Lyell C Read — 13.3 Homework — 11-27 odd, 39, 51-57 — 11/18/2018

11) $z = 4 - x^2 - y^2 \;\Big|\; \begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix} \;=\; z = 4 - r^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1}) = 4 - r^2$

$4 - r^2$ on $\{(r,\theta): 0 \le r \le 1;\ 0 \le \theta \le 2\pi\} = \int_0^{2\pi}\int_0^1 4 - r^2\, r\,dr\,d\theta$

1] $\int_0^1 (4 - r^2) r\,dr = \int_0^1 4r - r^3\,dr = 2r^2 - \frac{1}{4}r^4 \Big|_0^1 = 2 - \frac{1}{4} = \frac{7}{4}$

2] $\int_0^{2\pi} \frac{7}{4}\,d\theta = \frac{7\theta}{4}\Big|_0^{2\pi} = \frac{7 \cdot 2\pi}{4} = \boxed{\frac{7\pi}{2}}$

13) ignored, as practically the same as #11)

15) $z = 5 - \sqrt{1 + x^2 + y^2}$ on $\{(r,\theta): 0 \le r \le 2,\ 0 \le \theta \le 2\pi\}$

$z = 5 - \sqrt{1 + r^2\cos^2\theta + r^2\sin^2\theta} = z = 5 - \sqrt{1 + r^2} \rightsquigarrow A = \int_0^{2\pi}\int_0^2 5 - \sqrt{1 + r^2}\, r\,dr\,d\theta$

1] $\int_0^2 (5 - \sqrt{1 + r^2})\,r\,dr = \int_0^2 5r - r(1 + r^2)^{\frac{1}{2}}\,dr = \frac{5}{2}r^2 - \frac{2}{6}(1 + r^2)^{\frac{3}{2}}\Big|_0^2$

$= \frac{20}{2} - \frac{2}{6}(\sqrt{125}) + \frac{2}{6} = \frac{60 - 10\sqrt{5} + 1}{6} = \left[\frac{61 - 10\sqrt{5}}{6}\right]$

2] $\int_0^{2\pi} \left[\frac{61 - 10\sqrt{5}}{6}\right] d\theta = \left[\frac{61 - 10\sqrt{5}}{6}\right]\theta \Big|_{\theta = 0}^{2\pi} = \boxed{\left(\frac{61 - 10\sqrt{5}}{3}\right)\pi}$

17) same, if not messier, than 15), ignored

19) top: $z = 2 - x^2 - y^2 \rightsquigarrow \underline{z = 2 - r^2}$ | Bottom: $z = x^2 + y^2 \rightsquigarrow \underline{z = r^2}$

Curve C where top = bottom: $2 - r^2 = r^2 \to\ = \underline{r = 1}$

$A_{\text{between}} = \int_0^{2\pi}\int_0^1 ((2 - r^2) - r^2)\,r\,dr\,d\theta$

1] $\int_0^1 (2 - 2r^2) r\,dr = \int_0^1 2r - 2r^3\,dr = r^2 - \frac{1}{2}r^4\Big|_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$

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2] $\int_0^{2\pi} \frac{1}{2}\, d\theta = \frac{\theta}{2}\Big|_{\emptyset}^{2\pi} = \frac{2\pi}{2} = \boxed{\pi}$

21) TOP: $z = 2 - x^2 - y^2 = \underline{2 - r^2}$ | BOT: $z = 1$

Intersection curve C ⇒ ~~$z - x^2 - y^2$~~ $2 - r^2 = 1$ $\quad -r^2 = -1$ $\quad r^2 = 1$ $\quad \underline{r = 1}$

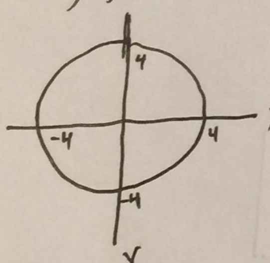
$$\int_0^{2\pi}\int_0^1 \left(\overset{TOP}{(2-r^2)} - \overset{BOT}{1}\right) r\, dr\, d\theta$$

NOTE: THE CURVE "C" IS THE UPPER LIMIT OF DIFFERENTIATION FOR THE DR INTEGRAL

1] $\int_0^1 ((2-r^2)-1)\, r\, dr = \int_0^1 r - r^3\, dr = \frac{1}{2}r^2 - \frac{1}{4}r^4 \Big|_{\emptyset}^{1} = \frac{1}{2} - \frac{1}{4} = \underline{\frac{1}{4}}$

2] $\int_0^{2\pi} \frac{1}{4}\, d\theta = \frac{\theta}{4}\Big|_{\emptyset}^{2\pi} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

23) ~~just a circle with rad 4 ...~~



$\iint_R \underbrace{(x^2+y^2)}_{r^2}\, dA \quad R = \{(r,\theta) : 0 \le r \le 4;\ 0 \le \theta \le 2\pi\}$

$\to \int_0^{2\pi}\int_0^4 (r^2)\, r\, dr\, d\theta$

1] $\int_0^4 r^3 = \frac{1}{4}r^4 \Big|_{\emptyset}^{4} = \underline{64}$ $\to$ 2] $\int_0^{2\pi} 64\, d\theta = 64\theta \Big|_{\emptyset}^{2\pi} = \boxed{128\pi}$

25) $\iint_R 2xy\, dA \quad R = \{(x,y) : x^2+y^2 \le 9,\ y \ge 0\}$

$r^2 \le 9 \qquad r\sin\theta \ge 0 \qquad$ ~~trig identity sin(2θ) = 2 sinθ cosθ~~

$r \le 3 \qquad \to \sin^{-1} \leftarrow \qquad$ USE GRAPH

$\qquad\qquad r \ge 0$

$\to \int_0^{2\pi}\int_0^3 2r^3 \underbrace{(\cos\theta\sin\theta)}_{\#}\, dr\, d\theta \qquad = \boxed{0}$

1] $\int_0^3 2r^3(\#)\, dr = r^2(\#)\Big|_{\emptyset}^{3} = \underline{9\#}$ $\quad$ 2] $\int_0^{2\pi} 9\cos\theta\sin\theta\, d\theta$

27) $\iint_R \frac{1}{\sqrt{16-x^2-y^2}}\, dA \quad R=\{(x,y): x^2+y^2\le 4,\ x\ge 0,\ y\ge 0\}$

$\hookrightarrow \int_0^{\pi/2}\int_0^2 \frac{r}{\sqrt{16-r^2}}\, dr\, d\theta = \int_0^{\pi/2}\int_0^2 (16-r^2)^{-1/2}\, r\, dr\, d\theta$

1] $\int_0^2 (16-r^2)^{-1/2}\, r = (16-r^2)^{1/2}\Big|_0^2 = \underbrace{\overset{\sqrt{12}}{\underset{2\sqrt{3}}{\cancel{3\sqrt{2}}}} - 4}$

2] $\int_0^{\pi/2} \underset{2\sqrt{3}}{\cancel{3\sqrt{2}}} - 4 = \overset{2\sqrt{3}}{\cancel{3\sqrt{2}}}\theta - 4\theta\Big|_0^{\pi/2} = \sqrt{3}\cdot\pi - 2\pi = \boxed{\pi\left(\sqrt{3}-2\right)}$

39) Annular Area of 1 where $R=\{(r,\theta): 1\le r\le 2,\ 0\le\theta\le\pi\}$

integral: $\int_0^{\pi}\int_1^2 1\, r\, dr\, d\theta = \int_0^{\pi}\int_1^2 r\, dr\, d\theta$

1] $\int_1^2 r\, dr = \frac{1}{2}r^2\Big|_1^2 = 2 - \frac{1}{2} = \underbrace{3/2}$

2] $\int_0^{\pi} 3/2\, d\theta = \frac{3\theta}{2}\Big|_0^{\pi} = \boxed{3\pi/2}$

51-57) just integrals.. might add later :)