

# MTH 231 LECTURE 18

## Division Rule

- If there are  $n$  ways to complete a proc from start to finish by first completing subtask 1 in  $a$  ways and subtask 2 in  $b$  ways, where each of the  $b$  ways is redundant, then there are  $a = \frac{n}{b}$  unique ways to complete the task

Ex:  $accc = 4!$  ways to place A, but  $3!$  ways to put  $ccc$  that are redundant.  $n = 4! \cancel{=}$

$$a = \frac{4!}{3!}$$

Ex: How Many Ways can we arrange  $aabb$

$4!$  ways to arrange all

$2!$  rearranging  $a$ ,  $2!$  arranging  $b$

$$= \frac{4!}{2!2!}$$

Ex: How Many Ways to Arrange

- 1]  $aaabbb = (5! / 2!3!)$
  - 2]  $aaabcc = (6! / 2!3!)$
  - 3]  $aabbcc = (6! / 2!2!2!)$
  - 4]  $aaabbbccdd = (10! / 3!3!2!2!)$
- multinomial coefficients  
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## Permutations

- Permutation is an ordered list of distinct elements.  $\{a, b, c, d\}$

- the number of permutations of  $k$  objects from  $n$  is  $P(n, k) = \frac{n!}{(n-k)!}$   
 $= \frac{n!}{(n-k)!}$

## Combinations

- number of combinations of size  $k$  from  $n$  objects is denoted  $C(n, k)$  or  $\binom{n}{k} \rightarrow n \text{ choose } k$   
 $= \binom{n}{k} = \frac{n!}{k!(n-k)!}$

Ex: How many 4-card hands are there?

$$\binom{52}{4} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot \cancel{48!}}{4! \cdot \cancel{(48!)}}$$

NOTE  $\binom{n}{k} = \binom{n}{n-k}$

NOTE: ALSO  $nPk, nCk$