MTH231 LEC12

Tricky Power Sets Note

- Note that $\emptyset \in P(A)$ because $\emptyset \subseteq A$

Set Operations

- Union of sets: $A \cup B$ = anything in a or/and b or logical or. $\{1,2\} \cup \{3,7,\emptyset\} = \{1,2,3,7,\emptyset\}$
- Intersection of sets: $A \cap B$: things only in a and b.
- Disjoint sets are sets where $A \cap B = \emptyset$ i.e. they do not share any elements
- Difference of A, B is "$A-B$" ~ "A/B" $= \{x \in A \mid x \notin B\}$
- Not, AKA "complement" $= \overline{A} = \{x \mid x \notin A\}$ ↳ required is the domain of discourse!

|| Note: $A - B = A \cap \overline{B}$ ↙↗ Ex

Set Equivalence

1] a set equality proof using equivalences and element chasing

2] $x \in A-B$ iff $x \in A$ and $x \notin B$

$\Leftrightarrow x \in A$ and $x \in \overline{B}$ } Def.'s

$\Leftrightarrow x \in (A \cap \overline{B})$

must check that maths work in both directions!

3] Thus, $A - B = A \cap \overline{B}$. $\square$

De Morgan's Law

- $\overline{A \cap B} = \overline{A} \cup \overline{B}$, $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Element Chasing Proof

NEXT COLUMN

$x \in \overline{A \cap B} \Leftrightarrow x \notin A \cap B$

$\Leftrightarrow \neg(x \in A \cap B)$

$\Leftrightarrow \neg(x \in A$ and $x \in B)$

$\Leftrightarrow (\neg x \in A)$ or $(\neg x \in B)$

$\Leftrightarrow x \notin A$ or $x \notin B$

$\Leftrightarrow x \in \overline{A}$ or $x \in \overline{B}$

$\Leftrightarrow x \in \overline{A} \cup \overline{B}$

$\overline{A \cap B} \Leftarrow \overline{A} \cup \overline{B}$ $\square$

Distribution Laws

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$