# MTH 341 4/4/2019

**Definition**

- If $AB = I = BA$, Then $B = A^{-1}$, $A = B^{-1}$
- Ex: $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

$$AB = \begin{pmatrix} 2-1 & 2-2 \\ 1-1 & 2-1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{I} = BA.$$

**Inverses**

- If $a \neq 0$, we have $1 \times 1$ matrix $(a)$.
  $(a)^{-1} = \left(\frac{1}{a}\right)$
- not every square matrix has an inverse.
- If the matrix is not square, NO INV!!
- Ex: if $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the inverse of $\begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix}$,
  then $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix} = I = \begin{pmatrix} 3a+c & 3b+d \\ 3a+c & 3b+d \end{pmatrix}$

  other order ☺

  but $3a+c$ is in both 1's and 0's - impossible.

↳ for $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, if
- $ad-bc = 0$ : NO INVERSE
- $ad-bc \neq 0$ : inverse $= \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

**DEF**

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \frac{1}{ad-bc}\underbrace{\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$

(more on this later)
1. switch
2. put "-"

**Chapter 2: matricies & sys of Lin Eq's**

EX: $3x+4y=5$ ①
$2x-y=0$ ②

1 solution: $4②+①$

$3x+4y=5$
$8x-4y=0$
$11x=5$
$x = 5/11 \ldots$
$y = \frac{10}{11}$

- $\underbrace{\begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}}_{\text{coefficient matrix}}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+4y \\ 2x-y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

- combine coeff and results matricies

$$\begin{array}{ccc} \begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 0 \end{pmatrix} \\ \uparrow\ \text{coeff of } x \quad \uparrow\ \text{coeff of } y \quad \uparrow\ \text{right side} \end{array}$$

} augmented matrix of the sys of lin eq's

- To solve ↑ which represents $\begin{cases} 3x+4y=5 & ① \\ 2x-y=0 & ② \end{cases}$

usually, you would do $\underbrace{②\cdot 4}_{\text{Part 1}} + \underbrace{①}_{\text{Part 2}}$

Part 1: $②\cdot 4$ $\begin{pmatrix} 3 & 4 & 5 \\ 8 & -4 & 0 \end{pmatrix}$

Part 2: $+①$ $\begin{pmatrix} 3 & 4 & 5 \\ 11 & 0 & 5 \end{pmatrix} \to 11x+0y=5$

- Definition:
  let $A_{2\times 2}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$, then $\begin{pmatrix} A & \begin{matrix} a \\ b \end{matrix} \end{pmatrix}$ is an augmented matrix of the sys.

- Another solving method for Eq pair above

$$\begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 0 \end{pmatrix} \xrightarrow{\text{replace ① by ①-②}} \begin{pmatrix} 1 & 5 & 5 \\ 2 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 5 & 5 \\ 2 & -1 & 0 \end{pmatrix} \xrightarrow{\text{replace ② by ②-2·①}} \begin{pmatrix} 1 & 5 & 5 \\ 0 & -11 & -10 \end{pmatrix}$$

**Applications of this stuff**

- Geometry $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$

$ax+by=u$
$cx+dy=v$
} 2 lines. solution is an intersection point

↳ 3 possible Outcomes

- $=1$: $\begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$
- $=0$: $\begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- $=\infty$ two lines are the same: $\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$3x+y=1$ $y = 1-3x$

let $x=t$

$\boxed{\text{General soln } \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} t \\ 1-3t \end{pmatrix}}$

## Moar Variables!!

- $\underbrace{\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}}_{\text{coeff matrix}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$

|| D.F = degree of freedom

$\left.\begin{matrix} ax+by+cz=u \\ dx+ey+fz=v \end{matrix}\right\}$ 2 planes

↳ ①: planes intersect at a line : DoF = 1

↳ ②: Planes intersect at a plane (they are equal) DoF = 2

↳ ③: Planes do not intersect

- Ex: $2x-4y+z=1$
  $4x+y-z=3$

$$\begin{pmatrix} 2 & -4 & 1 \\ 4 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

aggregated:

$$\begin{pmatrix} 2 & -4 & 1 & 1 \\ 4 & 1 & -1 & 3 \end{pmatrix}$$

order: x, y, z for elimination

replace ① with ①/2 : $\begin{pmatrix} 1 & -2 & \frac{1}{2} & \frac{1}{2} \\ 4 & 1 & -1 & 3 \end{pmatrix}$

replace ② with ② - 4·① $\begin{pmatrix} 1 & -2 & \frac{1}{2} & \frac{1}{2} \\ 0 & 9 & -3 & 1 \end{pmatrix}$

--- "solve"

replace ② with ②/9 $\begin{pmatrix} 1 & -2 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{9} \end{pmatrix}$

↳ Echelon Form

1's on Diag, with 0 below

replace ① with ①+2② $\begin{pmatrix} 1 & 0 & -\frac{1}{6} & \frac{13}{18} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{9} \end{pmatrix}$

↳ $\left.\begin{matrix} x+0y-\frac{1}{6}z=\frac{13}{18} \\ 0x+y-\frac{1}{3}z=\frac{1}{9} \end{matrix}\right\}$ result!

let $z=t$

$x = \frac{t}{6}+\frac{13}{18}$ $\quad y = \frac{t}{3}+\frac{1}{9}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{t}{6}+\frac{13}{18} \\ \frac{t}{3}+\frac{1}{9} \\ t \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{6} \\ \frac{1}{3} \\ 1 \end{pmatrix} t + \begin{pmatrix} \frac{13}{18} \\ \frac{1}{9} \\ 0 \end{pmatrix}}$$

vector
(Direction + magnitude)
also general form (eval at t)
also parametrization.

## Practice Problems

1. $\left\{\begin{matrix} 3x+2y-4z=3 \\ -x-2y+3z=4 \end{matrix}\right\}$ 2 planes
2. $\left\{\begin{matrix} 2x-4y=3 \\ 3x+2y=-1 \\ x-y=10 \end{matrix}\right\}$ 3 lines
3. $x+y+3z=4$

## Solutions

3. let $y=s$, $z=t$
   $x=-s-3t+4$

   General solution

   $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -s-3t+4 \\ s \\ t \end{pmatrix}$$

2. $\begin{pmatrix} 2 & -4 & 3 \\ 3 & 2 & -1 \\ 1 & -1 & 10 \end{pmatrix} \xrightarrow{\text{switch ① and ③}} \begin{pmatrix} 1 & -1 & 10 \\ 2 & -4 & 3 \\ 3 & 2 & -1 \end{pmatrix}$

   [replace ② by ② - 2·① AND rep ③ by ③ - 3·①]

   $\begin{pmatrix} 1 & -1 & 10 \\ 0 & -2 & -17 \\ 0 & 5 & -31 \end{pmatrix} \xrightarrow{\text{rep ② by ②/-2}} \begin{pmatrix} 1 & -1 & 10 \\ 0 & 1 & 17/2 \\ 0 & 5 & -31 \end{pmatrix}$

   [replace ③ by ③ - 5·②]

   $\begin{pmatrix} 1 & -1 & 10 \\ 0 & 1 & \frac{17}{2} \\ 0 & 0 & -\frac{147}{2} \end{pmatrix}$ Echelon Notation

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$$\begin{pmatrix} 1 & -1 & 10 \\ 0 & 1 & 17/2 \\ 0 & 0 & -147/2 \end{pmatrix} \xrightarrow[\text{① + ②}]{\text{replace ① with}} \begin{pmatrix} 1 & 0 & 37/2 \\ 0 & 1 & 17/2 \\ \boxed{0 \quad 0 \quad -147/2} \end{pmatrix} \rightarrow$$

$0x + 0y = -\frac{147}{2}$ = impossible

↳ these 3 lines do not share an intersection point.