# MTH 231 LECTURE 3

## Truth Tables

Ex: Evaluate $(p \to q) \wedge (q \to p)$ :

| p | q | $(p\to q) \wedge (q \to p)$ | | | | |
|---|---|---|---|---|---|---|
| T | T | T | ^ | T | = | T |
| F | T | T | ^ | F | = | F |
| T | F | F | ^ | T | = | F |
| F | F | T | ^ | T | = | T |

Note: the amount of rows in a TT with n "variables" is $2^n$ rows.

Logical Order of operations:

- Negation ---> And ---> Or ---> Impliation ---> BiDirect. Imp.
- Note: ( ) come first (as always). As such, use parenthesies

Associativity

- Doesn't matter where you put parenthisies – evaluates same.
- i.e. addition, multiplication...

## CH1.3

Equivalence / Compound Propositions

- A compound proposition $f(p_1, p_2 .. p_n)$ that is true [regardless of anything] for any propositions $p_1, p_2 ... p_n$ is called a tautology

Ex: Tautology: $p \vee \neg p$ (AKA P||!P) :

| p | $p \vee \neg p$ | | |
|---|---|---|---|
| T | T | F | $\equiv$ T |
| F | F | T | $\equiv$ T |

- A compound proposition that is always false is a controdiction

Ex: controdiction: $p \wedge \neg p$ ⟶ :

| p | $p \wedge \neg p$ | | |
|---|---|---|---|
| T | T | F | = F |
| F | F | T | = F |

- If compound proposition $f(p_1, p_2 ... p_n)$ and $f_a(p_1, p_2 .. p_n)$ have the same truth values for any choice of $p_1 .. p_n$, then $f$ and $f_a$ are logically equivalent or equivalent
  - → Their truth tables are the same
  - → This is the same as $f(...) \leftrightarrow f_a(..)$ is a tautology
  - → This is denoted by $f \equiv f_a$ or $f \Leftrightarrow f_a$

Ex: Prove $\underbrace{p \to q \equiv \neg p \vee q}_{\text{"claim"}}$

| $p$ | $q$ | $p \to q$ | $(\neg p) \vee q$ | |
|---|---|---|---|---|
| T | T | $\boxed{T}$ | F ∨ T = | $\boxed{T}$ |
| F | T | $\boxed{T}$ | T ∨ T = | $\boxed{T}$ |
| T | F | $\boxed{F}$ | F ∨ F = | $\boxed{F}$ |
| F | F | $\boxed{T}$ | T ∨ T = | $\boxed{T}$ |

Proved

$\therefore p \to q \equiv \neg p \vee q$

## Contrapositive & Converse

- Converse of $p \to q$ is $\underline{q \to p}$
- Inverse of $p \to q$ is $\underline{\neg p \to \neg q}$
- Contrapositive of $p \to q$ is $\underline{\neg q \to \neg p}$

| $p$ | $q$ | $p \to q$ | $q \to p$ | $\neg p \to \neg q$ | | | $\neg q \to \neg p$ | | |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | F | $\boxed{T}$ | F | F | $\boxed{T}$ | F |
| F | T | T | F | T | $\boxed{F}$ | F | F | $\boxed{T}$ | T |
| T | F | F | T | F | $\boxed{T}$ | T | T | $\boxed{F}$ | F |
| F | F | T | T | T | $\boxed{T}$ | T | T | $\boxed{T}$ | T |

- $p \to q \equiv \neg q \to \neg p$ ≈ Original ≡ contrapositive

## De Morgan's Law

1] $\neg(p \wedge q) \equiv \neg p \vee \neg q$

| $p$ | $q$ | $\neg(p \wedge q)$ | | $\neg p \vee \neg q$ | | |
|---|---|---|---|---|---|---|
| T | T | T | $\boxed{F}$ | F | $\boxed{F}$ | F |
| F | T | F | $\boxed{T}$ | T | $\boxed{T}$ | F |
| T | F | F | $\boxed{T}$ | F | $\boxed{T}$ | T |
| F | F | F | $\boxed{T}$ | T | $\boxed{T}$ | T |

2] $\neg(p \vee q) \equiv \neg p \wedge \neg q$

| $p$ | $q$ | $\neg(p \vee q)$ | | $\neg p \wedge \neg q$ | |
|---|---|---|---|---|---|
| T | T | $\boxed{F}$ | T | F ∧ F | $\boxed{F}$ |
| F | T | $\boxed{F}$ | T | T ∧ F | $\boxed{F}$ |
| T | F | $\boxed{F}$ | T | F ∧ T | $\boxed{F}$ |
| F | F | $\boxed{T}$ | F | T ∧ T | $\boxed{T}$ |

Ex: Let **T** be a tautology, **T** $\equiv p \vee \neg p$
let **F** be an arbitrary contradiction **F** Then **T** ∨ **F** = **T**

## Truths / Givens

- $p \wedge q \equiv q \wedge p$ - $p \vee q \equiv q \vee p$ - $p \leftrightarrow q \equiv q \leftrightarrow p$ - $\neg(\neg p) \equiv p$
- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- $p \to q \equiv \neg p \vee q$ - $p \to q \equiv p \wedge \neg q$ - $p \leftrightarrow q \equiv p \to q \wedge q \to p$
- $p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$