Lyell. C. Reed CH 13.1 Homework $^{11}/_{6}/2018$

Pr 11-35 odd

11) $\int_1^4 \int_0^4 \sqrt{uv}\, du\, dv$ $\quad \| \sqrt{uv} = \sqrt{u} \cdot \sqrt{v}$

1] $\int_0^4 \sqrt{v} \cdot (u)^{\frac{1}{2}} du \xrightarrow{\text{ANTI}} \frac{2}{3}\sqrt{v} \cdot u^{\frac{3}{2}} \Big|_0^4 = \frac{16}{3}\sqrt{v}$ 2] $\int_1^4 \frac{16}{3}\sqrt{v}\, dv \xrightarrow{\text{ANTI}} \frac{32}{9}(v)^{\frac{3}{2}} \Big|_1^4$

$= 8\left(\frac{32}{9}\right) - \frac{32}{9} = 7\left(\frac{32}{9}\right) = \boxed{\frac{224}{9}}$

13) $\int_0^{\ln 2} \int_0^1 6xe^{3y}\, dx\, dy$

1] evaluate $\int_0^1 6xe^{3y}\, dx \xrightarrow{\text{ANTI}} 3x^2 e^{3y} \Big|_0^1 = \left[3(1)e^{3y}\right] - 0$

2] evaluate $\int_0^{\ln 2} 3e^{3y}\, dy \xrightarrow{\text{ANTI}} e^{3y} \Big|_0^{\ln 2} = e^{3\ln 2} - 1 = \cancel{e^{\ln}} 2^3 - 1 = 8 - 1 = \boxed{7}$

15) $\int_1^{\ln 5} \int_0^{\ln 3} e^{x+y}\, dx\, dy$

1] $\int_0^{\ln 3} e^{x+y}\, dx \xrightarrow{\text{ANTI}} e^{x+y} \Big|_0^{\ln 3} = e^{\ln 3 + y} - e^{y}$ $\quad \| e^{a+b} = e^a \cdot e^b$

2] $\int_1^{\ln 5} e^{\ln 3 + y} - e^{y}\, dy \xrightarrow{\text{ANTI}} e^{\ln 3 + y} - e^{y} \Big|_1^{\ln 5} = \left[e^{\ln 3 + \ln 5} - e^{\ln 5}\right] - \left[e^{\ln 3 + 1} - e^{1}\right]$

$= \left[5 \cdot 3 - 5\right] - \left[3e - e\right]$

$= \boxed{10 - 2e}$

17) $\iint_R (x+2y)$ on $0 \le x \le 3,\ 1 \le y \le 4$ $= \int_1^4 \int_0^3 (x+2y)\, dx\, dy$

1] $\int_0^3 (x+2y)\, dx \xrightarrow{\text{ANTI}} \frac{1}{2}x^2 + 2xy \Big|_0^3 = \frac{9}{2} + 6y - 0$

2] $\int_1^4 \frac{9}{2} + 6y\, dy \xrightarrow{\text{ANTI}} \frac{9y}{2} + 3y^2 \Big|_1^4 = \left[18 + 48\right] - \left[\frac{9}{2} + 3\right] = \frac{132}{2} - \frac{15}{2} = \boxed{\frac{117}{2}}$

19) $\iint_R 4x^3 \cos y$ on $1 \le x \le 2,\ 0 \le y \le \pi/2$ $= \int_1^2 \int_0^{\pi/2} 4x^3 \cos y \, \partial y \, \partial x$

1] $\int_0^{\pi/2} 4x^3 \cos y \, \partial y \xrightarrow{\text{ANTI}} 4x^3 \sin y \Big|_0^{\pi/2} = \underline{4x^3(1)} - \cancel{4x^3(0)}$

2] $\int_1^2 4x^3 \, \partial x \xrightarrow{\text{ANTI}} x^4 \Big|_1^2 = 2^4 - 2^1 = 16 - 1 = \boxed{15}$

$\| \sqrt{\frac{a}{b}} = a^{\frac{1}{2}} \cdot b^{-1/2}$

21) $\iint_R \sqrt{\frac{x}{y}}$ on $0 \le x \le 1,\ 1 \le y \le 4$ $=$

1] $\int_0^1 x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \, \partial x \xrightarrow{\text{ANTI}} \frac{2}{3} x^{\frac{3}{2}} \cdot y^{-\frac{1}{2}} \Big|_{\cancel{0}}^1 = \underline{\frac{2}{3} y^{-\frac{1}{2}}}$

2] $\int_1^4 \frac{2}{3} y^{-1/2} \, \partial y \xrightarrow{\text{ANTI}} \frac{4}{3} y^{\frac{1}{2}} \Big|_1^4 = \frac{8}{3} - \frac{4}{3} = \boxed{\frac{4}{3}}$

23) $\iint_R e^{x+2y}$ on $0 \le x \le \ln 2,\ 1 \le y \le \ln 3$ $= \int_1^{\ln 3} \int_0^{\ln 2} e^{x+2y} \, \partial x \, \partial y$

1] $\int_0^{\ln 2} e^{x+2y} \, \partial x \xrightarrow{\text{ANTI}} e^{x+2y} \Big|_0^{\ln 2} = \underline{e^{\ln 2 + 2y} - e^{2y}}$

2] $\int_1^{\ln 3} e^{\ln 2 + 2y} - e^{2y} \, \partial y \xrightarrow{\text{ANTI}} \frac{1}{2} e^{\ln 2 + 2y} - \frac{1}{2} e^{2y} \Big|_1^{\ln 3} =$

$= \left[\frac{1}{2} e^{\ln 2 + 2\ln 3} - \frac{1}{2} e^{2\ln 3} \right] - \left[\frac{1}{2} e^{\ln 2 + 2} - \frac{1}{2} e^2 \right]$

$= \left[\frac{1}{2}(2 \cdot 3^2) - \frac{1}{2}(9) \right] - \left[\frac{1}{2}(2 \cdot e^2) - \frac{1}{2} e^2 \right] = \frac{9}{2} - \frac{e^2}{2} = \boxed{\frac{9 - e^2}{2}}$

25) $\iint_R (x^5 - y^5)^2$ $0 \le x \le 1,\ -1 \le y \le 1$ $\rightsquigarrow \int_{-1}^1 \int_0^1 (x^5 - y^5)^2 \, \partial x \, \partial y$

$= \int_{-1}^1 \int_0^1 x^{10} - 2x^5 y^5 + y^{10} \, \partial x \, \partial y$

Cont'd below

31) $\iint_R \frac{x}{(1+xy)^2}\, dx$ on $0 \le x \le 4$, $1 \le y \le 2$

1] $\int_1^2 \frac{x}{(1+xy)^2}\, dy \rightarrow \left\| \begin{array}{l} u = 1+xy \\ du = x\, dy \end{array} \right. \therefore \int_{1+x}^{1+2x} u^{-2}\, du \xrightarrow[\text{ANTI}]{} -u^{-1}\Big|_{1+x}^{1+2x} = \underbrace{\frac{1}{1+x} - \frac{1}{1+2x}}$

2] $\int_0^4 \frac{1}{1+x} - \frac{1}{1+2x}\, dx \xrightarrow[\text{ANTI}]{} \ln(1+x) - \frac{1}{2}\ln(1+2x)\Big|_{\cancel{0}}^{4} = \ln(5) - \frac{1}{2}\ln(9)$

$= \ln(5) - \ln(9^{\frac{1}{2}}) = \ln(5) - \ln(3) = \boxed{\ln\left(\frac{5}{3}\right)}$

33) $f(x,y) = e^{-y}$ $\quad 0 \le x \le 6,\ 0 \le y \le \ln 2 \rightarrow \int_0^{\ln 2}\int_0^6 e^{-y}\, dx\, dy$

1] $\int_0^6 e^{-y}\, dx \xrightarrow[\text{ANTI}]{} xe^{-y}\Big|_{\cancel{0}}^{6} = \underbrace{6e^{-y}}$

2] $\int_0^{\ln 2} 6e^{-y}\, dy \xrightarrow[\text{ANTI}]{} -6e^{-y}\Big|_0^{\ln 2} = -6 \cdot \frac{1}{2} - (-6) = \boxed{3}$

$A_R = 6 \ln 2$

$\bar{f}(x,y)$ on $0 \le x \le 6,\ 0 \le y \le \ln 2 = \frac{3}{6\ln 2} = \boxed{\frac{1}{2\ln 2}}$

35) Squared Distance $(-2 \le x \le 2,\ 0 \le y \le 2)$ and $(0,0)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad d = \sqrt{(-x)^2 + (-y)^2}$ $\quad d^2 = (-x)^2 + (-y)^2$

$= x^2 + y^2$

$d^2 = x^2 + y^2$ $\quad \int_{-2}^{2}\int_0^2 x^2 + y^2\, dy\, dx$

Average $= \frac{1}{A} \cdot \text{val} \rightarrow \frac{64}{3} \cdot \frac{1}{8} = \boxed{\frac{8}{3}}$

$A = 4 \cdot 2$

1] $\int_0^2 x^2 + y^2\, dy \xrightarrow[\text{ANTI}]{} yx^2 + \frac{1}{3}y^3\Big|_0^2 =$

$= \underbrace{2x^2 + \frac{8}{3}}$

2] $\int_{-2}^{2} 2x^2 + \frac{8}{3}\, dx \rightarrow \frac{2}{3}x^3 + \frac{8x}{3}\Big|_{-2}^{2}$

$= \frac{16}{3} + \frac{16}{3} - \left(-\frac{16}{3} - \frac{16}{3}\right) = \frac{64}{3}$

1] $\int_0^1 x^{10} - 2x^5y^5 - y^{10}\,dx \xrightarrow{\text{ANTI}} \frac{1}{11}x^{11} - \frac{2}{6}x^6y^5 + y^{10}x \Big|_0^1 =$

$= \underbrace{\frac{1}{11} - \frac{2}{6}y^5 + y^{10}}$

2] $\int_{-1}^1 \frac{1}{11} - 2/6\,y^5 + y^{10}\,dy \xrightarrow{\text{ANTI}} \frac{y}{11} - \frac{2}{12}y^6 + \frac{1}{11}y^{11} \Big|_{-1}^1 =$

$= \left[\frac{1}{11} - \cancel{\frac{2}{12}} + \frac{1}{11}\right] - \left[-\frac{1}{11} - \cancel{\frac{2}{12}} - \frac{1}{11}\right] = \boxed{\frac{4}{11}}$

27) $\iint_R (y+1)e^{x(y+1)}$ $\quad 0 \le x \le 1,\ -1 \le y \le 1 \quad ⇝ \int_{-1}^{1}\int_0^1 (y+1)\,e^{x(y+1)}\,dx\,dy$

NOTE, IT APPEARS BEST TO START WITH DX, AS THAT MAKES FOR: ne^{nx}

1] $\int_0^1 (y+1)e^{x(y+1)}\,dx \xrightarrow{\text{ANTI}} e^{x(y+1)}\Big|_0^1 = \underbrace{e^{y+1} - 1}$

2] $\int_{-1}^1 e^{y+1} - 1\,dy \xrightarrow{\text{ANTI}} e^{y+1} - y \Big|_{-1}^1 = \left[e^2 - 1\right] - \left[1 + 1\right] = \boxed{e^2 - 3}$

29) $\iint_R 6x^5e^{x^3y}$ $\quad 0 \le x \le 2,\ 0 \le y \le 2 \quad ⇝ \int_0^2\int_0^2 6x^5 e^{x^2y}\,dy\,dx$

NOTE: STARTING WITH DY AS THE EQUATION APPEARS TO SIMPLIFY...

1] $\int_0^2 6x^5 e^{x^3y}\,dy \xrightarrow{\text{ANTI}} 6x^2 e^{x^3y} \Big|_0^2 \}=y$

$= \underbrace{6x^2\left(e^{2x^3} - 1\right)}$

2] $\int_0^2 6x^2e^{2x^3} - 6x^2\,dx \xrightarrow{\text{ANTI}} e^{2x^3} - 2x^3 \Big|_0^2 = e^{16} - 16 - [1] = \boxed{e^{16} - 17}$

from e term