

Homework 3 / Lyell Read

4.10.11

Dependent? $\begin{pmatrix} 1 \\ 3 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 10 \\ 2 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 4 & 10 \\ -1 & -1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ Det = 0 because two identical lines.

↳ Linearly Dependent.

$$\begin{pmatrix} 1 \\ 10 \\ 2 \\ 1 \end{pmatrix} = -6 \downarrow + 4 \downarrow + 3 \downarrow \text{ Redundant.}$$

↳ $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 4 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ which is ^{NOT} indep b/c:

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 4 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} (3) - (1) \\ (4) - (1) \end{matrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{pmatrix} \text{ impossible.}$$

Linear indep set with same span: $\left\{ \begin{pmatrix} 1 \\ 3 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 0 \\ 1 \end{pmatrix} \right\}$

4.10.32

find basis, dim for: $\left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -2 \\ -2 \end{pmatrix} \right\}$

$$\begin{pmatrix} 2 & -1 & 5 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 3 & -2 \\ 1 & -1 & 3 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ 0 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 5 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \\ 1 & -1 & 3 \end{pmatrix} \xrightarrow{(2) - (3)} \begin{pmatrix} 2 & -1 & 5 \\ 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow 2 \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 3 \\ 3 \end{pmatrix}$$

↳ Basis = $\left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ -1 \end{pmatrix} \right\}$, dim = 2.

4.10.33

find dim, basis for $H = \text{span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 5 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ -2 \end{pmatrix} \right\}$

$$\begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -2 & 5 & 2 \\ -1 & 2 & -5 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -2 & 5 & 2 \\ \hline -2 & 5 & -2 \end{pmatrix}$$

\hookrightarrow
means lin dep, but in solving
 $\forall c \det = 0$.

$$\rightarrow \begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -2 & 5 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

\hookrightarrow impossible.

Therefore, Basis = $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 5 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ -2 \end{pmatrix} \right\}$

Dim = 4

10.4.47

is $S = \left\{ \begin{array}{l} 4v + v - 5w \\ 12v + 6v - 6w \\ 4v + 4v + 4w \end{array} \mid v, v, w \in \mathbb{R} \right\}$ subspace of \mathbb{R}^3

$\leftarrow = \text{span} \left(\begin{pmatrix} 4 \\ 21 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}, \begin{pmatrix} -5 \\ -6 \\ 4 \end{pmatrix} \right)$

$\hookrightarrow \det \begin{pmatrix} 4 & 1 & -5 \\ 21 & 6 & -6 \\ 4 & 4 & 4 \end{pmatrix} \neq 0 \rightarrow$ linearly independent.
(calc on review).

3 linearly independent vectors do form a basis for \mathbb{R}^3 ,

and $0 \in \text{Span } S$

2] closed under addition

3] closed under mult

Dim = 3
Yes

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$$\begin{pmatrix} 1 & 0 & 3 \\ 3 & 1 & 10 \\ 1 & 1 & 4 \\ 1 & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 4 \\ 1 & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Delta \Delta x$

Rank = 2

2 leading variables
1 free variable.

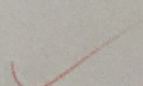
Basis for RowSpace:

$$\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Basis for Col Space:

~~$\begin{pmatrix} 3 \\ 10 \\ 4 \\ 2 \end{pmatrix}$~~
(free var, take from initial)

$$\begin{pmatrix} 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$



4.10.68

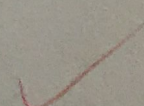
$$\begin{pmatrix} 1 & 0 & 3 & 0 \\ 3 & 1 & 10 & 0 \\ -1 & -2 & 1 & 1 \\ 1 & -1 & 2 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 3 & 1 & 10 & 0 \\ 1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$L L F F_{\text{row}}$

Rank = Leading = 3 if $-1 \neq 0$, else = 2

Col Basis: $\begin{pmatrix} 1 \\ 3 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}$

row Basis: $\begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$



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