15.076 Homework 2

Due: Feb. 28, 11:59 pm

Problem 1: Where to locate vaccination clinics

You are planning the deployment of vaccination clinics in response to an ongoing pandemic (totally made up scenario). Given available resources, you have capacity to build only 4 sites across the city. You have identified 10 potential sites, each with a fixed capacity (i.e., a maximum number of residents that can be vaccinated there). All residents will need to access one of the clinics you will construct. You aim to determine which sites to build to optimize the vaccination service provided to the population.

The city under consideration can be modeled as a 5×10 miles rectangle, with 50 major residential areas. We index the residential areas by $i = 1, \cdots, 50$ and the vaccination sites by $j = 1, \cdots, 10$. We consider the "Manhattan distance": if area i's coordinates are (x_i, y_i) and site j's coordinates are in (x_j, y_j) , the distance from i to j is given by $d_{ij} = |x_i - x_j| + |y_i - y_j|$.

You are provided with the following data:

Parameters

- r_i : number of residents in area $i = 1, \cdots, 50$
- C_j : capacity of vaccination site $j = 1, \cdots, 10$
- d_{ij} : distance between area $i = 1, \cdots, 50$ and site $j = 1, \cdots, 10$

and you have access to the following files:

- P1_areas.csv: A matrix of size 50x3 that indicates, for each area, (i) its x-coordinate, in miles (0 to 5), (ii) its y-coordinate, in miles (0 to 10) and (iii) its number of residents.
- P1_clinics.csv: A matrix of size 10×3 that indicates, for each candidate clinic, (i) its x-coordinate, in miles (0 to 5), (ii) its y-coordinate, in miles (0 to 10) and (iii) its capacity.

(a)

Formulate an integer optimization model that optimizes the selection of sites to minimize the total distance across all residents required to access their assigned clinic.

Hint : this essentially is a facility location problem.

(b)

Implement the model computationally in Julia. Report the average as well as the largest distance travelled, across all residents. Turn in a copy of your code.

Optional : make a plot with the location of the resident areas (based on their coordinates) and the chosen clinic locations.

(c)

Assume we also have a budget B for building the clinics and that it costs F_j to open each clinic j. Add a constraint to the previous model such that your plan is within budget.

Solution

(a)

Parameters

- r_i : number of residents in area $i = 1, \cdots, 50$
- C_j : capacity of vaccination site $j = 1, \cdots, 10$
- d_{ij} : distance between area $i = 1, \cdots, 50$ and site $j = 1, \cdots, 10$

Decision variables

- $y_j = \begin{cases} 1 & \text{if site } j = 1, \cdots, 10 \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$
- x_{ij} = number of residents from area $i = 1, \cdots, 50$ assigned to site $j = 1, \cdots, 10$

Formulation

$$
\begin{aligned}
\min_{x,y} \quad & \sum_{i=1}^{50} \sum_{j=1}^{10} d_{ij} \cdot x_{ij} & & & (1) \\
\text{s.t.} \quad & \sum_{j=1}^{10} y_j \leq 4 & & \text{Site limit} & (2) \\
& \sum_{i=1}^{50} x_{ij} \leq C_j \cdot y_j & j = 1, \cdots, 10 \quad & \text{Capacity} & (3) \\
& \sum_{j=1}^{10} x_{ij} = r_i & i = 1, \cdots, 50 \quad & \text{Demand} & (4) \\
& x_{i,j} \in \mathbb{Z}^+ \quad \forall i, j & & & (5) \\
& y_j \in \{0, 1\} \quad \forall j & & & (6)
\end{aligned}
$$

(b)

See python notebook for code details.

Average distance: 3.73

Max distance: 1.86

(c)

We add the following budget constraint:

$$\sum_{j=1}^{10} F_j \cdot y_j \leq B$$

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Problem 2: Radiotherapy Treatment

There are two treatments in IMRT radiotherapy: photon and proton therapy. The advantage of proton therapy is that it targets the tumor more precisely. However, it is also much more expensive and the capacity for proton therapy is very limited in many countries. Suppose a patient needs 15 fractions, which can be photon fractions, proton fractions, or a mix of photon and proton fractions, say e.g. 4 proton fractions and 11 photon fractions. Suppose that we have $n = 17$ patients. We would like to use the limited proton therapy capacity as best as possible. For each patient, we can calculate a score when p proton fractions and $15 - p$ photon fractions are used. This score is called the BED (Biological Equivalent Dose), more precisely $\text{BED}_i(p,15 - p)$, i.e. the BED when p proton and $15 - p$ photon fractions are delivered for patient i. The higher the score, the better.

Given a set of 17 patients, and given the $\text{BED}_i(p,15 - p)$ for each $0 \leq p \leq 15$ and each patient i. Suppose that the total maximal capacity is C proton fractions. In order to maximize the total BED scores for all the patients, which patients should get proton fractions and how many proton fractions should they get?

The data file "outcome_data.csv" on Canvas contains a matrix of 17 rows and 16 columns. In particular, the number at the (i, j) position is $\text{BED}_i(j,15 - j)$.

(a)

Formulate an integer linear optimization model to solve this problem.

(b)

For $C = 20, 40, 50$, solve the problem in Julia for the given data in "outcome_data.csv". Turn in a copy of your code.

Solution

(a)

Let $x_{i,p}$ be the decision variable, which is equal to 1 if patient i is treated with p proton fractions, and 0 otherwise. The optimization model reads

$$\begin{aligned}
\max_{x} \quad & \sum_{i=1}^{17}\sum_{p=0}^{15} \text{BED}_i(p, 15 - p)x_{i,p} && \text{(7a)}\\
\text{s.t.} \quad & \sum_{i=1}^{17}\sum_{p=0}^{15} p x_{i,p} \leq C && \text{(7b)}\\
& \sum_{p=0}^{15} x_{i,p} = 1, \quad \forall i = 1, \dots, 17 && \text{(7c)}\\
& x_{i,p} \in \{0, 1\} \quad \forall i, p. && \text{(7d)}
\end{aligned}$$

(b)

See python notebook for solution.