MTH 231 LECTURE 2
incl. 1.1

Product Notation

- $\prod_{K=a}^{b} K^2$ = evaluate each case on $K = a..b$, then multiply all together

Ex: $\prod_{K=1}^{4} K = 4! = 1 \cdot 2 \cdot 3 \cdot 4$

Product Notation Tricks

- Index shifting works with product notation
- Linearity does not.

Floor Function

- for a real number x, the floor of $x = \lfloor x \rfloor$ is the greatest int $\leq x$

Ex: $\lfloor 1.12 \rfloor = 1$ Ex: $\lfloor -2.1 \rfloor = -3$ Ex: $\lfloor 3 \rfloor = 3$

Ceiling function

- for a real number x, the ceiling of $x = \lceil x \rceil$ is the smallest int $\geq x$

Ex: $\lceil -1.5 \rceil = -1$; $\lceil 1 \rceil = 1$ Ex: $\lceil 1.5 \rceil = 2$ Ex: $\lceil -0.3 \rceil = 0$

CH 1.1: Propositional Logic

- A proposition is a phrase or sentence that is either True or false. Basically a statement that is T/F ~~AKA~~ a BOOLEAN / BIVALENT
- A Non-Proposition is something that can't be answered with a boolean.

Eg: Proposition: "Kangaroos are an animal" = True
Non-Proposition: "Where are we?", "$x + 3 = 4$" - this is a predicate/Prop. fxn

- p, q, r, s ... will be used to represent arbitrary propositions or Propositional Variables.

Operators (Logical)

$\neg$	$\wedge$	$\vee$	$\rightarrow$	$\leftrightarrow$
negation	and	or	implication	biconditional / iff

- Logical operators input propositions and output new propositions
- The truth value of an output proposition is completely determined by the logical operator(s) applied to it.

Truth Tables

Note: "$\equiv$" means logically equal.

- $\neg P \equiv$ "It is not the case that P." $\approx\ !P \approx$ not P

  Truth Table:

P	$\neg P$
T	F
F	T

"Truth assignment of p"

- $P \wedge Q \equiv$ "P and Q" $\equiv$ "P but Q"" $\approx$ "&&". Truth Table:

P	Q	$P \wedge Q$
T	T	T
F	T	F
T	F	F
F	F	F

  Also called ~~dis~~conjunction

- $P \vee Q \equiv$ "P or Q". "disjunction" $\approx$ "||" $\approx$. Truth Table

P	Q	$P \vee Q$
T	F	T
F	F	F
T	T	T
F	T	T

is false if exclusive (xor, $\veebar$)

- $P \rightarrow Q \equiv$ "If P, then Q" $\equiv$ "P implies Q" $\equiv$ "P is sufficient for Q"" $\equiv$ "Q is necessary for P". Truth

P	Q	$P \rightarrow Q$
T	T	T
F	T	T

P	Q	$P \rightarrow Q$
T	F	F
F	F	T

Expanded TT below

This can be thought of as a promise where if p is true, Q must be true...

This is called "Material Implication"

- $P \leftrightarrow Q \equiv$ "P iff Q" $\equiv$ "P is necessary & sufficient for Q"

Truth Table

P	Q	$P \leftrightarrow Q$
T	T	T
F	F	T
T	F	F
F	T	F

- $P \oplus Q \equiv$ "P xor Q"

truth table

P	Q	$P \oplus Q$
T	T	F
F	T	T
T	F	T
F	F	F

TT for $P \rightarrow Q$

P	Q	$P \rightarrow q$
T	T	T
F	T	T
T	F	F
F	F	T