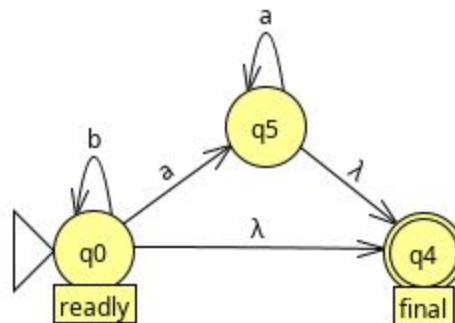


CS 321H HW1 - Lyell Read

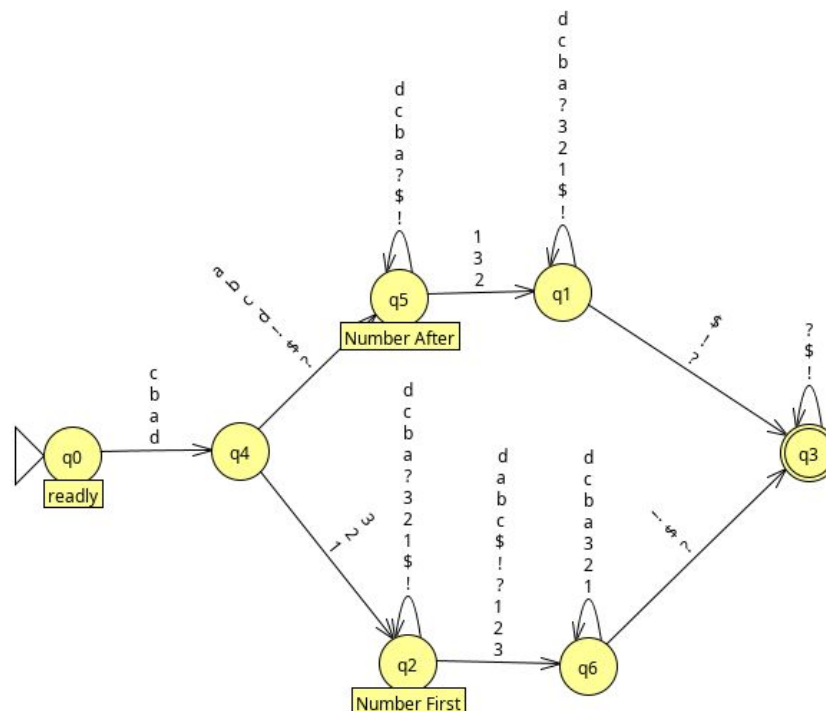
Submit written solutions to all questions as a pdf file in Canvas. Use a word processor or text editor, hand written assignments will receive a 0. For problems 1, 2, & 3 also submit JFLAP files separately in Canvas (do NOT zip).

1) (5 pts) Construct an NFA with three states that accepts the language

$$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}.$$

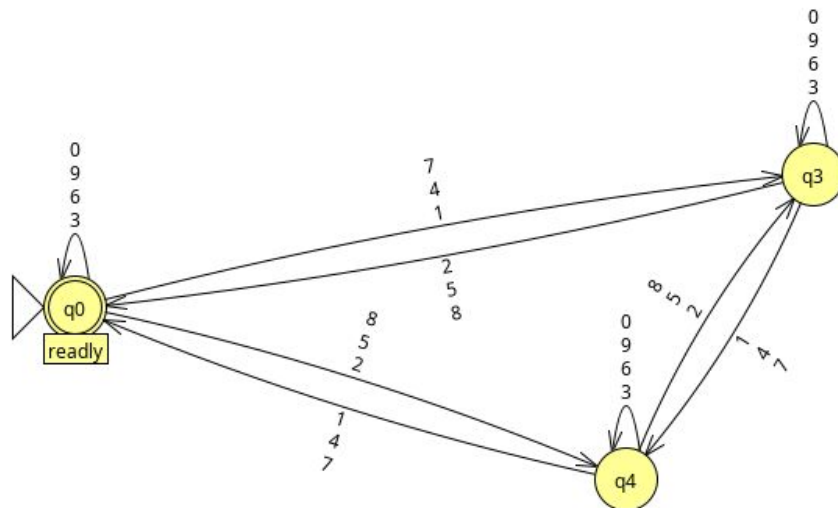


2) (5 pts) Suppose that a bank only permits passwords that are strings from the alphabet $\Sigma = \{a, b, c, d, 1, 2, 3, !, ?, \$\}$ that follow the rules: The length is at least four characters. It begins with a letter $\{a, b, c, d\}$. Must contain at least one digit $\{1, 2, 3\}$. Must end with a “special character” $\{!, ?, \$\}$. The set of legal passwords forms a regular language L . Construct an NFA or DFA for L . The table above contains some sample test cases.

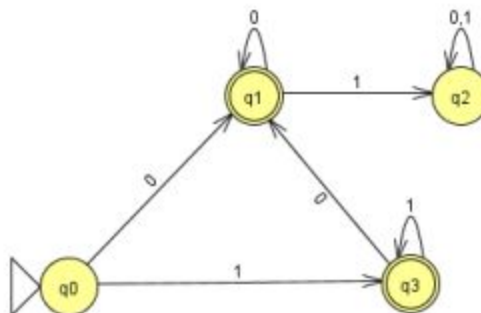


3) (5 pts) A number is divisible by 3 if the sum of its digits is divisible by 3. Construct a DFA M that accepts a base-10 number if it is divisible by 3.

That is $L(M) = \{ w \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^* : w \bmod 3 = 0 \}$. Hint: $\lambda \notin L(M)$.



4) (5 pts) For the DFA M below, give its formal definition as a quintuple. Verbally describe the language, $L(M)$, accepted by M.



Alphabet (Σ) = {0, 1}

States (Q) = {q0, q1, q2, q3}

Initial State (q_0) = q0

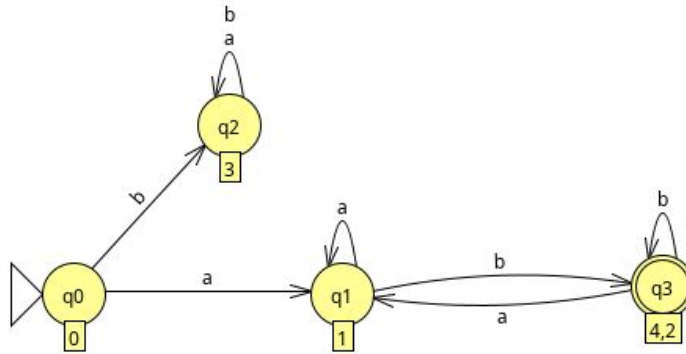
Final States (F) = {q1, q3}

Transition Function (δ) = { $\delta(q_0, 0) = q_1$, $\delta(q_0, 1) = q_3$, $\delta(q_1, 0) = q_1$, $\delta(q_1, 1) = q_2$, $\delta(q_2, 0) = q_2$, $\delta(q_2, 1) = q_2$, $\delta(q_3, 0) = q_1$, $\delta(q_3, 1) = q_3$ }

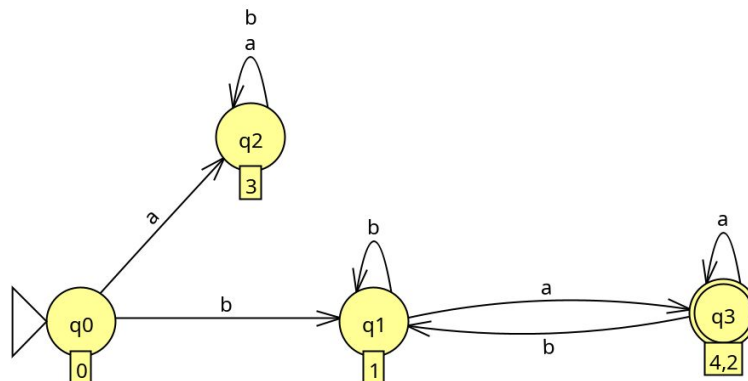
$M = ($
 $\{q_0, q_1, q_2, q_3\},$
 $\{0, 1\},$
 $\{\delta(q_0, 0) = q_1, \delta(q_0, 1) = q_3, \delta(q_1, 0) = q_1, \delta(q_1, 1) = q_2, \delta(q_2, 0) = q_2, \delta(q_2, 1) = q_2,$
 $\delta(q_3, 0) = q_1, \delta(q_3, 1) = q_3\},$
 $q_0,$
 $\{q_1, q_3\}$
 $)$

5) (5 pts) Let $L = \{w \in \{a, b\}^* : w \text{ begins with an "a" and ends with a "b"}\}$. a) Prove that L is a regular language. b) Prove that \bar{L} is a regular language.

Language L is a regular language if we can define DFA for L that accepts all strings in L :



Language \bar{L} is a regular language as we can define a DFA that accepts all strings in \bar{L} :



6) (5 pts) Prove that the class of regular languages is closed under complementation. That is if L is a regular language then \bar{L} is also a regular language. Hint: Use the DFA M that recognizes L to construct a DFA \bar{M} that recognizes \bar{L} .

Given a quintuple M representing a DFA for language L , where $M = (Q, \Sigma, \delta, q_0, F)$, we can generate a DFA that accepts everything not accepted by language L (creating a DFA that accepts language \bar{L}) by making all states final states, and making the final states normal states. This is achieved in quintuple $M_1 = (Q, \Sigma, \delta, q_0, Q-F)$, where all non-final states become final, and all final states become non-final. M_1 is a DFA for language \bar{L} , as it accepts everything that was not accepted in M , the DFA for L .