MTH231 QUIZ 3

1.8: 1, 3, 7, 14, 17, 30

1) Prove $n^2+1 \geq 2^n$ for $n \in \mathbb{Z}$ and n is between $1 \leq n \leq 4$.

1] Proof by cases. $n = \{1,2,3,4\}$

2] ...

3] $n=1$ $n=2$ $n=3$ $n=4$

$1+1 \geq 2$ $5 \geq 4$ $10 \geq 8$ $17 \geq 16$

$\equiv T$ $\equiv T$ $\equiv T$ $\equiv T$

5] $\therefore n^2+1 \geq 2^n$ for $n = \{1,2,3,4\}$ ▯

3) Prove if $x, y \in \mathbb{R}$, $\min(x,y) + \max(x,y) = x+y$

1] Proof by cases

Case 1: $x \geq y$

↳ $\min(x,y) = y$

$\max(x,y) = x$

$\therefore \min(x+y) + \max(x,y) = x+y$

Case 2: $x < y$

↳ $\min(x,y) = x$

$\max(x,y) = y$

$\therefore \min(x,y) + \max(x,y) = y+x$
$= x+y$

Case 3: $x = y$

Logically, this make sense.

$\therefore$ for $x, y \in \mathbb{R}$, $\min(x,y) + \max(x,y) = x+y$. ▯

7) Prove using 4 cases: $x \geq 0, y \geq 0$; $x<0$ $y \geq 0$, $x \geq 0$ $y<0$, $x<0$ $y<0$.

Case 1: $x \geq 0, y \geq 0$

$|x| + |y| = x+y = |x+y|$

Case 2: $x<0, y \geq 0$

$|x|+|y| = -x+y$

~~[illegible]~~ ... NEXT COLUMN

~~$|x|+|y| = -x+y$~~

~~If $-x>y$: $|x+y| = x+y$~~

~~because $-x > x$ (as $y<0$),~~

~~$|x|+|y| = -x+y > x+y = |x+y|$~~

~~If $-x$~~

$x=-6$
$y=4$

$|x|+|y| = -x+y$ as x is neg.

If $-x<y$:

$|x+y| = x+y$. as $-x>x$ $(x<0)$:

↳ $\left[|x+y| = x+y\right] < \left[|x|+|y| = -x+y\right]$.

If $-x>y$:

$|x+y| = -(x+y) = -x+-y$. as $-x<x$,

↳ $\left[|x|+|y| = -x+y\right] \geq \left[-x+-y = |x+y|\right]$

Case 3: same as ^ but swapped

Case 4: $x<0, y<0$.

$|x|+|y| = -x+-y$

$|x+y| = -(x+y) = -x+y$

$\therefore |x|+|y| = |x+y|$

$\therefore$ Across all cases, $|x|+|y| \geq |x+y|$

14) Prove/Disprove that if a, b are rational, a^b is aswell.

1] Proof by counter-example

2] assume $a, b \in \mathbb{Q}$, ~~such that~~ ~~a, b~~

3] NA

4] $2^{1/2}$ where $a=2$, $b=\frac{1}{2}$, both $\in \mathbb{Q}$

5] $\therefore$ It is not the case that if a and b are rational, a^b is rational.

17) Suppose a, b are odd integers, and
$a \neq b$. Show that there is a unique
integer, c, such that c makes
$|a-c| = |b-c|$

1] Direct Proof

2] a, b are odd, $a, b \in \mathbb{Z}$ $a \neq b$
$c \in \mathbb{Z}$. ~~there~~ c is unique

3] Assume $|x|^2 = x^2 \ldots$

4] $|a-c| = |b-c|$

$(a-c)^2 = (b-c)^2$

$a^2 - 2ac + \cancel{c^2} = b^2 - 2bc + \cancel{c^2}$

$a^2 - 2ac = b^2 - 2bc$

$a^2 - b^2 = 2ac - 2bc$

$(a+b)\cancel{(a-b)} = 2c\cancel{(a-b)}$ | because $a \neq b$, $a - b \neq 0$

$a + b = 2c$

$c = \left(\frac{a+b}{2}\right)$

Thus, c is unique, and
it is an int as:
- 2 odds make an even
- evens are divisible by 2 to form integers.

30)