MTH 341 NOTES 4/11/2019

Chapter 4

Algorithm for Gaussian Elimination

1] begin with (1,1)
- If $a \neq 0$, take ①$/a$
- If it is $a = 0$
  - interchange ① with ⓘ to get $a \neq 0$ (1,1)
  - If every entry in the column $= 0$, then go to column 2

2] You may have a leading 1 in some column. Use row ops to replace every entry below the '1' with a zero.

3] move (row+1, col+1) - one row down, one col over - and repeat trying to establish a leading value of 1, and

4] continue to echelon form.

—— echelon achieved ——

5] Use each leading 1 to blast out every nonzero element in that column

System of Equations

- a system of equations $Ax=y$ is <u>consistent</u> if they have at least one solution.

- $(A\ y) \to$ Echelon form
  - If there is a leading 1 in the last column, then no solutions exist, and the set is inconsistent
  - If there is <u>no</u> leading 1 in the last column, Eq's are consistent.

Leading Variables

- any column containing a leading 1 except the last column

Free Variable

- ! Leading Variable

↳ #(vars) = #(leading) + #(free)

Ex:
$$\begin{array}{c}\left(\begin{array}{cccc} 1 & 3 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right) \\ \begin{array}{cccc} x & y & z & = \end{array}\end{array}$$

x, z = leading y = Free

Solutions

- If a system does not have any free variables, then there is one solution
- If there is at least 1 free variable, then there are an infinite number of solutions.

↳ # of F.V's = Dimensions of solution space = Deg's of freedom

- to create solution
  - → Assign each free variable a var.
  - → rewrite other Eq's in terms of free variables.

Chapter 5

Homogenious Systems

- $Ax = 0$
- Always has solution $x = 0$ (can be others)
- If a solution to $Ax = 0$ is a nonzero vector, it is a non-trivial solution
  - If the system has no free variables then there is only one solution - the trivial solution
  - If there exist free variable(s), then there are infinite solutons to the system.
- if there are more variables than eq's, there will be non-trivial solutions.
- for a square $A_{n \times n} x = 0$
  - If $\det(A) = 0$: nontrivial solutions
  - If $\det(A) \neq 0$: $x = 0$.

Linear Combination

- defind $\underbrace{\text{vectors } u, v, \text{ scalors } s, t}$
- Linear Combination of ↑ :

  $su + tv$
- If u, v are solutions to $Ax = 0$, then $su + tv$ is too.