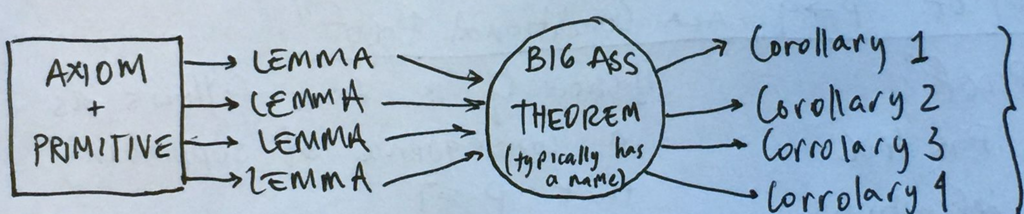
MTH 231 Lecture 7

1.7

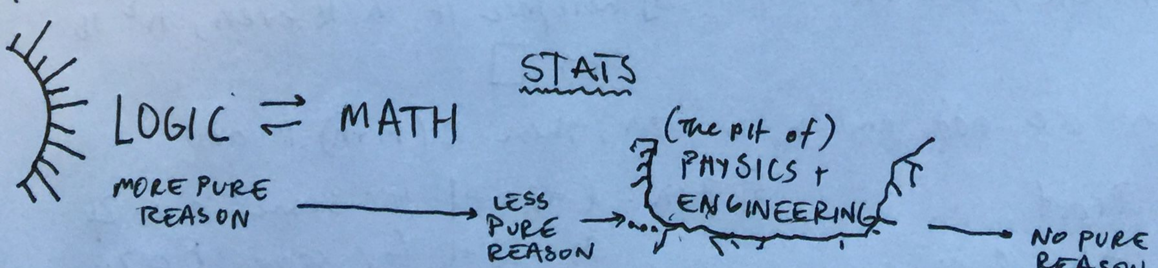
Vocabulary of Maths

- Primitive: an object of a system that cannot be broken into / defined by more granular elements.
- Axiom: a fundamental proposition that is assumed to be true, and cannot be deduced from other, more basic, propositions
- Theorems: A true proposition that can be deduced from axioms or other theorems
- Lemma: a minor theorem used to prove a larger theorem
- Corrolary: a theorem that follows easily from a larger theorem



Theorems

- Typically take the form $\underline{\forall x (P(x) \to Q(x))}$
- To prove a theorem, we'll generally establish that $P(x) \to Q(x)$ for an arbitrary x in domain. then we use arbitrary generalization that implies that $\forall x (P(x) \to Q(x))$.
- A simplification of $P(x) \to Q(x) \approx p \to q \mid p = P(x) ; q = Q(x)$



Takeaway

- as long as you work with the most general x, you'll be OK

Proofs

- E.g. Integer n is even provided there is an integer K such that $n = 2K$
- E.g. Integer n is odd provided there is an integer K such that $n = 2K+1$
- E.g. 4 is even because $4 = 2(2)$ and 2 is an integer
- Axioms Available for Use
  - $\longrightarrow x, y \in \mathbb{Z} \Rightarrow x + y \in \mathbb{Z}$
  - $\longrightarrow x, y \in \mathbb{Z} \Rightarrow x \cdot y \in \mathbb{Z}$

|| $\mathbb{Z}$ = ~~Is~~ integer (all ints from ($-\infty$ to ∞)

E.g: for an even integer n; if $\underbrace{n \text{ is even}}_{p}$, then $\underbrace{n^2 \text{ is even}}_{q}$

↳ show $p \to q$

|| NOTE: FOLLOW THESE STEPS

PROOF STEPS

Direct proof of $p \to q$ AKA "Conditional Proof"

1] "Direct Proof"

2] Suppose P is true

3] Unpack defenitions

4] Show q is true follows as a consequence of supposing that $p \equiv T$

5] Conclude $p \to q$ $\square$

1] Direct Proof

2] Suppose n is even

3] Because n is even, $n = 2K$ for some integer K

4] Then, $n = 2K$, $n^2 = 4K^2$

→ Since K is an integer, and $n^2 = 4K^2 = n^2 = 2(2K^2)$, by the axioms, then $2K^2$ is an integer, therefore n^2 must be even

5] Therefore if n is even, n^2 is even $\square$

E.g: l and m are odd, and n is even, then $l(m+n)$ is odd

1] Direct Proof

2] Suppose that l, m are odd and n is even

3] then: $l = 2a+1$ for some $a \in \mathbb{Z}$
$m = 2b+1$ for some $b \in \mathbb{Z}$
$n = 2c$ for some $c \in \mathbb{Z}$

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4] Then $\ell(m+n) = (2a+1)((2b+1)+2c)$

$$= (2a+1)(2b+2c+1)$$
$$= 4ab + 4ac + 2a + 2b + 2c + 1$$
$$= 2(2ab + 2ac + a + b + c) + 1$$

Since $a, b, c \in \mathbb{Z}$, then $(2ab + 2ac + a + b + c)$ is an integer by axioms. Then $\ell(m+n)$ is odd

5] $\therefore$ If ℓ, m are odd, then $\ell(m+n)$ is odd $\square$

"there exists forbidden knowledge to mankind"

"The other beings that have access to that knowledge have their own sets of forbidden knowledge"

"forbidden knowledge is only accessible through oracles, of varying godliness..."