MTH231 QUIZ 3

1.5: 1, 3, 9, 10, 27, 28, 33, 35, 39, 45

1.7: 1, 2, 4, 5, 6, 10–14, 27, 31, 38

1.5

1) a) for every real number, x, there exists a y such that $x < y$

3) $Q(x,y) =$ "x has emailed y", DOM: students

a) $\exists x \exists y$: there exists a student who has emailed another student.

b) $\exists x \forall y$: There exists a student, x, who has sent an email to all students

c) $\forall x \exists y$: Every student has sent another student an email.

d) $\exists y \forall x$: There exists a student who has been emailed by every other student

e) $\forall y \exists x$: Every student has been sent an email by one student

f) $\forall x \forall y$: Every student has sent an email to every student.

9) $L(x,y) =$ "x loves y". DOM: people

a) Every x loves Jerry : $\forall x\, L(x, \text{Jerry})$

b) Every x loves some y : $\forall x \exists y (L(x,y))$

c) There is an x loved by all y:
$\exists x \forall y L(y,x) \sim \exists y \forall x (L(x,y))$

d) Nobody loves everybody:
$\neg \exists x \forall y\, L(x,y) \equiv \forall x \exists y \neg L(x,y)$

e) There is someone Lyndia does not love
$\exists x \neg L(\text{Lyndia}, x)$

f) There is someone no-one loves
$\exists x \forall y \neg L(y,x)$

g) There is just 1 person everyone loves
$\exists x (\forall y (L(y,x)) \wedge \forall z (\forall w (L(w,z) \rightarrow z = x)))$ —— NEXT PAGE ——

h) There are exactly 2 people Lynn loves
$\exists x \exists y (L(\text{Lynn}, x) \wedge L(\text{Lynn}, y) \wedge$
$\forall z (L(\text{Lynn}, z) \rightarrow (z = x \vee z = y)$

i) Everyone loves themselves
$\forall x (L(x,x))$

j) there is someone who only loves them-selves
$\exists x (L(x,x) \wedge (\forall y\, L(x,y) \rightarrow x = y))$
$\exists x \forall y (L(x,y) \leftrightarrow x = y)$

27) Determine Truth Values: DOM: Int's

a) $\forall n \exists m\ (n^2 < m) \equiv T$
b) $\exists n \forall m\ (n < m^2) \equiv T$
c) $\forall n \exists m\ (n + m = 0) \equiv T$
d) $\exists n \forall m\ (nm = m) \equiv T$
e) $\exists n \exists m\ (n^2 + m^2 = 5) \equiv T$
f) $\exists n \exists m\ (n^2 + m^2 = 6) \equiv F$
g) $\exists n \exists m\ (n + m = 4 \wedge n - m = 1) \equiv F$
h) $\exists n \exists m\ (n + m = 4 \wedge n - m = 2) \equiv T$
i) $\forall n \forall m \exists p\ (p = (m+n)/2 \equiv F$

33) If $\neg$ passes by, $\exists x \overset{\text{SWAP}}{\Leftrightarrow} \forall x$

e) $\neg \forall x (\exists y \forall z P(x,y,z) \wedge \exists z \forall y P(x,y,z))$
↳ $\exists x (\forall y \exists z \neg P(x,y,z) \vee \forall z \exists y \neg P(x,y,z))$

NOTE AND → OR WHEN DIST NEG.

35) Find 2 DOM's such that $P(x,y,z,w)$ is both T and F:

$P(x,y,z,w) = \forall x \forall y \forall z \exists w ((\overbrace{w \neq x,\ w \neq y,\ w \neq z}^{\text{AND}})$

Domain of ≥ 4 unique digits $\equiv T$
Domain of ≤ 3 unique digits $\equiv F$

45) Real Numbers: number line

a) T b) F c) T

1.7 , Prove the sum of 2 odds is even

1) 1] Direct Proof

2] assume p and q are odd

3] $p = 2K+1 \quad q = 2L+1$

4] $p+q = 2K+1+2L+1$

$= 2(K+L+1)$

Because $K, L, 1 \in \mathbb{Z}$, $p+q$ is even

5] ∴ the sum of 2 odd numbers is even. BOOM!

11) Prove/Disprove the product of two irrationals is irrational

↳ $\sqrt{2} \cdot \sqrt{2} = 2$ LoL

13) Prove that if $\underbrace{x \text{ is irrational}}_{p}$, then $\underbrace{\frac{1}{x} \text{ is irrational}}_{q}$ as well

-1] Proof by Contraposition

2] [$\neg q$] ~~assume x is irrational~~, and $\frac{1}{x}$ is rational

3] $\frac{1}{x} = p/q$ where $p, q \in \mathbb{Z}, q \neq 0$

4] $x = 1/(1/x) = 1/(p/q)$

$= x = \frac{q}{p}$

hence, x is rational

5] ∴ if $\frac{1}{x}$ is rational x is rational

5.1] ∴ if x is ~~odd~~ irrational, $\frac{1}{x}$ is irrational

27) Prove that if n is a positive int, then $\underbrace{n \text{ is odd}}_{p}$ iff $\underbrace{5n+6 \text{ is odd}}_{q}$

1] 1] Prove $p \to q$: Direct Proof

2] assume n is odd

3] $n = 2K+1$

4] $5n+6 = 10K+11 = \underbrace{2(5K+5)+1}_{\text{odd}}$

5] ∴ if n is odd, $5n+6$ is odd

2] 1] Prove ~~Proof~~ ~~Directly~~ Contraposition

2] assume ~~5n+6 is odd~~ n is even

3] ~~$5n+6=2K+1$~~ $n = 2K$

4] If $n=2K$, then $5n+6 = 10K+6$

$= 2\underbrace{(5K+3)}_{\text{integer}}$

5] ∴ If n is even, $5n+6$ is even.

3] ∴ $5n+6$ is odd iff n is odd (and visa versa)

BOOM!

31) EVEN NUMBER : $2K$ ODD NUMBER : $2K+1$

i) $3x+2$ is even

ii) $x+5$ is odd

iii) x^2 is even

	$x = 2K$	$x = 2K+1$
(i)	$3(2K)+2 = 6K+2$ $= 2(3K+1)$ EVEN	$3(2K+1)+2 = 6K+5$ $= 2(3K+2)+1$ ODD
(ii)	$2K+5 = 2(K+2)+1$ ODD	$2K+1+5 = 2K+6$ $= 2(K+3)$ EVEN
(iii)	$2K^2 = 4K^2$ $= 2 \cdot 2K^2$ EVEN	$(2K+1)^2 = 4K^2+4K+1$ $= 2(2K^2+2K)+1$ ODD

38) Counterexample to: each int >0 can be written as the sum of 3 squares of ints

↳ $0+0+4$

um... not sure...