MTH 231 QUIZ 2

- 1.2: 1, 3, 5
- 1.3: 1, 2, 5-8, 9a, 9b, 13, 15, 18, 27-29
- 1.4: 1, 2, 5, 7, 11, 12, 13, 15, 17, 25, 31, 33, 35, 52, 59, 61

1.2

1) $e \to a$ (e only if a); (e because a) 5) $e \to (a \wedge ((p \wedge r) \vee b))$

3) $g \to (\neg m \wedge r \wedge \neg b)$ (g only when ...)

1.3

1) <just truth tables> 5) Draw massive TT, All, Profit?

2) $\neg(\neg p) \equiv p$:

p	¬p	¬¬p	p
T	F	T	T
F	T	F	F

8) a) x will take a job (p) or go to grad school (q): $(p \vee q)$. DML: $\neg p \wedge \neg q$ (negation)

b) y knows Java (a) and calculus (b): $(a \wedge b)$. NEG DML: $\neg p \vee \neg q$

9a) Prove $(p \wedge q) \to p$ is TAUT.

p	q	$(p \wedge q) \to p$
T	T	T → T ≡ T
T	F	F → T ≡ T
F	T	F → F ≡ T
F	F	F → F ≡ T

9b) Prove $\neg p \to (p \vee q)$

p	q	$\neg p \to (p \vee q)$
T	T	F → (T) ≡ T
T	F	F → (T) ≡ T
F	T	T → (T) ≡ T
F	F	T → (F) ≡ F

13) a)

p	q	$p \vee (p \wedge q)$	≡	p
T	T	T ∨ (T)	T	T
T	F	T ∨ (F)	T	T ✓
F	T	F ∨ (F)	F	F
F	F	F ∨ (F)	F	F

Prove $p \vee (p \wedge q) \equiv p$

15) Is $(\neg q \wedge (p \to q)) \to \neg p$ a Tautology

p	q	$((\to q) \wedge (p \to q)) \to (\neg p)$
T	T	(F) ∧ (T) ≡ F → (F) ≡ T
T	F	(T) ∧ (F) ≡ F → (F) ≡ T
F	T	(F) ∧ (T) ≡ F → (T) ≡ T
F	F	(T) ∧ (T) ≡ T → (T) ≡ T

It is!

18) <draw truth table to prove "logical equivalence">

27) <truth table> 28) <truth table> 29) <truth table>

1.4

1) If $P(x)$ is "$x \leq 4$", a) $P(0) \equiv T$ b) $P(4) \equiv T$ c) $P(6) \equiv F$

5) If $P(x)$ is "x spends > 5h/wkdy in class"

a) $\exists x\, P(x)$: There exists a student that spends > 5h/wkdy in class

b) $\forall x\, P(x)$: All students > 5h/wkdy in class

c) $\exists x \neg P(x)$: There is a student that does not > 5h/wkdy in class

d) $\forall x \neg P(x)$: ~~Not all~~ All students don't spend > 5h/wkdy in class

7) $C(x)$: "x is a comedian"; $F(x)$: "x is a funny"

a) $\forall x\,(C(x) \to F(x))$: for all people, if they are a com. they are funny

b) $\forall x\,(C(x) \wedge F(x))$: for all people, everyone is a funny comedian

c) $\exists x\,(C(x) \to F(x))$: there exists a person who, if they are a comedian then they are funny

d) $\exists x\,(C(x) \wedge F(x))$: There exists a funny comedian

11) $P(x)$ is "$x = x^2$". Truth values:

a) $P(0) \equiv T$ b) $P(1) \equiv T$ c) $P(2) \equiv F$ d) $P(-1) \equiv F$ e) $\exists x\,P(x) \equiv T$

f) $\forall x\,P(x) \equiv F$ (anti: $P(17) \equiv F$)

12) skip'd

13) a) $\forall n\,(n+1 > n) \equiv T$ b) $\exists n\,(2n = 3n)\ [n=0] \equiv T$ c) $\exists n\,(n = -n) \equiv T$

d) $\forall n\,(3n \not\leq 4n) \equiv T$

15) a) $\forall n\,(n^2 \geq 0) \equiv T$ b) $\exists n\,(n^2 = 2) \equiv F$ c) $\forall n\,(n^2 \geq n) \equiv T$

d) $\exists n\,(n^2 \not\leq 0) \equiv F$

17) $\text{DOM}(P(x)) = \{0,1,2,3,4\}$

a) $\exists x\,P(x) \equiv P(0) \vee \ldots \vee P(4)$ b) $\forall x\,P(x) \equiv P(0) \wedge \ldots \wedge P(4)$

c) $\exists x \neg P(x) \equiv \neg P(0) \vee \ldots \vee \neg P(4)$ d) $\forall x \neg P(x) \equiv \neg P(0) \wedge \ldots \wedge \neg P(4)$

e) $\neg \exists x\,P(x) \equiv \neg(P(0) \vee \ldots \vee P(4))$ f) $\neg \forall x\,P(x) \equiv \neg(P(0) \wedge \ldots \wedge P(4))$

25) $P(x)$: x is perfect, $F(x)$: x is your friend DOM: All people

a) No one is perfect: $\underline{\forall x \neg P(x)}$ b) Not everyone is perfect: $\underline{\neg \forall x\,P(x)}$

c) All your friends are perfect $\underline{\forall x\,(F(x) \to P(x))}$

d) At least one friend is perfect $\underline{\exists x\,(F(x) \wedge P(x))}$

e) everyone's your friend and is perfect: $\underline{\forall x\,(F(x) \wedge P(x))}$

f) Not everyone is perfect your friend or someone is not perfect: $\underline{\neg \forall x\,F(x) \vee \exists y\,(\neg P(y))}$

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31) <sub values ... simple-ish), sec 17)

33) a) Some old dogs can learn new tricks.
↳ Dom: ~~some~~ old dogs. P(x): x can learn new tricks.
↳ original: $\exists x\, P(x)$. Negation $\forall x \neg P(x)$: No old dogs can learn new tricks
...<for rest>

‖Note: $\neg(\exists x\, P(x)) \Rightarrow \forall x \neg P(x)$
$\neg(\forall x\, P(x)) \Rightarrow \exists x \neg P(x)$

35) find counterexamples:
a) $\forall x\,(x^2 \geq x)$ ≡TAUTOLOGY b) $\forall x\,(x>0 \vee x<0)$ ‖ x=0!
c) $\forall x\,(x=1)$ ‖ x=13.

52) Note: $\exists! x\, P(x)$ means "there exists a unique x such that P(x) ≡T"
a) $\exists! x\,(x>1) \equiv F$ b) $\exists! x(x^2=1) \equiv F$ ±1 c) $\exists! x\,(x+3=2x) \equiv T$
d) $\exists! x\,(x=x+1) \equiv F$ false!

59) P(x): x is a prof Q(x): x is ignorant R(x): x is vain
a) "no professors are ignorant": $\forall x\,(P(x) \rightarrow \neg Q(x))$
b) "All ignorant people are vain": $\forall x\,(Q(x) \rightarrow R(x))$
c) "no professors are vain": $\forall x\,(P(x) \rightarrow \neg R(x))$
d) does (c) follow from (a),(b): NO: as professors can be just vain...

61) same as 59.