Lyell C Read   CH 11.6 Homework   10/4/2018

n in [15..25] | n%2!=0, 47, 49
53, 55, 59, 61, 69, 71, 79, 81, 83

15) $r(t) = \langle t, 3t^2, t^3 \rangle$ @ $t=1$ $r'(t) = \langle 1, 6t, 3t^2 \rangle \mid t=1 = \boxed{\langle 1,6,3 \rangle}$

17) $r(t) = \langle t, \cos(2t), 2\sin t \rangle$ @ $t = \pi/2$ $r'(t) = \langle 1, -2\cancel{\cos}\sin(2t), 2\cos t \rangle$ @ $t = \pi/2$ $= \boxed{\langle 1, 0, 0 \rangle}$

19) $r(t) = 2t^4 i + 6t^{3/2} j + \frac{10}{t} k$ @ $t=1$ $= r(t) = \langle 2t^4, 6t^{3/2}, 10t^{-1} \rangle$

$r'(t) = \langle 8t^3, 9t^{1/2}, -10t^{-2} \rangle$ eval @ $t=1$ $= \boxed{\langle 8, 9, -10 \rangle}$

21) $r(t) = \langle 2t, 2t, t \rangle$ $U_{Tt} = \frac{r'(t)}{|r'(t)|}$ on $a \leq t \leq b$ $r'(t) = \langle 2, 2, 1 \rangle$

$|r'(t)| = \sqrt{4+4+1} = 3$ $U_{Tt} = \frac{\langle 2,2,1 \rangle}{3} = \boxed{\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle}$

23) $r(t) = \langle 8, \cos 2t, 2\sin 2t \rangle$ $U_{Tt} = \frac{r'(t)}{|r'(t)|}$ $r'(t) = \langle 0, -2\cancel{\cos}\sin 2t, 4\cos 2t \rangle$

$|r'(t)| = \sqrt{\cancel{0^2} + 4\sin^2 2t + 16\cos^2 2t} \rightarrow 2\sqrt{1 + 3\cos^2 2t}$

for Clarity: removing $4(\sin^2 n + \cos^2 n) = 4$

$$\boxed{U_{Tt} = \frac{\langle \cancel{0}, -\cancel{2}\sin 2t, \cancel{4}\cos 2t \rangle}{\cancel{2}\sqrt{1 + 3\cos^2 2t}}}$$

25) $r(t) = \langle t, 2, \frac{2}{t} \rangle$ $r'(t) = \langle 1, 0, \frac{-2}{t^2} \rangle$

$|r'(t)| = \sqrt{1 + \frac{4}{t^4}}$

47) $r(t) = \langle t^4 - 3t, 2t - 1, 10 \rangle$ $\boxed{\int r(t) = \langle \frac{t^5}{5} - \frac{3}{2}t^2, t^2 - t, 10t \rangle + C}$

49) $r(t) = \langle 2\cos t, 2\sin 3t, 4\cos 8t \rangle$ $\boxed{\int r(t) = \langle 2\sin t, \frac{-2}{3}\cos 3t, \frac{1}{2}\sin 8t \rangle + C}$

53) find $r(t) \mid r'(t) = \langle e^t, \sin(t), \sec^2 t \rangle$ $r(0) = \langle 2, 2, 2 \rangle$

$r(t) = \langle e^t + c, -\cos(t) + c, \tan(t) + c \rangle \therefore \boxed{r(t) = \langle e^t + 1, -\cos(t) + 3, \tan(t) + 2 \rangle}$

$e^0 = 1$ $-\cos(0) = -1$ $\tan(0) = 0$

55) $\langle 1, 2t, 3t^2 \rangle = r'(t)$ $r(1) = \langle 4, 3, -5 \rangle$

$r(t) = \langle t + C, t^2 + C, t^3 + C \rangle$

$\boxed{r(t) = \langle t+3, t^2+2, t^3-6 \rangle}$

59) $r(t) = \langle i + tj + 3t^2 k \rangle$

$\Rightarrow \langle 1, t, 3t^2 \rangle$

$\int_{-1}^{1} r(t) = \left\langle \int_{-1}^{1} 1, \int_{-1}^{1} t, \int_{-1}^{1} 3t^2 \right\rangle dt$

2 0 $\frac{1}{2}1^2 - \frac{1}{2}1^2 = 0$ $1^3 - -1^3 = 2$

$\boxed{\int_{-1}^{1} r(t)\,dt = \langle 2, 0, 2 \rangle}$

61) $\int_0^{\ln 2} \langle e^t, e^t \cos \pi e^t, 0 \rangle dt \Rightarrow \left\langle \int_0^{\ln 2} e^t, \int_0^{\ln 2} e^t \cos \pi e^t, 0 \right\rangle$

$e^{\ln 2} - e^0 = 1$ $\frac{1}{\pi}\cos \pi e^{\ln 2} - \frac{1}{\pi}\cos \pi e^0$

$-\frac{1}{\pi} - -\frac{1}{\pi} = 0$

$\boxed{\int_0^{\ln 2} r(t)\,dt = \langle 1, 0, 0 \rangle}$

69) $r(t) = \langle 2 + \cos t, 3 + \sin 2t, t \rangle$ $t_0 = \frac{\pi}{2}$ for point, $r(\frac{\pi}{2}) = \langle 2, 3, \frac{\pi}{2} \rangle$

$r'(t) = \langle -\sin t, 2\cos 2t, 1 \rangle$ $t_0 = \frac{\pi}{2}$ for direction, $r'(\frac{\pi}{2})$ eval'd below

$r'(\frac{\pi}{2}) = \langle -1, -2, 1 \rangle$. $T(t)$ = tangent line = $\boxed{\langle 2, 3, \frac{\pi}{2} \rangle + t\langle -1, -2, 1 \rangle}$

71) $r(t) = \langle 3t - 1, 7t + 2, t^2 \rangle$ point: $\langle 2, 9, 1 \rangle = r(1)$

$t_0 = 1$

$r'(t) = \langle 3, 7, 2t \rangle$

$r'(1) = \langle 3, 7, 2 \rangle$ line: $\boxed{\langle 2, 9, 1 \rangle + t\langle 3, 7, 2 \rangle = T(t)}$

79) $r(t) = \langle at^2 + 1, t \rangle$ $a > 0$

$r'(t) = \langle 2at, 1 \rangle$ $a > 0$

$2a^2t^3 + 2at + t = 0$

$t(2a^2t^2 + 2a + 1) = 0$

$\hookrightarrow \boxed{t = 0, r(0) = (1, 0)}$

when $r(t) \perp r'(t)$, $r(t)$ DOT $R'(t) = 0$

Therefore: $r'(t) \cdot r(t) = 0 \rightarrow 2at(at^2 + 1) + t = 0 \rightarrow$ cont'd above

81) $r(t) = \langle \cos(t), \sin(t), t \rangle \quad r'(t) = \langle -\sin(t), \cos(t), 1 \rangle$

$r(t) \cdot r'(t) = -\cos(t)\cancel{\sin}(t) + \cos\cancel{(t)}\sin(t) + t = 0 \quad \boxed{t = 0 \quad r(0) = \langle 0, 0, 1 \rangle}$

83) Lines through the origin: $r(t) = \langle n_1 t, n_2 t, n_3 t \rangle$

"Transformations" of those lines: $r(t) = \langle n_1 e^{kt}, n_2 e^{kt}, n_3 e^{kt} \rangle$