Lyell C Read — CH 13.3 Homework — pr. 11-27 odd, 39, 51-57 — 11/18/2018

11) $z = 4 - x^2 - y^2$ | $x = r\cos\theta$, $y = r\sin\theta$ $= z = 4 - r^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} = 4 - r^2$

$4 - r^2$ on $\{(r,\theta): 0 \le r \le 1;\ 0 \le \theta \le 2\pi\} = \int_0^{2\pi}\int_0^1 4 - r^2\, r\,dr\,d\theta$

1] $\int_0^1 (4 - r^2) r\,dr = \int_0^1 4r - r^3\,dr = 2r^2 - \frac{1}{4}r^4 \Big|_0^1 = 2 - \frac{1}{4} = \underline{\frac{7}{4}}$

2] $\int_0^{2\pi} \frac{7}{4}\,d\theta = \frac{7\theta}{4}\Big|_0^{2\pi} = \frac{7\cdot 2\pi}{4} = \boxed{\frac{7\pi}{2}}$

13) ignored, as practically the same as #11)

15) $z = 5 - \sqrt{1 + x^2 + y^2}$ on $\{(r,\theta): 0 \le r \le 2,\ 0 \le \theta \le 2\pi\}$

$z = 5 - \sqrt{1 + r^2\cos^2\theta + r^2\sin^2\theta} = z = 5 - \sqrt{1 + r^2} \rightsquigarrow A = \int_0^{2\pi}\int_0^2 5 - \sqrt{1 + r^2}\, r\,dr\,d\theta$

1] $\int_0^2 (5 - \sqrt{1 + r^2}) r\,dr = \int_0^2 5r - r(1 + r^2)^{\frac{1}{2}}\,dr = \frac{5}{2}r^2 - \frac{2}{6}(1 + r^2)^{\frac{3}{2}}\Big|_0^2$

$= \frac{20}{2} - \frac{2}{6}(\sqrt{125}) + \frac{2}{6} = \frac{60 - 10\sqrt{5} + 1}{6} = \underline{\left[\frac{61 - 10\sqrt{5}}{6}\right]}$

2] $\int_0^{2\pi} \left[\frac{61 - 10\sqrt{5}}{6}\right] d\theta = \left[\frac{61 - 10\sqrt{5}}{6}\right]\theta \Big|_{\theta = 0}^{2\pi} = \boxed{\left(\frac{61 - 10\sqrt{5}}{3}\right)\pi}$

17) same, if not messier, than 15), ignored

19) Top: $z = 2 - x^2 - y^2 \rightsquigarrow \underline{z = 2 - r^2}$ | Bottom: $z = x^2 + y^2 \rightsquigarrow \underline{z = r^2}$

Curve C where top = bottom: $2 - r^2 = r^2 \rightarrow = \underline{r = 1}$

$A_{between} = \int_0^{2\pi}\int_0^1 ((2 - r^2) - r^2) r\,dr\,d\theta$

1] $\int_0^1 (2 - 2r^2) r\,dr = \int_0^1 2r - 2r^3\,dr = r^2 - \frac{1}{2}r^4\Big|_0^1 = 1 - \frac{1}{2} = \underline{\frac{1}{2}}$

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2] $\int_0^{2\pi} \frac{1}{2}\, d\theta = \frac{\theta}{2}\Big|_{0}^{2\pi} = \frac{2\pi}{2} = \boxed{\pi}$

21) TOP: $z = 2 - x^2 - y^2 = \underbrace{2 - r^2}$ | BOT: $z = 1$

Intersection curve $C \Rightarrow$ ~~$z - x^2 - y^2$~~ $2 - r^2 = 1$ $\quad -r^2 = -1$ $\quad r^2 = 1$ $\quad \underbrace{r = \pm 1}$

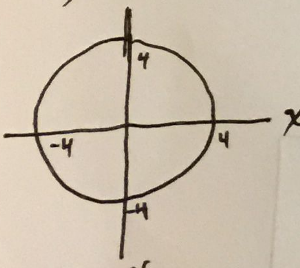
$$\int_0^{2\pi}\int_0^1 (\overset{TOP}{(2-r^2)} - \overset{BOT}{1})\, r \, dr\, d\theta$$

NOTE: THE CURVE "C" IS THE UPPER LIMIT OF DIFFERENTIATION FOR THE DR INTEGRAL

1] $\int_0^1 ((2-r^2)-1)\, r\, dr = \int_0^1 r - r^3\, dr = \frac{1}{2}r^2 - \frac{1}{4}r^4 \Big|_0^1 = \frac{1}{2} - \frac{1}{4} = \underbrace{\frac{1}{4}}$

2] $\int_0^{2\pi} \frac{1}{4}\, d\theta = \frac{\theta}{4}\Big|_0^{2\pi} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

23) ~~just a circle with rad 4~~



$\iint_R \underbrace{(x^2+y^2)}_{r^2}\, dA \qquad R = \{(r,\theta) : 0 \le r \le 4;\ 0 \le \theta \le 2\pi\}$

$\int_0^{2\pi}\int_0^4 (r^2)\, r\, dr\, d\theta$

1] $\int_0^4 r^3 = \frac{1}{4}r^4 \Big|_0^4 = \underbrace{64}$ $\rightarrow$ 2] $\int_0^{2\pi} 64\, d\theta = 64\theta \Big|_0^{2\pi} = \boxed{128\pi}$

25) $\iint_R 2xy\, dA \quad R = \{(x,y) : x^2 + y^2 \le 9,\ y \ge 0\}$

$r^2 \le 9$ $\qquad r\sin\theta \ge 0$

$r \le 3$ $\qquad \rightarrow \sin^{-1} \leftarrow$ $\qquad r \ge 0$

~~trig identity~~
~~$\sin(2\theta) = 2\sin\theta\cos\theta$~~

USE GRAPH

$= \boxed{0}$

$\int_0^{2\pi}\int_0^3 2r^3 \underbrace{(\cos\theta\sin\theta)}_{\#}\, dr\, d\theta$

1] $\int_0^3 2r^3(\#)\, dr = r^2(\#)\Big|_0^3 = \underbrace{9\#}$ $\quad$ 2] $\int_0^{2\pi} 9\cos\theta\sin\theta\, d\theta$

27) $\iint_R \frac{1}{\sqrt{16-x^2-y^2}}\, dA \quad R = \{(x,y) : x^2+y^2 \le 4,\ x \ge 0,\ y \ge 0$

$$\hookrightarrow \int_0^{\pi/2}\int_0^2 \frac{r}{\sqrt{16-r^2}}\, dr\, d\theta = \int_0^{\pi/2}\int_0^2 (16-r^2)^{-1/2}\, r\, dr\, d\theta$$

$$1]\ \int_0^2 (16-r^2)^{-1/2}\, r = (16-r^2)^{1/2}\Big|_0^2 = \underbrace{\cancel{3\sqrt{2}}}_{2\sqrt{3}}^{\sqrt{12}} - 4$$

$$2]\ \int_0^{\pi/2} \underset{2\sqrt{3}}{\cancel{3\sqrt{2}}} - 4 = \overset{2\sqrt{3}}{\cancel{3\sqrt{2}}}\theta - 4\theta \Big|_{\cancel{0}}^{\pi/2} = \sqrt{3}\cdot\pi - 2\pi = \boxed{\pi\left(\sqrt{3}-2\right)}$$

39) Annular Area of 1 where $R = \{(r,\theta) : 1 \le r \le 2,\ 0 \le \theta \le \pi\}$

integral: $\int_0^{\pi}\int_1^2 1\, r\, dr\, d\theta = \int_0^{\pi}\int_1^2 r\, dr\, d\theta$

$$1]\ \int_1^2 r\, dr = \frac{1}{2}r^2\Big|_1^2 = 2 - \frac{1}{2} = \underline{3/2}$$

$$2]\ \int_0^{\pi} 3/2\, d\theta = \frac{3\theta}{2}\Big|_{\cancel{0}}^{\pi} = \boxed{3\pi/2}$$

51-57) just integrals.. might add later :)