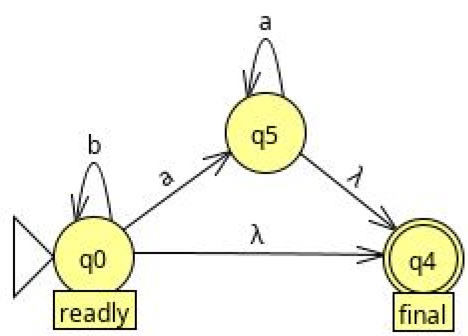
CS 321H HW1 - Lyell Read

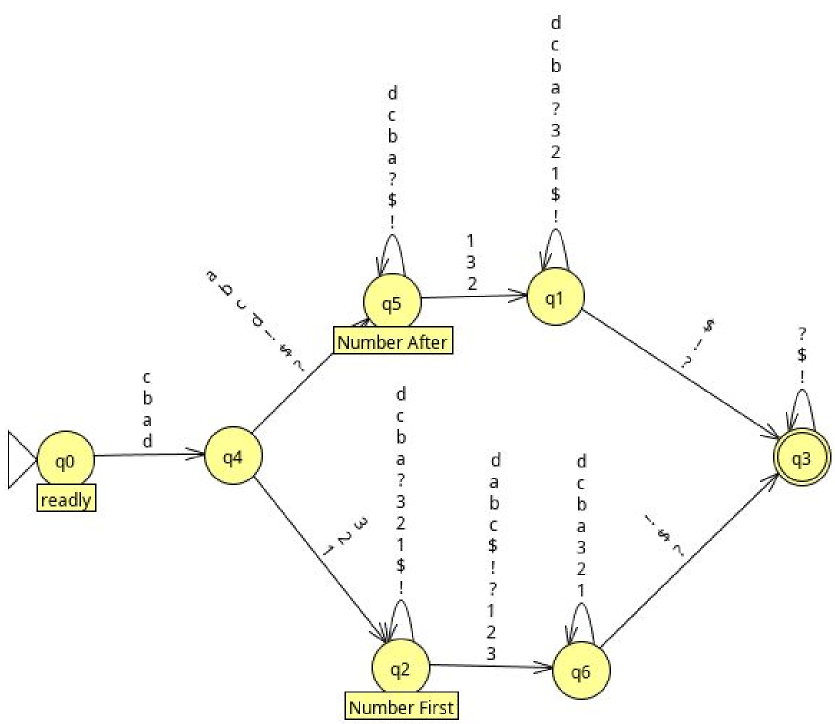
Submit written solutions to all questions as a pdf file in Canvas. Use a word processor or text editor, hand written assignments will receive a 0. For problems 1, 2, & 3 also submit JFLAP files separately in Canvas (do NOT zip).

1) (5 pts) Construct an NFA with three states that accepts the language

$$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}.$$

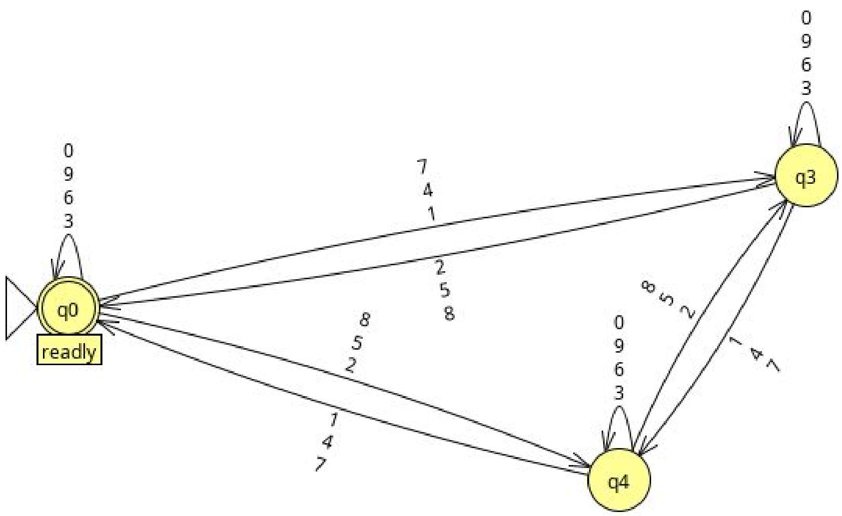


2) (5 pts) Suppose that a bank only permits passwords that are strings from the alphabet = {a, b, c, d, 1, 2, 3, !, ?, $} that follow the rules: The length is at least four characters. It begins with a letter {a, b, c, d}. Must contain at least one digit {1, 2, 3}. Must end with a “special character” { !, ?, $ }. The set of legal passwords forms a regular language L. Construct an NFA or DFA for L. The table above contains some sample test cases.

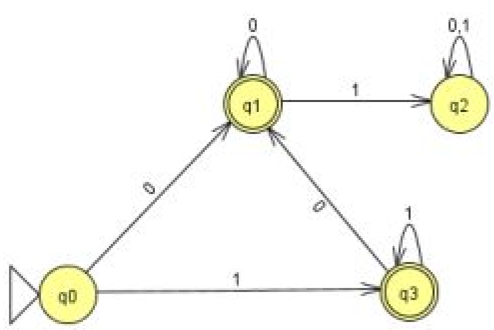


3) (5 pts) A number is divisible by 3 if the sum of its digits is divisible by 3. Construct a DFA M that accepts a base-10 number if it is divisible by 3.

That is $L(M) = \{ w \in \{0, 1,2, 3, 4, 5, 6, 7, 8, 9\}^* : w \bmod 3 = 0 \}$. Hint: $\lambda \notin L(M)$.



4) (5 pts) For the DFA M below, give its formal definition as a quintuple. Verbally describe the language, L(M), accepted by M.



Alphabet (Σ) = {0, 1}
States (Q) = {q0, q1, q2, q3}
Initial State (q_0) = q0
Final States (F) = {q1, q3}
Transition Function (δ) = {δ(q0,0)=q1, δ(q0,1)=q3, δ(q1,0)=q1, δ(q1,1)=q2, δ(q2,0)=q2, δ(q2,2)=q2, δ(q3,0)=q1, δ(q3,1)=q3}

M=(

{q0, q1, q2, q3},

{0, 1},

{δ(q0,0)=q1, δ(q0,1)=q3, δ(q1,0)=q1, δ(q1,1)=q2, δ(q2,0)=q2, δ(q2,2)=q2, δ(q3,0)=q1, δ(q3,1)=q3},

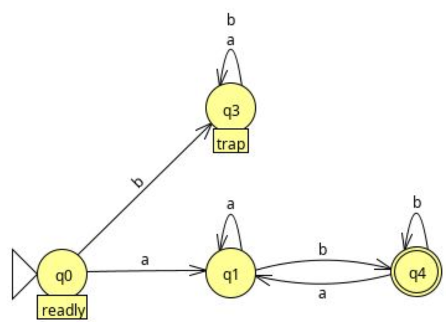
q0,

{q1, q3}

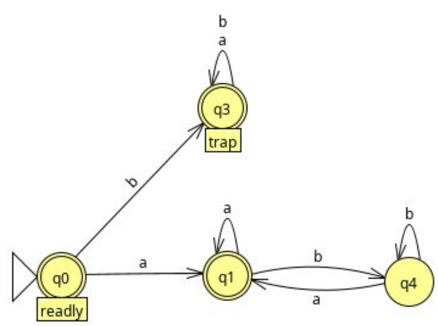
)

5) (5 pts) Let L = {w ∈ {a, b}*:w begins with an "a" and ends with a "b" }. a) Prove that L is a regular language. b) Prove that $\overline{L}$ is a regular language.

Language L is a regular language if we can define DFA for L that accepts all strings in L:



Language $\overline{L}$ is a regular language as we can define a DFA that accepts all strings in $\overline{L}$:



6) (5 pts) Prove that the class of regular languages is closed under complementation. That is if L is a regular language then $\overline{L}$ is also a regular language. Hint: Use the DFA M that recognizes L to construct a DFA $\overline{M}$ that recognizes $\overline{L}$.

Given a quintuple M representing a DFA for language L, where M = (Q, Σ, δ, q_0, F), we can generate a DFA that accepts everything not accepted by language L (creating a DFA that accepts language $\overline{L}$) by making all states final states, and making the final states normal states. This is achieved in quintuple M_1 = (Q, Σ, δ, q_0, Q-F), where all non-final states become final, and all final states become non-final. M_1 is a DFA for language $\bar{L}$, as it accepts everything that was not accepted in M, the DFA for L.