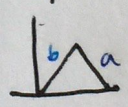
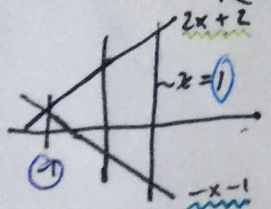


13.1 Double Integrals over Rectangular Areas

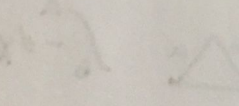
- To compute $\iint_R f(x,y) dx dy$ first evaluate $\int_a^b f(x,y) dx$, then substitute that into $\int_c^d \left[F(x,y) \right]_a^b dy$ where $F(x,y) = \int f(x,y)$
- Notation: $\iint_R f(x,y)$ on $\{(x,y) : a \leq x \leq b, c \leq y \leq d\}$ just means:
 $\rightarrow \int_a^b \int_c^d f(x,y) dy dx$ WHICH IS EQUAL TO $\int_c^d \int_a^b f(x,y) dx dy$
- Average Function Value: $\frac{1}{\text{Area of rectangle}} \cdot \iint_R f(x,y)$ on {rectangle}
- Average Square distance: $\frac{1}{\text{area of rectangle}} \cdot \iint_R f(x,y)$ on {rectangle} where
 $f(x,y) = d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ (Distance Function, Squared)

13.2 DOUBLE INTEGRALS OVER NON-RECTANGULAR AREAS

- Writing a double integral based on a graph, or functions.
 \rightarrow If you are doing $dx dy$ the first integral is the y range
 so $\int_a^b \dots dy$.
 \rightarrow The inside integral, the dx one, is from the "closer" function to the y axis to the "further".  $\int_b^a \dots dx$
- Evaluating an integral with equations as limits of integration: just do as you would a normal integral
- Example: $\iint_R y^2 dA$ bounded by $x=1$, $-x-1$, $2x+2$
 $\int_{-1}^1 \int_{-x-1}^{2x+2} y^2 dy dx \rightarrow \boxed{12}$

- Volume Calculation using double Integrals: Integrate over the range of the base, the value of the top. to find the range of the base, with a plane, set $z=0$, solve for $y=...$
- Changing the order of integration: Draw out the xy shape, and find the equations, then integrals, for the opposite variable.

1.2.2 Double Integrals over non-rectangular Areas

- Working a double integral based on a graph or function. If you are doing it by the first integral is the x range $\int \dots dx$
- The inside integral, the dy one, is from the "inner" function to the y axis to the "outer" function. 
- Evaluating an integral with equations as limits of integration. Just as we would a normal integral.

• Example: $\int_0^1 \int_0^{2-x} dy dx$ bounded by $x=0$, $y=0$, $x+y=2$

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