MTH341 Review Quiz1

DUE: 4/9/2019

CHAPTER 1

• Matrix: $\begin{pmatrix} a & b & z \\ 4 & \pi & x \\ -1 & 3.3 & 0.2 \end{pmatrix}_{3\times 3}$ Denoted rows X cols

• Real Numbers are 1x1 matricies.

• Vectors are 3 x 1 matricies such as $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

• matrix Addition $\begin{pmatrix} x & 2 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} -\pi & 0 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} x-\pi & 2 \\ 6 & 3 \end{pmatrix} := (A+B)_{ij} = a_{ij} + b_{ij}$

• Matrix Subtraction $\begin{pmatrix} 3 & 7 \\ x & 0 \end{pmatrix} - \begin{pmatrix} 2 & 18 \\ \pi & 0 \end{pmatrix} = \begin{pmatrix} 1 & -11 \\ x-\pi & 0 \end{pmatrix} := (A-B)_{ij} = a_{ij} - b_{ij}$

• Scalar (c) multiplication: multiply every element by constant $\langle c \rangle$.

• Zero matrix: If all values are zero, then 0_{mn} is the way to write that.

• Matrix Equality: $A = B$ if $a_{ij} = b_{ij}$ for all i,j

• Matrix Multiplication: if col (A) = row(B), then $(AB)_{ij} = \sum_{s=1}^{k} a_{is} b_{sj}$

Ex: $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 4 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} ((1\cdot -1) + \overset{11}{(2\cdot 4)} + (3\cdot 1)) & (\cancel{(1\cdot 0)} + \overset{12}{(2\cdot 2)} + (3\cdot 3)) \\ ((-1\cdot -1) + \overset{21}{\cancel{(0\cdot 4)}} + (4\cdot 1)) & (\cancel{(-1\cdot 0)} + \overset{22}{\cancel{(0\cdot 2)}} + (4\cdot 3)) \end{pmatrix} = \begin{pmatrix} 10 & 13 \\ 5 & 12 \end{pmatrix}$

• Multiplication: sizes: If $A_{m\times n} B_{n \times p}$ then $(AB)_{m\times p}$

• Multiplication: Notes: $AB \neq BA$ most often. (even when both are squares..)

• Multiplication Properties

If AB and AC are defined $\longrightarrow A(B+C) = AB + AC$

If AB is defined, and c is a scalar $\longrightarrow A(cB) = c(AB)$

• Transpose: Transposing makes col_n of A row_n of B:

$a^t_{ij} = a_{ji}$ or $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}^t = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

- Transpose-Multiplication Property: $(AB)^t = B^t A^t$ $\left[\underline{\textbf{NOT}}\ A^t B^t\right]$
- Square Matrix: rows = cols
- Identity Matrix: square matrix with 1's on main diag $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- Identity Matrix Properties: if $A_{m \times n}$ then $\underline{I_m A = A}$ $\underline{A I_n = A}$
- Inverses: $AB = I = BA$ (and A, B are square, same size) $\longrightarrow A = B^{-1};\ B = A^{-1}$; "is inverse of"

EX: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = B$ $\underline{\text{Iff}}$ $AB = I = BA$

Inverses: if $ad - bc = 0$, then there exists no inverse

Note: must meet the requirements - some square matricies dont have inverses

- Rotation Matrix:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \text{"}R_M\text{"}$$ Usage: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} R_m \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

CHAPTER 2

- Linear Equations to Matricies: $\begin{cases} 3x + 4y = 5 \\ 2x - y = 0 \end{cases} \longrightarrow \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$
- Augmented Matrix: $\begin{cases} 3x + 4y = 5 \\ 2x - y = 0 \end{cases} \longrightarrow \begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 0 \end{pmatrix}$ x y after '='
- Solutions: must consider solutions to the proposed set of Eq's

EX: two lines $ax + by = c$ where $a, b, c \in \mathbb{R}$ have:
1] X intersection 2] = parallel 3] / same-line

- General Solution: set $z = t$, solve all equations for t / in terms of t
- Solutions in Vector form: parametrize for t, then substitute into vector form: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$
- Operations against augmented matricies: AKA <u>elementary row ops</u>
  - multiply by a nonzero scalar c
  - replace any "row" or "equation" by the original eq +/- a scalar mult of another Eq
  - Interchange two rows.