

SI 211: Numerical Analysis

Homework 2

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Deadline: October 17, 2018

1. Assume that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(0) = 1$, $f(1) = 3$, and $f(2) = 19$. Construct a polynomial of the form $p(x) = a_0 + a_1x + a_2x^2$ such that p interpolates f at $x \in \{0, 1, 2\}$. What are a_0, a_1, a_2 ?

2. Assume that a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ satisfies

$$f(0, 0) = 1, \quad f(0, 1) = 3, \quad f(0, 2) = 19, \quad f(1, 0) = 3, \quad f(2, 0) = 19, \quad f(1, 1) = 0$$

Construct a polynomial $p : \mathbb{R}^2 \rightarrow \mathbb{R}$ of the form

$$p(x) = a_0 + a_1x_1 + a_2x_1^2 + a_3x_2 + a_4x_2^2 + a_5x_1x_2$$

such that p interpolates f at all 6 points. What are $a_0, a_1, a_2, a_3, a_4, a_5$?

3. Implement a computer program that interpolates a function $f(x)$ at the points

$$x_1 = -5, \quad x_2 = -4, \quad x_3 = -3, \quad \dots, \quad x_{10} = 4, \quad x_{11} = 5$$

with a polynomial p of order 10. Test your program for

(a) the function $f(x) = \sin(x)$ and

(b) the function $f(x) = \frac{1}{1+x^2}$.

Plot the functions as well as their interpolating polynomials. How big are the approximation errors?