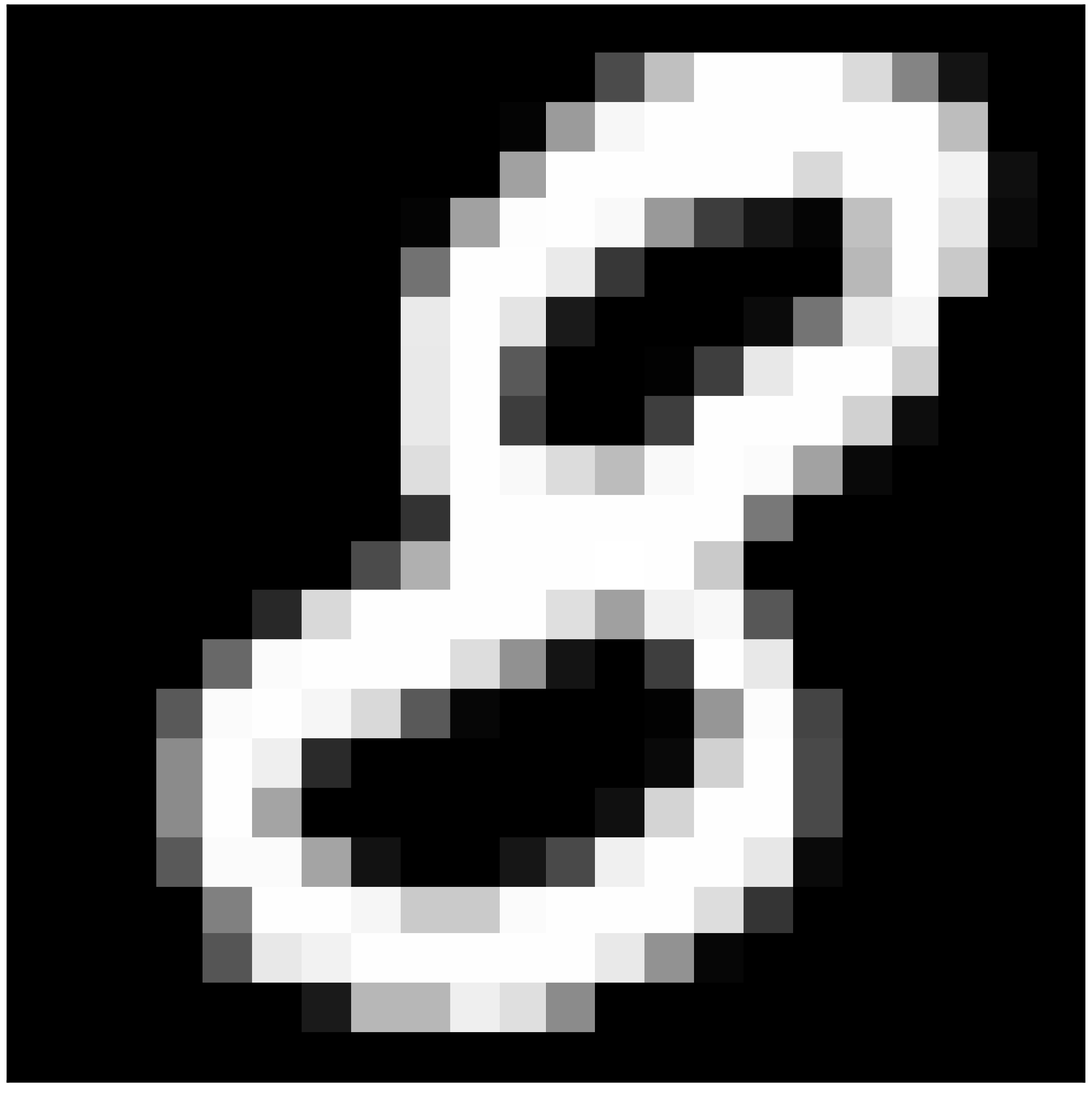
- May think of vector b as a template or filter (will visualize to make clear)
- If x_i is aligned/matched with b, then $x_i \odot b$ will be large
- The parameter b_0 is a bias to correct for class prevalences

RECOGNIZING NUMERICAL DIGITS

The MNIST Dataset

- The Modified National Institute of Standards and Technology (MNIST) contains pictures of handwritten digits (0,1,2,…)
- Want to be able to tell what digit each image is (e.g. optical character recognition)

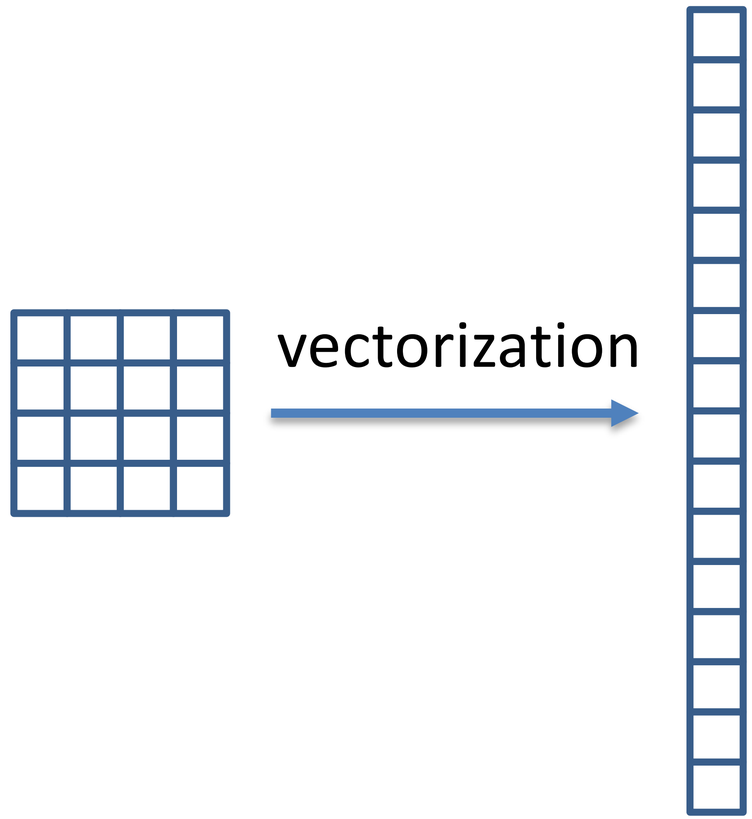
Images are Encoded as Numbers



0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	118	200	223	155	155	23	0	0	0	0	0
0	0	0	0	0	0	0	0	6	124	214	253	253	253	253	254	229	213	67	0	0	0
0	0	0	0	0	0	0	43	198	253	254	253	247	175	175	236	253	253	108	0	0	0
0	0	0	0	0	0	9	212	253	253	224	58	19	0	0	16	139	247	247	71	0	0
0	0	0	0	0	0	58	254	254	186	14	0	0	0	0	0	0	139	254	118	0	0
0	0	0	0	0	0	58	253	183	8	0	0	0	0	0	0	36	222	253	118	0	0
0	0	0	0	0	0	54	250	128	2	0	0	0	6	71	192	237	253	247	71	0	0
0	0	0	0	0	0	0	213	253	50	0	18	123	198	253	254	253	247	85	0	0	0
0	0	0	0	0	0	0	69	241	227	136	200	253	253	253	254	192	34	0	0	0	0
0	0	0	0	0	0	0	0	175	254	256	254	254	254	149	59	0	0	0	0	0	0
0	0	0	0	0	0	0	101	253	253	254	253	253	253	42	0	0	0	0	0	0	0
0	0	0	0	0	8	138	247	253	243	159	196	243	253	199	0	0	0	0	0	0	0
0	0	0	0	12	183	253	253	253	50	0	0	49	253	214	0	0	0	0	0	0	0
0	0	0	0	180	254	253	213	50	2	0	0	71	253	214	0	0	0	0	0	0	0
0	0	0	0	234	217	97	10	0	0	0	23	207	254	215	0	0	0	0	0	0	0
0	0	0	174	253	156	0	0	0	0	45	215	253	253	95	0	0	0	0	0	0	0
0	0	9	210	253	163	5	19	49	130	244	253	251	137	4	0	0	0	0	0	0	0
0	0	13	229	253	254	192	253	253	253	254	253	137	0	0	0	0	0	0	0	0	0
0	0	0	160	253	254	253	253	253	253	239	132	4	0	0	0	0	0	0	0	0	0
0	0	0	18	112	194	254	254	254	163	59	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

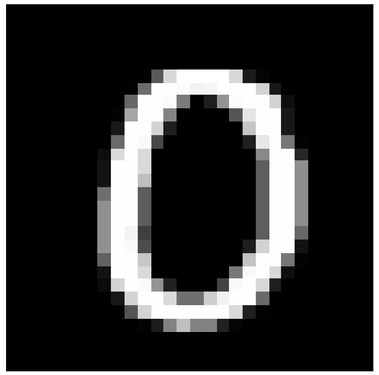
Vectorization

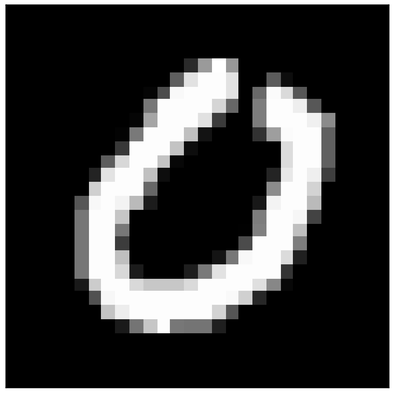
- We will start talking about deep learning *without* using the structure of the image
- We will return to this in a future lecture
- To convert an image into an unstructured set of numbers, we *vectorize it*

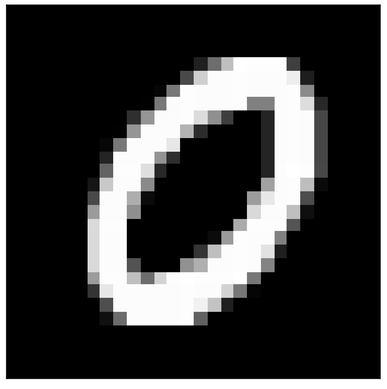


Start With The Binary Case

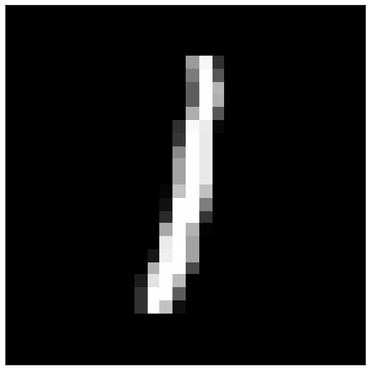
Zeros

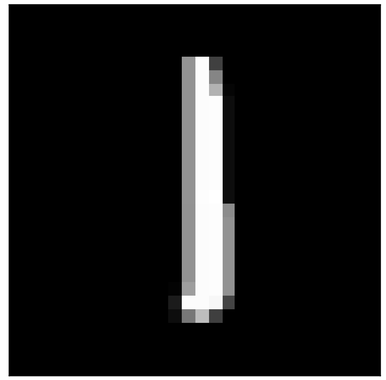


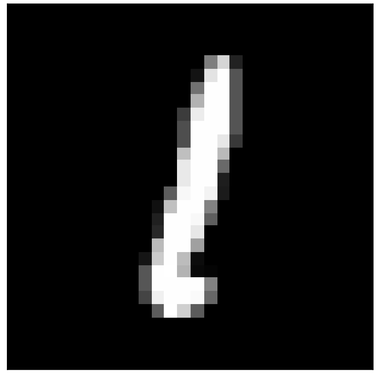




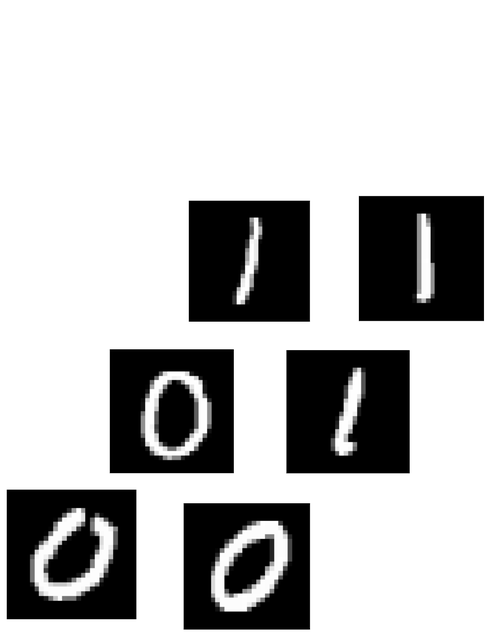
Ones



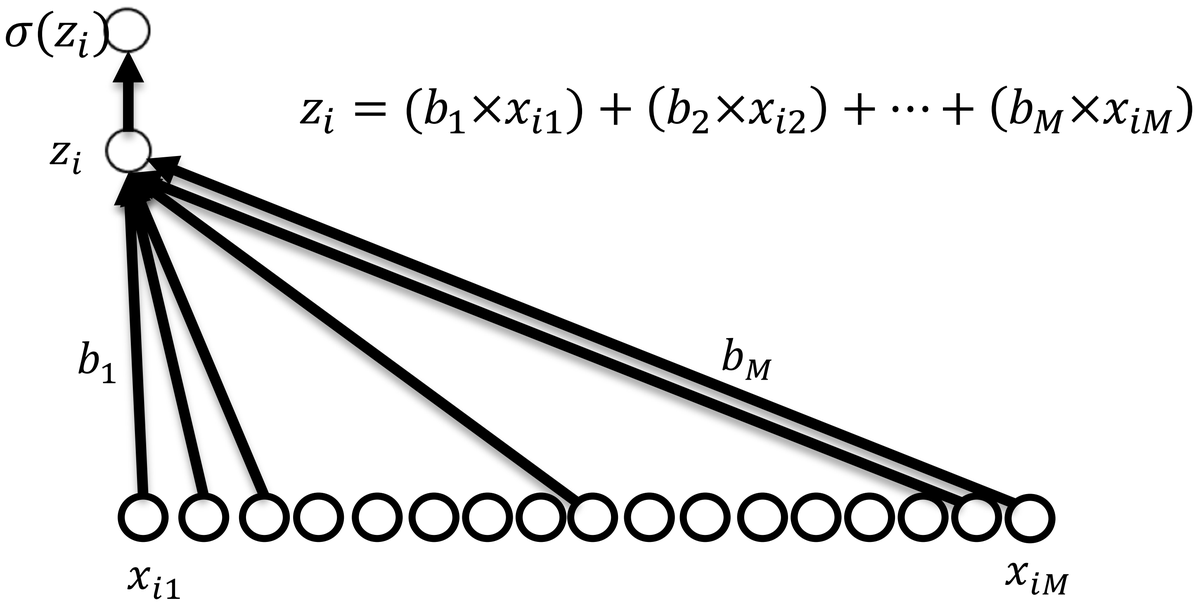




Learning on MNIST



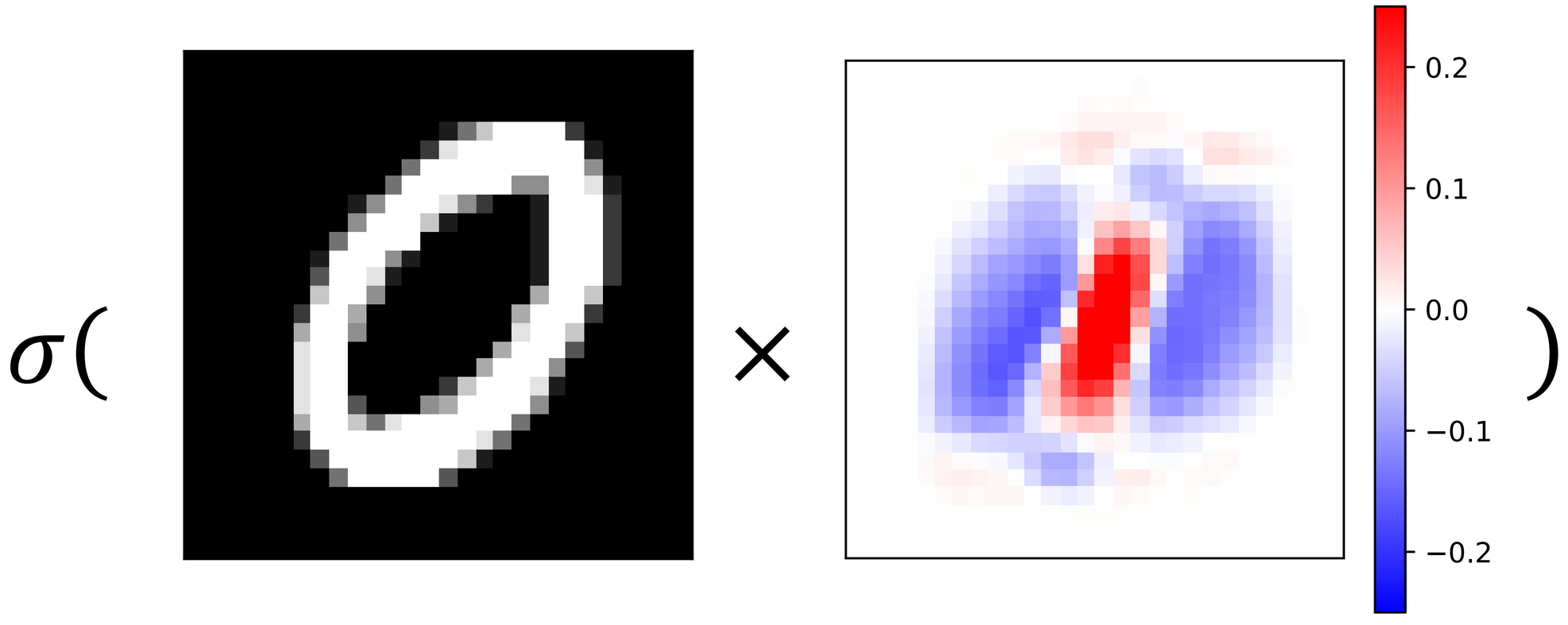
Vectorization



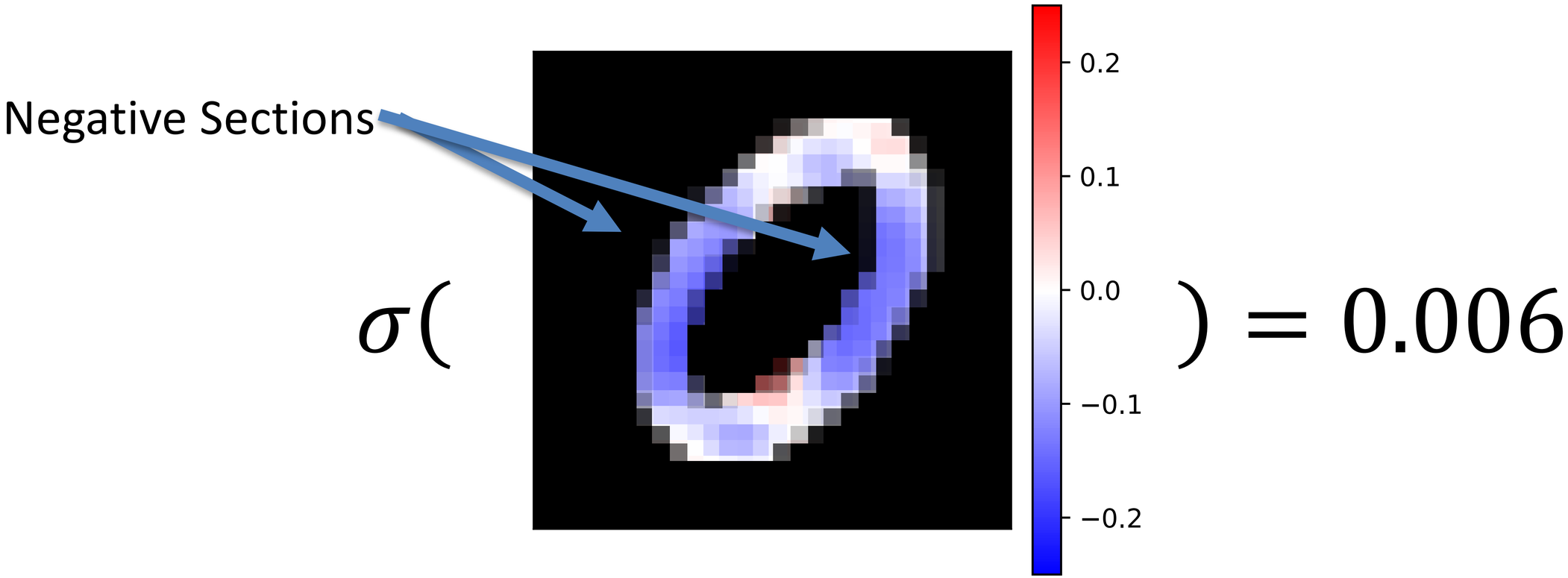
Logistic Regression Model (or “Network”)

Learned Parameters $(b_0, b_1, \ldots b_N)$

Zooming in on 0/1

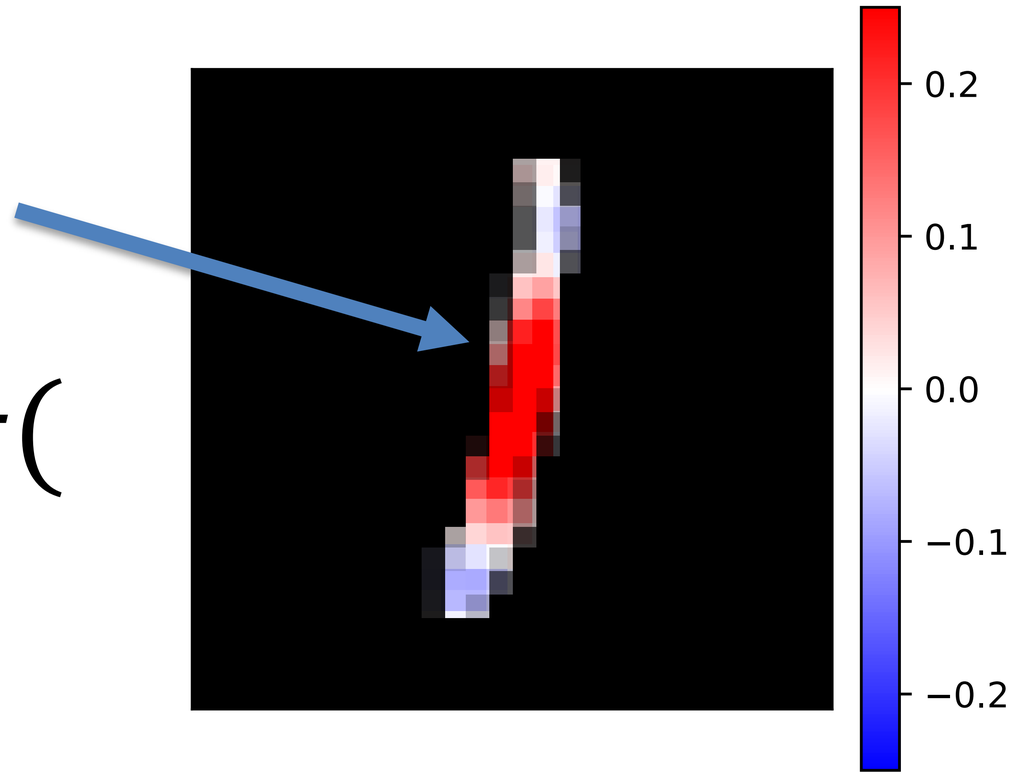


Zooming in on 0/1



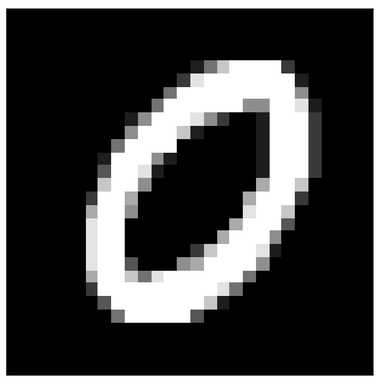
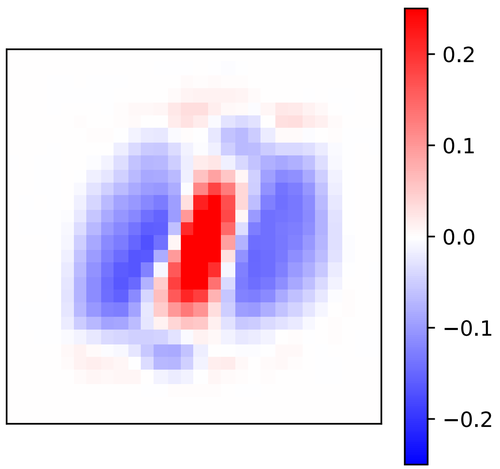
Zooming in on 0/1

Positive Section



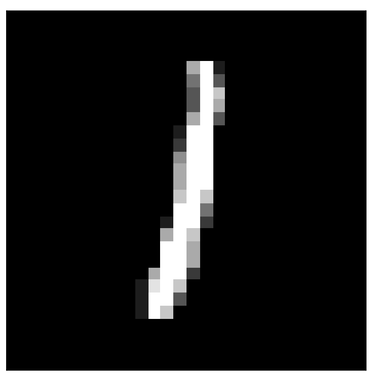
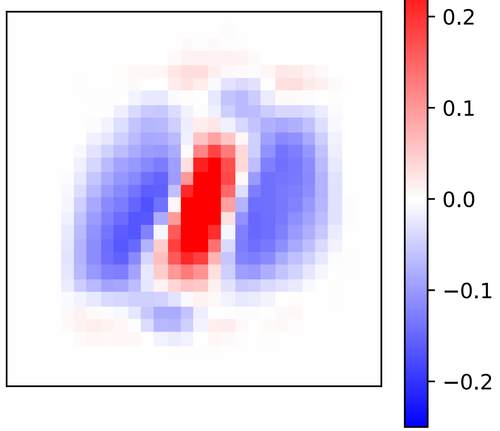
$\sigma($ $) = .991$

Learned Weights for 0/1

$\sigma($  $\times$ 

$)$ = .006

We think that this is a “zero” (.6% chance it is a “one”)

$\sigma($  $\times$ 

$)$ = .991

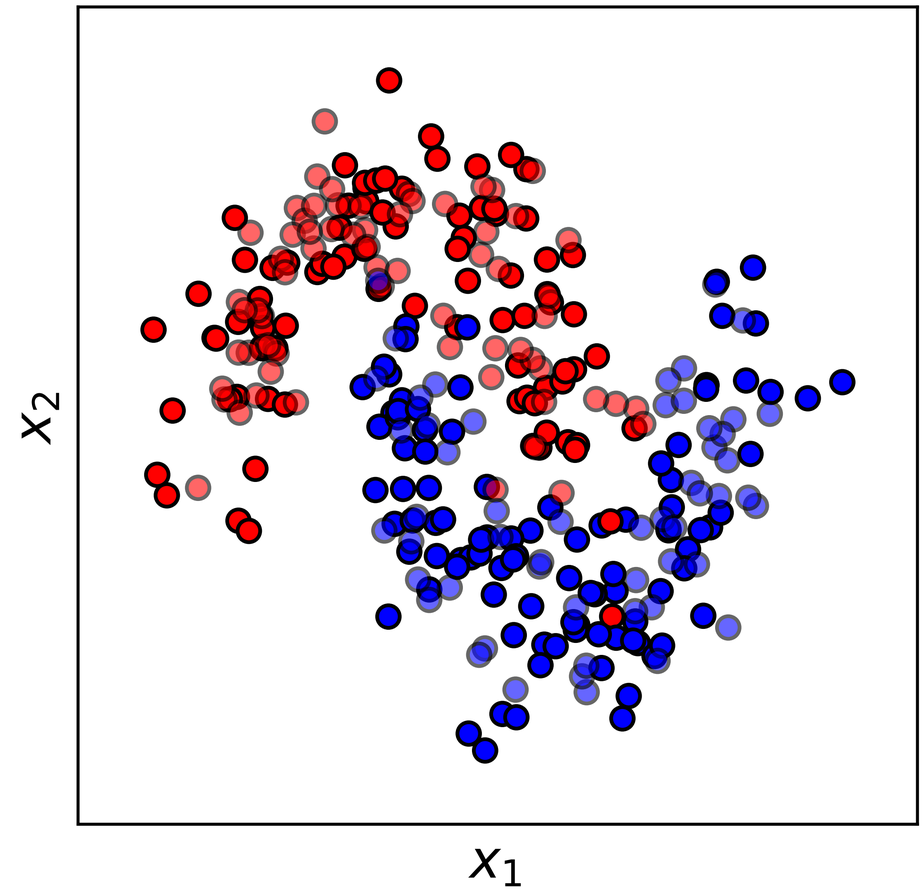
We think that this is a “one” (99.1% chance it is a “one”)

From Shallow to Deep Learning

GENERALIZING LOGISTIC REGRESSION

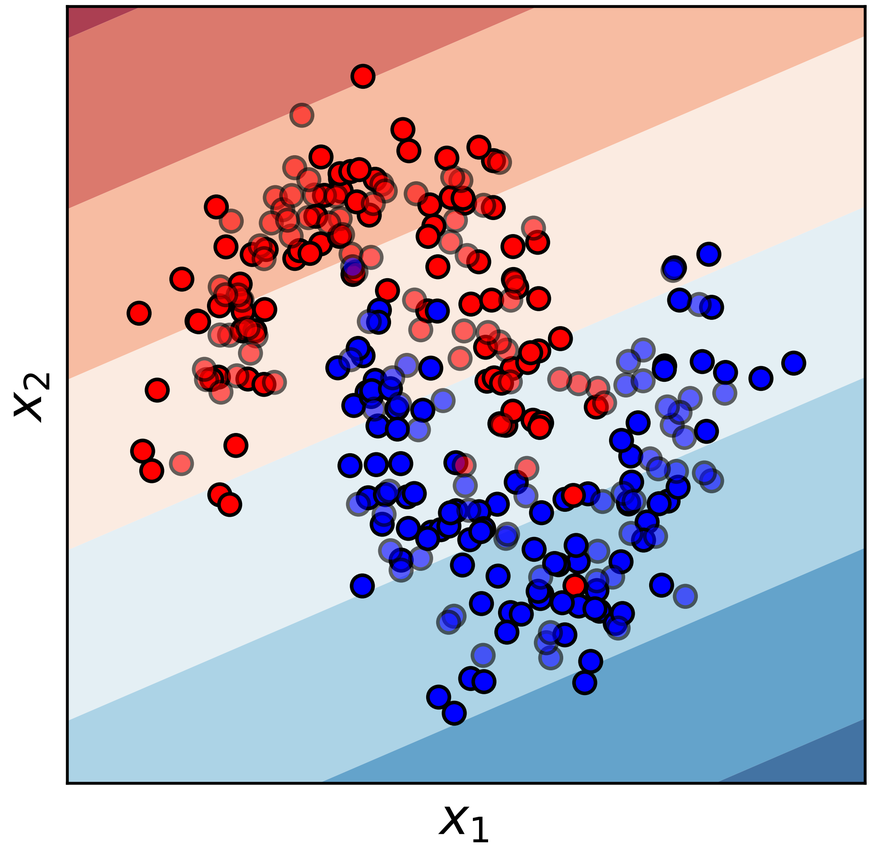
Logistic Regression is a "Linear" Classifier

- Also referred to as a generalized linear model
- Can only split data by linear trends



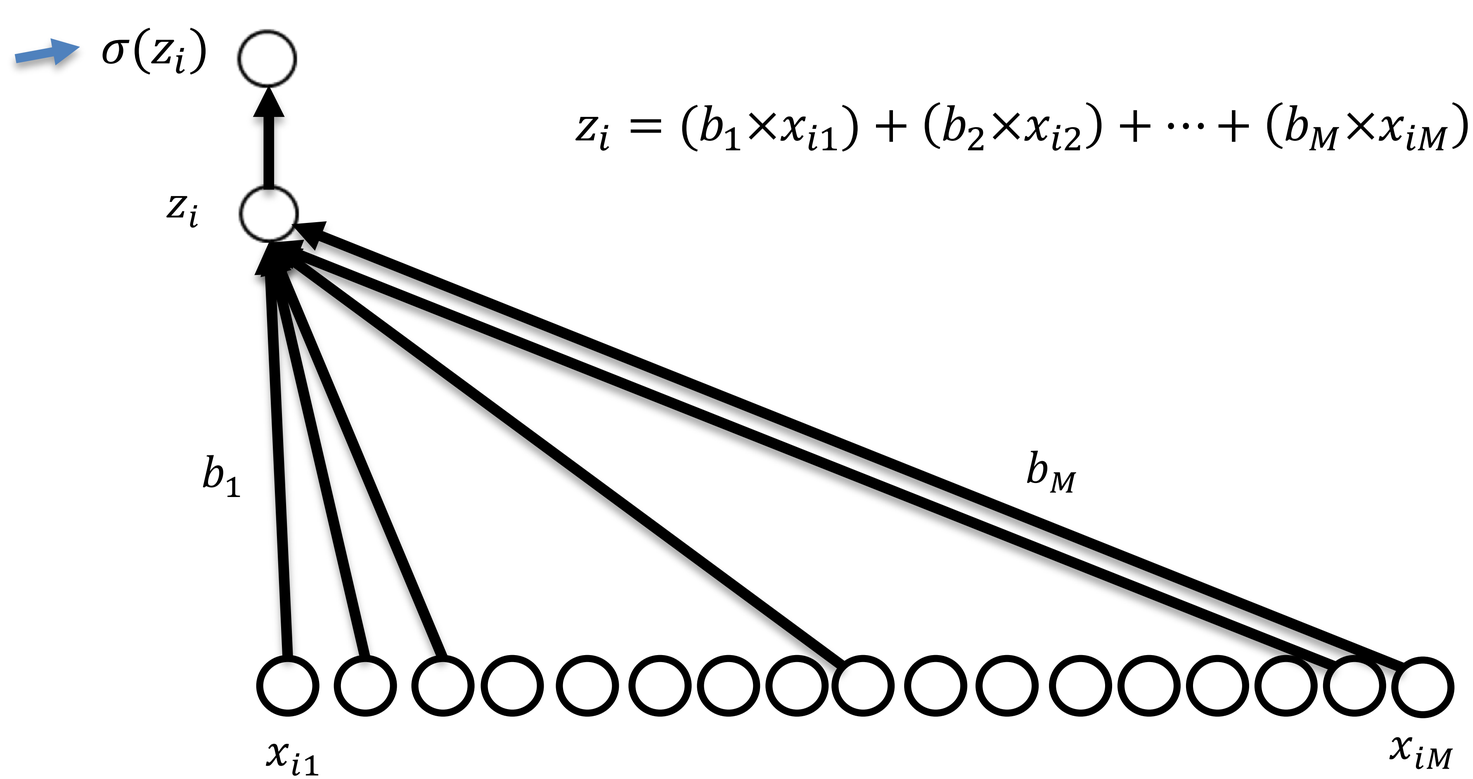
Non-linear classifier

- Linear classifiers can only represent limited relationships
- Often want to use a classifier that can handle non-linearities.
- Many different ways of creating non-linear classifiers



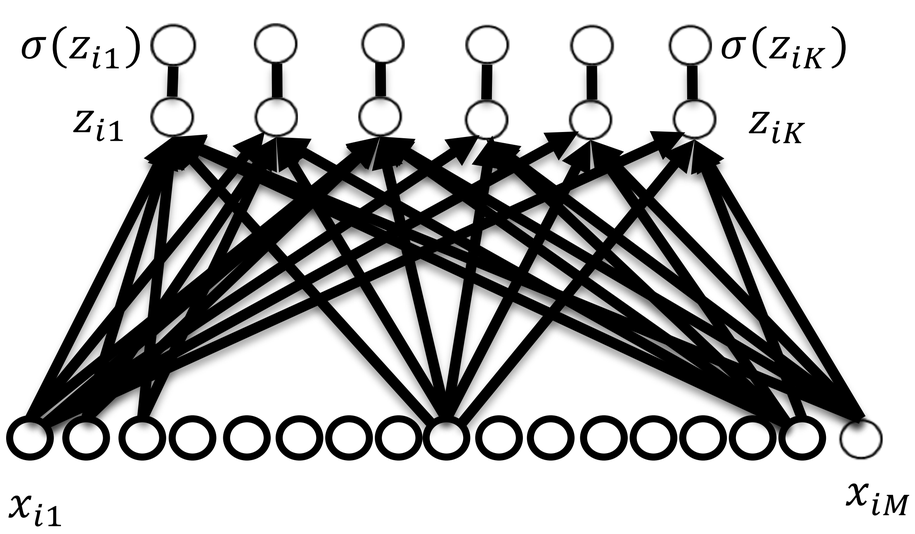
Can we modify Logistic Regression to learn complex structures?

Probability of a single outcome



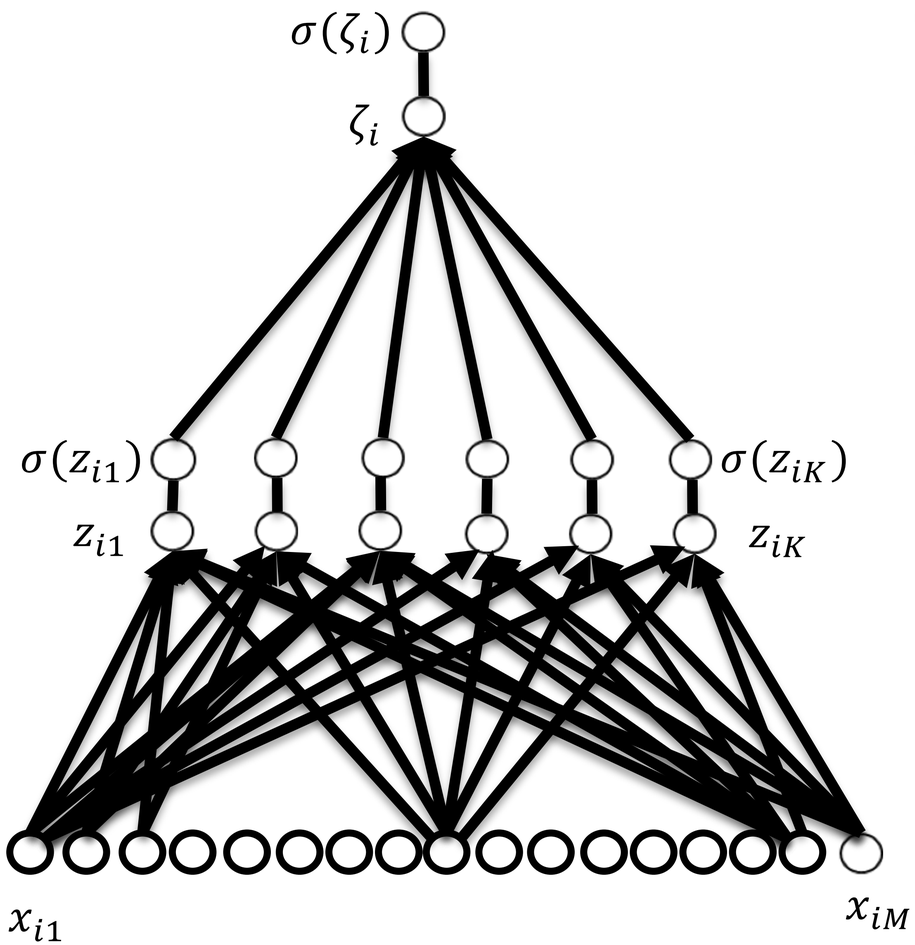
$$z_i = (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM})$$

Generalization of Logistic Regression: Learned Features



Probability of K latent processes/features

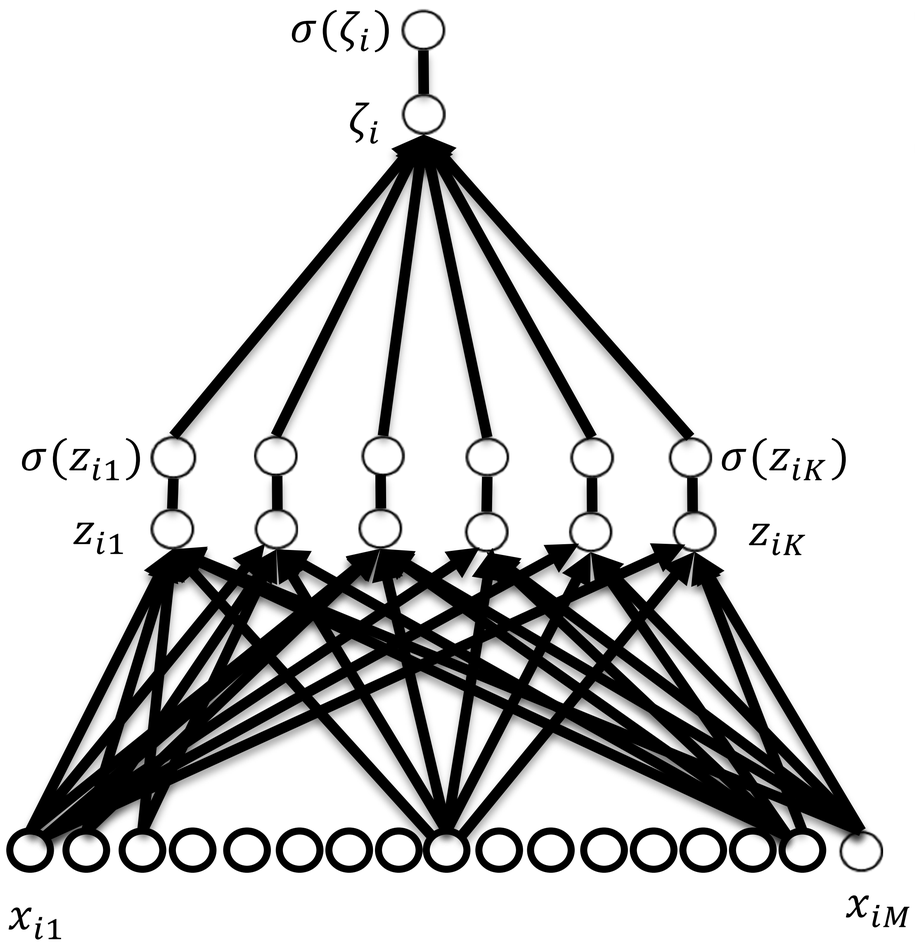
Extended Logistic Regression



Probability of particular outcome

Probability of K latent processes

Extended Logistic Regression



Probability of particular outcome

Probability of K latent processes

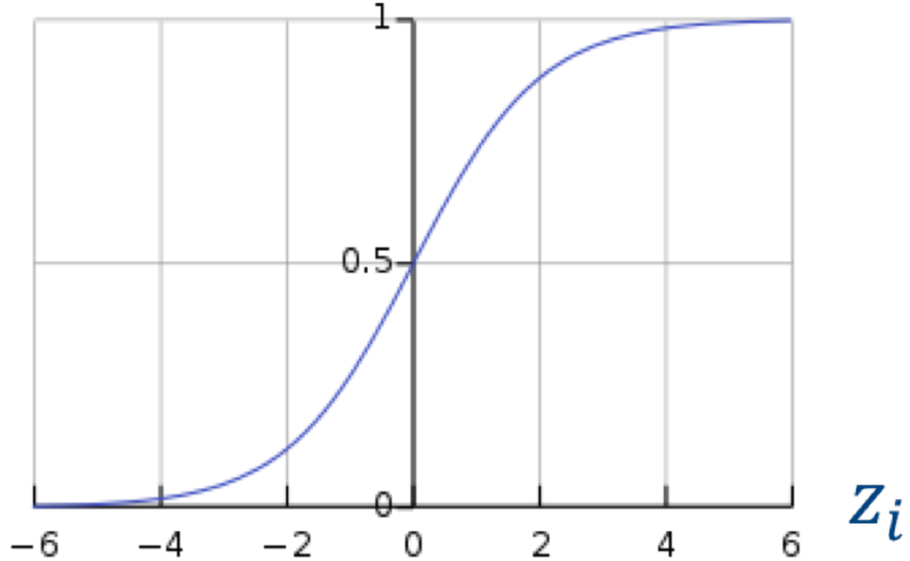
May be viewed as logistic regression on K latent features, rather than directly on the M components of raw data

Recall Logistic Regression

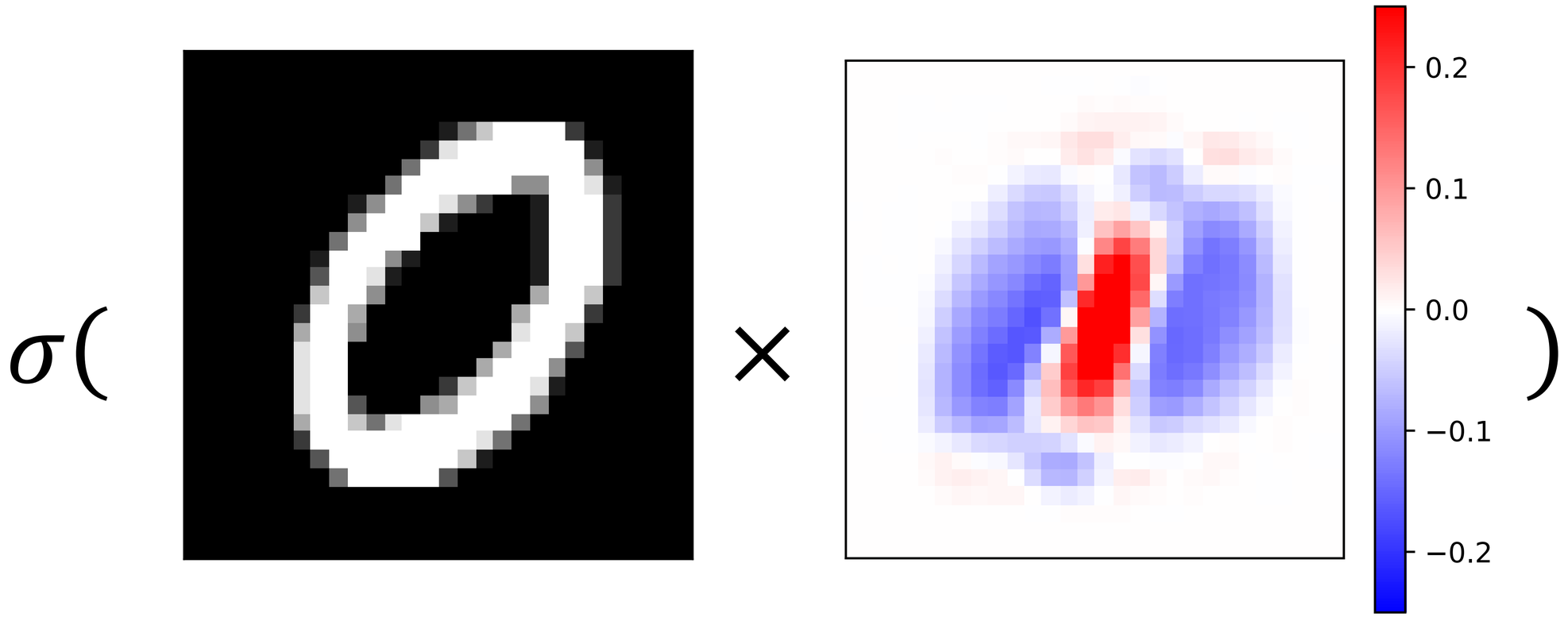
$$z_i = b_0 + (b_1 \times x_{i1}) + (b_2 \times x_{i2}) + \cdots + (b_M \times x_{iM})$$

$$= b_0 + x_i \odot b$$

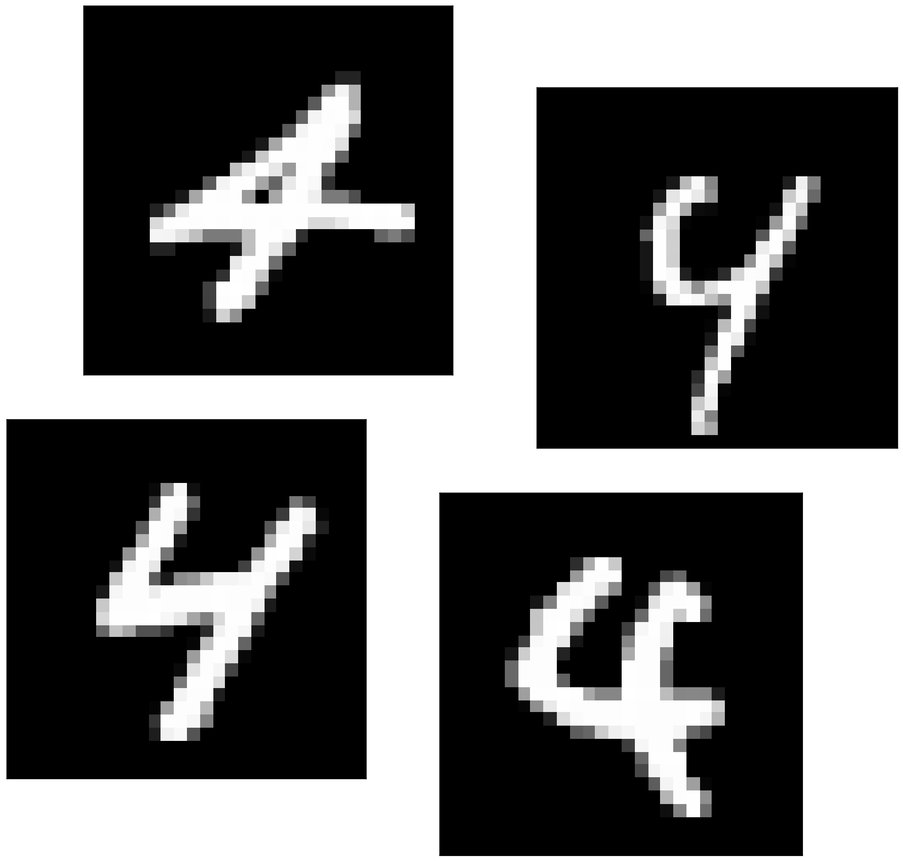
$$p(y_i = 1 | x_i) = \sigma(z_i)$$



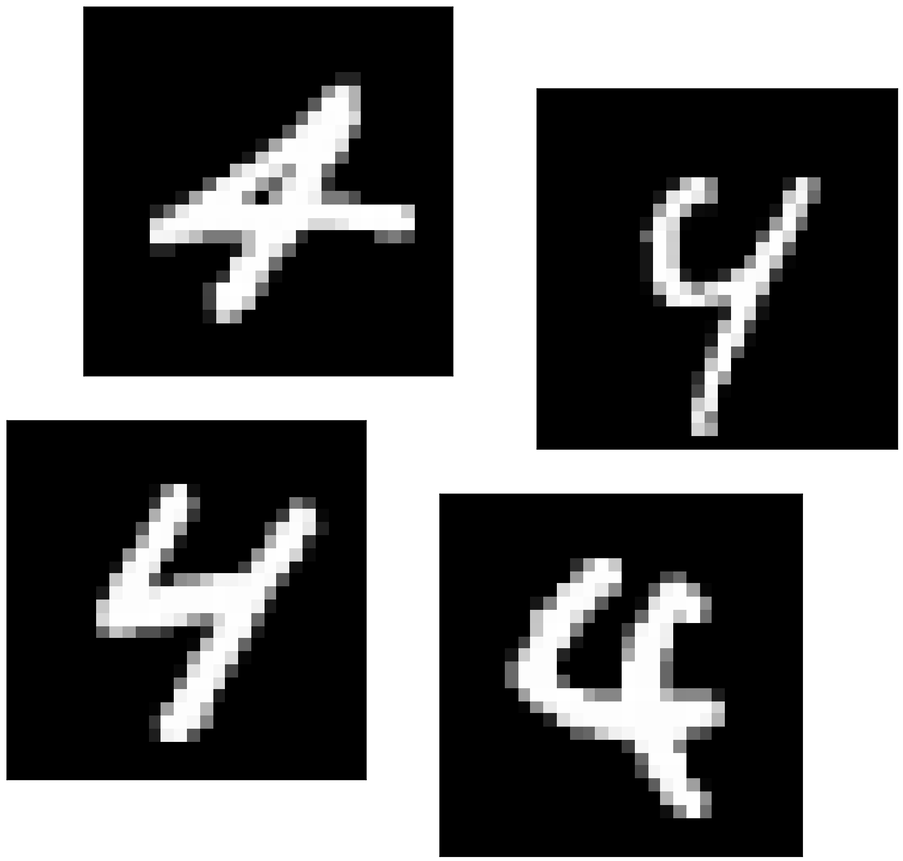
Why Limit Ourselves to Only One Filter?

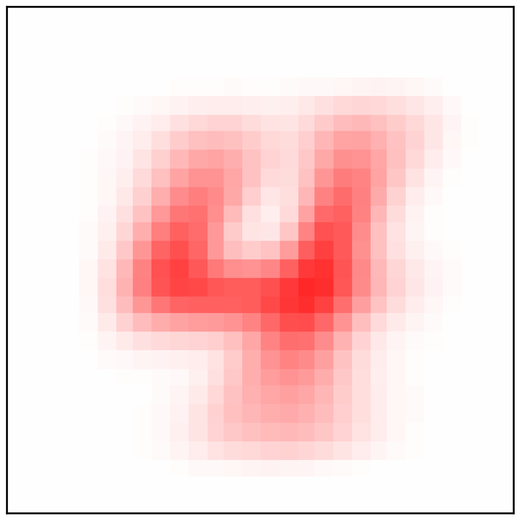


Return to MNIST:
Many ways of writing “4”



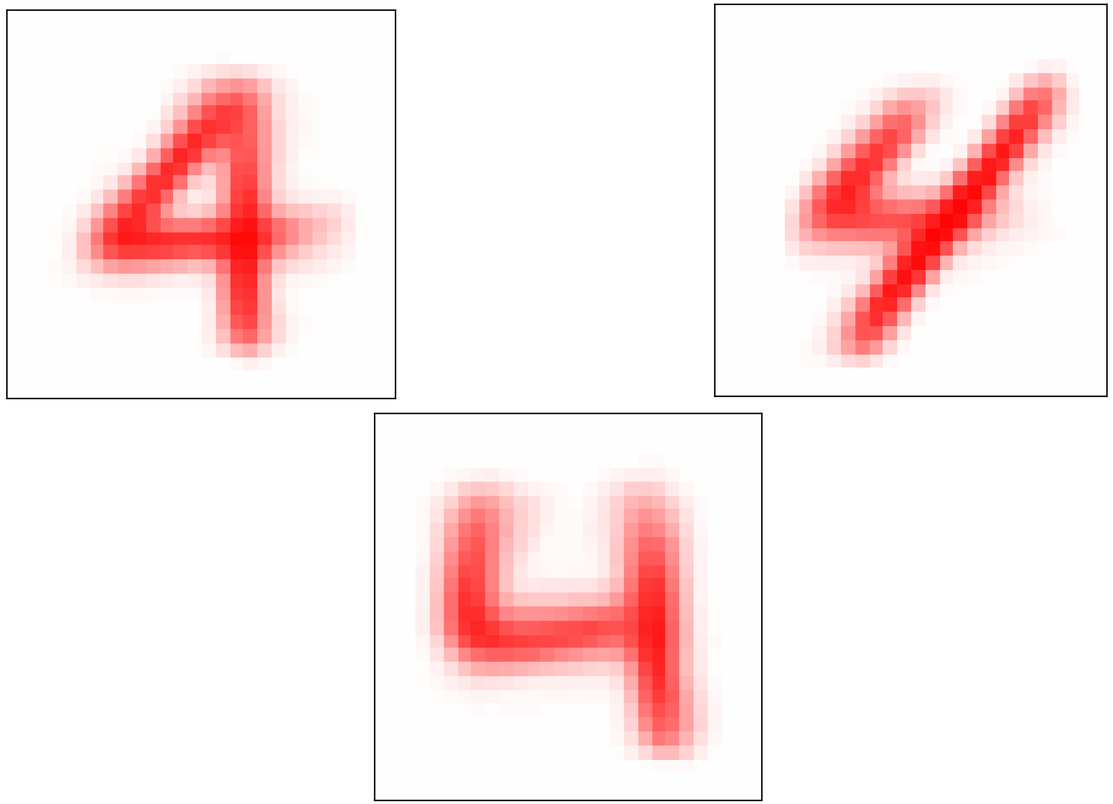
Return to MNIST:
Many ways of writing “4”





Single Filter (e.g. Logistic Regression/ “Shallow Learning”) only uses one filter, looks for the average shape

Return to MNIST:
Many ways of writing “4”



Multiple filters can look for *subtypes* indicative of different ways of writing “4”

Why Limit Ourselves to Only One Filter?

Introduce *K* Filters

$$z_{i1} = b_{01} + x_i \odot b_1$$
$$z_{i2} = b_{02} + x_i \odot b_2$$
$$\vdots$$
$$z_{iK} = b_{0K} + x_i \odot b_K$$

Project data x_i onto *K* filters: $b_1, \ldots, b_K$

Why Limit Ourselves to Only One Filter?

Introduce *K* Filters

$$z_{i1} = b_{01} + x_i \odot b_1$$
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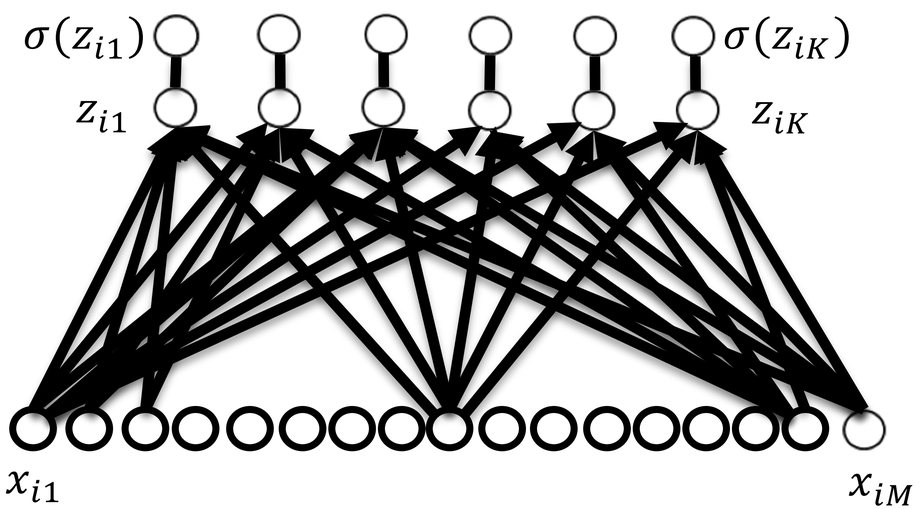
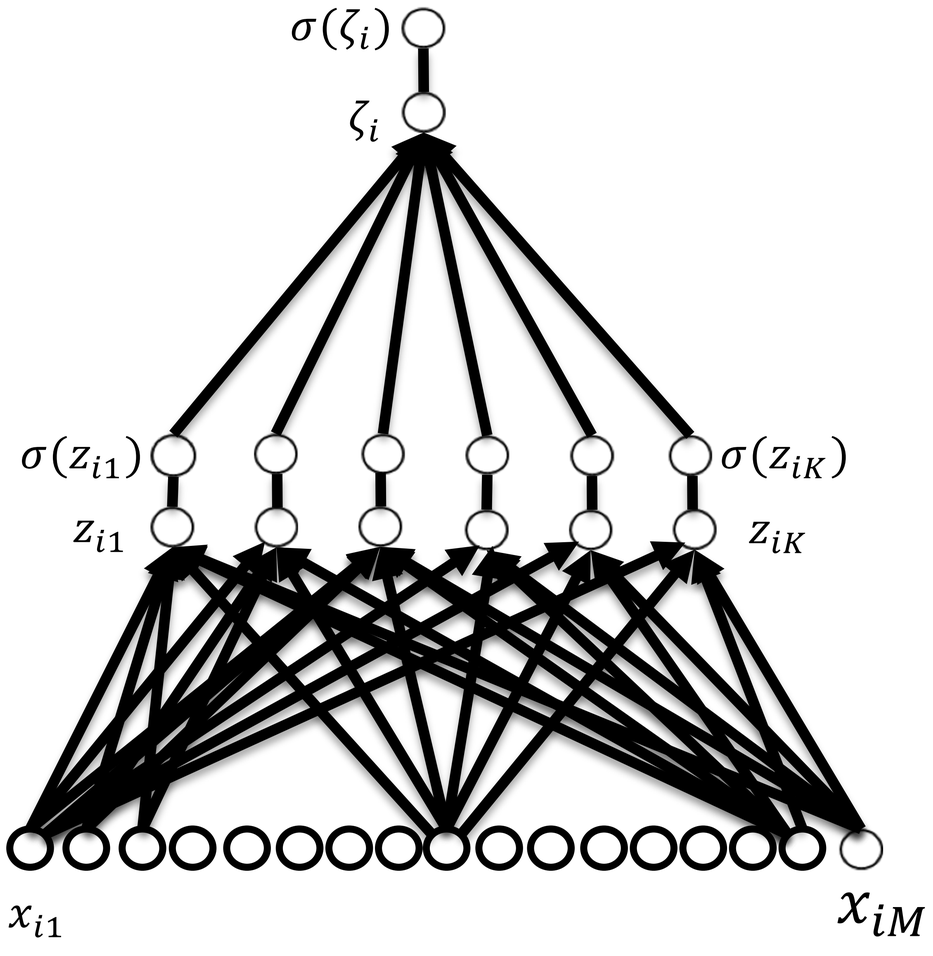
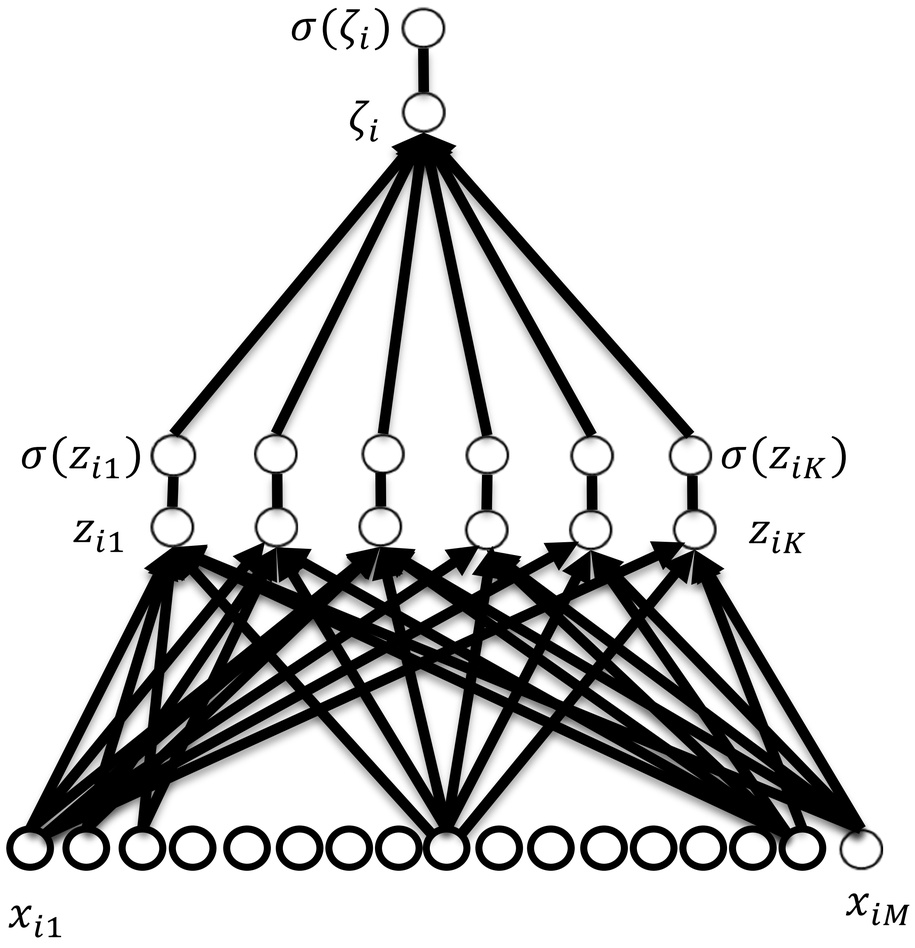
Project data x_i onto *K* filters: $b_1, \dots, b_K$



Extended Logistic Regression



Extended Logistic Regression

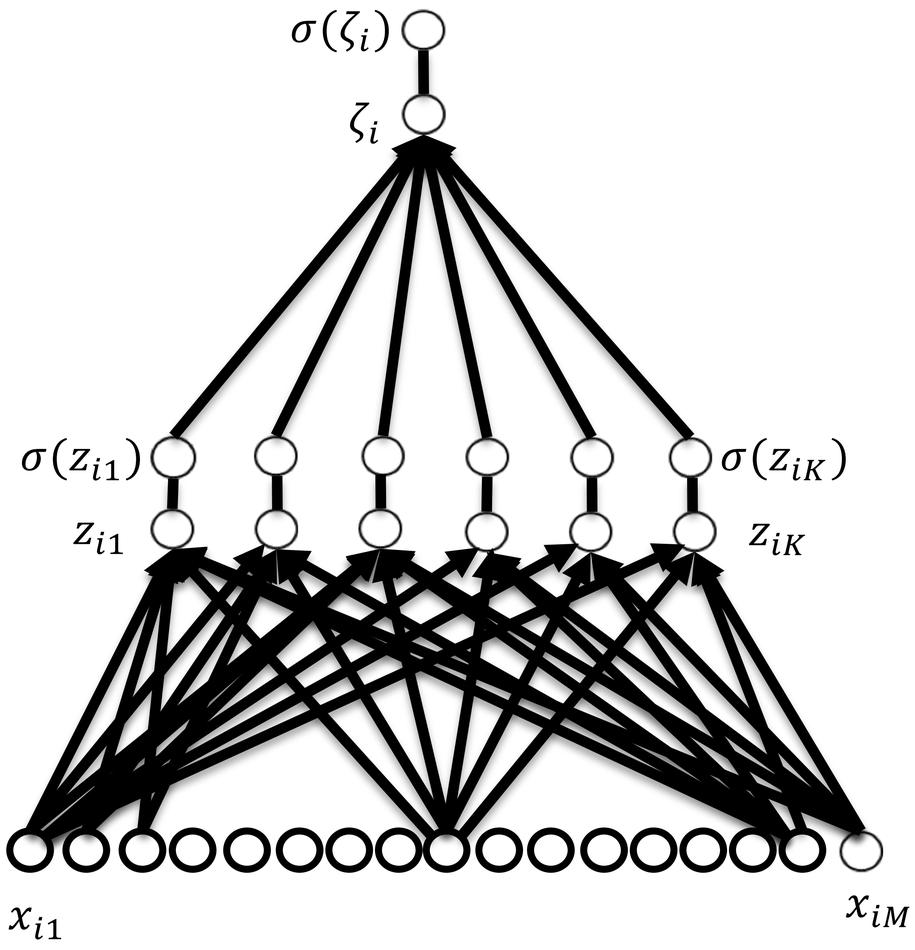


$$z_{i1} = b_{01} + x_i \odot b_1$$
$$z_{i2} = b_{02} + x_i \odot b_2$$
$$\vdots$$
$$z_{iK} = b_{0K} + x_i \odot b_K$$

K Features output from Layer-1

K Layer-1 filters

Extended Logistic Regression



Single Layer-2 Filter

$$\zeta_i = c_0 + \sigma(z_i) \odot c$$

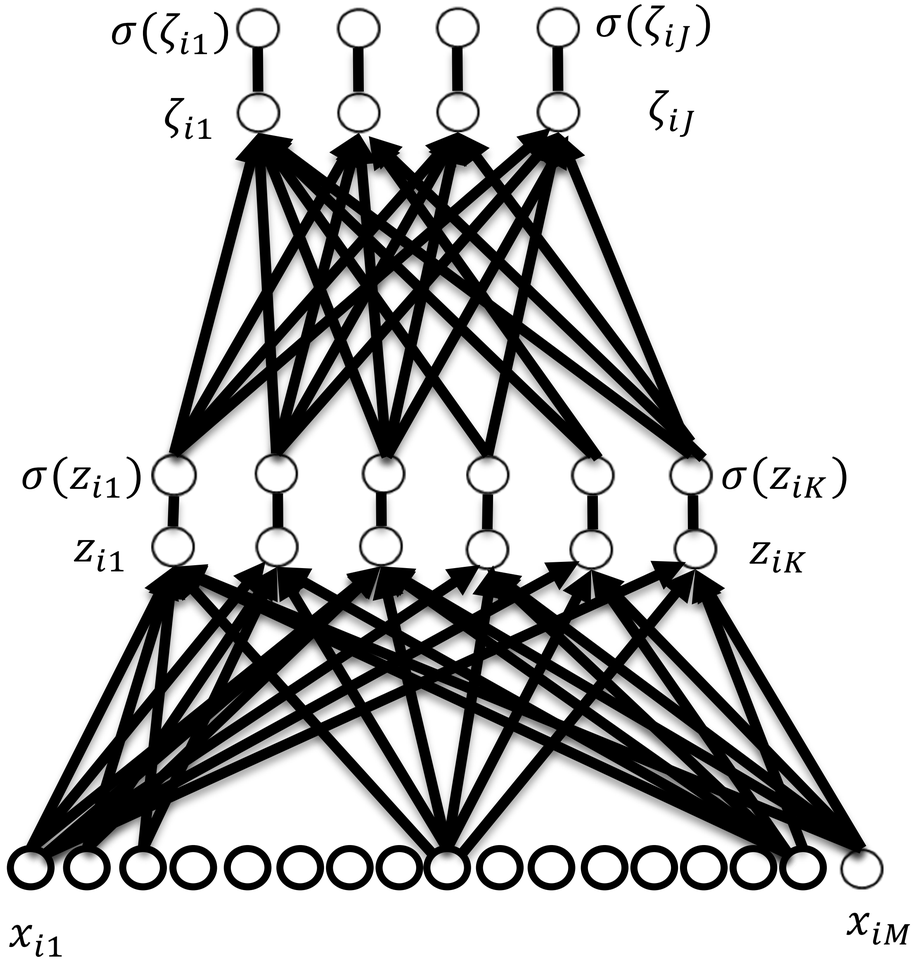
K-Dimensional Feature Vector Output from Layer-1

$$z_{i1} = b_{01} + x_i \odot b_1$$
$$z_{i2} = b_{02} + x_i \odot b_2$$
$$\vdots$$
$$z_{iK} = b_{0K} + x_i \odot b_K$$

K Features output from Layer-1

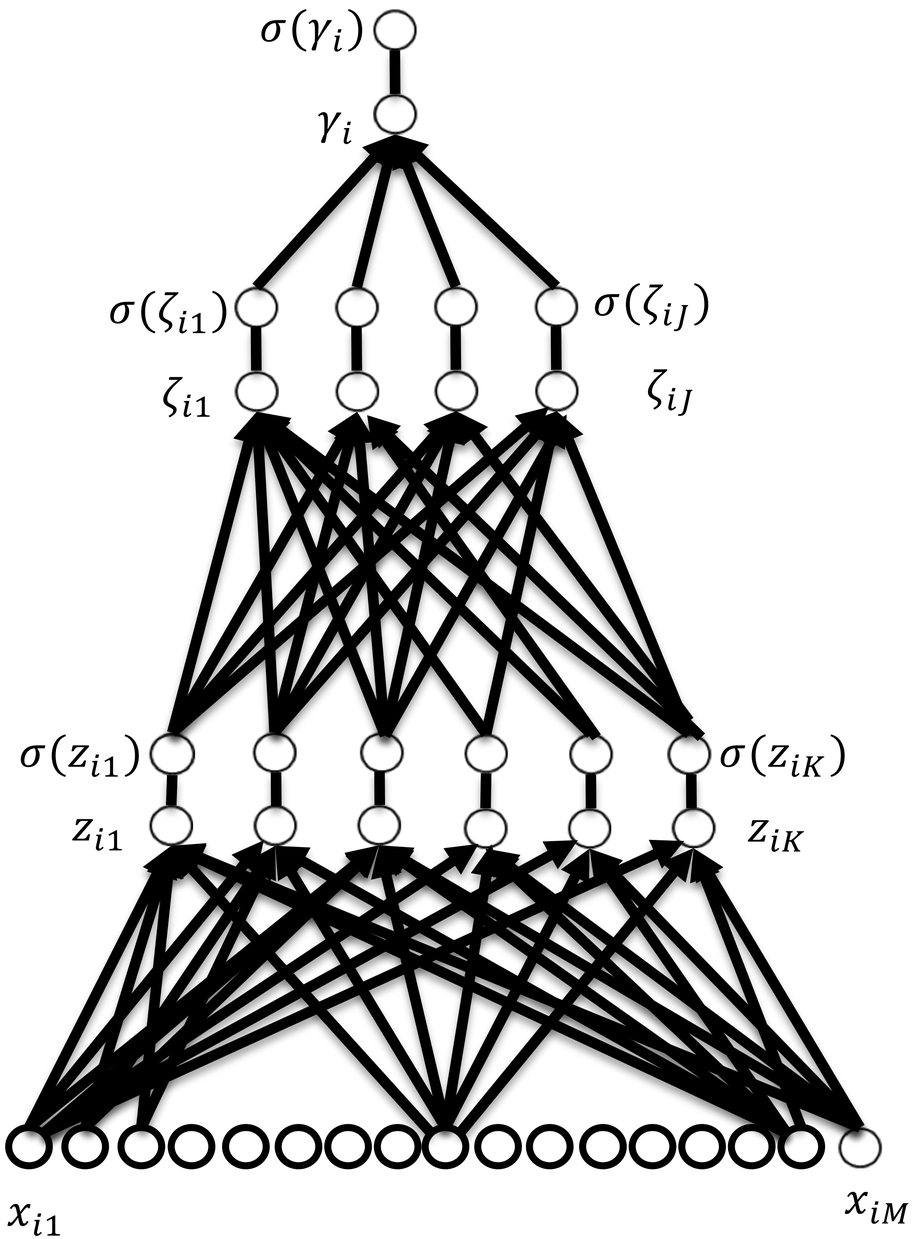
K Layer-1 filters

Going Deeper



Probability of J Layer-2 latent processes

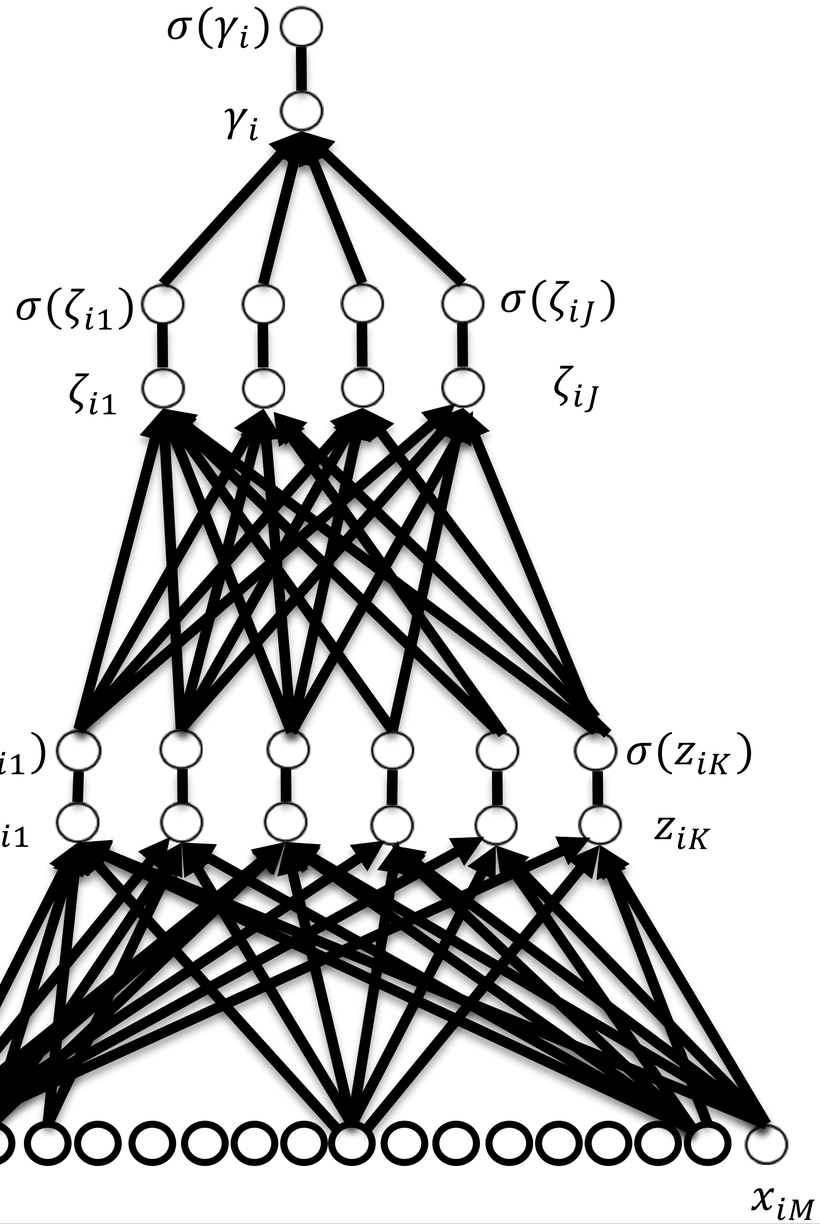
Probability of K Layer-1 latent processes



Probability of particular outcome

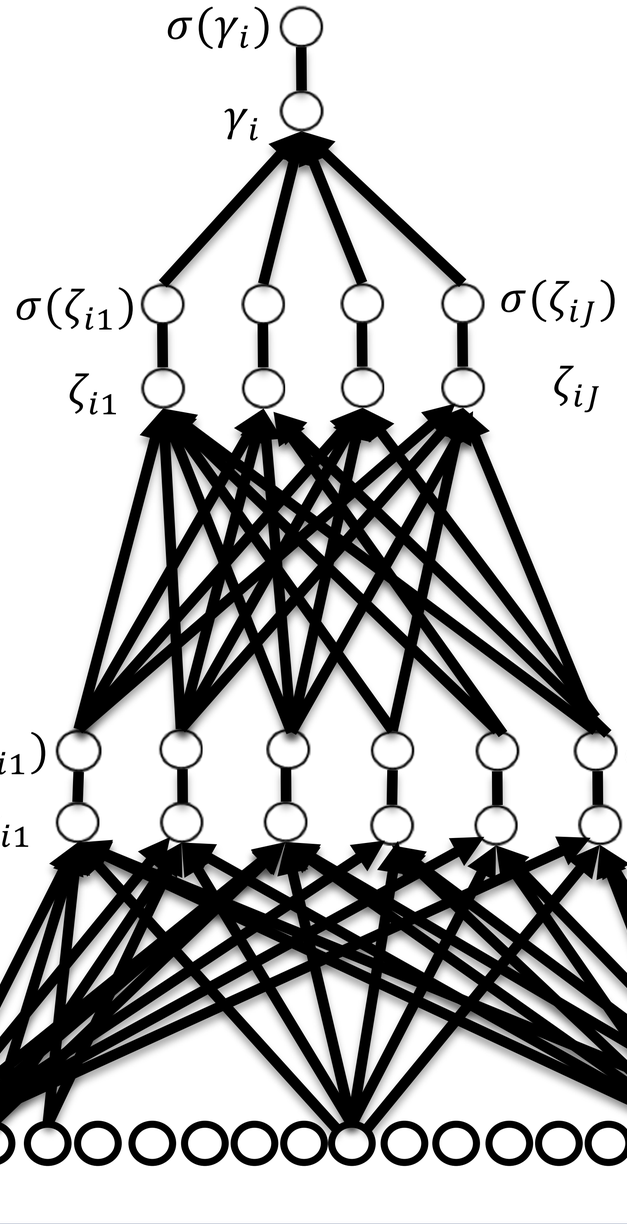
Probability of J Layer-2 latent processes

Probability of K Layer-1 latent processes



$$z_{i1} = b_{01} + x_i \odot b_1$$
$$z_{i2} = b_{02} + x_i \odot b_2$$
$$\vdots$$
$$z_{iK} = b_{0K} + x_i \odot b_K$$

Features of the Data with Layer-1 Filters

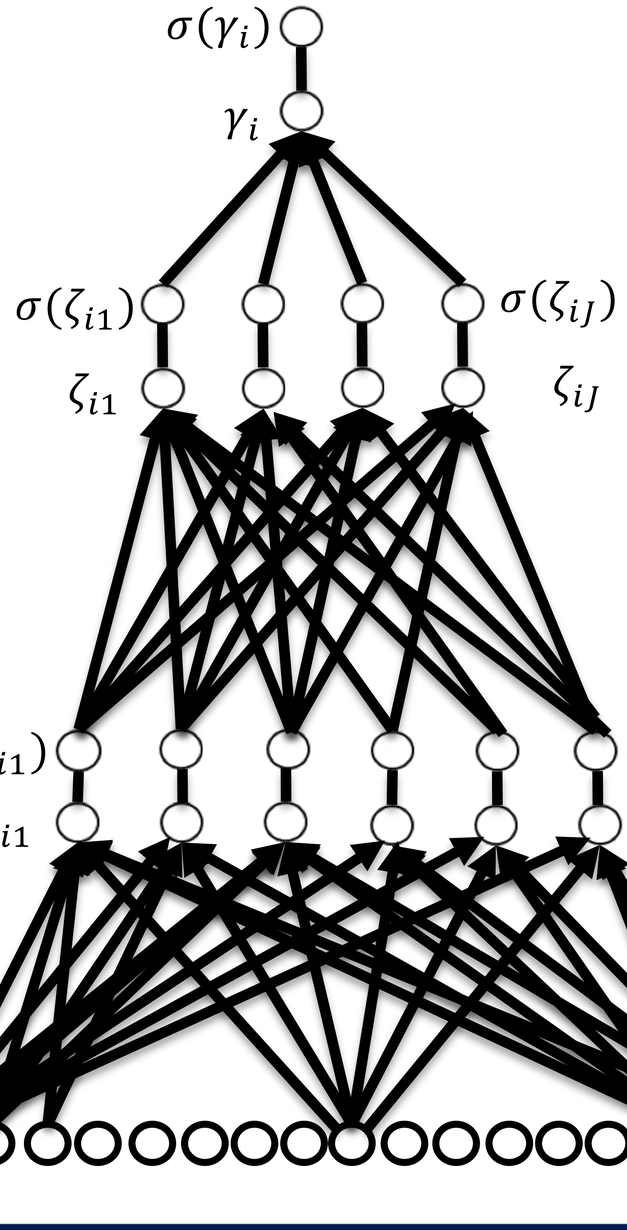


$$\zeta_{i1} = c_{01} + \sigma(z_i) \odot c_1$$
$$\zeta_{i2} = c_{02} + \sigma(z_i) \odot c_2$$
$$\vdots$$
$$\zeta_{iK} = c_{0J} + \sigma(z_i) \odot c_J$$

Features of the Layer-1 Features, with Layer-2 Filters

$$z_{i1} = b_{01} + x_i \odot b_1$$
$$z_{i2} = b_{02} + x_i \odot b_2$$
$$\vdots$$
$$z_{iK} = b_{0K} + x_i \odot b_K$$

Features of the Data with Layer-1 Filters



$$\gamma_i = d_0 + \sigma(\zeta_i) \odot d$$

Logistic Regression Based on the Layer-2 Features

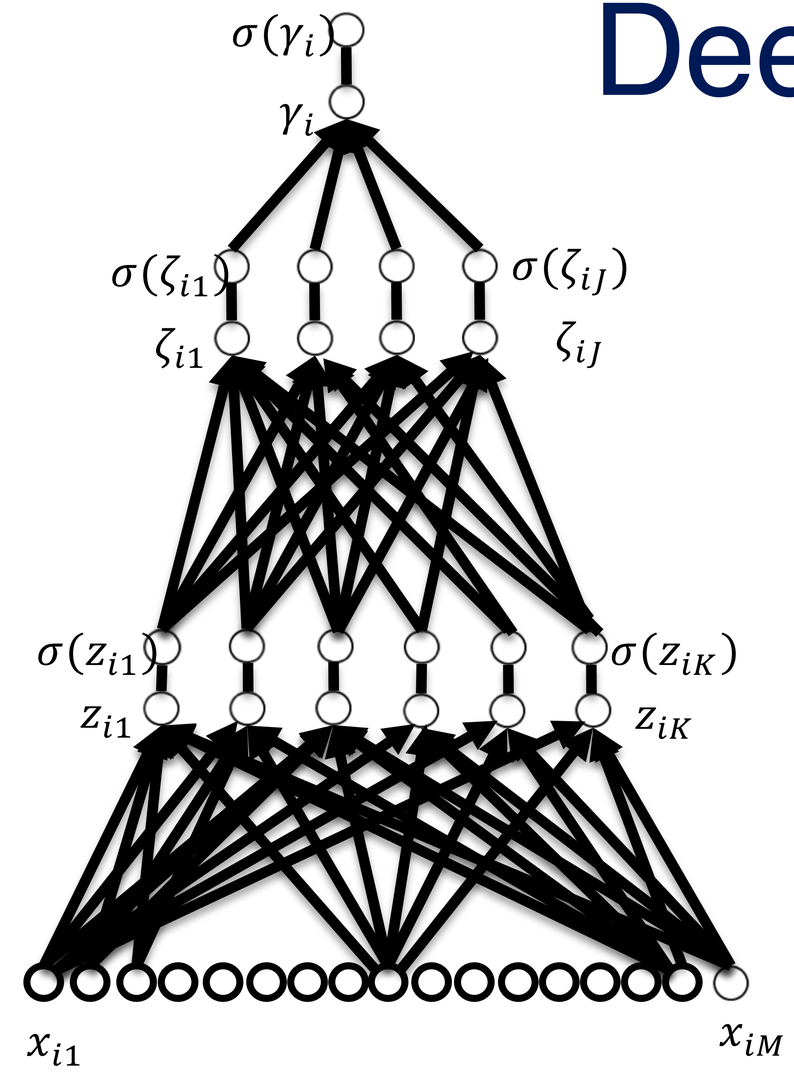
$$\zeta_{i1} = c_{01} + \sigma(z_i) \odot c_1$$
$$\zeta_{i2} = c_{02} + \sigma(z_i) \odot c_2$$
$$\vdots$$
$$\zeta_{iK} = c_{0J} + \sigma(z_i) \odot c_J$$

Features of the Layer-1 Features, with Layer-2 Filters

$$z_{i1} = b_{01} + x_i \odot b_1$$
$$z_{i2} = b_{02} + x_i \odot b_2$$
$$\vdots$$
$$z_{iK} = b_{0K} + x_i \odot b_K$$

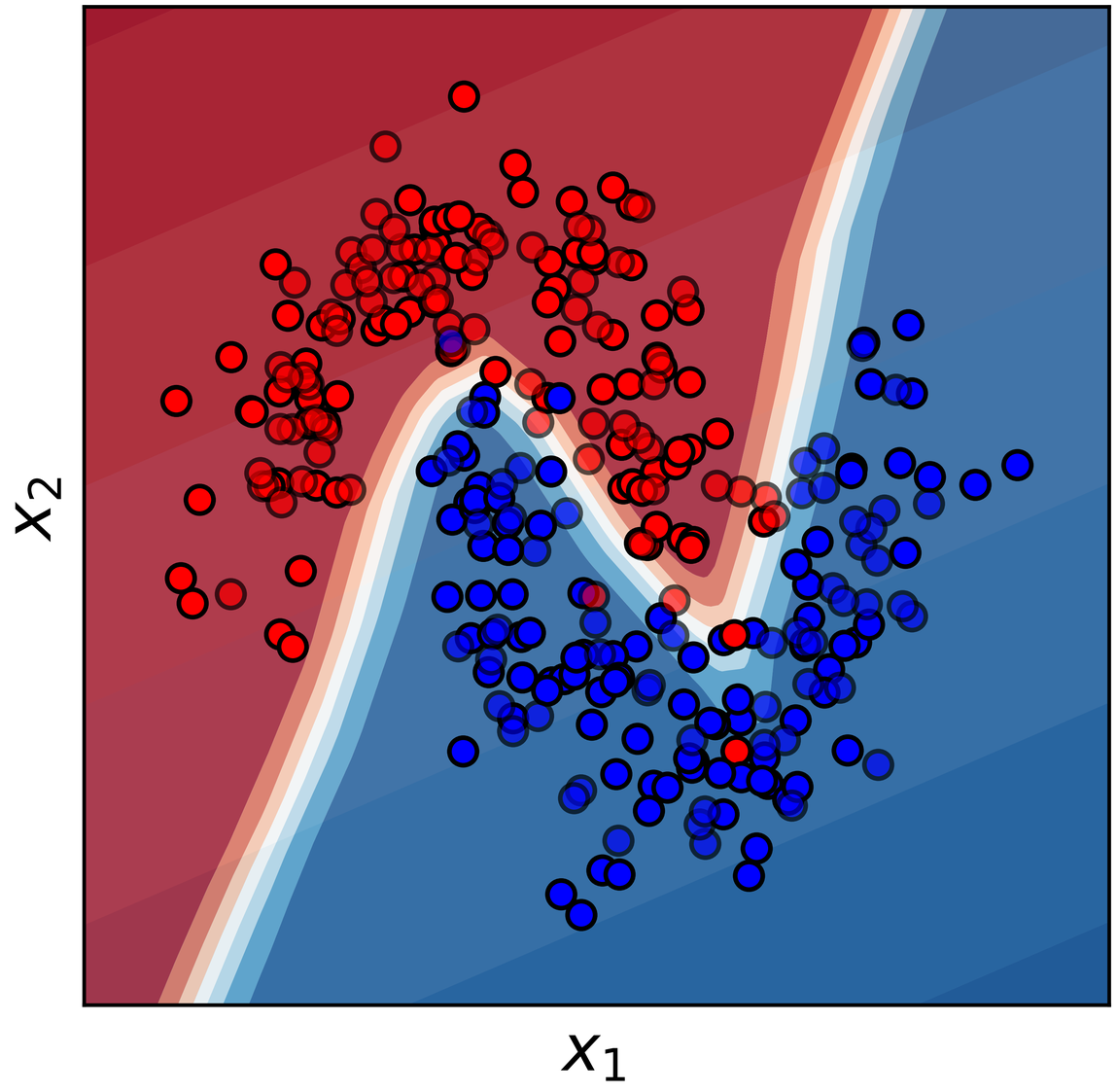
Features of the Data with Layer-1 Filters

Multilayer Perceptron: Neural Network Deep Learning

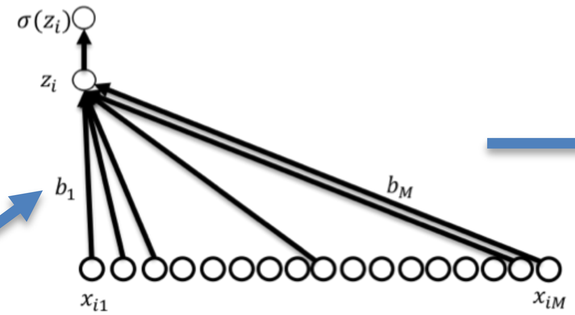


Can learn non-linear classifications

By learning multiple layers and transformation, it is possible to have a non-linear classifier capable of more accurately capturing the data.

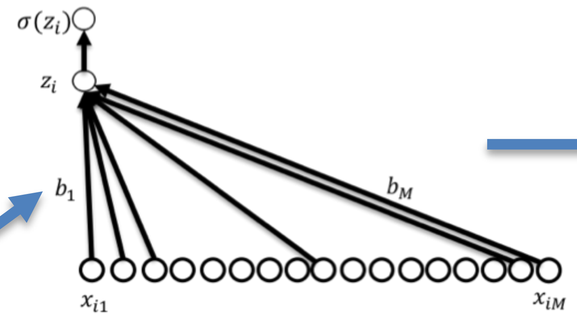


Does this work with MNIST?

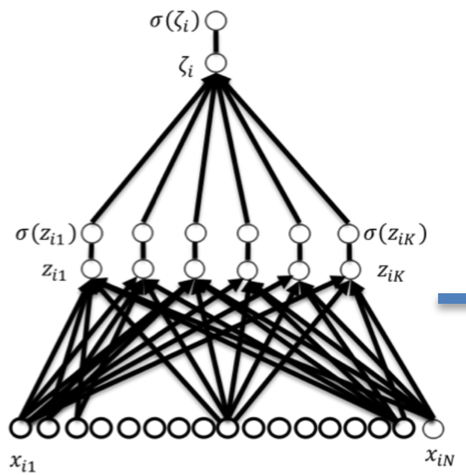


~91% Accurate

Does this work with MNIST?

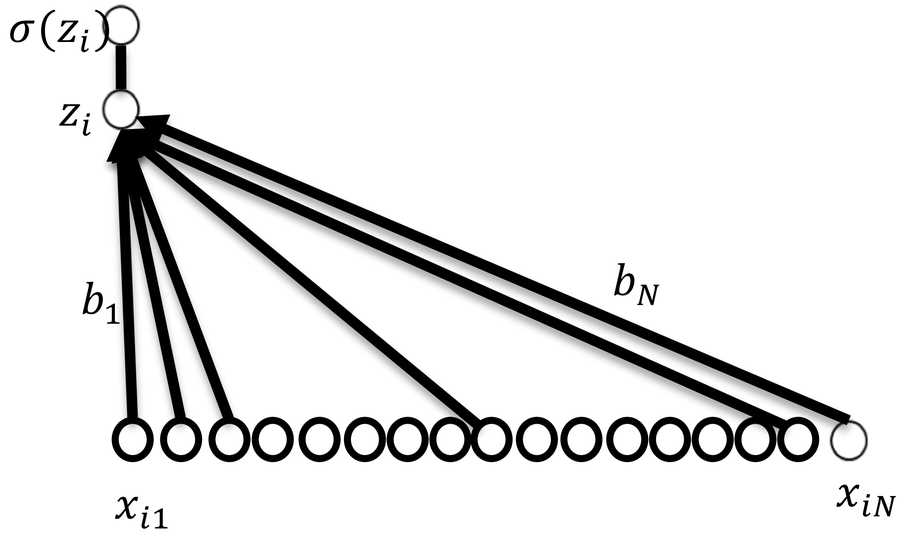


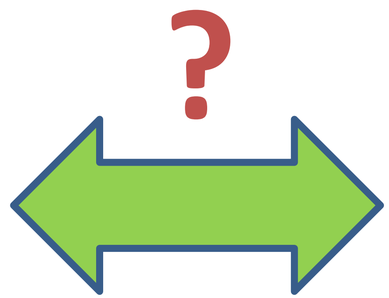
~91% Accurate

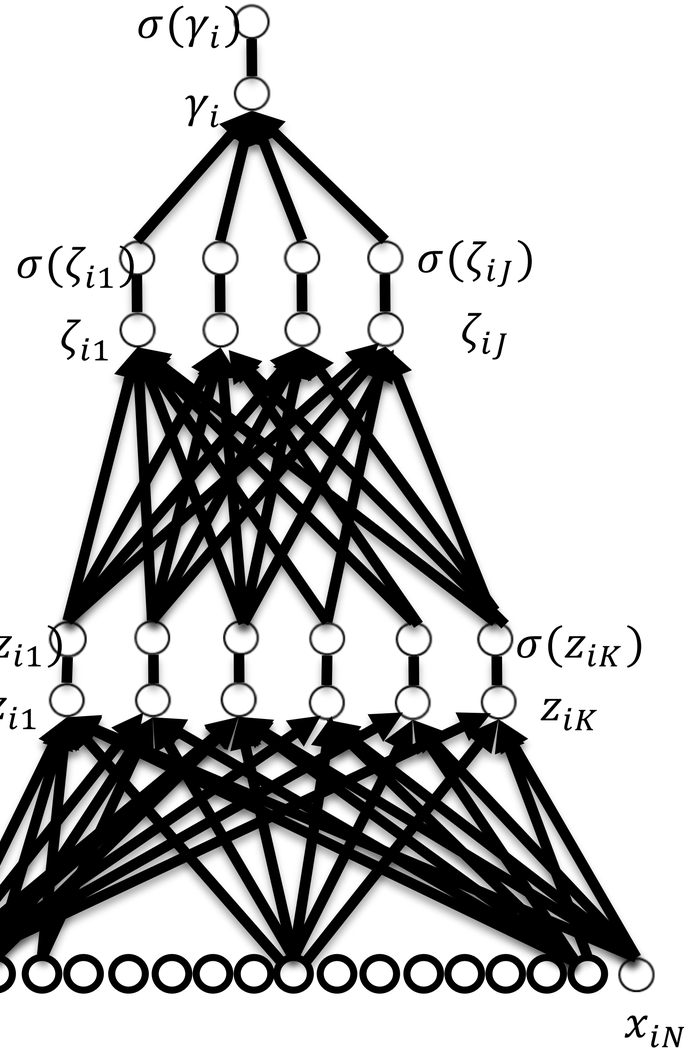


~96% Accurate

Do we always want to increase complexity?







Thanks for your attention!

BREAK; WHEN WE RETURN WE'LL DISCUSS HOW THE NETWORK IS LEARNED