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MATH 254H, Fall 2018 Test #1

FOR EACH PROBLEM SHOW ALL ESSENTIAL STEPS.

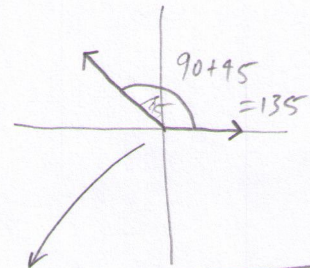
✓ 1. Calculate $Proj_{\mathbf{v}}\mathbf{u}$ if $\mathbf{u} =< 1, 4, 7 >$ and $\mathbf{v} =< 2, -4, 2 >$.

$$Proj_V U = \left(\frac{U \cdot V}{V \cdot V}\right) V = \left(\frac{2-16+14}{4+16+4}\right)\langle 2,-4,2\rangle = \frac{0}{24}\langle 2,-4,2\rangle = \boxed{\vec{0}}$$

✓ 2. For the vectors $\mathbf{u} =< 10, 0 >$ and $\mathbf{v} =< -5, 5 >$,

✓ (a) compute the dot product, and

$$U \cdot V = -50 + 0 = \boxed{-50}$$



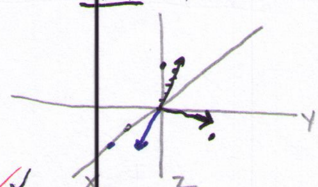
✓ (b) find the angle between the vectors.

$$U \cdot V = |U||V|\cos\theta \quad \therefore \quad \cos\theta = \frac{U \cdot V}{|U||V|}$$

$$\cos\theta = \frac{-5\cancel{0}}{1\cancel{0}\sqrt{50}} \quad \frac{-5}{\sqrt{50}} \cdot \frac{\sqrt{50}}{\sqrt{50}} \quad \frac{-5\sqrt{50}}{50}$$

$\boxed{\theta_{between} = 135^\circ \text{ Found geometrically}}$

✓ 3. Find the volume of the parallelepiped determined by the position vectors $\mathbf{u} =< 3, 1, 0 >$, $\mathbf{v} =< 2, 4, 1 >$, and $\mathbf{w} =< 1, 1, 5 >$.



$$▱ = \underbrace{U \times V}$$

$$h = \cdot W = \cdot W\cos\theta \quad = \boxed{48}$$

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✓ 4. Find a vector perpendicular to $< 1, 2, 3 >$ and $< -2, 4, -1 >$.

Vector perpendicular $= \pm[\langle 1,2,3\rangle \times \langle -2,4,-1\rangle]$

SEE SCRATCH PAPER $= \boxed{\langle -14, -5, 8\rangle}$

✓ 5. Find an equation for the line through $< 1, 2, 3 >$ that is perpendicular to the lines $\mathbf{r}_1(t) =< 3 - 2t, 5 + 8t, 7 - 4t >$ and $\mathbf{r}_2(t) =< -2t, 5 + t, 7 - t >$.

$$r_1(t) = \langle 3, 5, 7\rangle + t\overbrace{\langle -2, 8, -4\rangle}^{a}$$

"Direction"

Points

$$r_2(t) = \langle 0, 5, 7\rangle + t\overbrace{\langle -2, 1, -1\rangle}^{b}$$

Direction of $r_x(t) = a \times b$

$a = \langle -2, 8, -4\rangle$

$b = \langle -2, 1, -1\rangle$

$$\boxed{r_x(t) = \langle 1,2,3\rangle + t\langle -4, \cancel{10}\ 6, 14\rangle}$$

$$a \times b = \begin{vmatrix} i & -j & k \\ -2 & 8 & -4 \\ -2 & 1 & -1 \end{vmatrix} = i\begin{vmatrix} 8 & -4 \\ 1 & -1 \end{vmatrix} - j\begin{vmatrix} -2 & -4 \\ -2 & -1 \end{vmatrix} + k\begin{vmatrix} -2 & 8 \\ -2 & 1 \end{vmatrix} = \langle -4, \cancel{10}\ 6, 14\rangle$$

$-8+4 \quad -(2 \cancel{-} 8) \quad -2+16$

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✓

6. Find the function $\mathbf{r}(t)$ that satisfies $\mathbf{r}'(t) = < 1, 2t, 3t^2 >$ and $\mathbf{r}(1) = < 4, 3, -5 >$.

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$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r(1) = \langle 1 + c, 1 + c, 1 + c \rangle$

$c = 4 \quad c = 3 \quad c = -5$?

$r(t) = \langle t + c, t^2 + c, t^3 + c \rangle$

$\boxed{r(t) = \langle t + 4, t^2 + 3, t^3 - 5 \rangle}$

T

7. Let the x-axis point east, the y-axis north, and the z-axis vertical upward. The ground is level in the xy-plane and the gravitational acceleration of magnitude $g = 9.8 m/s^2$ is downward. At the initial time $t = 0$ a golf ball is hit east down a fairway with an initial velocity of $< 50, 0, 30 >$ m/s. A crosswind blowing south produces an acceleration of the ball of $-0.8 m/s^2$.

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(a) Find the velocity vector for the ball for $t \geq 0$.

✓

$v(t) = \langle 50, -0.8t, -9.8t + 30 \rangle$

(b) Find the position vector for the ball for $t \geq 0$.

✓

$x(t) = \langle 50t, -0.4t^2, -4.9t^2 + 30t \rangle$

×✓

8. Find the length of the curve $\mathbf{r}(t) = < 4\cos(t), 4\sin(t), 3t >$, $0 \leq t \leq 6\pi$.

✓

$r(t) = \langle 4\cos t, 4\sin t, 3t \rangle$ on $[0, 6\pi]$

$r'(t) = \langle -4\sin t, 4\cos t, 3 \rangle$

$|r'(t)| = \sqrt{16\sin^2 t + 16\cos^2 t + 9} = \sqrt{16 + 9} = \sqrt{25} = 5$

$\therefore L = \int_0^{6\pi} 5\, dt = 5t \Big|_0^{6\pi} = 30\pi - 0 = \boxed{30\pi}$

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$Proj_u v = \left(\frac{v \cdot u}{u \cdot u}\right) u$ $\quad Scal_u v = \left(\frac{v \cdot u}{|u|}\right)$

PROBLEM WORK L. Read

QUESTION 3

$u = \langle 3, 1, 0 \rangle \quad v = \langle 2, 4, 1 \rangle \quad w = \langle 1, 1, 5 \rangle$

$$u \times w = \begin{vmatrix} i & -j & k \\ 3 & 1 & 0 \\ 2 & 4 & 1 \end{vmatrix} = i \underbrace{\begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix}}_{(1-0)} - j \underbrace{\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}}_{-(3-0)} + k \underbrace{\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix}}_{+(12-2)}$$

$$= \boxed{\langle 1, -3, 10 \rangle} = \square$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\square \cdot w = \langle 1, -3, 10 \rangle \cdot \langle 1, 1, 5 \rangle = 1 - 3 + 50 = \boxed{48}$$

QUESTION 4

$a = \langle 1, 2, 3 \rangle \quad b = \langle -2, 4, -1 \rangle$

$$a \times b = \begin{vmatrix} i & -j & k \\ 1 & 2 & 3 \\ -2 & 4 & -1 \end{vmatrix} = i \begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix} - j \begin{vmatrix} 1 & 3 \\ -2 & -1 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} = \langle -14, -5, 8 \rangle$$

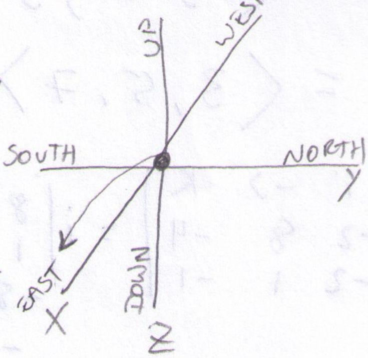
$(-2-12) \quad -(-1+6) \quad 4 - -2$

$-14 \quad -5 \quad 8$

QUESTION 7

$$a(t) = \langle 0, \overbrace{-0.8}^{X\text{-wind}}, \overbrace{-9.8}^{\text{Gravity}} \rangle \qquad v_0 = \langle 50, 0, 30 \rangle$$

$$x_0 = \langle 0, 0, 0 \rangle$$



$$v(t) = \langle C, -0.8t + C, -9.8t + C \rangle \qquad v_0 = \langle 50, 0, 30 \rangle$$

$$\boxed{v(t) = \langle 50, -0.8t, -9.8t + 30 \rangle}$$

$$x(t) = \langle 50t \cancel{+C}, -0.4t^2 \cancel{+C} -4.9t^2 + 30t \cancel{+C} \rangle \qquad x_0 = \langle 0, 0, 0 \rangle$$

$$\boxed{x(t) = \langle 50t, -0.4t^2, -4.9t^2 + 30t \rangle}$$