12.6 GRADIENTS AND DIRECTIONAL DERIVATIVES

- Gradients: $\nabla f(x,y)$ = "gradient" = $\langle f_x, f_y \rangle$. If evaluated at a point, substitute x, y from point, solve as a vector.

- Directional Derivative:
  1) find if $\vec{v}$ provided is unit vector, else make it one
  2) find $\nabla f(x,y)$ (see above for help there :)
  3) find $D_u f(x,y) = u \cdot \nabla f(x,y) = u_x \cdot f_x + u_y \cdot f_y$
  4) Substitute Point "p".

  NOTE: STEPS 3 and 4 CAN BE DONE INTERCHANGEABLY

- Directions of steepest change, no change
  - → Steepest ascent $= \left(\nabla f(x,y) / \|\nabla f(x,y)\|\right)$ AKA unit in dir of $\nabla f(x,y)$
  - → Steepest descent = − [steepest ascent]
  - → No change = $\pm$ [swapped variables and one made neg, i.e. $\langle 8,1\rangle \to \langle -1,8\rangle$ from pos steepest or neg steepest.]

- Tangent Line to Level Curves: $f(x,y)$ at point $P=(a,b)$
  - → $y'(x) = -\dfrac{f_x(a,b)}{f_y(a,b)}$: eval, if UND = VERT TAN,
  - → CHECK: should be $\perp$ to $\nabla f(x,y)\big|_P$ ...

12.7 TANGENT CURVES, ASSOCIATED STUFF

- Tangent Plane to $f(x,y,z)$ at (a,b,c)
  - → $= f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$
- Tangent Plane to $z = f(x,y)$ at (a,b,c)
  - → $= z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$

Lyell C. Read CH 12.4 – 12.8 REVIEW 10/24/2018

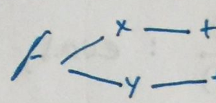
NOTE: KNOW YOUR DERIVATIVES (i.e. A^x, x^n, $\ln(x)$ ... and product, chain rules!

$\boxed{12.4}$ PARTIAL DERIVATIVES

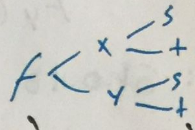
- First Degree Partial Derivative: for $f(x,y)$:
  - → Calc f_x = Derivative of $f(x,y)$ treating all non-x var's as const.
  - → Calc f_y = Derivative $f(x,y)$ where all non-y var's are const.
  - → NOTE: $f_x = \frac{\partial f}{\partial x}$, $f_y = \frac{\partial f}{\partial y}$
- First Deg. P. D. Using Limits: for $f(x,y)$
  - → $f_x = f_x(a,b) = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$
  - → $f_y = f_y(c,d) = \lim_{h \to 0} \frac{f(c,d+h) - f(c,d)}{h}$
- Second Degree P.D.'s: for function $f(x,y)$:
  - → Perform FDPD for the first var, then again on result for second: f_{xy} would be f_x derived with y ...

$\boxed{12.5}$ DA CHAIN RULE

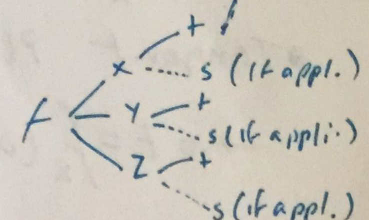
- f' for $f \in x \dots y$ where x is a function of another var, and so is y
  - → $f' = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$



- f' for $f(x,y)$ where $x(s,t)$, $y \in s,t)$



  - → $f'_s = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$, $f'_t = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$
- If $f(x,y,z)$: add another term, $\frac{\partial f}{\partial z}$, and chain rule
- solution process: evaluate $f'(\dots)$, then substitute $x=$, $y=$, $z=$ ... and simplify
- Implicit Differentiation $\frac{dy}{dx} = -\frac{F_x}{F_y}$



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- Linear Approximation : Find the tangent plane, then substitute the ("approximate for (h, i, k)") (In that case, use (h, j, k) or (h, j)...) in the plane equation.

- Approximate Function Change for $f(x,y) = z$ on $(a,b) \to (c,d)$
  → $dx = c - a$; $dy = d - b$
  → $\Delta z =$ "appro. fxn chg" $= f_x(a,b)\,dx + f_y(a,b)\,dy$

- Find Horizontal tangent Planes : These occurr where either $\left(\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0\right)$ or where $\nabla f(x, y, \ldots) = \langle 0, 0, \ldots \rangle$
  → Hint: list all the values that would make f_x ~~and~~ or f_y zero, and choose all that work for both $= 0$.