MTH 341 REVIEW PART TWO

(15) for $A = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{pmatrix}$ find eigenvalues & vec's

$0 = \det \begin{pmatrix} -\lambda & 2 & -1 \\ 2 & 3-\lambda & -2 \\ -1 & -2 & -\lambda \end{pmatrix}$

* pull a common factor from a row out to in front.

↳ $\lambda = 5, -1, -1$

$\lambda = 5 \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

$\lambda = -1 \rightarrow \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

It is diagonallzeable

* Diagonaltzeable if
alg mult (λ_A) = geog mult (λ_A)
for all eigenvalues

(16) Diagonallze the symmetric matrix

$A = \begin{pmatrix} -2 & 2 & 8 \\ 2 & 7 & 10 \\ 8 & 10 & 4 \end{pmatrix}$ properties $\boxed{A^T = A}$

... a ~~shitton~~ of work

(17) $A = \begin{pmatrix} 5+i & -2i \\ 2 & 4+2i \end{pmatrix}$ find eigen {val, vec}

$0 = \det(A - \lambda) = (\lambda - 6)(\lambda - (3+3i))$

$\lambda = 6$ $\qquad$ $\lambda = 3+3i$

$\begin{pmatrix} 1-i \\ 1 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} \frac{1-i}{2} \\ 1 \end{pmatrix}$

(18) $A = \begin{pmatrix} 7/5 & 1/5 \\ -1 & 1/2 \end{pmatrix}$

find $\lim_{n \to \infty} A^n$

- find eigenvalues → eigenvectors

$$\underbrace{Q = \begin{pmatrix} -1 & -2 \\ 2 & 5 \end{pmatrix} \quad Q^{-1}AQ = \begin{pmatrix} 1 & 0 \\ 0 & 9/10 \end{pmatrix}}$$

$(Q^{-1}AQ)^2 = Q^{-1}A\cancel{QQ^{-1}}AQ$
$= Q^{-1}A^2Q$

$\begin{pmatrix} (1)^n & 0 \\ 0 & (9/10)^n \end{pmatrix} =$

→ need to diagonallze first so that only diagonal exists

(matrix)^n only acts on diagonal values, thus need for diagability

Now for the limit

$$\lim_{n \to \infty} A^n \underbrace{Q^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} Q}$$

$$\boxed{\begin{pmatrix} 5 & 2 \\ -10 & -4 \end{pmatrix}}$$

Cool fact (no exam)

If A is an nxn diagonallzeable matrix,

$\cos^2 A + \sin^2 A = I$ Yeet!