MTH 231 Lecture 11

Common Sets

- Naturals: $\mathbb{N} = \{0,1,2,3\ldots\}$
- Rationals: $\mathbb{Q} = \{\frac{m}{n} \mid m,n \in \mathbb{Z} \wedge n \neq 0\}$
- Integers: $\mathbb{Z} = \{-1,0,1,2\ldots\}$
- Reals: $\mathbb{R}$ = All real numbers
- Complex $\mathbb{C} = \{(x,y) \mid x,y \in \mathbb{R}\}$

Subsets

- Set A is a subset of set B iff $x \in A$ then $x \in B$.
  - ↳ This is denoted $A \subseteq B$
  - ↳ This requires a proof!

Ex: Prove $\mathbb{Z} \subseteq \mathbb{Q}$

1] Direct Proof

2] Suppose $x \in \mathbb{Z}$

3] N/A

4] Then $x = \frac{x}{1}$. Since $1, x \in \mathbb{Z}$ and $1 \neq 0$, $x \in \mathbb{Q}$

5] ∴ If $x \in \mathbb{Z}$, then $x \in \mathbb{Q}$, so $\mathbb{Z} \underset{\text{subset}}{\subseteq} \mathbb{Q}$

- To show A is not a subset of B, you give a counter-exampe to "$\forall x\ (x \in A \rightarrow x \in B)$".

Set Equallty

- $A = B$ iff $A \subseteq B \underset{\Leftrightarrow}{\wedge} B \subseteq A$
  - ↳ If they're both subsets of eachother

Ex: Prove $A = B$ if
$A = \{2K+1 \mid K \in \mathbb{Z}\}$
$B = \{2K-1 \mid K \in \mathbb{Z}\}$

1] 1] Direct

2] Suppose $x \in A$

3] Then $x = 2K+1$ for $K \in \mathbb{Z}$

4] Then $x = 2K+1-1+1$
$= 2K+2-1$
$= 2(K+1)-1$
Since $K+1 \in \mathbb{Z}$, $x \in B$

5] ∴ If $x \in A$, then $x \in B$
∴ $A \subseteq B$

2] 1] Direct

2] Suppose $x \in B$

3] $x = 2K-1$ for some integer K

4] So $x = 2K-1+1-1$
$x = 2K-2+1 = 2(K-1)+1$
because K is $\in \mathbb{Z}$, then $x \in A$

5] ∴ $x \in B$ implies $x \in A$
∴ $B \subseteq A$

3] ∴ Since $A \subseteq B$ and $B \subseteq A$, $A = B$ Boom!

Proper Subsets

- $A \subset B$ iff $A \subseteq B$ but $A \neq B$
- other notation $A \subsetneq B$

Special Sets

- Empty Set: $\emptyset = \{\ \}$ NOTHING IN HERE
- Cardinality of a set → Denoted $|A|$
  - ↳ number of distinct elements || Repeated elements don't count

- Power set: the set of all subsets of A:

$\mathbb{P}(A) = \{B \mid B \subseteq A\}$

Ex: $P(\{1,2,3\}) =$

$\{\emptyset, \{1\}, \{2\}, \{1,2\}, \{3\}, \{1,2,3\}, \{2,3\}\}$

- Note: $|\mathbb{P}(A)| = 2^{|A|}$