MTH 231 Lecture 4

1.4

Predicates and Qualifiers

- If $\underbrace{\text{all a's are b's}}_{p}$, if $\underbrace{\text{all b's are c's}}_{q}$, then $\underbrace{\text{all a's are c's}}_{r}$ } Prop (1)

$\therefore (p \wedge q) \rightarrow r$ } Prop (2) ‖ Prop 1 is a tautology, while (2) is not

- Predicates and qualifiers improve the resolution of our logical structure. They improve it enough to describe all of mathmatics
- A predicate is a propositional function that typically assert some property, or assert some class to its input resulting in a proposition whose truth value depends on the truth value of the input.
  - ↳ Convention: use capital letters "P", "Q"...
  - ↳ Use: Ex: $P(x) =$ "x is a cat" Ex: $P_A(x) =$ "$4x > 10$" Ex: $P_A(1)\ 4 > 10$ {F}
    Ex: $P(x,y) =$ "$x \cdot y > 10$"

‖ Note: "Dom (f)" = the domain of f
‖ (definition) "Range (f)" = the range of f

- Domain of a predicate, "DOM (P)", also known as (domain | universe) of discourse, or universe, may mean the objects involved in a discussion.

  Ex: in MTH 111, $(-1)^{\frac{1}{2}}$ = DNE, as Dom of Discourse is Real Numbers
  in MTH 306, $(-1)^{\frac{1}{2}} = \pm i$, as DoD is Imaginary numbers

‖ NOTE: in MTH231 (this class), DoD is just gonna be the Dom of the pred.

- The truth set of a predicate is the domain that makes Predicate true: $T(P) = \{x \in Dom(P) : P(x) \text{ is true}\}$
- Quantifiers: the universal qualifier $\forall x\, P(x)$ asserts that $P(x)$ always true for all x. Can be disproven with 1 counter example.

- Existential Qualifier : $\exists P(x)$ asserts that there is at least one ~~set~~ x in dom(P) that makes P true.

- Translations from English Qualifiers
  - ↳ $\forall x \Rightarrow$ "for every x", "for any x", "for an arbitrary x" ...
  - ↳ $\exists x \Rightarrow$ "There is an x", "for some x", "there is at least $1 \cdot x$" ...

  || NOTE: Mathmaticians generally use/mean $\forall x$, if their language is ambiguous

- Quantifiers and Domains:
  - → If Dom(P) = $\{x_1, x_2 .. x_n\}$, then $\forall x P(x) = P(x_1) \wedge P(x_2) \wedge ... \wedge P(x_n)$ which also can be described as $\bigwedge_{K=1}^{n} P(x_K)$ ← same
  - → If Dom(P) = $\{x_1, x_2, ... x_n\}$, then $\exists x P(x) = \bigvee_{K=1}^{n} P(x_K)$
  - → $\neg \forall x P(x) \equiv \exists x \neg P(x)$
  - → $\neg \exists x P(x) \equiv \forall x \neg P(x)$