MTH 231 LECTURE 15

Induction Proof

- You want to show that $\forall n \in \mathbb{N}$, $P(n) \equiv$ True
- Use Template from lecture 13.

<u>Ex</u>: $\forall n \geq 1 \quad B \cap \left(\bigcup_{k=1}^{n} A_k \right) = \bigcup_{k=1}^{n} (B \cap A_k)$

NOTE: $B \cap (C \cup D) = (B \cap C) \cup (B \cap D)$

1] By Induction

2] B.C. $n=1$: $B \cap \left(\bigcup_{k=1}^{1} A_k \right) = B \cap A_1$

$= \bigcup_{k=1}^{1} (B \cap A_1)$

3] Suppose $B \cap \left(\bigcup_{k=1}^{n} A_k \right) = \bigcup_{k=1}^{n} (B \cap A_k)$

4] Then $B \cap \left(\bigcup_{k=1}^{n+1} A_k \right) = B \cap \left(\bigcup_{k=1}^{n} A_k \cup A_{n+1} \right)$

$= \left(B \cap \left(\bigcup_{k=1}^{n} A_k \right) \right) \cup (B \cap A_{n+1})$

I.H. $\downarrow$

$= \left(\bigcup_{k=1}^{n} (B \cap A_k) \right) \cup (B \cap A_{n+1})$

$= \bigcup_{k=1}^{n+1} (B \cap A_k)$

5] By P.M.I: $\forall n \geq 1 \quad B \cap \bigcup_{k=1}^{n} A_k = \bigcup_{k=1}^{n} (B \cap A_k)$

BOOM

(5) PMI

(5.1) PMI

show: $\forall n \in \mathbb{N}, P(n) \equiv T$

Idea:

1] show $P(0) \equiv T$

2] Show $P(n) \Rightarrow P(n+1)$

Then:

$P(0) \Rightarrow P(1) \cdots$

(5.2) SPMI

Show: $\forall n \in \mathbb{N}, P(n) \equiv T$

Idea:

1] show $P(0) \equiv T$

2] show that

$P(0) \wedge P(1) \wedge \ldots \wedge P(n-1) \wedge P(n) \Rightarrow P(n+1)$

(5) PMI Templates

PMI	SPMI (strong PMI)
1] PMI	1] SPMI
2] Base Case $P(0)$	2] Base Case $P(0)$
3] Suppose $P(n) \equiv T$	3] Suppose $P(0) \ldots P(n) \equiv T$
4] show $P(n+1) \equiv T$	4] show $P(n+1) \equiv T$
5] $\therefore$ By PMI	$\therefore$ By SPMI

<u>Ex</u>: $\forall n \geq 12$, $\exists a \geq 0, b \geq 0 \in \mathbb{N}$ such that

$n = 4a + 5b$

i.e.:

$12 = 4(3) + 5(0)$

$13 = 4(2) + 5(1)$

$14 = 4(1) + 5(2)$

$15 = 4(0) + 5(3)$

$16 = 4(4) + 5(0)$

1] SPMI

2] Base Case : $12 = 4(3) + 5(0)$

Also need: 13, 14, 15

$13 = 4(2) + (5)(1)$

$14 = (4)(1) + (5)(2)$

$15 = (4)(0) + (5)(3)$

3] S.I.H: Suppose for some n, and all K such that $15 \leq K \leq n$, there exists $a, b \in \mathbb{N} \rightarrow K = 4a + 5b$

4] Then $n+1 = n+1-3+3$

$= (n-3) + 4$

S.I.H $\downarrow$

$= (4a + 5b) + 4$

note can't be <12

$= 4(a+1) + 5b$

— since $a, b \geq 0$, $a+1, b \geq 0$

5] $\therefore$ spmi $\forall n \geq 12 \quad n = 4a + 5b$