Lyell C Reed CH 13.1 – REVIEW 11/6/2018

13.1 Double Integrals over Rectangular Areas

- To compute $\int_c^d \int_a^b f(x,y)\, dx, dy$ first evaluate $\int_a^b f(x,y)\, dx$, then substitute that into $\int_c^d \boxed{F(x,y)}\Big|_a^b\, dy$ where $F(x,y) = \int f(x,y)$
- Notation: $\iint_R f(x,y)$ on $\{(x,y) : a \le x \le b,\ c \le y \le d\}$ just means:

  $\rightarrow \int_a^b \int_c^d f(x,y)\, dy\, dx$ WHICH IS EQUAL TO $\int_c^d \int_a^b f(x,y)\, dx\, dy$
- Average Function Value: $\frac{1}{\text{Area of rectangle}} \cdot \iint_R f(x,y)$ on {rectangle}
- Average Square distance: $\frac{1}{\text{area of rectangle}} \cdot \iint_R f(x,y)$ on {rectangle} where

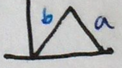
  $f(x,y) = d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ (Distance Function, Squared)

13.2 DOUBLE INTEGRALS OVER NON-RECTANGULAR AREAS

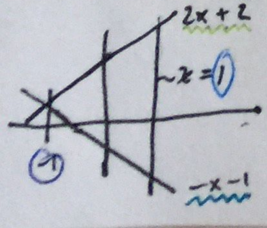
- Writing a double integral based on a graph, or functions.

  → If you are doing $dx\, dy$ the first integral is the y range so $\int_a^b \cdots dy$.

  → the inside integral, the dx one, is from the "closer" function to the y axis to the "further".

  $\int_b^a \cdots dx$
- Evaluating an integral with equations as limits of integration: just do as you would a normal integral
- Example: $\iint_R y^2\, dA$ bounded by $x=1$, $-x-1$, $2x+2$

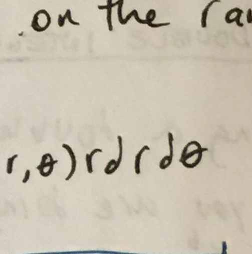


$\int_{-1}^{1} \int_{-x-1}^{2x+2} y^2\, dy\, dx \longrightarrow \boxed{12}$

- Volume Calculation using double Integrals: Integrate over the range of the base, the value of the top. to find the range of the base, with a plane, set $z=0$, solve for $y = \dots$.
- Changing the order of integration; Draw out the xy shape, and find the equations, then integrals, for the opposite variable.

$\boxed{13.3}$ INTEGRALS X2 IN POLAR COORDINATES

- Cartesian to polar: $x = r\cos\theta$; $y = r\sin\theta$ $r^2 = x^2 + y^2$
- Solving an integral pair in polar:
  - → where in Cart. you would do $\int_a^b \int_c^d f(x,y)\,dx\,dy$,
  - you instead use $\int_a^b \int_c^d f(r,\theta)\,r\,dr,d\theta$ {a,b,c,d} arent the same as }
- Annular Region: simply evaluate the integral on the range provided.
- Converting from $\underbrace{\int_a^b \int_c^d f(x,y)\,dx\,dy}_{\text{integral 1}} \dashrightarrow \underbrace{\int_e^f \int_g^h f(r,\theta)\,r\,dr\,d\theta}_{\text{integral 2}}$
  - → Draw out area of integral 1, then find values of r, θ that will draw shape. convert $f(x,y) \to f(r,\theta)$ and plug into integral 2.
- Volume between upper and lower curves



  - → find function C where A = B where A,B are in polar. should result in $C = (r = n)$ where n ~~is~~ maybe an integer
  - → find integral over area C of $\int_a^b \int_c^d (\text{top} - \text{bottom})\,r\,dr\,d\theta$