Lyell C. Read CH 12.4 – REVIEW 10/24/2018

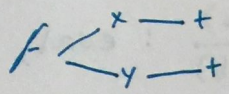
NOTE: KNOW YOUR DERIVATIVES (i.e. A^x, x^n, $\ln(x)$ ... and product, chain rules!

12.4 PARTIAL DERIVATIVES

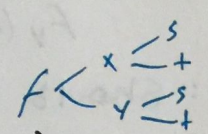
- First Degree Partial Derivative: for $f(x,y)$:
  - → Calc f_x = Derivative of $f(x,y)$ treating all non-x var's as const.
  - → Calc f_y = Derivative $f(x,y)$ where all non-y var's are const.
  - → NOTE: $f_x = \frac{\partial f}{\partial x}$, $f_y = \frac{\partial f}{\partial y}$
- First Deg. P. D. Using Limits: for $f(x,y)$
  - → $f_x = f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$
  - → $f_y = f_y(c,d) = \lim_{h\to 0} \frac{f(c,d+h) - f(c,d)}{h}$
- Second Degree P. D.'s: for function $f(x,y)$:
  - → Perform FDPD for the first var, then again on result for second: f_{xy} would be f_x derived with y ...

12.5 DA CHAIN RULE

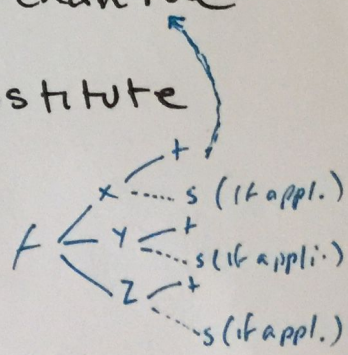
- f' for $f \in x \dots y$ where x is a function of another var, and so is y
  - → $f' = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$



- f' for $f(x,y)$ where $x(s,t)$, $y \in (s,t)$
  - → $f'_s = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}$, $f'_t = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$



- If $f(x,y,z)$: add another term, $\frac{\partial f}{\partial z}$, and chain rule
- solution process: evaluate $f'(\dots)$; then substitute $x=$, $y=$, $z=\dots$ and simplify
- Implicit Differentiation $\frac{dy}{dx} = -\frac{F_x}{F_y}$



12.6 GRADIENTS AND DIRECTIONAL DERIVATIVES

- Gradients: $\nabla f(x,y)$ = "gradient" = $\langle f_x, f_x \rangle$. If evaluated at a point, substitute x, y from point, solve as a vector.

- Directional Derivative:
  1) find if $\vec{v}$ provided is unit vector, else make it one
  2) find $\nabla f(x,y)$ (see above for help there :)
  3) find $D_u f(x,y) = u \cdot \nabla f(x,y) = u_x \cdot f_x + u_y \cdot f_y$
  4) Substitute Point "P".

  NOTE: STEPS 3 and 4 CAN BE DONE INTERCHANGEABLY

- Directions of steepest change, no change
  → steepest ascent = $\left(\nabla f(x,y) \,/\, \|\nabla f(x,y)\|\right)$ AKA unit in dir of $\nabla f(x,y)$
  → steepest descent = $-$ [steepest ascent]
  → NO change = $\pm$ [swapped variables and one made neg, i.e. $\langle 8,1 \rangle \to \langle -1, 8 \rangle$ from pos steepest or neg steepest.

- Tangent Line to Level Curves: $f(x,y)$ at point $P = (a,b)$
  → $y'(x) = -\frac{f_x(a,b)}{f_y(a,b)}$: eval, if UND = VERT TAN, ~~IE~~
  → CHECK: should be $\perp$ to $\nabla f(x,y) \,|\, P$ ...