Lyell C. Read   CH 12.4 – 12.8 REVIEW   10/24/2018

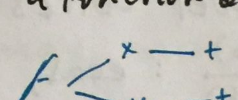
> NOTE: KNOW YOUR DERIVATIVES (i.e. A^x, x^n, $\ln(x)$ ... and product, chain rule!

$\boxed{12.4}$ PARTIAL DERIVATIVES

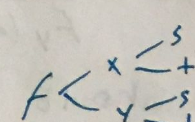
- First Degree Partial Derivative: for $f(x,y)$:
  - → Calc f_x = Derivative of $f(x,y)$ treating all non-x var's as const.
  - → Calc f_y = Derivative $f(x,y)$ where all non-y var's are const.
  - → NOTE: $f_x = \frac{\partial f}{\partial x}$, $f_y = \frac{\partial f}{\partial y}$
- First Deg. P. D. Using Limits: for $f(x,y)$
  - → $f_x = f_x(a,b) = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$
  - → $f_y = f_y(c,d) = \lim_{h \to 0} \frac{f(c,d+h) - f(c,d)}{h}$
- Second Degree P.D.'s: for function $f(x,y)$:
  - → Perform FDPD for the first var, then again on result for second: f_{xy} would be f_x derived with y ...

$\boxed{12.5}$ DA CHAIN RULE

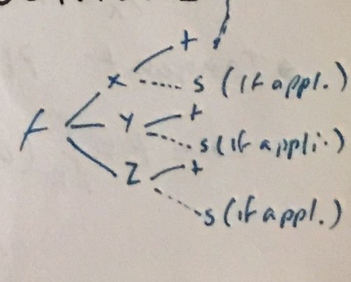
- f' for $f(x \cdots y)$ where x is a function of another var, and so is y
  - → $f' = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$



- f' for $f(x,y)$ where $x(s,t)$, $y(s,t)$



  - → $f'_s = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$, $f'_t = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$
- If $f(x,y,z)$: add another term, $\frac{\partial f}{\partial z}$, and chain rule
- solution process: evaluate $f'(\ldots)$, then substitute $x=$, $y=$, $z=$... and simplify



- Implicit Differentiation $\frac{dy}{dx} = -\frac{F_x}{F_y}$
-

$\boxed{12.6}$ GRADIENTS AND DIRECTIONAL DERIVATIVES

- Gradients: $\nabla f(x,y)$ = "gradient" = $\langle f_x, f_y \rangle$. If evaluated at a point, substitute x, y from point, solve as a vector.

- Directional Derivative:
  1) find if $\vec{v}$ provided is unit vector, else make it one
  2) find $\nabla f(x,y)$ (see above for help there :)
  3) find $D_u f(x,y) = u \cdot \nabla f(x,y) = u_x \cdot f_x + u_y \cdot f_y$
  4) Substitute Point "p".

  NOTE: STEPS 3 and 4 CAN BE DONE INTERCHANGEABLY

- Directions of steepest change, no change
  → Steepest ascent $= \left(\nabla f(x,y) / \|\nabla f(x,y)\|\right)$ AKA unit in dir of $\nabla f(x,y)$
  → Steepest descent = − [steepest ascent]
  → NO change = $\pm$ [swapped variables and one made neg, i.e. $\langle 8,1\rangle \to \langle -1,8\rangle$ from pos steepest or neg steepest.]

- Tangent Line to Level Curves: $f(x,y)$ at point $P = (a,b)$
  → $y'(x) = -\dfrac{f_x(a,b)}{f_y(a,b)}$: eval, if UND = VERT TAN,
  → CHECK: should be $\perp$ to $\nabla f(x,y) \big|_P$ ...

$\boxed{12.7}$ TANGENT CURVES, ASSOCIATED STUFF

- Tangent Plane to $f(x,y,z)$ at (a,b,c)
  → $= f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$
- Tangent Plane to $z = f(x,y)$ at (a,b,c)
  → $= z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$

- Linear Approximation: Find the tangent plane, then substitute the ("approximate for (h,i,K)") (In that case, use(h,j,K) or (h,j)...) in the plane equation.

- Approximate Function Change for $f(x,y)=z$ on $(a,b)\to(c,d)$
  → $dx = c-a$; $dy = d-b$
  → $\Delta z =$ "appro. fxn chg" $= f_x(a,b)\,dx + f_y(a,b)\,dy$

- Find Horizontal tangent Planes: These occur where either $\left(\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0\right)$ or where $\nabla f(x,y,..) = \langle 0,0,\cdots\rangle$
  → Hint: list all the values that would make f_x ~~and~~ or f_y zero, and choose all that work for both $=0$.

12.8 CRITICAL POINTS, MIN, MAX

- Finding Critical Points: find all values of x,y where $f_x = f_y = 0$,
- Determining If a CP is a max, min, saddle, or unknown
  → use the "D" equation: $D(x,y) = f_{xx}f_{yy} - (f_{xy})^2$, your pt $=(a,b)$

Sign / val of D	Val of f_{xx}	Outcome
$D(a,b)>0$	$f_{xx}<0$	MAXIMUM
$D(a,b)>0$	$f_{xx}>0$	MINIMUM
$D(a,b)<0$	—	SADDLE
$D(a,b)=0$		UNDECIDED ☺

- Absolute Minimums and Maximums on range R
  → find critical points, and keep as long as they are in R.
  → Generate equations for the edges of the range (CONT'D)

→ If the range is circular (i.e. $\{x, y : x^2 + y^2 \le r^2\}$) parametrize $x = r\cos\theta$, $y = r\sin\theta$.

→ NOTE. If you see $\{x, y : (x-1)^2 \dots\}$ then that shifts the circle's center, and in the above case, $x = r\cos\theta \underline{\pm 1}$!

→ otherwise, generate line equations (i.e. $y=1$, $x=10\dots$) for the edges, ~~sim~~

then, simplify the equation along each edge, or with the parametrization, and find derivative of this new equation, find where that is $= 0$, then plug those points back into your simplified equation.

make a table of heights, and find |max|, |min|.