MTH 231 LEC 14

Induction

Ex: $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$ for $\forall n \geq 0$

1] Proof by PMI

2] Base Case: $n=0$

LHS: $\sum_{k=0}^{0} k = 0$

RHS: $\frac{0(0+1)}{2} = 0$

$\hookrightarrow \sum_{k=0}^{0} k = \frac{0(0+1)}{2}$ ✓

3] Induction Hypothesis: suppose

$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$ for an arbitrary n

4] Then $\sum_{k=0}^{n+1} k = \sum_{k=0}^{n} k + (n+1)$

given IH, $\hookrightarrow = \frac{n(n+1)}{2} + (n+1)$

$= \left(\frac{n(n+1)}{2}\right) + (n+1)\cdot\frac{2}{2}$

$= \frac{n(n+1) + 2(n+1)}{2} = \frac{(n+2)(n+1)}{2}$

$\frac{(n+2)(n+1)}{2} = \sum_{k=0}^{n+1} k = \frac{n+1(n+2)}{2}$

5] $\therefore \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$

Ex #2: $\sum_{k=1}^{n} k\cdot k! = (n+1)! - 1$ for $n \geq 1$

1] Proof by PMI

2] Base Case: $n=1$

Left Side: $\sum_{k=1}^{1} k\cdot k! = 1\cdot 1! = 1$

Right Side: $((1)+1)! - 1 = 2-1 = 1$

3] IH: Suppose $\sum_{k=1}^{n} k\cdot k! = (n+1)! - 1$

for some n

4] Then $\sum_{k=1}^{n+1} k\cdot k! = \underbrace{\sum_{k=1}^{n} k\cdot k!}_{} + (n+1)(n+1)!$

$= (n+1)! - 1 + (n+1)(n+1)!$

$= ((n+1)!)(n+1+1) - 1$

$= (n+2)(n+1)! - 1 = (n+2)!$

$= ((n+1)+1)! - 1$

5] $\therefore$ by PMI, $\sum_{k=1}^{n} k\cdot k! = (n+1)! - 1$

Eval that sum's RHS @ $(n+1)$

$(n+1)! - 1 \Big|^{n=n+1} = \boxed{(n+1+1)! - 1}$

Ex # 3: $\sum_{k=0}^{n} a^k = \frac{a^{n+1} - 1}{a-1}$ $a \neq 1$, $n \geq 0$

1] Proof by PMI

2] Base Case $n=0$

$\sum_{k=0}^{0} a^k = a^0 = \boxed{1}$

$\Updownarrow$

$\frac{a^{0+1} - 1}{a-1} = \frac{a-1}{a-1} = \boxed{1}$

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3] $\underline{\text{I.H.}}$ Suppose $\sum_{k=0}^{n} a^k = \boxed{\dfrac{a^{n+1}-1}{a-1}}$

4] Then $\sum_{k=0}^{n+1} a^k = \underbrace{\sum_{k=0}^{n} a^k}_{\text{use IH}} + a^{n+1}$

$$= \left(\frac{a^{n+1}-1}{a-1}\right) + \frac{a^{n+1}(a-1)}{a-1}$$

When evald at (n+1)

$$= \frac{a^{n+1} - 1 + a^{n+2} - a^{n+1}}{a-1}$$

this is equal to

$$= \frac{a^{(n+1)+1} - 1}{a-1}$$

5] ∴ By PMI, solved yeet □

Ex#4: for $h > -1$: $1 + nh \leq (1+h)^n$

1] P B PMI

2] $\underline{\text{B.C}}$: $n=0$

$$1+(0)h \leq (1+h)^0$$

$$1 \leq 1 \checkmark$$

3] I.H.: Suppose $1 + nh \leq (1+h)^n$:

4] Then $(1+h)^{n+1} = (1+h)^n \cdot (1+h)^1$

trying to get to $\rightarrow \left[1 + (n+1)h \leq (1+h)^{n+1}\right]$

$$\geq (1+nh)(1+h)$$

$$= 1 + h + nh + nh^2$$

$$= 1 + (n+1)h + \cancel{nh^2}$$

since $h^2 \geq 0$

$$\geq 1 + (n+1)h$$

∴ By PMI

5] $1 + (n+1)h \leq (1+h)^{n+1}$