Lyell C Reed CH 13.1 – REVIEW 11/6/2018

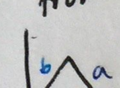
13.1 Double Integrals over Rectangular Areas

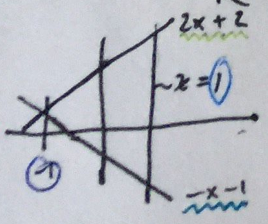
- To compute $\int_c^d\int_a^b f(x,y)\,dx,dy$ first evaluate $\int_a^b f(x,y)\,dx$, then substitute that into $\int_c^d \boxed{F(x,y)}\Big|_a^b\,dy$ where $F(x,y)=\int f(x,y)$
- Notation: $\iint_R f(x,y)$ on $\{(x,y): a \le x \le b,\ c \le y \le d\}$ just means:

$$\rightarrow \int_a^b\int_c^d f(x,y)\,dy\,dx \quad \text{WHICH IS EQUAL TO} \quad \int_c^d\int_a^b f(x,y)\,dx\,dy$$

- Average Function Value: $\frac{1}{\text{Area of rectangle}} \cdot \iint_R f(x,y)$ on {rectangle}
- Average Square distance: $\frac{1}{\text{area of rectangle}} \cdot \iint_R f(x,y)$ on {rectangle} where $f(x,y) = d^2 = (x_1-x_2)^2 + (y_1-y_2)^2$ (Distance Function, Squared)

13.2 DOUBLE INTEGRALS OVER NON-RECTANGULAR AREAS

- Writing a double integral based on a graph, or functions.
  - → If you are doing $dx\,dy$ the first integral is the y range so $\int_a^b \cdots dy$.
  - → the inside integral, the dx one, is from the "closer" function to the y axis to the "further".  $\int_b^a \cdots dx$
- Evaluating an integral with equations as limits of integration: just do as you would a normal integral
- Example: $\iint_R y^2\,dA$ bounded by $x=1$, $-x-1$, $2x+2$



$$\int_{-1}^{1}\int_{-x-1}^{2x+2} y^2\,dy\,dx \longrightarrow \boxed{12}$$