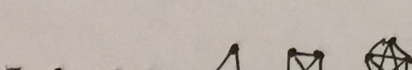
MTH 231 LEC 22

Directed Graphs

- If $(u, v) \in E$, we say "u is adjacent to v" or "u connects to v" or "v is adjacent from u" or "u is the initial vertex" "v is the (terminal | end | final) vertex"
- The In-Degree of v is the number of edges leading into v $= \deg^-(v)$
- The Out Degree of vertex v is the number of edges leading out of v $= \deg^+(v)$
- $\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$

Complete Graphs

- all possible edges are there
- Simple graphs = no self loops

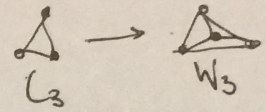


- sides per K : $\boxed{\sum_{k=0}^{n-1} = \text{num sides at } K_n}$

Cycles

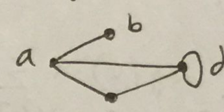
- $V = \{v_1, v_2 \ldots v_n\}$
- $E = \{\{v_1, v_2\}, \{v_2, v_3\} \ldots \{v_{n-1}, v_n\}, \{v_n, v_1\}\}$
- C_n (shape with n verticies) has n edges

- wheel is where you add one vertex that connects to all others



Total Degree

- sum of all lines leaving all points and entering all points. represented as $\deg(V)$
- ex: $\deg(V)$:



deg d = 4
deg b = 1
deg a = 3
deg c = 2

$\therefore \deg(V) = 10$

Handshake Theorem ✱FINAL✱

- In an undirected graph, $g = (V, E)$
  - ↳ $\sum_{v \in V} \deg v = 2|E|$
  - ↳ $|E| = \frac{1}{2} \deg(v)$
  - ↳ $\deg(V) = 2|E|$
- Proof: every new edge (item in E) adds exactly 2 to the degree of the point(s) it connects to.

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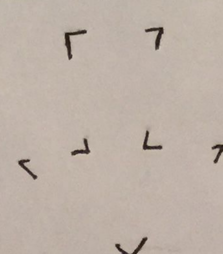

Prove theorem: An undirected graph has an even number of odd degree'd verticies

] Direct Proof

2] .. 5] :

-let V_o be all verticies with odd degrees, and V_e to be the set of all verticies with even degree.

Then: $2|E| = \sum_{v \in V} \deg(v)$ // handshake Principle

$$= \sum_{v \in V_o} \deg(v) + \sum_{v \in V_e} \deg(v)$$

$$= \sum_{i=\phi}^{|V_o|} (2K+1) + \sum_{j=1}^{|V_e|} (2K) \quad \text{// for some integers } i, j, K \in \mathbb{Z}$$

$$= 2\sum_{i=1}^{|V_o|} K + \sum_{i=1}^{|V_o|} 1 + 2\sum_{j=1}^{|V_e|} K$$

$$|V_o| = 2|E| - 2\sum_{i=\phi}^{|V_o|} K - 2\sum_{j=1}^{|V_e|} K$$

$$|V_o| = 2\underbrace{\left(|E| - \sum_{i=1}^{|V_o|} K - \sum_{j=1}^{|V_e|} K\right)}_{\in \mathbb{Z}}$$

$\downarrow$

$\therefore$ There are an even number of odd degree'd verticies in an undirected graph