SI 211: Numerical Analysis
Homework 4

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Deadline: Nov 13, 2019

1. Implement Gauss Quadrature for $n = 2, 3, 4$ in JULIA and compute the integral
$$I = \int_1^2 \frac{1}{x}\,\mathrm{d}x$$
with all three methods and compare your results with the exact value for the integral.

2. The exact value of the integral
$$I(\omega) = \int_0^{\frac{\pi}{4}} \cos(\omega x) dx$$
is given by $I(\omega) = \frac{1}{\omega}\sin(\omega x)$ for any $\omega > 0$ In the following, we test how accurate a Gauss-Quadrature of the form
$$I_1(\omega) = \sum_{i=0}^{1} \alpha_i \cos(\omega x_i)$$
can approximate this integral. Explain how to compute the approximation $I_1(\omega) \approx I(\omega)$. You may use that the second order Legendre polynomial of order 2 on the interval $[-1, 1]$has roots at $\pm\sqrt{\frac{1}{3}}$. How large is the approximation error $|I(1) - I_1(1)|$? What happens for large ω?

3. In this exercise we would like to construct an integration formula of the form
$$\int_a^b f(x)e^{-x}\,\mathrm{d}x \approx \frac{b-a}{2}\left(\alpha_1 f(x_1) + \alpha_2 f(x_2)\right) \ .$$
How do you need to choose x_1, x_2 and α_1, α_2 if our goal is to construct an intergration formula with maximum convergence order (for small $|b-a|$)?