Lyell. C. Read

CHAPTER 12.4 Homework
pr. 7-37 odd

10/20/2018

7) $f(x,y) = 5xy$

$f_x = f_x(a,b) = \lim_{h\to 0} \dfrac{f(a+h,b) - f(a,b)}{h}$

$f_y = f_y(ab) = \lim_{h\to 0} \dfrac{f(a,b+h) - f(a,b)}{h}$

$f_x(a,b) = \lim_{h\to 0} \dfrac{5y(x+h) - \cancel{\#}5xy}{h} = \dfrac{5xy + 5hy - 5xy}{h} = \boxed{5y}$

→ same process just reversed for $f_{\cancel{x}}$ - - - - - - - - - → $= \boxed{5x = f_y}$

9) $f(x,y) = \dfrac{x}{y}$ $\quad f_x(a,b) = \lim_{h\to 0} \dfrac{\frac{x+h}{y} - \frac{x}{y}}{h} = \dfrac{\frac{h}{y}}{h} = \dfrac{h}{y}\cdot\dfrac{1}{h} = \boxed{\dfrac{1}{y} = f_x}$

$f_y(a,b) = \lim_{h\to 0} \dfrac{\frac{x}{y+h} - \frac{x}{y}}{h} = \dfrac{\frac{xy}{y^2+yh} - \frac{xy+xh}{y^2+yh}}{h}$

$= \dfrac{\frac{xh}{y^2+yh}}{h} \lim_{h\to 0} = \dfrac{\cancel{h}\left(\frac{x}{y^2+yh}\right)}{\cancel{h}} \lim_{h\to 0} = \dfrac{x}{y^2+\cancel{0y}} = \boxed{\dfrac{x}{y^2} = f_y}$

11) $f(x,y) = 3x^2 + 4y^3$ $\quad \boxed{f_x(x,y) = 6x}$ $\quad \boxed{f_y(x,y) = 12y^2}$

13) $f(x,y) = 3x^2y + 2$ $\quad \boxed{f_x(x,y) = 6xy}$ $\quad \boxed{f_y(x,y) = 3x^2}$

15) $f(x,y) = xe^y$ $\quad \boxed{f_x(x,y) = e^y}$ $\quad \boxed{f_y(x,y) = xe^y}$

17) $f(x,y) = \cos 2xy$ $\quad \boxed{f_x(x,y) = -2y\sin(2xy)}$ $\quad \boxed{f_y(x,y) = -2x\sin 2xy}$

19) $f(x,y) = e^{x^2y}$ $\quad \boxed{f_x(x,y) = 2yxe^{x^2y}}$ $\quad \boxed{f_y(x,y) = x^2e^{x^2y}}$

21) $f(x,y) = \dfrac{x}{x^2+y^2}$ $\quad \dfrac{edh - hde}{e^2}$ $\Rightarrow f_x(x,y) = \dfrac{(x^2+y^2)(1) - (x)(\cancel{x^2})^{2x}}{(x^2+y^2)^2} = \boxed{\dfrac{y^2-x^2}{(x^2+y^2)^2}}$

$f_y = \dfrac{edh - hde}{e^2} = \dfrac{(0)(\dots) - x(2y)}{(x^2+\cancel{2}y^2)^2} = \boxed{\dfrac{2xy}{(x^2+y^2)^2}}$

23) $f(x,y) = \underbrace{y^2}_{a}\underbrace{\tan(\cancel{x}y)}_{b}$ $\quad f_x = \cancel{\#}\; ab' + a'b = (y^2)(y\sec^2(\cancel{x}y)) + (0)(\dots)$

$= \boxed{y^3\sec^2(xy)}$

$f_y = ab' + a'b = \boxed{(y^2)(x\sec^2(xy)) + (2y)(\tan xy)}$

25) $G(s,t) = \frac{\sqrt{st}}{s+t} \dashrightarrow f(x,y) = \frac{\sqrt{xy}}{x+y} = \frac{(xy)^{\frac{1}{2}}}{x+y}$

$f_x = \frac{l\,dh - h\,dl}{l^2} = \frac{(x+y)((xy)^{-\frac{1}{2}} \cdot y) - ((xy)^{\frac{1}{2}}(1))}{(x+y)^2}$ $\underset{\text{should}}{\sim}$ $\frac{\sqrt{xy}\,(y-x)}{2x(x+y)^2}$

$f_y = \frac{l\,dh - h\,dl}{l^2} = \frac{(x+y)(x(xy)^{-\frac{1}{2}}) - (xy^{\frac{1}{2}})(1)}{(x+y)^2}$ $\underset{\text{should}}{\sim}$ $\frac{\sqrt{xy}\,(x-y)}{2y(x+y)^2}$

27) $F(x,y) = x^{2y}$ $\quad f_x = x^{\langle \text{constant} \rangle} = \boxed{2y\,x^{2y-1}}$

$f_y = \underbrace{x}_{\text{const}}{}^{2y} = \boxed{\ln(x) \cdot x^{2y} \cdot 2}$

29) $f(x,y) = x^3 + xy^2 + 1$ $\quad f_x(x,y) = 3x^2 + y^2$

$f_y(x,y) = \cancel{x^3} + 2xy$
0

$f_{xx} = \frac{d}{dx}\, 3x^2 + \underbrace{\cancel{3}y^2}_{\text{const}} = \boxed{6x}$ $\quad f_{xy} = \boxed{2y}$

$f_{yx} = \cancel{3x^2} + \boxed{2y}$ $\quad f_{yy} = \boxed{2x}$

31) $f(x,y) = x^2y^3$ $\quad f_x = 2y^3x$ $\quad f_y = 3x^2y^2$

$f_{xx} = \boxed{2y^3}$ $\quad f_{xy} = \boxed{6y^2x}$ $\quad f_{yx} = \boxed{6y^2x}$ $\quad f_{yy} = \boxed{6x^2y}$

33) $f(x,y) = y^3\sin(4x)$ $\quad f_x = 4y^3\cos(4x)$ $\quad f_y = 3y^2\sin(4x)$

$f_{xx} = \boxed{-16\sin(4x) \cdot y^3}$ $\quad f_{xy} = \boxed{12y^2\cos 4x}$ $\quad f_{yx} = \boxed{12y^2\cos(4x)}$ $\quad f_{yy} = \boxed{6y\sin 4x}$

35) $f(x,y) = \ln(x^2+y^2+4)$ $\quad f_x = \frac{2x}{x^2+y^2+4}$ $\quad f_y = \frac{2y}{x^2+y^2+4}$

$f_{xx} = \frac{l\,dh - h\,dl}{l^2} = \frac{(2)(x^2+y^2+4) - (2x)(2x)}{(x^2+y^2+4)^2} = \frac{2x^2+2y^2+8-4x^2}{''}$

$= \boxed{\frac{-2x^2+2y^2+8}{(x^2+y^2+4)^2}}$

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35) cont'd from last page $\quad f_x = \dfrac{2x}{x^2+y^2+4} \quad f_y = \dfrac{2y}{x^2+y^2+4}$

$$f_{xy} = \frac{\ell dh - hd\ell}{\ell^2} \quad x = \text{const} = \frac{(x^2+y^2+4)(0) - (2x)(2y)}{\ell^2}$$

$$= \boxed{\frac{4xy}{(x^2+y^2+4)^2}}$$

$$f_{yx} = \frac{\ell dh - hd\ell}{\ell^2} \quad y = \text{const} = \frac{0 \ldots - (2y)(2x)}{\ell^2} = \boxed{\frac{4xy}{(x^2+y^2+4)^2}}$$

$$f_{yy} = \frac{\ell dh - hd\ell}{\ell^2} \quad x = \text{const} = \frac{(x^2+y^2+4)(2) - (2y)(2y)}{\ell^2}$$

$$= \frac{2x^2 + \cancel{2y^2} + 8 - \cancel{4}y^2}{(x^2+y^2+4)^2} = \boxed{\frac{2x^2 - 2y^2 + 8}{(x^2+y^2+4)^2}}$$

37) $f(x,y) = xe^y \qquad f_x = e^y \qquad f_y = xe^y$

$$f_{xx} = \boxed{0} \quad f_{xy} = \boxed{e^y} \quad f_{yx} = \boxed{e^y} \quad f_{yy} = \boxed{xe^y}$$