Lyell C Read

CH 12.6 Homework

12.5: 31-36 12.6: 9-31 odd 43,45

10/25/2018

12.5

31) $x^2 \pm 2y^2 = 1 \dashrightarrow 2x \pm 2y\frac{\partial y}{\partial x}\cdot 2 = 0$ $\quad 2x \pm 4y\frac{\partial y}{\partial x} = 0$

$2x = 4y\frac{\partial y}{\partial x}$ $\quad \boxed{\frac{x}{2y} = \frac{\partial y}{\partial x}}$

33) $2\sin xy \dashrightarrow -\frac{F_x}{F_y} = \cancel{-\frac{x}{y}} \quad -\frac{2y}{2x} = \boxed{-\frac{y}{x}}$

35) $\sqrt{x^2 + 2xy + y^4} = 3$
$x^2 + 2xy + y^4 = 9 \dashrightarrow -\frac{F_x}{F_y} = -\frac{2x+2y}{2y+4y^3} = \boxed{-\frac{x+y}{y+2y^3}}$

~~37)~~

12.6

9) $f(x,y) = 2 + 3x^2 - 5y^2$ $\quad f_x = 6x \quad f_y = -10y \quad P = (2,-1)$

$\nabla f(x,y) = \langle 6x, -10y\rangle \quad \nabla f(x,y)\Big|_{\substack{x=2\\y=-1}} = \boxed{\nabla f(2,-1) = \langle 12, 10\rangle}$

11) $f(x,y) = x^2 - 4x^2y - 8xy^2 \quad P = (-1,2)$ $\quad \nabla f(-1,2) = \langle -2+16-32, -4+32\rangle$

$f_x = 2x - 8yx - 8y^2 \quad f_y = -4x^2 - 16xy$ $\quad = \boxed{\langle -18, 28\rangle}$

$\therefore \nabla f(x,y) = \langle 2x - 8yx - 8y^2, -4x^2 - 16xy\rangle$

13) $f(x,y) = xe^{2xy} \quad P = (1,0) \quad f_x = 2xye^{2xy} + e^{2xy} \quad f_y = 2x^2e^{2xy}$

$\nabla f(x,y) = \langle 2xye^{2xy} + e^{2xy}, 2x^2e^{2xy}\rangle = e^{2xy}\langle 2xy+1, 2x^2\rangle$

$\nabla f(1,0) = e^0\langle 1,2\rangle = \boxed{\langle 1,2\rangle = \nabla f(1,0)}$

15) $f(x,y) = \underbrace{e^{-x^2-2y^2}}_{\text{"}\sigma\text{"}} \quad P = (-1,2) \quad f_x = -2x\sigma \quad f_y = -4y\sigma$

$\nabla f(x,y) = \langle -2x, -4y\rangle\sigma = e^{-x^2-2y^2}\langle -2x, -4y\rangle$

$\nabla f(-1,2) = e^{-1-8}\langle -2\cdot -1, -4\cdot 2\rangle = \boxed{e^{-9}\langle 2,-8\rangle = \nabla f(-1,2)}$

17) $f(x,y) = x^2 - y^2$, $P = (-1,-3)$ $\vec{v} = \langle \frac{3}{5}, -\frac{4}{5} \rangle$

1] find unit vector of v : $|\vec{v}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9+16}{25}} = 1$ $\sqrt{1} = 1$ it is unit v.

2] find $\nabla f(x,y) = \langle f_x, f_y \rangle$ $f_x = 2x$ $f_y = -2y$ $\nabla f(x,y) = \langle 2x, -2y \rangle$

3] find $D_u f(x,y) = u \cdot \nabla f(x,y) = u_x f_x + u_y f_y = \frac{3}{5} \cdot 2x + -\frac{4}{5} \cdot 2y = \frac{6x}{5} + \frac{8y}{5}$

4] substitute P : $\frac{-6}{5} - \frac{24}{5} = \frac{-30}{5} = \boxed{-6}$

19) $f(x,y) = 10 - 3x^2 + \frac{y^4}{4}$ $P = (2,-3)$ $v = \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$

1] v is unit? $\sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{\frac{4}{4}} = 1$ ✓ it is unit!

2] $\nabla f(x,y) = \langle f_x, f_y \rangle$ $f_x = -6x$ $f_y = y^3$ $\therefore$ $\nabla f(x,y) = \langle -6x, y^3 \rangle$

3] substitute P into $\nabla f(x,y)$: $\nabla f(x,y) = \langle -12, -27 \rangle$

4] Dot $\nabla f(x,y)$ with u : $-12 \cdot \frac{\sqrt{3}}{2} + -27 \cdot -\frac{1}{2} = \boxed{-6\sqrt{3} + \frac{27}{2}}$

21) $f(x,y) = \sqrt{4 - x^2 - 2y}$ $P = (2,-2)$ $v = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

1] is v a unit vector? $\sqrt{\frac{1}{5} + \frac{4}{5}} = 1$ ✓ it is unit!

2] $\nabla f(x,y) = \langle f_x, f_y \rangle$ $f_x = \frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2-2y}} = \frac{-x}{\sqrt{4}} = -1$

$f_y = \frac{-\cancel{2}}{\cancel{2}\sqrt{4}} = -\frac{1}{2}$

3] $\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle \cdot \langle -1, -\frac{1}{2} \rangle = -\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{5}} = \boxed{-\frac{2}{\sqrt{5}}}$

25) 23 skipped... $f(x,y) = \ln(4 + x^2 + y^2)$ $P = (-1,2)$ $v = \langle 2,1 \rangle$

1] make v unit $\sqrt{4+1} = \sqrt{5}$ $u_v = \langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle$ ← unit

2] $\nabla f(x,y) = \langle f_x, f_y \rangle$ $f_x = \frac{2x}{4+x^2+y^2}$ $f_y = \frac{2y}{4+x^2+y^2}$

$\nabla f(x,y) = \left\langle \frac{2x}{4+x^2+y^2}, \frac{2y}{4+x^2 y^2} \right\rangle \Big|_{\substack{x=-1 \\ y=2}} = \left\langle \frac{-2}{4+1+4}, \frac{4}{4+1+4} \right\rangle$

3] $\left\langle \frac{-2}{9}, \frac{4}{9} \right\rangle \cdot \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle = \boxed{\frac{-4}{9\sqrt{5}} + \frac{4}{9\sqrt{5}} = 0}$

27) $f(x,y) = x^2 - 4y^2 - 9 \quad P = (1,-2)$

Direction remains unchanged

$$\nabla f(x,y) = \left\langle f_x = 2x,\ f_y = -8y \right\rangle \Big|_{\substack{x=1 \\ y=-2}} = \langle 2, --16\rangle = \cancel{2}\langle 1,8\rangle$$

$$\nabla f(x,y) \underset{\text{DIR}}{\sim} \langle 1,8\rangle \xrightarrow[\text{VEC}]{\text{UNIT}} \boxed{\frac{1}{\sqrt{65}} \cdot \langle 1,8\rangle = \text{STEEPEST ASCENT}}$$

$$\boxed{-\left(\frac{1}{\sqrt{65}}\langle 1,8\rangle\right) = \text{STEEPEST DESCENT}} \quad \boxed{\pm\langle -8,1\rangle \cdot \frac{1}{\sqrt{65}} = \text{NO CHANGE}}$$

29) $f(x,y) = x^4 - x^2y + y^2 + 6 \quad P = (-1,1)$

$$\nabla f(x,y) = \left\langle f_x = 4x^3 - 2xy,\ f_y = -x^2 + 2y \right\rangle \Big|_{\substack{x=-1 \\ y=1}} = \langle -4+2, -1+2\rangle$$

$$\nabla f(x,y) = \langle -2, 1\rangle \xrightarrow[\text{vector}]{\text{Unit}} \boxed{\frac{1}{\sqrt{5}}\langle -2,1\rangle = \text{STEEP. ASC.}}$$

$$\boxed{-\left(\frac{1}{\sqrt{5}}\langle -2,1\rangle\right) = \text{ST. DEC}} \quad \boxed{\pm \frac{1}{\sqrt{5}}\langle -1,-2\rangle = \text{NO CHG}}$$

31) $f(x,y) = e^{\overbrace{-\frac{1}{2}x^2 - \frac{1}{2}y^2}^{\rho}} \quad P = (-1,1) \quad \rho|P = -\frac{1}{2} - \frac{1}{2} = -1$

$$\nabla f(x,y) = \left\langle f_x = -xe^{\rho},\ f_y = -ye^{\rho} \right\rangle \Big|_P = \langle e^{-1}, -e^{-1}\rangle$$

$$\nabla f(x,y) = \cancel{e^{-1}} \boxed{\langle 1,-1\rangle \cdot \frac{1}{\sqrt{2}} = \text{STE. ASC}} \quad \boxed{\text{ST. DEC} = \frac{1}{\sqrt{2}}\langle -1,1\rangle}$$

$$\boxed{\text{NO CHANGE} = \pm\frac{1}{\sqrt{2}} \cdot \langle -1,-1\rangle}$$

43) $f(x,y) = 16 - x^2/4 - y^2/16,\ P = (0,16)$

$$f_x = \frac{-x}{2} \quad f_y = \frac{-y}{8}$$

$$y'(x) = -\frac{\frac{\cancel{-}0}{2}}{\frac{\cancel{-}16}{8}} = \boxed{0}$$

45) $f(x,y) = \sec 43),\ P = 4,0$

$$f_x = \frac{-x}{2} \quad f_y = \frac{-y}{8}$$

$$y' = -\frac{\frac{-4}{2}}{\frac{-0}{8}} = +\frac{2}{0}$$

$= \text{UNDEFINED}$

$\boxed{= \text{VERTICAL TANGENT}}$