# CS 321 Activity 4

Submit one per group.

1. Construct a context-free grammar for the following language.

   $L = \{ a^n b^m c^{2n} : m \geq 0, n > 0\} \cup \{b^n a^n : n > 0\}$

   $S \to S1 \mid S2$

   $S1 \to aBcc$

   $B \to aBcc \mid C$

   $C \to bC \mid b \mid \lambda$

   $S2 \to bXa$

   $X \to bXa \mid ba \mid \lambda$

2. Consider the following grammar G = ({S, A}, {a, b}, S, P} where P is defined below

   $S \to SS \mid AAA \mid \lambda$

   $A \to aA \mid Aa \mid b$

   a. Describe the language generated by this grammar.
      a. L = { w in {a, b}* : $N_b(w)$ % 3 = 0 }
   b. Give a left-most derivation for the terminal string abbaba.
      a. $S \to AAA \to aAAA \to abAA \to abbA \to abbaA \to abbaAa \to abbaba$
   c. Show that the grammar is ambiguous by exhibiting two distinct derivation trees for some string $w \in L(G)$.
      a. baabb
      b. $S \to AAA \to bAA \to baAA \to baaAA \to baabA \to baabb$
      c. $S \to AAA \to AaAA \to baAA \to baaAA \to baabA \to baabb$

3. Let $L = \{ a^n b^n : n \geq 0 \}$

   a. Show that $L^2$ is a context-free language.

   $L^2 = \{ a^n b^n a^m b^m : n \geq 0, m \geq 0\}$

   $S \to XX$

   $X \to aXb \mid \lambda$

Now that we have defined a context free grammar for $L^2$, we may say that it is a context free language.

b. Show that $L^*$ is a context-free language.

$L^* = \{ (a^n b^n : n \geq 0)^* \}$

$S \rightarrow SS \mid X \mid \lambda$

$X \rightarrow aXb \mid \lambda$

EXTRA CREDIT: Prove that context-free languages are closed under union.

- Each context free language has an associated context free grammar.
- For two context free languages $L_1$, $L_2$, we defined context free grammars $G_1(v_1, t_1, G1, p_1)$, $G_2(v_2, t_2, G2, p_2)$.
- To show that $L_1$, $L_2$ are closed under union, we must generate $G_{12}$ that is a context free grammar for $L_1$ and $L_2$.
- We do this as follows:

  $S \rightarrow G1 \mid G2$
- Therefore, we can assert that any two context free languages are closed under union, by creating a context free grammar that describes them both at once, creating thereby a context free language $L_{12}$ that is the union of context free languages $L_1$ and $L_2$.