Lyell C Read

CH 12.5 Homework

K= 7-25 | K%.2!=0

10/22/2018

7) $z = x^2 + y^3$ $\quad x = t^2$ $\quad y = t$ $\quad \frac{dz}{dt} = \frac{dz}{dx}\cdot\frac{dx}{dt} + \frac{dz}{dy}\cdot\frac{dy}{dt}$

$\frac{dz}{dt} = (2x)(2t) + (3y^2)(1) = 4t^3 + 3t^2 = \boxed{4t^3 + 3t^2}$

$x = t^2$ $\quad y = t$

9) $z = x\sin y$ $\quad x = t^2$ $\quad y = 4t^3$ $\quad \frac{dz}{dt} = \frac{dz}{dx}\cdot\frac{dx}{dt} + \frac{dz}{dy}\cdot\frac{dy}{dt}$

$\frac{dz}{dt} = (\sin y)(2t) + (x\cos y)(12t^2)$ $\quad \frac{dz}{dt} = \boxed{2t\sin 4t^3 + 12t^4\cos 4t^3}$

$y = 4t^3$ $\quad x = t^2$ $\quad y = 4t^3$

11) $z = \cos 2x \sin 3y$ $\quad x = \frac{1}{2}t$ $\quad y = t^4$

$\frac{\partial z}{\partial t} = (-2\sin 2x \sin 3y)(\frac{1}{2}) + (3\cos 3y \cos 2x)(4t^3)$

$x = \frac{1}{2}t$ $\quad y = t^4$ $\quad y = t^4$ $\quad x = \frac{1}{2}t$

$= \boxed{-\sin t \sin 3t^4 + 12t^3 \cos 3t^4 \cos t}$

13) $z = xy\sin z$ $\quad x = t^2$ $\quad y = 4t^3$ $\quad z = t + 1$

$\frac{\partial z}{\partial t} = (y\sin z)(2t) + (x\sin z)(12t^2) + \frac{\partial z}{\partial z} = xy\cos z\cdot(1)$

$y = 4t^3$ $\quad z = t+1$ $\quad x = t^2$ $\quad z = t+1$ $\quad x = t^2$ $\quad y = 4t^3$ $\quad z = t+1$

NOTE: 3 VAR'S

$= \boxed{20t^4\sin(t+1) + 4t^5\cos(t+1)}$ $\quad \frac{\partial z(t)}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial z}{\partial z}\cdot\frac{\partial z}{\partial t}$

15) $U = \ln(x + y + z)$ $\quad x = t$ $\quad y = t^2$ $\quad z = t^3$

$\frac{dU}{dt} = \frac{dU}{dx}\cdot\frac{dx}{dt} + \frac{dU}{dy}\cdot\frac{dy}{dt} + \frac{dU}{dz}\cdot\frac{dz}{dt}$

$= \frac{1}{x+y+z}\cdot(1) + \frac{1}{x+y+z}\cdot(2t) + \frac{1}{x+y+z}\cdot(3t^2) = \boxed{\frac{1 + 2t + 3t^2}{t + t^2 + t^3}}$

17) $V = \pi r^2 h$ $\quad r$ is $r(t)$, h is $h(t)$

a) $\frac{\partial V}{\partial t} = \frac{\partial V}{\partial r}\cdot\frac{\partial r}{\partial t} + \frac{\partial V}{\partial h} + \frac{\partial h}{\partial t} = (2h\pi r)\cdot(r'(t)) + (\pi r^2)\cdot(h'(t))$

$= \boxed{2\pi r(t)h(t)r'(t) + \pi r(t)^2 h'(t)}$

cont'd on next page!

17) cont'd from last page!

b) $V'(t) = 2\pi r(t)\,h(t)\,r'(t) + \pi r(t)^2 h'(t)$ | $r(t) = e^t \quad h(t) = e^{-2t}$

$= 2\pi(e^t)(e^{-2t})(e^t) + \pi(e^{2t})(-2e^{-2t})$

$= 2\pi(e^{2t-2t}) + -2(\pi)(e^{2t-2t})$

$= 2\pi - 2\pi = \boxed{0}$

c) Constant as the derivative is constant (0)

19) $z = x^2 \sin y \qquad x = s - t \qquad y = t^2$

$z'_s = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = (2x\sin y)(1) + \cancel{(x^2\cos y)(0)}$

$x = s - t \quad y = t^2$

$= \boxed{2(s-t)\sin t^2}$

$z'_t = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = (2x\sin y)(-1) + (x^2\cos y)(2t)$

$x = s - t \quad y = t^2 \qquad x = s - t \quad y = t^2$

$= -2(s-t)\sin t^2 + 2t(s-t)^2\cos t^2 = \boxed{2(s-t)(t(s-t)\cos t^2 - \sin t^2)}$

21) $z = xy - x^2 y \qquad x = s + t \qquad y = s - t$

$z'_s = (y - 2xy)(1) + (x - x^2)(1) = s - t - (2)(s+t)(s-t) + s + t - (s+t)^2$

$y = s - t \quad x = s + t \qquad x = s + t$

$s - \cancel{t} - 2s^2 + 2t^2 + s + \cancel{t} - s^2 - 2st - t^2$

$= \boxed{2s - 3s^2 - 2st + t^2}$

$z'_t = (y - 2xy)(1) + (x - x^2)(-1) = s - t - (2)(s+t)(s-t) - ((s+t) - (s+t)^2)$

$\cancel{s} - t - 2s^2 + 2t^2 - \cancel{s} - t + s^2 + 2st + t^2$

$= \boxed{-s^2 + 2st + 3t^2 - 2t}$

25) skipped 23) $\quad w = \frac{x - z}{y + z} \qquad w'_s = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial s} \qquad x = s + t \quad y = st \quad z = s - t$

$w'_s = \left(\frac{1}{y+z}\right)(1) + -\left(\frac{(x-z)}{(y+z)^2}\right)\cdot(t) + \left(\frac{-y-x}{(y+z)^2}\right)\cdot(1)$

repeat for w'_t ... quite a bit of work ...