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MATH 254H, Fall 2018

Test #2

FOR EACH PROBLEM SHOW ALL ESSENTIAL STEPS.

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1. Use the chain rule to find the derivatives $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at the point (s,t), where $z = xy - x^2y,\ x = s+t,\ y = s-t.$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} \quad ✓ \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \quad ✓$$

$$\frac{\partial z}{\partial s} = (y-2xy)(1) + (x-x^2)(1) \Rightarrow \left[(s-t)-2(s+t)(s-t)\right] + \left[(s+t)-(s+t)^2\right]$$

solved on reverse

✓ $y = s-t \quad x = s+t$

$$\frac{\partial z}{\partial t} = (y-2xy)(1) + (x-x^2)(-1) \Rightarrow \left[(s-t)-2(s+t)(s-t)\right] - \left[(s+t)-(s+t)^2\right]$$

✓

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2. Evaluate $\frac{dy}{dx}$ if $y = y(x)$ is defined implicitly by $\sqrt{x^2+2xy+y^4} = 3$. $= (x^2+2xy+y^4)^{\frac{1}{2}}$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y}$$

$$= -\frac{F_x}{F_y} = -\left[\frac{\frac{2x+2y}{2\sqrt{x^2+2xy+y^4}}}{\frac{2x+4y^3}{2\sqrt{x^2+2xy+y^4}}}\right] = -\left[\frac{\cancel{2}x+\cancel{2}y}{\cancel{2}x+\cancel{4}y^3}\right] = \boxed{-\frac{x+y}{x+2y^3}} \quad ✓$$

-?

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3. Compute the directional derivative of $f(x,y) = \sqrt{4-x^2-2y}$ at the point $P(2,-2)$ in the direction $\mathbf{u} = < \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} >$.

Grad dot U(u) $\quad \|u\| = \sqrt{\frac{1}{5}+\frac{4}{5}} = \sqrt{5/5} = 1$ ✓ u is unit

$$\nabla f(x,y) = \langle f_x, f_y \rangle = \left\langle \frac{-2x}{2\sqrt{4-x^2-2y}}, \frac{-2}{2\sqrt{4-x^2-2y}} \right\rangle \Bigg|_{\substack{x=2 \\ y=-2}}$$

$$= \left\langle \frac{-4}{2\sqrt{\cancel{4}-\cancel{4}+4}}, \frac{-2}{2\sqrt{\cancel{4}-\cancel{4}+4}} \right\rangle = \left\langle -1, -\frac{1}{2} \right\rangle = \nabla f(x,y)$$

$$\underline{\left\langle -1, -\frac{1}{2} \right\rangle \cdot \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle} = \boxed{\left\langle -\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle} \quad \text{ack}$$

✓ $\quad = \quad ??$

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10 4. Find an equation of the plane tangent to the surface $x^2+y+z=3$ at the point $P(2,0,-1)$.

Tan Pl. $= F_x(x-a) + F_y(y-b) + F_z(z-c)$

$= \left(\overset{x=a}{2x}(x-a)\right) + (y-b) + (\cancel{z}-c)$

$a=2$ $b=0$ $c=-1$

$= 4x-8+y+z+1$

?? $= \boxed{4x+y+z-7=0}$

5. For the function $f(x,y)=x^4-x^2y+y^2+6$ at the point $P(-1,1)$,

10 (a) find the unit vector that gives the direction of steepest ascent, and

10 (b) find the slope of the graph in that direction.

$\nabla f(x,y)$ points in dir. of steepest ascent

$\nabla f(x,y) = \langle f_x, f_y \rangle = \langle 4x^3-2xy, -x^2+2y \rangle \Big|_P$

$= \langle -4+2, -1+2 \rangle = \boxed{\dfrac{\langle -2, 1\rangle}{\sqrt{4+1}}}$ = steepest ascent unit v ✓

Directional Derivative. This is "v" · "Grad"

$U = \left\langle \frac{-2}{\sqrt5}, \frac{1}{\sqrt5} \right\rangle$ $\nabla f(x,y) = \langle -2, 1\rangle$ $\boxed{\vec{U}\cdot\vec{\nabla f(x,y)} = \frac{4}{\sqrt5} + \frac{1}{\sqrt5} \text{ Directional, } \vec{U}}$ ✓

10 6. Find the linear approximation of the function $f(x,y)=xy+x-y$ at the point $(2,3)$. Use it to estimate $f(2.1, 2.99)$.

Derivative Plane at 2,3 : $f_x(x-a) + f_y(y-b) + f(a,b)$

$= (\overset{b}{y}+1)(x-a) + (\overset{a}{x}-1)(y-b) + (6+2-3)$

$4x - \underbrace{4(2)}_{-8} + \underbrace{1(y-3)}_{-3} + 5$

$\boxed{L(x,y) = 4x+y-6 = \text{Linear Approx}}$

8.4
2.99
11.39

$L(2.1, 2.99) =$

$4(2.1) + (2.99) - 6$

$8.4 + 2.99 - 6$

98 $11.39 - 6 = \boxed{5.39}$

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$D = f_{xx} f_{yy} - (f_{xy}^2)$

$D > 0$ $f_{xx} < 0$ MAX
$D > 0$ $f_{xx} > 0$ MIN
$D < 0$ SAD $D = 0$ UND

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7. For the function $f(x,y) = x^2 + y^2 - 4x + 5$, find the critical points, and determine whether each is a local maximum, local minimum, or saddle point.

CRITICAL IS WHERE $F_x = F_y = 0$

$\underbrace{F_x = 2x - 4 \quad F_y = 2y}$

critical where $(x=2, y=0)$

X	Y	D	f_{xx}	VAL
2	0	4	>0	MINIM

$f_{xx} = 2$
$f_{yy} = 2$ $\quad f_{xy} = f_{yx} = 0 \quad D = (2)(2) - 0^2 = 4$

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8. Find the absolute maximum and minimum values of the function $f(x,y) = 4 + 2x^2 + y^2$ on the region $R = \{(x,y) : -1 \le x \le 1,\ -1 \le y \le 1\}$.

1] find, eval all crit inside Dom R

$f_x = 4x \quad f_y = 2y$ } CRITICAL $x=0$ $y=0$

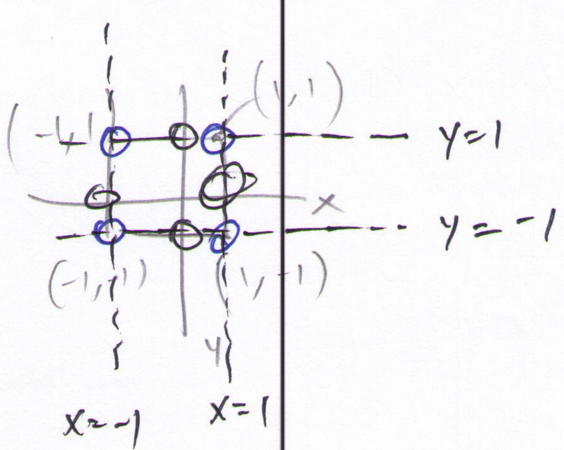
2] Evaluate the corners of the sq. Domain

3] Eval where min max on lines that make edges

x	y	VAL=f(x,y)
0	0	4
1	1	7
-1	-1	7
-1	1	7
1	-1	7
0	1	5
0	-1	5
1	0	6
-1	0	6

∴ MIN $(0,0)$ val: 4
∴ MAX $(\pm1, \pm1)$, val 7

On $y=1$ $\quad g_1(x) = 4 + 2x^2 + 1 \quad g'(x) = 4x, = 0 : x=0, y=1$
On $y=-1$ " " " " $= 4x, = 0 \nearrow$
On $x = \pm 1$, $g(y) = 6 + y^2$, $g'(y) = 2y$, $= 0 \rightarrow x = \pm 1, y = 0$



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