Summation: $\sum_{a}^{b} c = (b-a+1) \cdot c$ **Floor Function:** $\lfloor 1.1 \rfloor = 1$; $\lfloor -2.1 \rfloor = -3$ **Ceiling Function:** $\lceil -1.5 \rceil = -1$, $\lceil 1.5 \rceil = 2$ **Proposition:** Bool statement **Associativity:** when () don't matter. **Order of Logical Op's:** $\neg \wedge \vee \rightarrow \leftrightarrow$. **Tautology:** Always T. **Contradiction:** Always F. **Converse** $p \rightarrow q \Rightarrow q \rightarrow p$; **Inverse:** $p \rightarrow q \Rightarrow \neg p \rightarrow \neg q$ **Contrap:** $p \rightarrow q \Rightarrow \neg q \rightarrow \neg p$. contrapositive $\equiv$ original **DeMorgans Law:** $\neg(p \wedge q) \equiv \neg p \vee \neg q$ **Predicate:** $P(x) \mid P(x) = x$ is pos... or... **Dom(P):** Domain of discourse, for predicate P. **Qualifiers:** $\forall x, \exists x$. $\neg \forall x \Leftrightarrow \exists x$. $\neg \exists x \Leftrightarrow \forall x$. **Truth Table Rows:** $2^n \mid n =$ number of propositions **Primitive:** can't be broken down further. **Axiom:** fundamental true proposition. cant be derived. **Lemma:** minor theorem used to prove larger one. **Correlary:** Theorem that follows from larger one **Natural Numbers:** $\mathbb{N} = \{1, 2, 3..\}$ **Rationals** $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}$ and $b \neq 0\}$ **Integer:** $\mathbb{Z} = \{-2, -1, 0, 1...\}$ **Reals:** $\mathbb{R} = \{$all reals$\}$ **Complex** $\mathbb{C} = \{(x, y) \mid x, y \in \mathbb{R}\}$ **Subset:** $A \subseteq B$ if $x \in A \rightarrow x \in B$ **Set Equality** $A = B$ iff $A \subseteq B \wedge B \subseteq A$ **Proper Subsets:** $A \subset B$: $A \subseteq B \wedge A \neq B$ also written as $A \subsetneq B$. **Empty Set:** $\emptyset = \{\}$ **Cardinality:** $|A|$ number of distinct elements in set A. **Power Sets:** $\mathbb{P}(A)$ is all the subsets of A. there are $|\mathbb{P}(A)| = 2^{|A|}$ **Set Unions:** anything in a or b, combine both sets. **Set Intersection:** things in a and b **Disjoint Sets:** $A \cap B = \emptyset$. **Difference of sets:** $A - B$ or A / B: remove all elements of B in A: $\{x \mid x \in A$ and $x \notin B\}$

Truths, Givens... $p \rightarrow q \equiv \neg p \vee q \equiv$ ~~$\neg q \rightarrow \neg p$~~ $p \leftrightarrow q = p \rightarrow q \wedge q \rightarrow p$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$\emptyset \subseteq$ Pretty much anything...

6"

"there is 1 person everyone loves" $L(x,y) = x \heartsuit y$:
$\exists x (\forall y (L(y,x)) \wedge \forall z (\forall w (L(w,z) \rightarrow z = x)))$

There is one person that only ♡ themselv.
$\exists x \forall y (L(x,y) \leftrightarrow x = y)$

5"

I ~~will not use this on test~~ Robert

Not Operator for sets: $\bar{A} = \{x^{\epsilon} \mid x \notin A\}$ requires DoD. **DML set:** $\overline{A\cap B} = \bar{A} \cup \bar{B}$; $\overline{A\cup B} = \bar{A} \cap \bar{B}$ **Distribution Laws:** $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$; $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

Mass-Operators for sets: $\bigcup_{k=1}^{n} \{k\} = \{1\} \cup \{2\} \ldots \{n\} = \{1,2,..n\}$

$\bigcap_{k=1}^{n} \{k\} = \{1\} \cap \{2\} \cap \ldots \cap \{n\} = \emptyset$ **Multiset:** set where multiplicity doesn't matter (multiple identical elemets allowed)

Tuples: (...) ordered (yes), multiplicative (yes). 2-tuples = $(\cdot,\cdot)$ [1,2]

n-tuple = n-long tuple. **Cartesian (Cross) Product:** $A \times B = \{(x,y) \mid (x\in A, y\in B)\}$

1] PMI: $\forall n \geq 1$ $B\cap\left(\bigcup_{k=1}^{n} A_k\right) = \bigcup_{k=1}^{n} (B\cap A_k)$

2] BC $n=1$, $B\cap A_k = B\cap A_k$

3] IH $B\cap\left(\bigcup_{k=1}^{n} A_k\right) = \bigcup_{k=1}^{n} (B\cap A_k)$

4] // show both halves of IH $\equiv$ T when eval'd @ $n=n+1$

$B\cap\left(\bigcup_{k=1}^{n+1} A_k\right)$ || pop term off top

$= B\cap\left(\bigcup_{k=1}^{n} A_k \cup A_{n+1}\right)$

$= \left(B\cap \bigcup_{k=1}^{n} A_k\right) \cup (B\cap A_{n+1})$

- Apply Induction Hypothesis

$= \left(\bigcup_{k=1}^{n} (B\cap A_k)\right) \cup (B\cap A_{n+1})$

$= \bigcup_{k=1}^{n+1} (B\cap A_k)$ 5] ∴ by pmi ...BOOM!

1] SPMI: $\forall n \geq 12$, $\exists a \geq 0, b \geq 0 \in \mathbb{N}$ such $n = 4a + 5a$

2] Base Cases need 4

$12 = 4(3) + 0(5)$

$13 = 4(2) + 5(1)$

$14 = 4(1) + 5(2)$

$15 = 4(0) + 5(0)$

3] S.I.H Suppose for all k and some n, $15 \leq k \leq n$, there exists $a,b \in \mathbb{N}$ such that $k = 4a + 5b$

4] $n+1 = n+1+3-3$

$= (n-3)+4$ // use S.I.H

$= (4a+5b)+4$ | given $a,b \geq 0$.

$= 4(a+1) + 5b$

5] ∴ By SPMI, $\forall n \geq 12$ ... Boom

1] Contradiction $p \to q$

2] Suppose p and $\neg q$

3] unpack def's

4] Calculate, prove false... but we supp'd a contra.

5] we done

1] contrapos $p \to q$

2] Suppose $\neg q \equiv T$

4] Show $\neg p \equiv T$ follows from $\neg q$

5.1] Show $\neg p \to \neg q$

5] conclude $p \to q$

6''

5''