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MATH 254H, Fall 2018

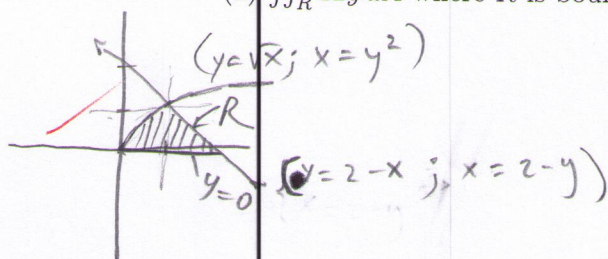
FOR EACH PROBLEM SHOW ALL ESSENTIAL STEPS.

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QUIZ #4

1. Sketch the region of integration and evaluate the integral:

(a)  $\iint_R 12y \, dA$  where  $R$  is bounded by  $y = 2 - x$ ,  $y = \sqrt{x}$ , and  $y = 0$ .



$$\iint_R 12y \, dA = \int_0^1 \int_0^{2-x} 12y \, dy \, dx + \int_1^2 \int_{y^2}^{2-x} 12y \, dy \, dx$$

$$= \int_0^1 \left[ 6y^2 \right]_0^{2-x} dx + \int_1^2 \left[ 6y^2 \right]_{y^2}^{2-x} dx$$

$$= \int_0^1 (6(2-x)^2) dx + \int_1^2 (6(2-x)^2 - 6y^4) dx$$

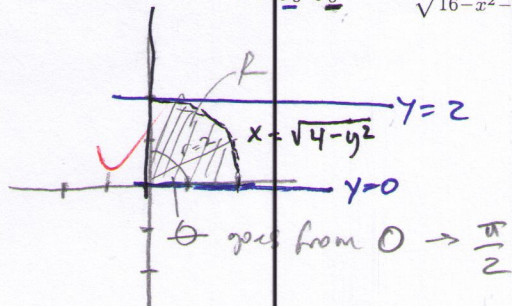
$$= \int_0^1 (24 - 24x + 6x^2) dx + \int_1^2 (24 - 24x + 6x^2 - 6y^4) dx$$

$$= \left[ 24x - 12x^2 + 2x^3 \right]_0^1 + \left[ 24x - 12x^2 + 2x^3 - \frac{6}{5}y^5 \right]_1^2$$

$$= (24 - 12 + 2) - (24 - 12 + 2 - \frac{6}{5}) = 10 - \frac{24}{5} + \frac{12}{5} - \frac{2}{5} + \frac{6}{5} = 10 - \frac{10}{5} = 8$$

$$\int_0^1 \left( 12xy \Big|_{x=y^2}^{x=2-y} \right) dy = \int_0^1 (24y - 12y^2 - 12y^3) dy = 12y^2 - 4y^3 - 3y^4 \Big|_0^1 = 12 - 4 - 3 = 5$$

(b)  $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{dx \, dy}{\sqrt{16-x^2-y^2}}$



$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{dx \, dy}{\sqrt{16-x^2-y^2}}$$

$$-(x^2+y^2) = -r^2$$

$$\int_0^{\pi/2} \int_0^2 \frac{r}{\sqrt{16-r^2}} dr \, d\theta = \int_0^{\pi/2} \left[ -\sqrt{16-r^2} \right]_0^2 d\theta$$

$$\int_0^{\pi/2} \left( -\sqrt{16-r^2} \Big|_0^2 \right) d\theta = \int_0^{\pi/2} (-\sqrt{12} + 4) d\theta$$

$$\int_0^{\pi/2} (-\sqrt{12} + 4) d\theta = \left[ -\theta\sqrt{12} + 4\theta \right]_0^{\pi/2} = -\frac{\sqrt{12} \cdot \pi}{2} + 2\pi$$

197.8

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