Activity 3

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1. Formal languages can be used to describe a variety of two-dimensional figures. Chain-code languages are defined on the alphabet $\Sigma = \{ u, d, r, l \}$, where these symbols stand for unit-length straight lines in the directions up, down, right, and left, respectively. An example of this notation is *urdl*, which stands for a square with sides of unit length. Let *L* be the set of all $w \in \{u, r, l, d\}^*$ that describe rectangles. Show that *L* is not a regular language.

$L_{urdl} = \{u^a r^b d^a l^b : a > 0, b > 0\}$

Pumping Lemma

1. Assume for contradiction that L_{urdl} is a regular language.
2. Given that L_{urdl} is infinite, we can apply Pumping Lemma to prove that L_{urdl} is regular.
3. Choose Pumping Lemma integer m>0
4. For string w in L, $u^{m+a} r^{m+b} d^{m+a} l^{m+b}$
5. For string w, we can define w = xyz such that $x = u^k$, $y = u^g$, $z = u^{((k+g)-(m+a))} r^{m+b} d^{m+a} l^{m+b}$, such that (k + g) <= m, g >= 1
6. $x = u^k$, $y = u^g$, and |xy|<=m, |y| >= 1
7. Thus, xy^jz in L for j = 3
8. Thus $u^{(2k + 3g - m - a)} r^{m+b} d^{m+a} l^{m+b}$ in L
9. But $u^{(2k + 3g - m - a)} r^{m+b} d^{m+a} l^{m+b}$ **not** in L, therefore contradiction.
10. By contradiction of the Pumping Lemma, we can assert that L is not a regular language.
11. In the same manner, each of L_{dlur}, L_{rdlu}, L_{lurd} … are also not regular.

2. Prove or disprove the following statement: If L_1 and L_2 are nonregular languages, then $L_1 \cup L_2$ is also nonregular. A counter example is sufficient to disprove the statement.

Disproof by counterexample: Take language $L_1 = \{a^n : n = k^3\}$, which is not regular. Take the complement of that language L_{1_bar}. L_{1_bar} is also non regular by closure. However, when we take $L_1 \cup L_{1\_bar}$, we get every string possible with the alphabet of the languages. Therefore, we can state that $L_1 \cup L_{1\_bar}$ $\{a^*\}$, which is regular.