MTH 341 Notes from 4/23/2019

A couple Lemmas about determinants

- $M \xrightarrow{\textcircled{i}\textcircled{j}} \tilde{M}$ (row swap) $\rightarrow$ $\det(\tilde{M}) = -\det(M)$
- $M \xrightarrow{\textcircled{i}\cdot\alpha} \tilde{M} \rightarrow \det(\tilde{M}) = \alpha \det(M)$
- Elementary row op rep ⓙ by ⓙ + 3ⓘ Does not change Det.

Review: Det (2x2 matrix)

- ad - bc Ex $\det\begin{pmatrix} 1 & 5 \\ 3 & 6 \end{pmatrix} \rightarrow \boxed{6-15}$

Using ERO's and ECO's to make determinant calculation more simple

- Ex $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix} \xrightarrow{\text{ERO + ECO's}} \cdots \longrightarrow \begin{pmatrix} * & * & * \\ * & * & * \\ 0 & & 0 \end{pmatrix}$

  ↳ Note it does not have to be this format, any row or col with the most 0's makes it easiest

Specific Matrix Determinant

- $\det\begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & & x_n^{n-1} \end{pmatrix}$

$$= \prod_{\text{if } j>i} (x_j - x_i)$$

- Ex $\begin{pmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x_4 & x_4^2 & x_4^3 \end{pmatrix} = \begin{array}{l} (x_4 - x_1)(x_3 - x_1)(x_2 - x_1) \cdot \\ (x_4 - x_2)(x_3 - x_2) \cdot \\ (x_4 - x_3) \cdot \end{array}$

Invertability

- A matrix is only invertible when $\det(M) \neq 0$.

Theorem 6

- If A and B are nxn Matricies then $\det(AB) = \det(A)\det(B)$.

Inverse

- $\det(A^{-1}) = \dfrac{1}{\det(A)}$

Cramer's Rule

- linears system $Ax = b$
  - A = nxn Matrix
  - $b \in \mathbb{R}^n$
- If $\det(A) \neq 0$, then

  $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = A^{-1}b$

- further $x_i = \dfrac{\det A_i}{\det A}$

  where A_i is obtained by replacing the ith row of A with b