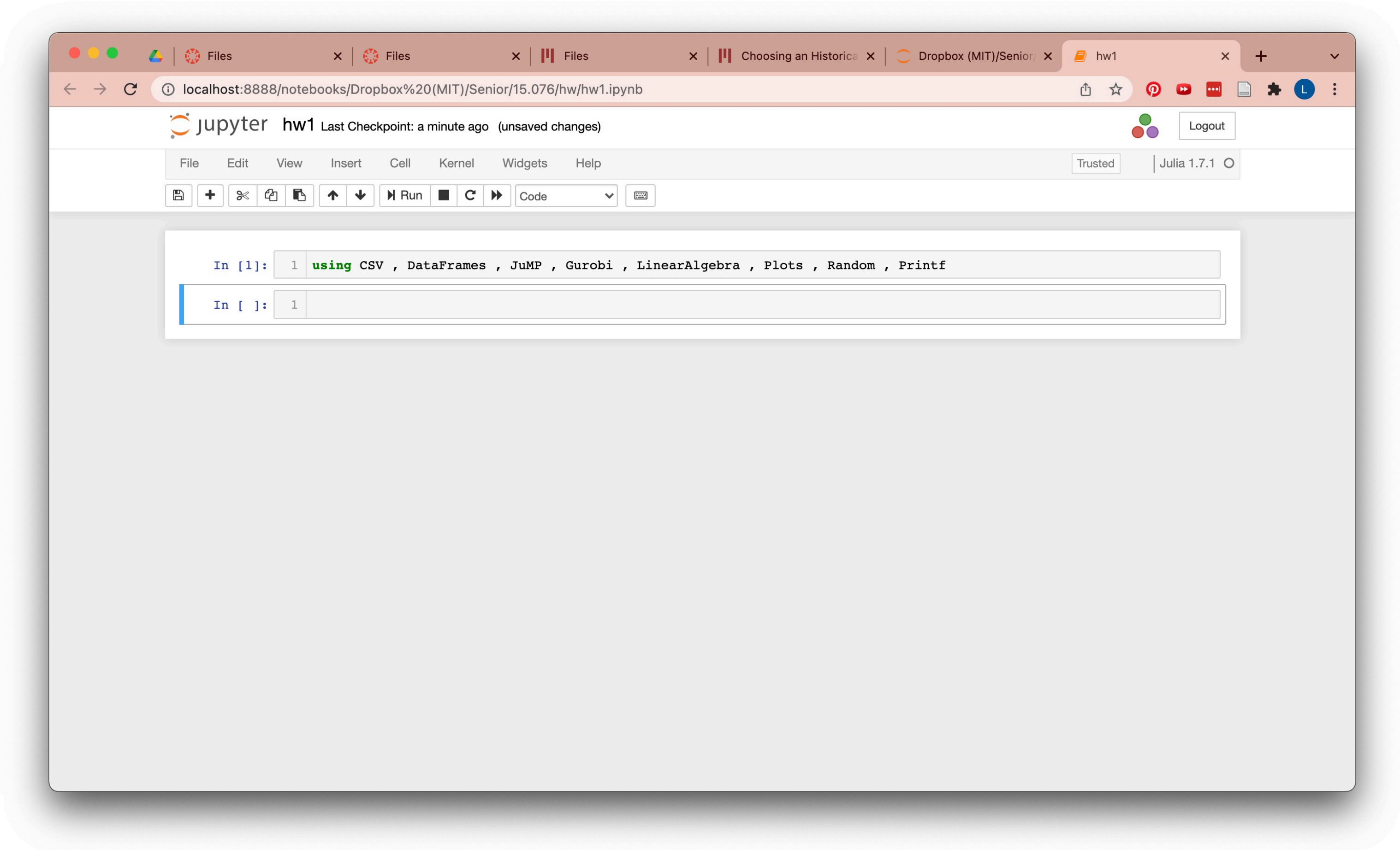
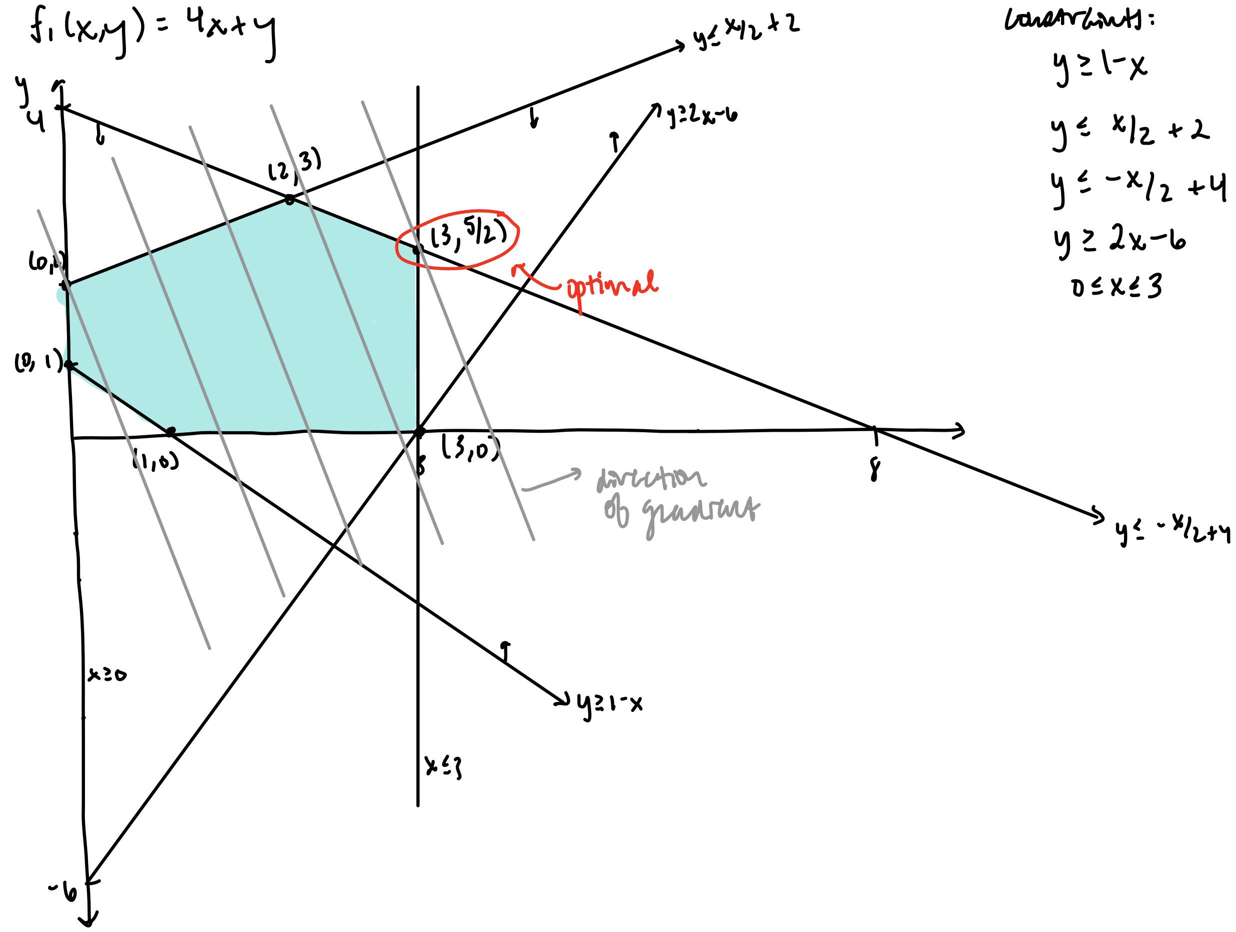
Lydia Yu 15.076 HW1

Problem 1



Problem 2

1. $f_1(x,y) = 4x+y$



Constraints:

$y \geq 1 - x$

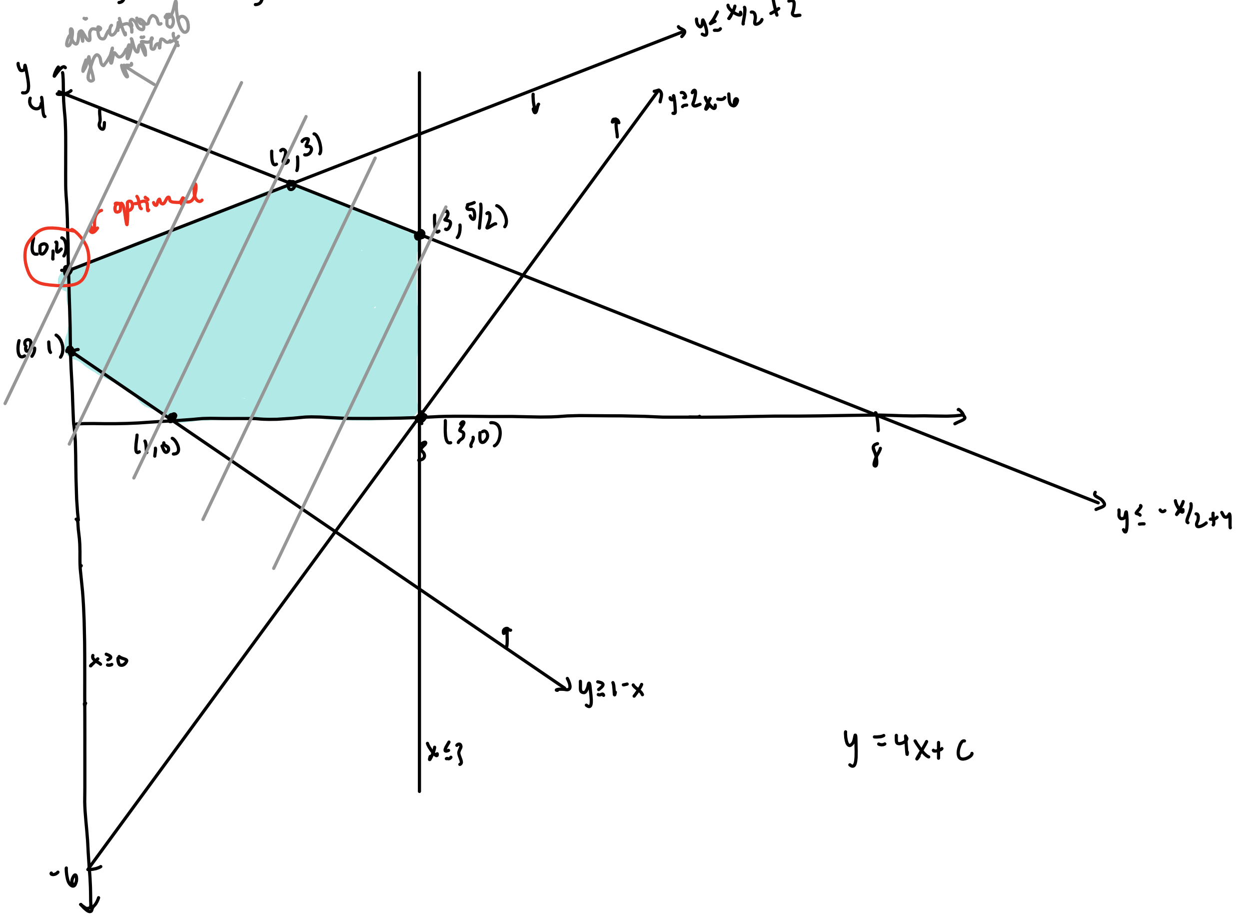
$y \leq x/2 + 2$

$y \leq -x/2 + 4$

$y \geq 2x - 6$

$0 \leq x \leq 3$

$f_2(x,y) = -4x+y$



$f_3(x,y) = x^2 \exp(y)$

not linear - is it useful to think about how you can change certain moments of your life?

2. Code for f_1:

$f_1(x, y) = 4x + y$

In [14]:
```
m1 = Model(Gurobi.Optimizer);
```

In [15]:
```
@variable(m1, x);
@variable(m1, y);
```

In [16]:
```
@constraint(m1, x + y >= 1);
@constraint(m1, -x + 2*y <= 4);
@constraint(m1, x + 2*y <= 8);
@constraint(m1, -2*x + y >= -6);
@constraint(m1, 0 <= x <= 3);
```

In [17]:
```
@objective(m1, Max, 4*x + y);
```

In [18]:
```
print(m1)
```

$$
\begin{aligned}
\max \quad & 4x + y \\
\text{Subject to} \quad & x + y \geq 1.0 \\
& -2x + y \geq -6.0 \\
& -x + 2y \leq 4.0 \\
& x + 2y \leq 8.0 \\
& x \in \backslash[0.0, 3.0\backslash]
\end{aligned}
$$

In [20]:
```
optimize!(m1)
```

In [21]:
```
@show value(x)
@show value(y)
@show objective_value(m1)
```

```
value(x) = 3.0
value(y) = 2.5
objective_value(m1) = 14.5
```

Out[21]: 14.5

Code for f_2:

$f_2(x, y) = -4x + y$

In [22]:
```
m2 = Model(Gurobi.Optimizer);
```

In [23]:
```
@variable(m2, x);
@variable(m2, y);
```

In [24]:
```
@constraint(m2, x + y >= 1);
@constraint(m2, -x + 2*y <= 4);
@constraint(m2, x + 2*y <= 8);
@constraint(m2, -2*x + y >= -6);
@constraint(m2, 0 <= x <= 3);
```

In [25]:
```
@objective(m2, Max, -4*x + y);
```

In [26]:
```
print(m2)
```

$$
\begin{aligned}
\max \quad & -4x + y \\
\text{Subject to} \quad & x + y \geq 1.0 \\
& -2x + y \geq -6.0 \\
& -x + 2y \leq 4.0 \\
& x + 2y \leq 8.0 \\
& x \in \backslash[0.0, 3.0\backslash]
\end{aligned}
$$

In [27]:
```
optimize!(m2)
```

In [28]:
```
@show value(x)
@show value(y)
@show objective_value(m2)
```

```
value(x) = 0.0
value(y) = 2.0
objective_value(m2) = 2.0
```

Out[28]: 2.0

When the sign of the coefficient of x is flipped e.g. from positive to negative, the direction of the new gradient becomes orthogonal to the previous one. This is because now, with a negative coefficient, lower values of x result in a higher objective value (while higher values of y still yield higher objective values). This is why the objective value for f_2 occurs at the lowest value of x in the feasible region and the highest value of y among those points at the lowest value of x (x is weighted more than y in the objective function).

Problem 3

a) The objective is to minimize the total costs of producing and transporting each commodity *k* between all possible *i,j* supplier and demand locations. These costs include procurement costs of producing *k* at supplier *i* as well as transportation costs of taking commodity *k* from *i* to *j*.

The first constraint states that for all commodities, the total amount of commodity *k* (in units of 100 grams) transported to location *i* across all suppliers *j* must be equal to the total amount in grams of commodity *k* required to feed all the people at location *i* for the entire time period, which is given by the Ration amount per day.

The second constraint states that for each nutrient, the total nutritional value for nutrient *l* across all Rations of commodity *k* must be at least the nutritional requirement per person per day for nutrient *l*.

The third constraint states that the amount of commodity *k* transported from *i* to *j* as well as the Ration of commodity *k* must be non-negative.

b) This constraint says that the total cost of procuring and transporting commodities that were produced by regional suppliers to various locations must be at least half the total cost of procuring and transporting the commodities produced across all suppliers -- i.e. most of the money spent in shipping commodities must be associated with regional suppliers. We might want most of the costs concentrated regionally if, for example, we feel that for equity reasons it is good to support regional suppliers as opposed to international ones.