CS 321 HW3 - Lyell Read

Submit a pdf in Canvas. Use a word processor and/or text editor. (30 pts, 6 pts each)

Determine whether or not the following languages are regular. If the language is regular then give a regular expression for the language. Otherwise, use the pumping lemma for regular languages or closure properties to prove the language is not regular.

1) L = { $a^n b^k$: k ≤ n ≤ 2k}

Not Regular.

1. Assume for contradiction that L is a regular language
2. Given that L is infinite, we can apply Pumping Lemma to prove that L is regular.
3. Choose Pumping Lemma integer m>0
4. For string w in L, $a^n b^{m+k}$
5. For string w, we can define w = xyz such that x = a^q, y = $a^{(n-q)}$, z = b^{m+k}, such that |xy| <=m and |y|>=1 (this implies that q < n).
6. Thus, xy^jz in L for j = (4 * (m+k))
7. Thus a in L
8. But $a^{(q+4m+4k)}b^{(m+k)}$ not in L as it is not the case that (m+k) <= (q+4m+4k) <= 2(m+k), therefore contradiction.
9. By contradiction of the Pumping Lemma, we can assert that L is not a regular language.

2) L = { $b^n a^k$: n > 0, k > 0 } ∪ { $a^n b^k$: k > 0, n > 0}

This language is regular, regular expression for the language:

(bb*aa*) + (aa*bb*)

3) L = { a^n : n=3k for some k ≥ 0}

This language is regular, regular expression for the language:

(aaa)*

4) L = { a^n : n=k^3 for some k ≥ 0}

Not Regular.

1. Assume for contradiction that L is a regular language
2. Given that L is infinite, we can apply Pumping Lemma to prove that L is regular.
3. Choose Pumping Lemma integer m>0

4. For string w in L, a^{n+m}
5. For string w, we can define w = xyz such that x = a, y = a, z = $a^{(n+m-2)}$, such that |xy| <=m and |y|>=1 (this implies that n >= 1). In this case, we specify that m<=($\sqrt[3]{(n)}$+1)^3-1.
6. Thus, xy^jz in L for j = 1
7. Thus $a^{(n+m+1)}$ in L
8. But $a^{(n+m+1)}$ not in L as it is not the case that $\sqrt[3]{(n+m+1)}$ is an integer, because $\sqrt[3]{(n)}$ is and (m+1)<($\sqrt[3]{(n)}$+1)^3 (m+1 will always be less than the next cube), therefore contradiction.
9. By contradiction of the Pumping Lemma, we can assert that L is not a regular language.

5) L = { w : $n_a(w) > n_b(w)$, w ∈ {a, b}* }

Not Regular.

1. Assume for contradiction that L is a regular language
2. Given that L is infinite, we can apply Pumping Lemma to prove that L is regular.
3. Choose Pumping Lemma integer m>0
4. We find string w in L
5. For string w, we can define w = xyz such that x = w[0], y = w[1], z = w[2:], such that |xy| <=m and |y|>=1, and y = b.
6. Thus, w' = xy^jz in L for j = ($n_a(w)-n_b(w)+1$)
7. Thus w' in L
8. But w' not in L as the number of 'b' exceeds the number of 'a's, as we pumped enough times to make $n_b(w') > n_a(w')$, therefore contradiction.
9. By contradiction of the Pumping Lemma, we can assert that L is not a regular language.