MTH 231 LECTURE 6

Commutation

Statement	T when	F when
$\forall x \forall y\, P(x,y) \equiv \forall y \forall x\, P(x,y)$	$P(x,y) \equiv T$ on all x, y	$P(x,y)$ has a false case
$\exists x \exists y\, P(x,y) \equiv \exists y \exists x\, P(x,y)$	If there is some x,y that makes $P(x,y)$ True	If $P(x,y) \equiv F$ for all x,y
$\exists x \forall y\, P(x,y)$	Find a single x for which all y are T	If for every x there is some y that makes $P(x,y) \equiv F$
$\forall y \exists x\, P(x,y)$	If for every y there is an x that makes $P(x,y) \equiv T$	There is some y for which $P(x,y)$ fails for all x

NOTE: $\exists x\, Ay\, P(x,y) \Rightarrow \forall y \exists x\, P(x,y)$
but <u>not</u> $\forall y \exists x\, P(x,y) \Rightarrow \exists x \forall y\, P(x,y)$

Negation of Qualifiers

1] ~~GA~~ $\neg \forall x \forall y\, P(x,y) \equiv \exists x \exists y \neg P(x,y)$

2] $\neg \forall x \exists y\, P(x,y) \equiv \exists x \forall y \neg P(x,y)$

3] $\neg \exists x \forall y\, P(x,y) \equiv \forall x \exists y \neg P(x,y)$

4] $\neg \exists x, \exists y\, P(x,y) \equiv \forall x \forall y \neg P(x,y)$

NOTE:
$\neg \forall x\, P(x) \Leftrightarrow \exists x \neg P(x)$
$\neg \exists x\, P(x) \Leftrightarrow \forall x \neg P(x)$