Lyell C Read

CH 11.8 Homework

4, 8, 12, 16, 20, 24, 41, 45, 49

10/9/2018

4) $r(t) = \langle x(t), y(t), z(t) \rangle$ from $t = [a, b]$, Length

$$L = \int_a^b |r'(t)|\, dt = \ldots$$

8) Yes, as $s(t) = t$ (try $\frac{\pi}{2}$) $= T$.

12) $r(t) = \langle 4\cos 3t, 4\sin 3t \rangle$ on $[0 .. \frac{2\pi}{3}]$

$r'(t) = \langle -12\sin 3t, 12\cos 3t \rangle$ " "

$$|r'(t)| = \sqrt{12^2 \sin^2 3t + 12^2 \cos^2 3t} = \sqrt{144} = 12$$

$$\int_0^{2\pi/3} 12\, dt = \frac{12 \cdot 2\pi}{3} = \boxed{8\pi = L}$$

16) $r(t) = \langle 4\cos t, 4\sin t, 3t \rangle$ on $[0 .. 6\pi]$

$r'(t) = \langle -4\sin t, 4\cos t, 3 \rangle$ " "

$$|r'(t)| = \sqrt{[16(1) + 9] = [25]} = 5 \qquad \int_0^{6\pi} 5\, dt = \boxed{30\pi = L}$$

20) $r(t) = \langle t^2, t^3 \rangle$ on $[0 .. 4]$

$r'(t) = \langle 2t, 3t^2 \rangle$

$$|r'(t)| = \sqrt{4t^2 + 9t^4}$$

$$\int_0^4 \sqrt{4t^2 + 9t^4} = \int_0^4 t\sqrt{9t^2 + 4}$$

$$= \int_0^4 t(9t^2 + 4)^{\frac{1}{2}} = \frac{2}{54}(9t^2 + 4)^{\frac{3}{2}} \Big|_0^4 = \boxed{\frac{2}{54}(9 \cdot 16 + 4)^{\frac{3}{2}} - \frac{2}{54}(4)^{\frac{3}{2}}}$$

24) $r(t) = \langle 5\cos t^2, 5\sin t^2, 12t^2 \rangle$ on $0..2$

$r'(t) = \langle -10t \sin t^2, 10t \cos t^2, 24t \rangle$

$|r'(t)| = \text{speed} = \sqrt{100t^2 \cancel{\sin^2 t^2} + 100t^2 \cancel{\cos^2 t^2} + 24^2 t^2}$

$= \boxed{\sqrt{100t^2 + 24^2 t^2}}$

$L = \int_0^2 \sqrt{100t^2 + 24^2 t^2} = \int_0^2 26\sqrt{t^2}\, dt = \int_0^2 26t\, dt \quad 13t^2 \Big|_0^2$

$= \boxed{52}$

41) Yes, as it is a circle in the YZ plane.

45) $r(t) = \langle 2\cos t, 2\sin t \rangle \quad 0,, 2\pi \quad |r'(t)| = \sqrt{4\sin^2 t + 4\sin^2 t}$

$r'(t) = \langle -2\sin t, 2\cos t \rangle \quad 0,, 2\pi \quad = \sqrt{4} = 2$

$s(t) = \int |r(\tau)| d\tau \quad s(t) = 2 \quad s = 2t \quad t = s/2$

$\boxed{\begin{array}{l} \cancel{s(t)} = \langle 2\cos\frac{s}{2}, 2\sin\frac{s}{2} \rangle \quad 0, 4\pi \\ f(s) = \end{array}}$

49) $r(t) = \langle e^t, e^t, e^t \rangle$ for $1..4$

$r'(t) = \langle e^t, e^t, e^t \rangle \quad |r'(t)| = \sqrt{3e^{2t}} = \sqrt{3} \cdot e^t$

$s(\tau) = \int_0^\tau \sqrt{3} e^t\, dt = \sqrt{3} e^\tau - \sqrt{3} e^0 = \sqrt{3}(e^\tau - 1)$

$s(\tau) = \sqrt{3}(e^\tau - 1) \rightarrow \frac{1}{\sqrt{3}} s = e^t - 1$

$s = \sqrt{3}(e^t - 1) \quad \frac{s}{\sqrt{3}} + 1 = e^t$

$\boxed{\begin{array}{l} f(s) = \langle 1, 1, 1 \rangle \left(\frac{s}{\sqrt{3}} + 1\right) \\ s \geq 0 \end{array}}$