SI 211: Numerical Analysis
Homework 3

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Deadline: October 24, 2018

1. Write a computer code in JULIA, Matlab, Python, or C++, which returns a natural spline that intepolates the function $f : [x_0, x_N] \to \mathbb{R}$ at the equidistant points

$$x_i = x_0 + hi \qquad \text{with} \qquad h = \frac{x_N - x_0}{N} .$$

2. Use your computer code from the first exercise in order to compute a natural spline of the function

$$f(x) = \frac{1}{1 + x^2}$$

   on the interval $[x_0, x_N] = [-5, 5]$. You may set $N = 10$. Plot the function f as well as the natural spline that interpolates f.

3. Use your compute code to compute a natural spline of the function

$$f(x) = x^2$$

   on the interval $[x_0, x_N] = [0, 1]$ with $N = 10$. What is the exact value for the integral

$$\int_0^1 [f''(x)]^2 \ \mathrm{d}x \ = ?$$

   Also compute the value

$$\int_0^1 [p''(x)]^2 \ \mathrm{d}x \ = ?$$

   for the interpolating spline. Explain how you compute this integral numerically. Which value is bigger, $\int_0^1 [f''(x)]^2 \ \mathrm{d}x$ or $\int_0^1 [p''(x)]^2 \ \mathrm{d}x$?