Lyell. C. Reed CH 13.1 Homework Pr 11-35 odd $^{11}/6/2018$

11) $\int_1^4 \int_0^4 \sqrt{uv}\ du\ dv$ $\qquad \| \sqrt{uv} = \sqrt{u} \cdot \sqrt{v}$

1] $\int_0^4 \sqrt{v} \cdot (u)^{\frac{1}{2}}\ du \xrightarrow{\text{ANTI}} \frac{2}{3}\sqrt{v} \cdot u^{\frac{3}{2}} \Big|_{\cancel{0}}^{4} = \frac{16}{3}\sqrt{v}$

2] $\int_1^4 \frac{16}{3}\sqrt{v}\ dv \xrightarrow{\text{ANTI}} \frac{32}{9}(v)^{\frac{3}{2}} \Big|_1^4$

$= 8\left(\frac{32}{9}\right) - \frac{32}{9} = 7\left(\frac{32}{9}\right) = \boxed{\frac{224}{9}}$

13) $\int_0^{\ln 2} \int_0^1 6xe^{3y}\ dx\ dy$

1] evaluate $\int_0^1 6xe^{3y}\ dx \xrightarrow{\text{ANTI}} 3x^2e^{3y}\Big|_0^1 = \left[3(1)e^{3y}\right] - 0$

2] evaluate $\int_0^{\ln 2} 3e^{3y}\ dy \xrightarrow{\text{ANTI}} e^{3y}\Big|_0^{\ln 2} = e^{3\ln 2} - 1 = \cancel{e}^{\cancel{\ln} 2^3} - 1 = 8 - 1 = \boxed{7}$

15) $\int_1^{\ln 5} \int_0^{\ln 3} e^{x+y}\ dx\ dy$

1] $\int_0^{\ln 3} e^{x+y}\ dx \xrightarrow{\text{ANTI}} e^{x+y}\Big|_0^{\ln 3} = e^{\ln 3 + y} - e^{y}$ $\qquad \| e^{a+b} = e^a \cdot e^b$

2] $\int_1^{\ln 5} e^{\ln 3 + y} - e^{y}\ dy \xrightarrow{\text{ANTI}} e^{\ln 3 + y} - e^{y}\Big|_1^{\ln 5} = \left[e^{\ln 3 + \ln 5} - e^{\ln 5}\right] - \left[e^{\ln 3 + 1} - e^{1}\right]$

$= [5 \cdot 3 - 5] - [3e - e]$

$= \boxed{10 - 2e}$

17) $\iint_R (x+2y)$ on $0 \le x \le 3,\ 1 \le y \le 4$ $= \int_1^4 \int_0^3 (x+2y)\ dx\ dy$

1] $\int_0^3 (x+2y)\ dx \xrightarrow{\text{ANTI}} \frac{1}{2}x^2 + 2xy \Big|_0^3 = \frac{9}{2} + 6y - 0$

2] $\int_1^4 \frac{9}{2} + 6y\ dy \xrightarrow{\text{ANTI}} \frac{9y}{2} + 3y^2 \Big|_1^4 = \left[18 + 48\right] - \left[\frac{9}{2} + 3\right] = \frac{132}{2} - \frac{15}{2} = \boxed{\frac{117}{2}}$

19) $\iint_R 4x^3 \cos y$ on $1 \le x \le 2,\ 0 \le y \le \pi/2$ $= \int_1^2 \int_0^{\pi/2} 4x^3 \cos y \, \partial y \, \partial x$

1] $\int_0^{\pi/2} 4x^3 \cos y \, \partial y \xrightarrow{\text{ANTI}} 4x^3 \sin y \Big|_0^{\pi/2} = \underline{4x^3(1)} - \cancel{4x^3(0)}$

2] $\int_1^2 4x^3 \, dx \xrightarrow{\text{ANTI}} x^4 \Big|_1^2 = 2^4 - 2^1 = 16 - 1 = \boxed{15}$

$\left\| \sqrt{\frac{a}{b}} = a^{\frac{1}{2}} \cdot b^{-1/2} \right.$

21) $\iint_R \sqrt{\frac{x}{y}}$ on $0 \le x \le 1,\ 1 \le y \le 4$ $=$

1] $\int_0^1 x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \, \partial x \xrightarrow{\text{ANTI}} \frac{2}{3} x^{\frac{3}{2}} \cdot y^{-\frac{1}{2}} \Big|_{\cancel{0}}^1 = \underline{\frac{2}{3} y^{-\frac{1}{2}}}$

2] $\int_1^4 \frac{2}{3} y^{-1/2} \, \partial y \xrightarrow{\text{ANTI}} \frac{4}{3} y^{\frac{1}{2}} \Big|_1^4 = \frac{8}{3} - \frac{4}{3} = \boxed{\frac{4}{3}}$

23) $\iint_R e^{x+2y}$ on $0 \le x \le \ln 2,\ 1 \le y \le \ln 3$ $= \int_1^{\ln 3} \int_0^{\ln 2} e^{x+2y} \, dx \, \partial y$

1] $\int_0^{\ln 2} e^{x+2y} \, dx \xrightarrow{\text{ANTI}} e^{x+2y} \Big|_0^{\ln 2} = \underline{e^{\ln 2 + 2y} - e^{2y}}$

2] $\int_1^{\ln 3} e^{\ln 2 + 2y} - e^{2y} \, \partial y \xrightarrow{\text{ANTI}} \frac{1}{2} e^{\ln 2 + 2y} - \frac{1}{2} e^{2y} \Big|_1^{\ln 3} =$

$= \left[\frac{1}{2} e^{\ln 2 + 2\ln 3} - \frac{1}{2} e^{2 \ln 3} \right] - \left[\frac{1}{2} e^{\ln 2 + 2} - \frac{1}{2} e^2 \right]$

$= \left[\frac{1}{2}(2 \cdot 3^2) - \frac{1}{2}(9) \right] - \left[\frac{1}{2}(2 \cdot e^2) - \frac{1}{2} e^2 \right] = \frac{9}{2} - \frac{e^2}{2} = \boxed{\frac{9 - e^2}{2}}$

25) $\iint_R (x^5 - y^5)^2$ $0 \le x \le 1,\ -1 \le y \le 1$ $\rightsquigarrow \int_{-1}^{1} \int_0^{1} (x^5 - y^5)^2 \, \partial x \, \partial y$

$= \int_{-1}^{1} \int_0^{1} x^{10} - 2x^5 y^5 + y^{10} \, dx \, dy$

Cont'd below

1] $\int_0^1 x^{10} - 2x^5y^5 - y^{10}\, dx \xrightarrow{\text{ANTI}} \frac{1}{11}x^{11} - \frac{2}{6}x^6y^5 + y^{10}x \Big|_0^1 =$

$= \underbrace{\frac{1}{11} - \frac{2}{6}y^5 + y^{10}}$

2] $\int_{-1}^1 \frac{1}{11} - 2/6y^5 + y^{10}\, dy \xrightarrow{\text{ANTI}} \frac{y}{11} - \frac{2}{12}y^6 + \frac{1}{11}y^{11} \Big|_{-1}^1 =$

$= \left[\frac{1}{11} - \frac{\cancel{2}}{\cancel{12}} + \frac{1}{11}\right] - \left[-\frac{1}{11} - \frac{\cancel{2}}{\cancel{12}} - \frac{1}{11}\right] = \boxed{\frac{4}{11}}$

27) $\iint_R (y+1)e^{x(y+1)} \quad 0 \le x \le 1,\ -1 \le y \le 1 \rightsquigarrow \int_0^1\int_{-1}^1 (y+1)e^{x(y+1)}\, dx\, dy$

NOTE, IT APPEARS BEST TO START WITH DX, AS THAT MAKES FOR: ne^{nx}

1] $\int_0^1 (y+1)e^{x(y+1)}\, dx \xrightarrow{\text{ANTI}} e^{x(y+1)} \Big|_0^1 = \underbrace{e^{y+1} - 1}$

2] $\int_{-1}^1 e^{y+1} - 1\, dy \xrightarrow{\text{ANTI}} e^{y+1} - y \Big|_{-1}^1 = \left[e^2 - 1\right] - \left[1 + 1\right] = \boxed{e^2 - 3}$

29) $\iint_R 6x^5 e^{x^2y} \quad 0 \le x \le 2,\ 0 \le y \le 2 \rightsquigarrow \int_0^2\int_0^2 6x^5 e^{x^2y}\, dy\, dx$

NOTE: STARTING WITH DX AS THE EQUATION APPEARS TO SIMPLIFY...

1] $\int_0^2 6x^5 e^{x^2y}\, dx \xrightarrow{\text{ANTI}}$

31) $\iint_R \frac{x}{(1+xy)^2}\,dx$

33) $f(x,y)=e^{-y}$ $\quad 0\le x\le 6,\ 0\le y\le \ln 2 \quad \to \int_0^{\ln 2}\int_0^6 e^{-y}\,dx\,dy$

1] $\int_0^6 e^{-y}\,dx \to xe^{-y}\Big|_0^6 = \underline{6e^{-y}}$

ANTI

2] $\int_0^{\ln 2} 6e^{-y}\,dy \to -6e^{-y}\Big|_0^{\ln 2} = -6\cdot\frac{1}{2}-(-6) = \boxed{3}$

ANTI

$A_R = 6\ln 2$

$\bar{f}(x,y)$ on $0\le x\le 6,\ 0\le y\le \ln 2 = \frac{3}{6\ln 2} = \boxed{\frac{1}{2\ln 2}}$

35) Squared Distance $(-2\le x\le 2,\ 0\le y\le 2)$ and $(0,0)$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad d=\sqrt{(-x)^2+(-y)^2}$ $\quad d^2=(-x)^2+(-y)^2$

$= x^2+y^2$

$d^2=x^2+y^2$ $\quad \int_{-2}^{2}\int_0^{2} x^2+y^2\,dy\,dx$

Average $= \frac{1}{A}\cdot$ val $\to \frac{64}{3}\cdot\frac{1}{8} = \boxed{\frac{8}{3}}$

$A = 4\cdot 2$

1] $\int_0^2 x^2+y^2\,dy \to yx^2+\frac{1}{3}y^3\Big|_0^2 =$

ANTI

$= \underline{2x^2+\frac{8}{3}}$

2] $\int_{-2}^{2} 2x^2+\frac{8}{3}\,dx \to \frac{2}{3}x^3+\frac{8x}{3}\Big|_{-2}^{2}$

$= \frac{16}{3}+\frac{16}{3}-\left(-\frac{16}{3}-\frac{16}{3}\right) = \frac{64}{3}$