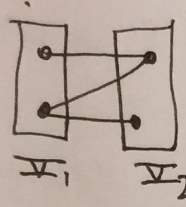
MTH 231 LEC 23

Bipartite Graph

- No edges within V_1 or V_2
- Ex:



$V_1 \cap V_2 = \emptyset$

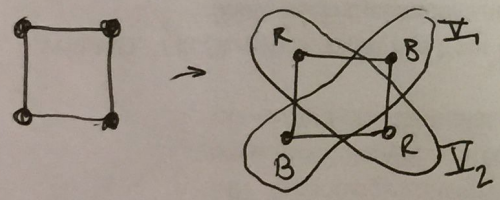
~~FINAL~~

Graph Coloring Theorem

- A simple graph is bipartite if it's possible to color the vertices either red or blue so that no red vertices are connected and no blue vertices are connect.

↳ "connected" = "share an edge"

Ex:



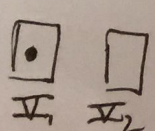
Theorem

- If C_n n = odd $(C_2 =$ —, $C_4 =$ □$)$ then C_n is not bipartite.

Theorem

- If K_n n = anything except K_1, K_2 $(K_2 =$ —$)$ then K_n is not bipartite

- K_1:



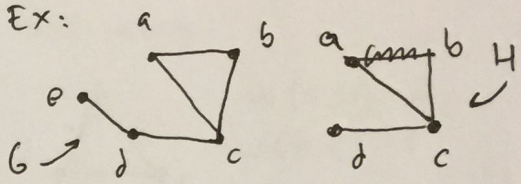
$V_1 \cup V_2 = V$

$V_1 \cap V_2 = \emptyset$

Subgraphs

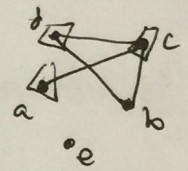
- Subgraph of a graph $G = (V, E)$ is a graph $(W, F) = H$ where $W \subseteq V$ and $F \subseteq E$.

Ex:



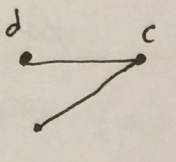
Induced Subgraph

- The subgraph induced by $W \subseteq V$ for $G = (V, E)$ is the graph $H = (W, F)$ where $\{u, v\} \in F$ iff $v, u \in W$ and $\{u, v\} \in E$



$W = \{a, d, c\}$

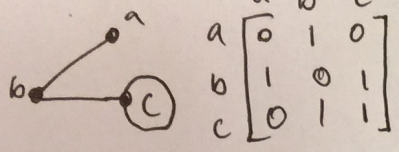
INDUCED SUBGRAPH



Graph Theory convention

- If you remove an vertex, you should remove all the edges that are incident to it.

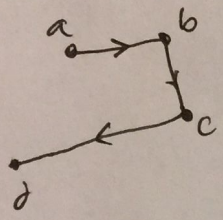
Adjacency Matrix



	a	b	c
a	0	1	0
b	1	0	1
c	0	1	1

- Put a 1 if the ~~vertices~~ verticies connect
- put a 0 if the verticies don't connect

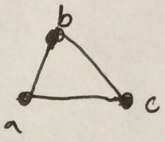
Adjacency Matrix for Directed Graph



	a	b	c	d
a	0	1	0	0
b	0	0	1	0
c	0	0	0	1
d	0	0	0	0

Def: (for undirected's that are simple
- A path is a list of verticies $v_1, v_2 v_3 \ldots v_n$ where the edges are in E

Weights
- assigning:

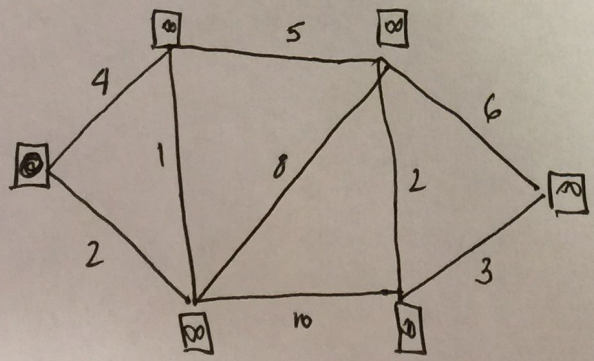


$w(a,b) = 2$
$w(b,c) = 3$
$w(c,d) = 4 \ldots$

Dikstra's Algorithm

- finds the path with the least total weight between 2 verticies. Computation time $O(|V|^2)$

✱FINAL



- for i..n
  label all verticies = ∞ (in box)
- Label starting vertex = 0 (a)
- Define $S = \emptyset$
- while $z \notin S$:

  u = a vertex not in S with minimal label

  $S = S \cup \{u\}$

then find all paths from current point and label them with their total weight

find the lowest labeled vertex not already passed through