

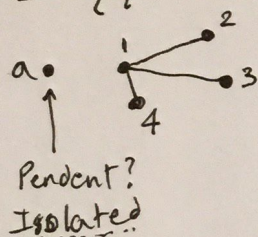
Graph Taxonomy

- A graph is an ordered pair of sets $G = (V, E)$, where $V \neq \emptyset$ is called the vertex set, and E is a set of ordered or unordered pairs of elements from V , called an edge set. Can be multiset. The elements of V are called vertices or nodes

$$\text{ex: } G = (V, E)$$

$$V = \{1, 2, 3, 4, a\}$$

$$E = \{\{1, 2\}, \{1, 3\}, \{1, 4\}\}$$

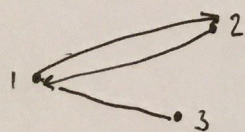


$$\text{ex: if } E = \{\{a, v\} \mid v \in V, v \neq a\}$$

- Simple Directed Graph is a graph whose edge set, E , is a subset of $V \times V$ and $(v, v) \notin E$ for any $v \in V$.

$$\text{ex: } V = \{1, 2, 3\}$$

$$E = \{(1, 2), (2, 1), (1, 3)\}$$



Graph Type Chart

Graph	Edges	multiedg allow	loops allow
Simple undirected	Undirected	N	N
Multi-graph	Undirected	Y	N
Pseudograph	Undirected	YES	YES
Simple Directed	Directed	N (7 indiff dir)	N
multigraph - Directed	Directed	Y	Y
MIXED Graph	*	*	*

Undirected Graphs

- Two vertices, u, v , are called connected if $\{u, v\} \in E$ (i.e. E connects u, v). this makes them neighbors. E is called incident with u, v

- The neighborhood of v in G is the set $N(v) := \{u \in V \mid \{v, u\} \in E\}$
 \hookrightarrow all the other vertices connected to one vertex form that vertex's neighborhood

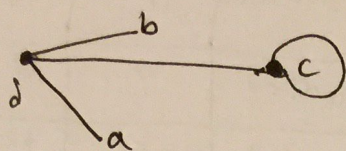
$$\text{ex: } \begin{array}{l} \text{graph with vertices } a, b, c \text{ and edges } \{a, b\}, \{b, c\} \\ N(b) = \{a, c\} \\ N(\{a, c\}) = \{b\} \end{array}$$

$$\text{ex: } \text{graph with vertices } d, e \text{ and edge } \{d, e\} \quad N(e) = \{d\}$$

- Neighborhood of a set

$$N(A) = \bigcup_{v \in A} N(v)$$

- Degree of a vertex = the number of edges leading into the node.
this is denoted $\deg(\text{vertex})$.



$$\deg(a) = 1$$

$$\deg(b) = 3$$

$$\deg(c) = 3$$