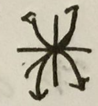
Lyell C Read

12.8 Homework
Pr: 19–27 odd, 43–51

11/1/2018

19) $f(x,y) = 4 + 2x^2 + 3y^2$ This is obviously a "parabola" up, critical point is a $\boxed{\text{min at } x=0,\ y=0.}$

21) $f(x,y) = -4x^2 + 8y^2 + 3$ $f_x = -8x = 0$ at $x=0$ $f_y = 16y = 0$ at $y=0$

Graphing $f(x,y)$ shows a saddle  at $\boxed{x,y = 0,0. \text{ saddle}}$

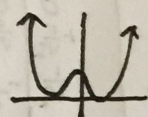
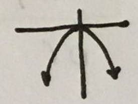
23) $f(x,y) = x^4 + 2y^2 - 4xy$

$f_x = 4x^3 - 4y$ $\Rightarrow$ $x^3 = y$ & (and) $x = y$ $\Rightarrow$ All points where $x = x^3$ $[\pm 1, 0]$

$f_y = 4y - 4x$

$x = y$

x	y	D(x,y)	f_{xx}	Res.
1	1	32	48	MIN
-1	-1	32	48	MIN
0	0	-16	—	SAD

$y = x^4$:  $y = -x^2$  $D(x,y) = f_{xx} f_{yy} - f_{xy}^2$

$f_{xx} = 12x^2$ $f_{xy} = -4$ $f_{yx} = -4$ $f_{yy} = 4$

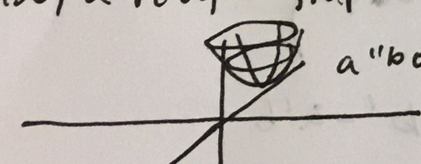
25) $f(x,y) = \sqrt{x^2 + y^2 - 4x + 5}$ $f_x = \dfrac{2x-4}{2(x^2+y^2-4x+5)^{\frac{1}{2}}}$ $f_y = \dfrac{2y}{2(x^2+y^2-4x+5)^{\frac{1}{2}}}$

f_x is never undefined as the bottom cannot be $=< 0$, top $= 0$ at $x = 2$

f_y is never undefined. top $= 0$ when $y = 0$. $\therefore$ Pont is $(2,0)$

Instead of doing the $D(x,y)$ test, I'll just plug in a point... or two

Also, a rough graph is:



$P_1 = (0, 100, \cancel{z})$ $f(0,100) = \approx 100 > 0$

$P_2 = (10, 0, \cancel{z})$ $f(10,0) = \sqrt{60+5} > 0$

Therefore, I think it is sketchy but OK to say that $\boxed{x = 2,\ y = 0 \text{ is a minimum}}$, not a saddle

27) $f(x,y) = 2xye^{-x^2-y^2}$

$f_x = \underbrace{2y(x)}_{a}\underbrace{(e^{-x^2-y^2})}_{b}$ DERIV OF $\Rightarrow ab' + a'b = (2yx)(-2xe^{-x^2-y^2}) + (2y)(e^{-x^2-y^2})$

$= e^{-x^2-y^2}(-4yx^2 + 2y) = 0$ when $\boxed{y=0}$ or $\cancel{4}yx^2 = \cancel{2}y$

$x = \mathbb{R}$

$2yx^2 = y$

$\cancel{y}x^2 = \frac{\cancel{4}}{2} \cdot \frac{1}{\cancel{y}}$ $y \neq 0$

$x^2 = \frac{1}{2}$ $\cancel{x = \pm\frac{1}{4}}$

or

$\boxed{x = \pm\frac{1}{\sqrt{2}} \quad y = \mathbb{R}}$

$f_y = \underbrace{(2(y)x)}_{a}\underbrace{(e^{-x^2-y^2})}_{b} \Rightarrow ab' + a'b$

$= (2xy)(-2y(e^{-x^2-y^2})) + (2x)(e^{-x^2-y^2})$

$= e^{-x^2y^2}(-4xy^2 + 2x) \xrightarrow{=0}$ $\boxed{x=0,\ y=\mathbb{R}}$ or $\boxed{x=\mathbb{R},\ y=\pm\frac{1}{\sqrt{2}}}$

$\therefore$ Test Points either $x,y=0$ or $x,y = \pm 1/\sqrt{2}$

X	Y	D	f_{xx}	R
0	0	-4	—	SAD
$+\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	+	—	MAX
$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	+	+	MIN
$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	+	+	MIN
$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	+	—	MAX

$D(x,y) = f_{xx}f_{yy} - f_{xy}^2$

$f_{xx} = \frac{\partial f_x}{\partial x} \Big| f_x = \underbrace{e^{-x^2-y^2}}_{a}\underbrace{(-4yx^2+2y)}_{b} \Rightarrow ab' + a'b$

$= (-2xe^{-x^2-y^2})(-4yx^2+2y) + (-8yx\cancel{4})(e^{-x^2-y^2})$

Eval at Points, fill in Chart

$f_{yy} = \frac{\partial f}{\partial x}\cancel{\partial} f_y = \underbrace{(\cancel{2yx})}_{a}\underbrace{(e^{-x^2-y^2})}_{b} \Rightarrow ab' + a'b$ $-4xy^2+2x$

$= (\cancel{2xy})(-2ye^{-x^2-y^2}) + (-8xy\cancel{x}e^{-x^2-y^2})$ $-4xy^2+2x$

$f_{xy} = \frac{\partial f}{\partial y}\underbrace{e^{-x^2-y^2}}_{a}\underbrace{(-4yx^2+2y)}_{b} \Rightarrow ab' - a'b$

$= (-2ye^{-x^2-y^2})(\cancel{-4x^2+2}\ -4yx^2+2y) + (e^{-x^2-y^2})(-4x^2+2)$

43) $f(x,y) = x^2 + y^2 - 2y + 1 \qquad R = \{x, y : x^2 + y^2 \leq 4\}$

1] find critical pts + values for $f(x,y)$

$f_x = 2x \rightsquigarrow = 0$ when $x = 0 \qquad f_y = 2y - 2 = 0 \to y = 1 \qquad f(0,1) = 0$

2] find criticals on boundary

boundary is circle with $r = 2$. $\quad x = 2\cos\theta \quad y = 2\sin\theta$

$g(\theta) = \left(4\cos^2(\theta) + 4\sin^2(\theta)\right) - 4\sin\theta + 1 \;=\; -4\sin\theta + 5$

$-4\sin\theta + 5$ is largest when $\theta = \frac{3\pi}{2}$ (ignore smallest b/c would be > 0)

$g\left(\frac{3\pi}{2}\right) = 9\;;\; \underset{2\cos\frac{3\pi}{2}}{x = 0} \quad \underset{2\sin\frac{3\pi}{2}}{y = -2}$

↳ $\boxed{\text{Max at } x=0,\ y=-2,\ \text{val}=9,\ \text{Min at } x=0,\ y=1,\ \text{val}=0}$

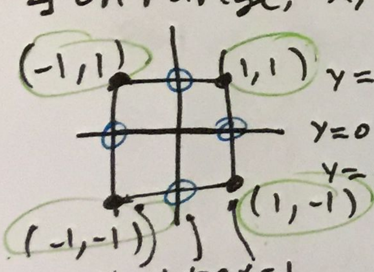
45) $f(x,y) = 4 + 2x^2 + y^2$ on $R = \{(x,y) \mid x,y \text{ in } [-1,1]\}$

X	Y	Z
0	0	4
1	1	7
-1	-1	7
1	-1	7
-1	1	7
0	1	5
0	-1	5
1	0	6
-1	0	6

1] Find critical points + values for $f(x,y)$

$f_x = 4x = 0 \quad x = 0 \qquad f_y = 2y \mid y = 0$

2] on range, x, y in $[-1, 1]$



$y = 1 \longrightarrow g(x) = 4 + 2x^2 + 1 \quad g' = 0 = 4x \quad x = 0, \; g = 4 + 1$

$y = -1 \longrightarrow$ " " " $= 5$ @ $x = 0$

$\longrightarrow$ $x = 1$ same as $x = -1$ $\quad 4 + 2 + y^2 = g(x) \quad g'(x) = 2y \quad y = 0,\ g = 6$

↳ $\boxed{\text{Max's at } (\pm 1, \pm 1) = 7,\ \text{mins at } (0,0) = 4}$