

MTH 231 LECTURE 1

Summation Notation

$\sum_{k=a}^b f(k)$ // sum everything from $k=a$ to $k=b$ while applying the rule $f(k)$

EX: $\sum_{k=10}^{k=12} 2k = \frac{20}{10} + \frac{22}{11} + \frac{24}{12} = \boxed{66}$

EX: $\sum_{k=1}^3 (k^3 + 2k) = \frac{(1+2)}{1} + \frac{(8+4)}{2} + \frac{(27+6)}{3}$

EX: $\sum_{k=a}^{k=b} (c) = (b-a+1)c$

Linearity Property

1] $\sum_{k=a}^b c f(k) = c \cdot \sum_{k=a}^b f(k)$

2] $\sum_{k=a}^b f(k) + g(k) = \sum_{k=a}^b f(k) + \sum_{k=a}^b g(k)$

Index Shifting on Summations

1] $\sum_{k=a}^b f(k) = \sum_{k=a-c}^{b-c} f(k+c)$

2] $\sum_{k=a}^b f(k) = \sum_{k=a+c}^{b+c} f(k-c)$

SET Summation Notation

Let $A = \{1, 7, 3\}$; $\sum_{k \in A} k^2 = 1^2 + 7^2 + 3^2$

NOTE: useful when the set of k -values aren't part of an incrementing series (i.e. 1, 2, 3, ...).

Double Sums

$\sum_{i=1}^2 \sum_{k=2}^3 i \cdot k = \sum_{i=1}^2 i \cdot \sum_{k=2}^3 k = \boxed{15}$
linearity

"Popping Terms off the top"

Consider... $\sum_{k=3}^6 k^3 = \frac{3^3}{3} + \frac{4^3}{4} + \frac{5^3}{5} + \frac{6^3}{6}$

"Popping the 6 off the top"
 $\Rightarrow \left(\sum_{k=3}^5 k^3 \right) + 6^3$

NOTE: can also be taken off bottom

Factorials

$n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$