MTH 231 Lecture 8

Ex: Prove that if n is odd, 3n + 2 is odd

1] Direct Proof

2] Suppose n is odd

3] $n = 2K+1 \quad K \in \mathbb{Z} \qquad 3n+2 = 3(2K+1)+2$

4] $6K+3+1 = 6K+4+1 = 6K+5$

$$3n+2 = 6k+4+1$$
$$= 2(3k+2)+1$$

Since 3K+2 is an integer (because K is int)
then 3n+2 must be odd

5] $\therefore$ n odd $\Rightarrow$ 3n + 2 odd

NOTE: "$\Rightarrow$" is the "TRUE" version of "$\rightarrow$" symbol

NOTE: contraposition means $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Ex: Prove that if n^2 is even, n is even

1] Direct Proof

2] Suppose n^2 is even

3] If n^2 is even, then $n^2 = 2K$ for some integer K

4] $n = \sqrt{2K} \quad | \; 2K \geq 0$

... Cannot prove!!

1] Proof Contraposition (see nxt pg)

2] Suppose n is odd

3] Then $n = 2K+1$ for $K \in \mathbb{Z}$

4] [goal: Show $n^2 = 2x+1$]

So if $n = 2K+1$, $n^2 = (2k+1)^2$

$n^2 = 4k^2+4k+1$

$n^2 = 2(2k^2+2k)+1$

Since $K \in \mathbb{Z}$, $(2k^2+2k \in \mathbb{Z})$

5] $\therefore$ If n is odd, n^2 is odd

5] Then, by Contraposition if n^2 is even, n is even $\square$

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Proof By Contraposition

1] Contraposition Proof

2] Suppose $\neg q$ is True

3] Unpack Defenitions

4] Argue that $\neg p$ follows from $\neg q$

5] $\therefore p \Rightarrow q$ <u>OR</u> $\neg q \Rightarrow \neg p$ $\square$

Ex: Using Contraposition, prove that if n is even, $n-1$ is odd

1] Contraposition Proof

2] Suppose that [$\neg q$] $n-1$ is even

3] If $n-1$ is even, then:

$n-1 = 2K$ for $K \in \mathbb{Z}$

$n = 2K+1$

4] then $n = 2K+1$

5'] $\therefore$ If $n-1$ is even, n is odd

5] By Contraposition, that means that if $n-1$ is odd, n is even $\square$

Axiom: any (every) rational, x, can be written in reduced form a/b such that a and b have no common Divisor

Numbers

- a real number, ~~a~~ x is <u>rational</u> if
  - → 1] $x = \frac{m}{n}$ for $m, n \in \mathbb{Z}$
  - → 2] $n \neq 0$

  $\}$ → $\mathbb{Q}$
- a real number is <u>irrational</u> if:
  - → 1] It is not rational $\}$ $\overline{\mathbb{Q}}$ or $\mathbb{Q}^c$
- Some Examples

1] $2/3 \in \mathbb{Q}$

2] $0.1 = \frac{1}{10} \in \mathbb{Q}$

3] $5 \in \mathbb{Q}$