MTH 231 LEC 13

Summation Things for sets

- $\bigcup_{K=1}^{n} \{K\} = \{1\} \cup \{2\} \cup \dots \cup \{n\} = \{1,2,3,\dots n\}$
- $\bigcap_{K=1}^{n} \{K\} = \{1\} \cap \{2\} \cap \dots \cap \{n\} = \{\ \} = \emptyset$

Ex: $\bigcap_{K=1}^{n} [-K,K] = [-1,1]$ || [..] is a range, not set.

Adding Some Logic.

- $x \in \bigcup_{K=1}^{\infty} A_K :\Leftrightarrow$ x is in at least one of the A_K's.
  $\Leftrightarrow \exists x \geq 1$ such that $x \in A_K$
- $x \in \bigcap_{K=1}^{\infty} A_K :\Leftrightarrow$ x is in every single A_K
  $\Leftrightarrow \forall_K \geq 1$ such that $x \in A_K$
- $\bigcup_{K=1}^{\infty} A_K = A_1 \cup A_2 \cup \dots$ to ∞ || $\bigcup_{K=0}^{\infty} [-K,K] = \mathbb{R}$
- $\bigcap_{K=1}^{\infty} A_K = A_1 \cap A_2 \cap \dots$ to ∞ || $\bigcup_{K=0}^{\infty} \{K\} = \mathbb{N}$

|| 2-tuple is an ordered pair.
|| n-tuple is a usual tuple.

Ex Proof

I] 1] Direct Proof

2] Suppose $x \in \bigcup_{K=0}^{\infty} [-K,K]$

3] Then, $\exists K > 0$ such that $x \in [-K,K]$

4] Thus, x is a $\mathbb{R}$ such that $-K \leq x \leq K$
  ↳ in particular, $x \in \mathbb{R}$

5] Then $\bigcup_{K=0}^{\infty} [-K,K] \subseteq \mathbb{R}$

II] 1] Direct Proof

2] Suppose $x \in \mathbb{R}$

3] N/A

4] Then $K = \lceil x \rceil$
Then $x \in [-\lceil x \rceil, \lceil x \rceil]$
Since $\lceil x \rceil$ is some integer, $K \geq 0$,
$x \in \bigcup_{K=0}^{\infty} [-K,K]$

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5] Therefore, $\mathbb{R} \subseteq \bigcup_{K=0}^{\infty} [-K,K]$

3] $\therefore \mathbb{R} = \bigcup_{K=0}^{\infty} [-K,K]$ ☒

CH 2.1 Further Notes

Object	Sym	Ord?	Multiplicity matters?
SET	{...}	N	N
Multiset	$\{..\}_m$	N	Y
Tuples	(....)	Y	Y
n-Tuple	(n-obj)	Y	Y
2-Tuple	(a,b)	Y	Y

|| Note: ordered Pairs, and tuples use same Notation so read context.

Cartesian Product

- C.P. of A, B is "A×B"
  = A cross B
  $= A \times B := \{(x,y) \mid x \in A, y \in B\}$

Ex: $\mathbb{R} \times \mathbb{R} =$ x-y plane, also $= \mathbb{C}$

N-fold Cartesian Product

- $A_1 \times A_2 \times \dots \times A_n = \{(x_1, x_2, \dots, x_n) \mid x_i \in A_i,\ 1 \leq i \leq n\}$

Ex: $\{1\} \times \{a,b\} \times \{5\} = \{(1,a,b),(1,b,5)\}$

NEXT BAGE

PMI

- Principle of Mathematical Induction

Ex: What if we want to prove that $\forall n\, P(n)$ is true where $DOM(P) = \mathbb{N} = \{0, 1 \ldots \infty\}$

Basically what you do:

1] Show $P(0)$ is true

2] Prove that $\forall n\, (P(n) \rightarrow P(n+1))$

3] That generalizes to the whole set.

- Proof Template

1] PMI | PMI means Proof By Mathematical Induction

2] Base Case: show $P(0)$ true, manually

3] ~~Suppo~~ IH: Induction Hypothesis: Suppose $P(n) \equiv T$

4] Show that $P(n+1)$ follows from supposing $P(n)$

5] Conclude with a fancy ending like:

"$\therefore \forall n\, (P(n))$ is True <u>Boom!</u>"