SI 211: Numerical Analysis
Homework 1

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Deadline: Sep 25, 2019

1. What is the bit representations of the floating point number 6.25 using the IEEE standard for double precision numbers? Please explain all intermediate derivation steps. Use the JULIA command `bitstring(6.25)` (or any other programming language) to verify your result.

2. We want to evaluate the function
$$f(x) = \frac{\sin(10^5 \sin(10^4 x))}{x}$$
for different values of x.
   (a) Evaluate the above function at $x = \pi$ by using `Matlab`, `Julia`, `C++` or any other programming language of your choice. How big is the numerical approximation error? Can you explain why you observe this error?
   (b) Evaluate the above function at $x = 10^{-10}$. How big is the numerical evaluation error?

3. Numeric differentiation based on central differences:
   (a) Implement a function (for example in Python, Julia, or Matlab) that uses numeric differentiation based on central differences,
$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \ .$$
   Here, the inputs of the differentiation routine are the scalar function f that we want to differentiate, the point x at which the derivative should be evaluated, and the finite perturbation $h > 0$. Use the syntax
```
diff(f,x,h) = ...
```
   (b) Evaluate the derivative of the function $f(x) = \exp(x)$ at $x = 0$ using the above routine `diff`. Plot the numerical differentiation error in dependence on $h \in [10^{-15}, 10^{-1}]$ and interpret the result. Use logarithmic scales on both axis!

4. Let f be a 5-times continuously differentiable function. Consider the numerical difference formula
$$f'(x) \approx \frac{f(x-4h) - 2f(x-h) + 2f(x+h) - f(x+4h)}{120h}$$
What is the mathematical approximation error? How would you choose h?

5. In order to evaluate the factorable function $f(x) = \sin(\cos(x)) * \cos(x)^2$ we write an evaluation algorithm of the form

$$\begin{aligned} a_0 &= x \\ a_1 &= \cos(x) \\ a_2 &= a_1 * a_1 \\ a_3 &= \sin(a_1) \\ a_4 &= a_2 * a_3 \\ f(x) &= a_4 \ . \end{aligned}$$

What is the corresponding algorithm for evaluating the derivative of $f(x)$ using the forward mode of algorithmic differentiation (AD)? What is the order of magnitude of the numerical error that is associated with evaluating the derivative of f at $x = 0$ using this AD code?