MTH 341 NOTES 5/14/2019

'Mapping'

- consider $f(x)$ whose domain is $\mathbb{R}^n$ and who accepts values in $\mathbb{R}^m$.

$$y = f(x) \equiv f: \mathbb{R}^n \mapsto \mathbb{R}^m$$

- Ex's

$f: \mathbb{R} \to \mathbb{R}$: $f(x) = 3\cos(x) + xe^x$

$f_1: \mathbb{R} \to \mathbb{R}^3$: $f(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$

$f_2: \mathbb{R}^3 \to \mathbb{R}$: $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dfrac{3xz + 2yxz}{zy}$

$f_3: \mathbb{R}^2 \to \mathbb{R}^3$: $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ 3\cos(xy) \\ x-y \end{pmatrix}$

- Ex:

$f_A: \mathbb{R}^n \to \mathbb{R}^m$

$x \in \mathbb{R}^n \to xA \in \mathbb{R}^m$ -

Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ $[2 \times 3]$

If $x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$

$f_A(x) = Ax = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$= \begin{pmatrix} x + 2y + 3z \\ 4x + 5y + 6z \end{pmatrix}$

note $\mathbb{R}^3 \to \mathbb{R}^2$ has dimensions 2×3

Linearity of Functions

- a function is linear if

1] $f(x_1 + x_2) = f(x_1) + f(x_2)$ AND

2] $f(cx) = f(x) \cdot c$ for all $x_1, x_2 \in \mathbb{R}^n$ and $c \in \mathbb{R}$

- a linear function f is also known as a linear transformation

Ex:

$f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3x - 2y + z$

for $x_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ $x_2 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$

$f\begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{pmatrix} = 3(x_1 + y_2) - 2(y_1 + y_2) + (z_1 + z_2)$

$= \cancel{3(x_2+z)}$

$(3x_1 - 2y_1 + z_1) + (3x_2 - 2y_2 + z_2)$

↳ it is linear.

- If $A_{m \times n}$ is a matrix, then $f_A: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation.

Kernel

- If $f: \mathbb{R}^n \to \mathbb{R}^m$ is linear, then the kernel of f:

$$\text{Ker}(f) = \{ v \in \mathbb{R}^n \mid f(v) = 0 \}$$

Range

- If $f: \mathbb{R}^n \to \mathbb{R}^m$, then range

$$\text{Range}(f) = \{ y \in \mathbb{R}^m \mid y = f(x) \text{ for some } x \in \mathbb{R}^n \}$$

Rank Nullity Theorem Cont'd

- for $A_{m \times n}$, $n = R(A) + N(A)$

↳ now think of A as $f_A: \mathbb{R}^n \to \mathbb{R}^m$

NEXT PG

if $A = f_A : \mathbb{R}^n \to \mathbb{R}^m$

then :

domain $(A) = \mathbb{R}^n$

$\ker(f_A) =$ Null space (A)

Range $(f_A) =$ Col space (A)

$\dim(\text{domain } f) = (\dim(\ker(f)) + \dim(\text{Range}(f)))$