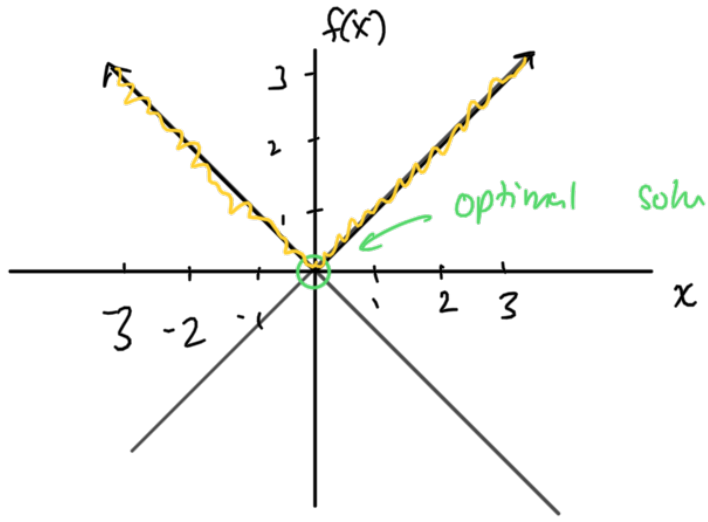
Recitation 3 Notes

① $\min\limits_{x} \max\{-x, x\}$

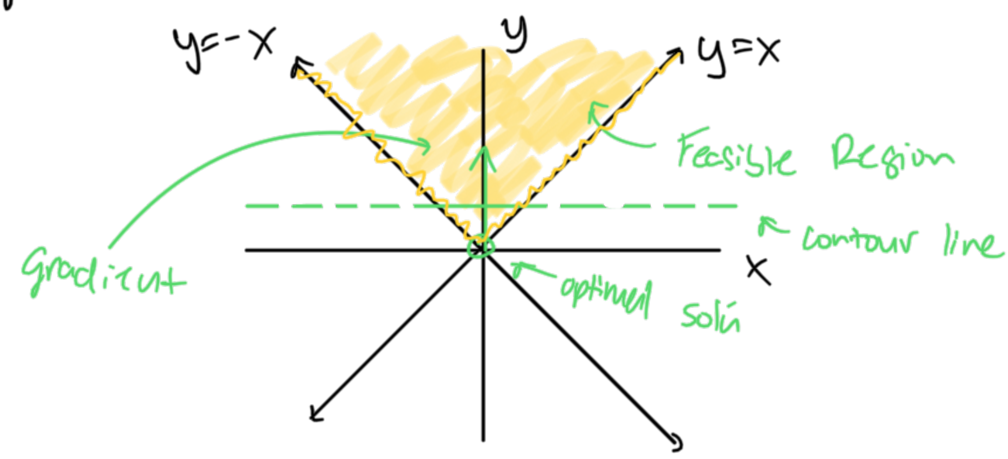
Graph of $f(x) = \max\{-x, x\}$:



Reformulation is that

$$\min_{x} \max\{-x, x\} = \begin{array}{ll} \min\limits_{x} & y \\ \text{s.t.} & y \geq x \\ & y \geq -x \end{array}$$

Drawing of Linear Optimization problem is:

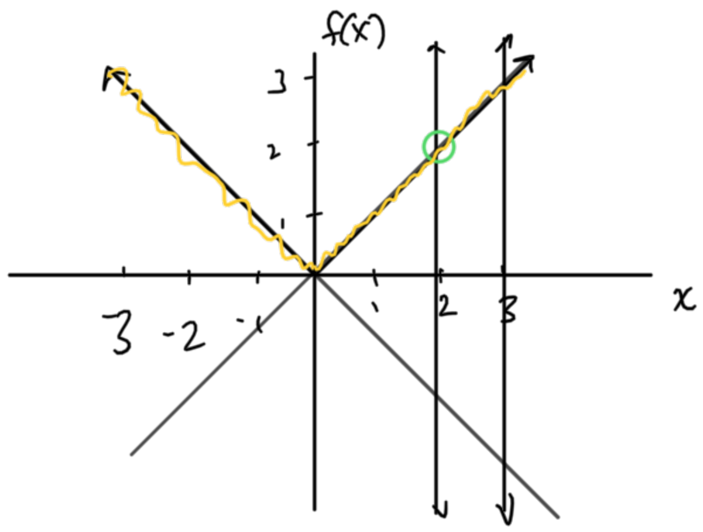


* the objective ensures any solution will be on the original set: $\{(x, f(x)) : x \in \mathbb{R}\}$!

② If you have any additional linear constraints on x to start w/ you are still okay:

$$\begin{array}{ll} \min\limits_{x} & \max\{-x, x\} \\ \text{s.t.} & x \leq 3 \\ & x \geq 2 \end{array}$$

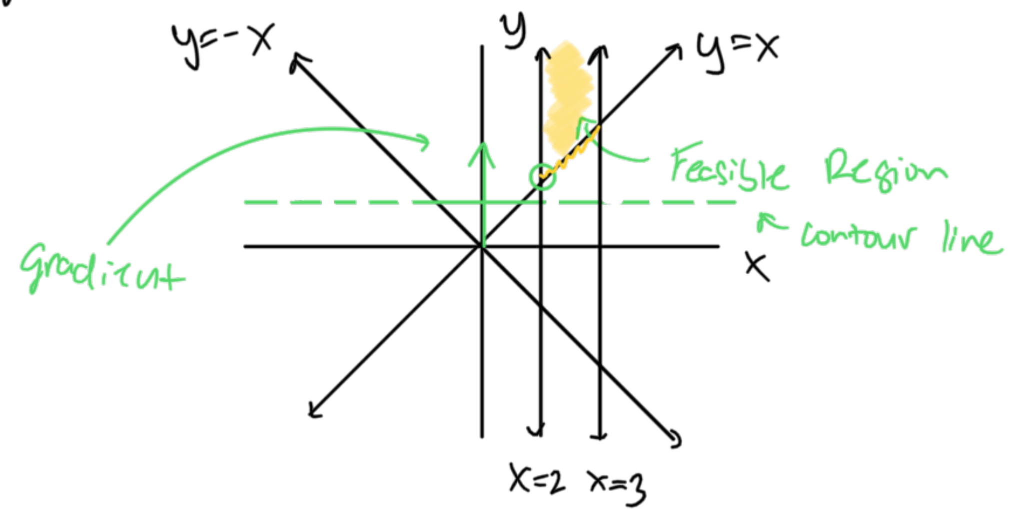
Graph of $f(x) = \max\{-x, x\}$:



Reformulation is that

$$\min_x \max\{-x, x\} = \min_x y \quad \text{s.t. } y \ge x,\ y \ge -x,\ x \le 3,\ x \ge 2$$

Drawing of Linear Optimization reformulation is:



③ The general case:

$$\begin{aligned}\min_x \ &\max_{k=1,\dots,K} \{a_k' x + b_k\} & = \quad \min_{x,y} \ & y \\ \text{s.t.} \ & x \in \text{Polyhedron} & \text{s.t.} \ & y \ge a_k' x + b_k \quad \forall k \\ & & & x \in \text{Polyhedron}\end{aligned}$$

or

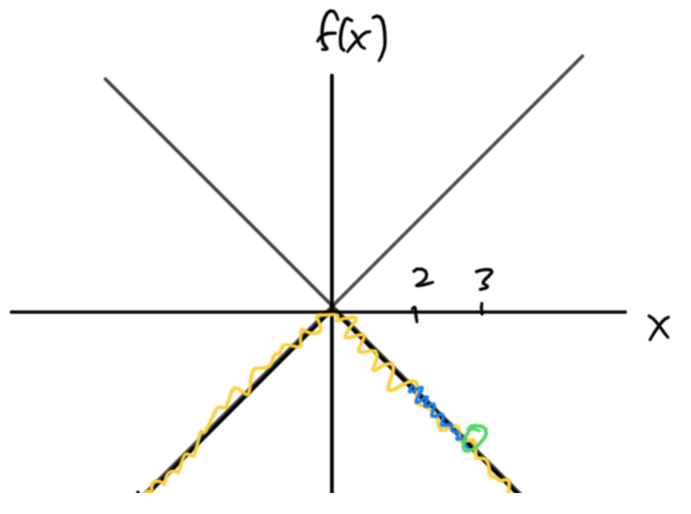
$$\begin{aligned}\max_x \ &\min_{k=1,\dots,K} \{a_k' x + b_k\} & = \quad \max_{x,y} \ & y \\ \text{s.t.} \ & x \in \text{Polyhedron} & & y \le a_k' x + b_k \quad \forall k \\ & & & x \in \text{Polyhedron}\end{aligned}$$

does the same thing.

④ Doesn't quite work for min-min or max-max:

$$\min_x \min\{x, -x\}$$

s.t. $2 \le x \le 3$

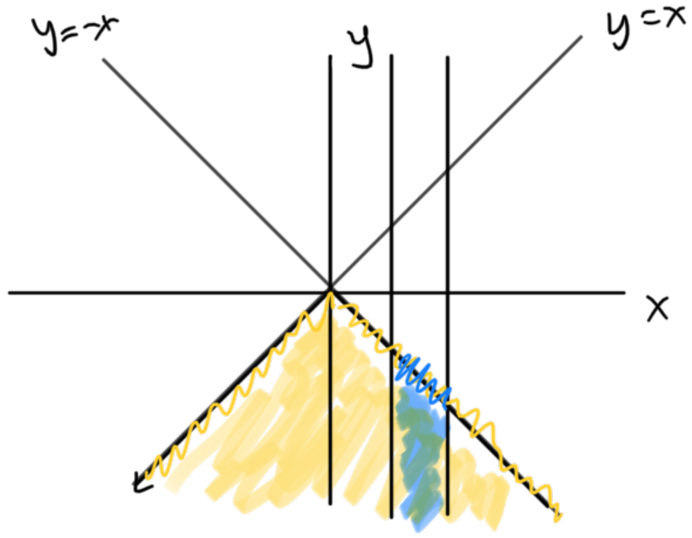


Unbounded

$x=3$ $f(x)=-3$

$$\min_{x} \ \min\{x, -x\} \overset{?}{=} \begin{array}{ll} \min\limits_{x,y} & y \\ \text{s.t.} & y \le x \\ & y \le -x \\ & 2 \le x \le 3 \end{array}$$

not a good reformulation



unbounded

unbounded