SI 211: Numerical Analysis
Homework 5

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Deadline: Dec 2, 2019

1. *LU decomposition.* Write a computer code, which returns an LU decomposition of a given input matrix, $A = LU$. Use your code to solve the linear equations $Ax = b_i,\ i = 1, 2, 3$, with

$$A = \begin{bmatrix} 17 & 24 & 1 & 8 & 15 \\ 23 & 5 & 7 & 14 & 16 \\ 4 & 6 & 13 & 20 & 22 \\ 10 & 12 & 19 & 21 & 3 \\ 11 & 18 & 25 & 2 & 9 \end{bmatrix},\ b_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{bmatrix},\ b_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 3 \\ 6 \end{bmatrix} \text{ and } b_3 = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \\ 8 \end{bmatrix}$$

   as efficiently as possible.

2. *Cholesky Decomposition.* Write a computer code function in Julia (or Matlab/Python), which returns a Cholesky decomposition, $A = LDL^\top$, of the matrix

$$A = \begin{bmatrix} 34 & 47 & 5 & 18 & 26 \\ 47 & 10 & 13 & 26 & 34 \\ 5 & 13 & 26 & 39 & 47 \\ 18 & 26 & 39 & 42 & 5 \\ 26 & 34 & 47 & 5 & 18 \end{bmatrix}.$$

3. *Perturbation analysis.* Consider the linear system $Ax = b$ with

$$A_1 = \frac{1}{2}\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix},\ A_2 = \frac{1}{2}\begin{bmatrix} 1.000 & 1.000 \\ 1.001 & 0.999 \end{bmatrix}$$

   Compute the condition numbers of A_1 and A_2. What happens if you solve the equations $Ax = b$ and $Ax = \bar{b}$ with

$$b = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \bar{b} = \frac{1}{2}\begin{bmatrix} 0.99 \\ 1.01 \end{bmatrix}.$$

   Interpret your results.