Lyell Read   CH 12.~~?~~ Homework   10/30/2018
odd 9-33, 47-53

9) $f(x,y,z) = x^2 + y + z \quad P_1 = (1,1,1) \quad P_2 = (2,0,-1)$

Tan P_1: $f_x(P_1)(x-a) + f_y(P_1)(y-b) + f_z(P_1)(z-c)$

$2(1)(x-1) + (1)(y-1) + (1)(z-1) = 2x-2-1-1 = -y-z$

$= \boxed{2x+y+z=4}$

Tan P_2: $f_x(P_2)(x-a) + f_y(P_2)(y-b) + f_z(P_2)(z-c)$

$= 2(2)(x-2) + 1(y-0) + 1(z+1) = 4x-8+y+z+1$

$= \boxed{4x+y+z=7}$

11) $f(x,y,z) = xy + yz + xz - 12 = 0 \quad P_1 = (2,2,2) \quad P_2 = (2,0,6)$

Tan $P_1 = f_x(P_1)(x-a) + f_y(P_1)(y-b) + f_z(P_1)(z-c)$

$= (y+z)(x-a) + (x+z)(y-b) + (y+x)(z-c) \; |P_1 = (2,2,2)$

$= (2+2)(x-2) + (2+2)(y-2) + (2+2)(z-2)$

$= 4x-8+4y-8+4z-8 = x-2+y-2+z-2$

$= \boxed{x+y+z=6}$

Tan $P_2 = f_x(P_2)(x-a) + f_y(P_2)(y-b) + f_z(P_2)(z-c)$

$= (y+z)(x-a) + (x+z)(y-b) + (y+x)(z-c)$

$= (0+6)(x-2) + (2+6)(y-0) + (0+2)(z-6)$

$= 6(x-2) + 8(y-0) + 2(z-6) = 6x-12+8y+2z-12$

$= 6x+8y+2z = 24 = \boxed{3x+4y+z=12}$

18) $f(x,y,z) = xy\sin z = 1 \quad P_1 = (1, 2, \pi/6) \quad P_2 = (-2, -1, \frac{5\pi}{6})$

$\text{Tan } P_1 = f_x(P_1)(x-a) + f_y(P_1)(y-b) + f_z(P_1)(z-c)$

$= (2\sin\frac{\pi}{6})(x-1) + (1\sin\frac{\pi}{6})(y-2) + (1\cdot 2\,\cancel{\sin\frac{\pi}{6}})(z-\frac{\pi}{6})\cdot\cos\frac{\pi}{6}$ $\frac{\sqrt{3}\pi}{6}$

$= (1)(x-1) + (\frac{1}{2})(y-2) + (2)(\frac{\sqrt{3}}{2})(z-\frac{\pi}{6}) = x - 1 + \frac{y}{2} - 1 + \sqrt{3}z$

$= \boxed{x + \frac{y}{2} + \sqrt{3}z = 2 + \frac{\sqrt{3}\pi}{6}}$

$\text{Tan } P_2 = f_x(P_2)(x-a) + f_y(P_2)(y-b) + f_z(P_2)(z-c)$

$= (y\sin z)(x-a) + (x\sin z)(y-b) + (xy\cos z)(z-c)$

$= (-1)(\frac{1}{2})(x+2) + (-2)(\frac{1}{2})(y+1) + (-2)(-1)(\frac{-\sqrt{3}}{2})(z - \frac{5\pi}{6})$

$= -\frac{x}{2} - 1 - y - 1 (-\frac{2\sqrt{3}}{2})(z - \frac{5\pi}{6}) = \frac{x}{2} + 1 + y + 1 + \sqrt{3}z - \frac{5\sqrt{3}\pi}{6}$

$= \boxed{\frac{x}{2} + y + \sqrt{3}z = \frac{5\sqrt{3}\pi}{6} - 2}$

17) (15 skipp'd) $f(x,y) = 4 - 2x^2 - y^2 \quad P_1 = (2, 2, -8) \quad P_2 = (-1, -1, 1)$

$\text{Tan } P_1: \ z = f_x(x-a) + f_y(y-b) + f(a,b)$

$z = -4(2)(x-2) + -2(2)(y-2) + [4 - 8 - 4]$

$z = -8x + 16 - 4y \cancel{+8} \cancel{-8} \quad \boxed{z = -8x - 4y + 16}$

$\text{Tan } P_2: \ z = f_x P_2 (x-a) + f_y P_2 (y-b) + f(a,b)$

$z = -4(-1)(x+1) + -2(-1)(y+1) + \cancel{[4 - 2 - 2]}$ corr = 1

$z = 4x + 4 + 2y + 2 + 1 \quad \boxed{z = 4x + 2y + 7}$

21) (skip 19) $f(x,y) = \underbrace{x^2}_{a}\,\underbrace{e^{x-y}}_{b}$ $P_1 = (2,2,4)$ $P_2 = (-1,-1,1)$

Tan P_1: $z = f_x(P_1)(x-a) + f_y(P_1)(y-b) + f(a,b)$

$z = f_x(a,b)\cdot(x-a) + f_y(a,b)\cdot(y-b) + f(a,b)\ \cancel{\cdot(z-c)}$

$z = (8)(x-a) + (-4)(y-2) + 4$

$= 8x - 16 - 4y + 8 + 4 = \boxed{8x - 4y - 4}$

$f_x = ab' + a'b$
$= (x^2)(e^{x-y}) +$
$(2x)(e^{x-y})$

$f_y = -x^2 e^{x-y}$

Tan P_2: $z = f_x(P_2)(x-a) + f_y(P_2)(y-b) + f(P_2)$

$z = (1-2)(x-a) + (-1)(y-b) + [f(-1,-1) = 1]$

$z = -x - 1 - y - 1 + 1$ $\boxed{z = -x - y - 1}$

31) $z = 2x - 3y - 2xy$ $(\underbrace{1}_{a},\underbrace{4}_{b}) \to (1.1, 3.9)$ $\therefore dy = -.1$ $dx = .1$

$dz \approx \Delta z = f_x(a,b)dx + f_y(a,b)dy = (2 - 2\overset{b}{y})(0.1) + (-3 - 2\overset{a}{x})(-0.1)$

$\Delta z = -0.6 + 0.5 = \boxed{-0.1}$

33) $z = e^{x+y}$ $(0,0) \to (0.1, -0.05)$ $dx = 0.1$ $dy = -0.05$

$\Delta z = f_x(0,0)\,dx + f_y(0,0)\,dy = e^{0+0}dx + e^{0+0}dy = 0.1 - 0.05$

$\boxed{\Delta z = 0.05}$

47) $z = \tan^{-1} xy$ $\left[\text{NOTE: } \frac{\partial}{\partial x}\tan^{-1}x = \frac{1}{1+x^2}\right]$ $P_1 = (1, 1, \frac{\pi}{4})$

Tan P_1: $= z = f_x(P_1)(x-a) + f_y(P_1)(y-b) + \boxed{f(a,b) = z}$

$z = \frac{y}{1+x^2y^2} + \frac{x}{1+x^2y^2} + \frac{\pi}{4} = \frac{1}{2}x - \frac{1}{2} + \frac{1}{2}y - \frac{1}{2} + \pi/4 = \boxed{\frac{1}{2}x + \frac{1}{2}y + \frac{\pi}{4} - 1}$

49) $\sin xyz = \frac{1}{2} = f(x,y,z)$ $P = (\pi, 1, \frac{1}{6})$

Tan $p_1 = f_x(P_1)(x-a) + f_y(P_1)(y-b) + f_z(P_1)(z-c)$

$= (yz)(\cancel{\cos\frac{\pi}{6}})(x-a) + (xz)(\cancel{\cos\frac{\pi}{6}})(y-b) + (xy)(\cancel{\cos\frac{\pi}{6}})(z-c)$

$= \frac{1}{6}(x-a) + \frac{\pi}{6}(y-b) + \pi(z-c) = \boxed{\frac{1}{6}(x-\pi) + \frac{\pi}{6}(y-1) + \pi(z-\frac{1}{6}}$

51) find Horiz. Tan ~~at~~ $f(x,y,z) = x^2 + y^2 - z^2 - 2x + 2y + 3 = 0$

Horizontal tangent is where gradient $= \langle 0, 0, k \mid k \in \mathbb{R} \rangle$

$\therefore f_x = 0 = 2x - 2 \rightarrow x = 1 ; f_y = 0 = 2y + 2 \rightarrow y = -1$

$f(1,-1,z) = \cancel{1} + \cancel{1} - z^2 \cancel{-2} - 2 + 3 = 0 = z^2 = 1$

$z^2 = 1 \rightarrow z = \pm 1 \quad \therefore$ solutions: $\boxed{(1,-1,1) \cup (1,-1,-1)}$

53) $z = \cos(2x)\sin(y)$ on $\{x; y; \in [-\pi, \pi]\}$ → AKA $-\pi \le x \le \pi$, $-\pi \le y \le \pi$:)

$\partial z/\partial x = 0 = -2 \underbrace{\sin(2x)}\underbrace{\sin(y)}$ $\quad \underbrace{x = \pm \frac{\pi}{4} \quad \pm\frac{3\pi}{4}}_{\text{"a"}} \quad \underbrace{y = \pm \frac{\pi}{2} \quad \pm \cancel{\frac{3\pi}{2}}}_{\text{"c"}}$

$\partial z/\partial y = 0 = \underbrace{\cos(y)}\underbrace{\cos(2x)}$ $\quad \underbrace{y = 0, \pm\pi \cancel{\pm 2\pi}}_{\text{"d"}} \quad \underbrace{x = 0, \pm\frac{\pi}{2} \quad \pm\pi}_{\text{"b"}}$

Explanation: If you choose a for x, then $\frac{\partial z}{\partial x} = 0$ no matter what y =. therefore, you need to use d to make $\partial z/\partial y = 0$. Therefore answers are (a and d) ~~(~~b or c) (or)