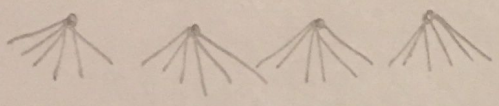
MTH231 LECTURE 16

Counting

Ex: if there are 4 floors in a building and 6 offices per floor, how many offices are there in the building?

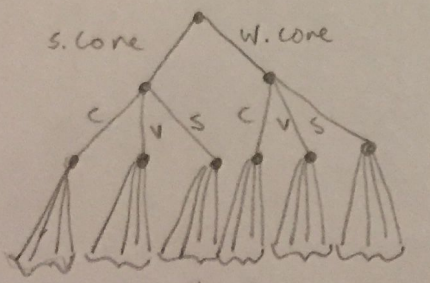


Product Rule

If a procedure can be broken up into two subtasks with m ways to do the first subtask, and for each of those m ways there are n ways to do the second subtask, there are $m \cdot n$ ways to complete the procedure.

Ex: Ice cream Shop

Flav: Choc, Van, Sbry
Top: Oreo, GB's, Carmel, Sprinkla
Cone: Waffle, Sugar



$= 24$ ‖ 1-flav, 1-cone, 1-topping

$4 \cdot 3 \cdot 2 = 24$

Now, for 2-Scoop

Cone Scoop Scoop topp
• 2 3 3 4 $= 2 \cdot 9 \cdot 4$

String

an ordered (tuple) list of symbols
ex: 1 a b * 7 ? # h e 7
↳ alphabet = {all char's}

Bit String = Binary Words
a string of 0's and 1's
alphabet = {0, 1}
Ex: 1 0 1 0 1 1 0 1 0 0 1

Empty String
"λ" = nothing listed. Len = 0

"* Operator"
001 * 10 = 00110

Word about answers
- use 2^3 rather than 8 as it gives insight about work

How Many ... of ... are there
- alphabet$^{\text{num_chrs}}$ (unless repetition forbidden)

Identifying String Count Probs

$|\mathbb{P}(A)| = 2^{|A|}$

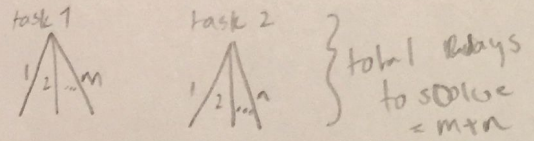
Identify all subsets of A with n-letter binary strings.

{a,b,c} → ∅ = 000
...
$\bar{a}\ \bar{b}\ \bar{c}$ {a,b} = 110
...
{a,b,c} = 111

Sum Rule

If a procedure can be completed in ways by task 1 or in n ways by task 2 →

such that tasks 1, 2 aren't redundant (or overlapping), Then there are $m+n$ ways to complete that procedure



EX: How many ice creams have ≤ 3 scoops and 1 topping and 1 cone

1 scoop
cone flav top
$2 \cdot 3 \quad 4$

2 scoop
$2 \cdot 3 \cdot 3 \cdot 4$

3 scoop
$2 \cdot 3 \cdot 3 \cdot 3 \cdot 4$

total =
$2 \cdot 3 \cdot 4$
$+ 2 \cdot 3^2 \cdot 4$
$+ 2 \cdot 3^3 \cdot 4$

How many $\leq n$ ice creams?

$$\left\lceil \sum_{k=1}^{n} 8 \cdot 3^k \right\rceil$$