MTH431 notes from 4/18/2019

Determinants of ≥3x3 Matricies

- Note $A = (a) \to A^{-1} = \left(\frac{1}{a}\right)$ if $a \neq 0$
- Note $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \to A^{-1} = \left(\frac{1}{ad-bc}\right)\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ if $ad-bc \neq 0$.
- Define $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

$$\mathrm{Det}(A) = a_{11} \cdot \det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12} \cdot \det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} + a_{13} \cdot \det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

- If $A = (a_{ij})_{n \times m}$, then, the "ij-th minor", M_{ij} is the $(n-1) \times (n-1)$ matrix formed by deleting the row and column containing a_{ij}
- The Cofactor $A_{ij} = -1^{(i+j)} \det(M_{ij})$

↳ $\mathrm{Det}(A) = a_{ij} \cdot A_{ij} \ldots$

↳ $\mathrm{Det}(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.

- for an $n \times n \mid n \geq 2$:

$$\mathrm{Det}(A) = a_{11}A_{11} + a_{12}A_{12} \cdots a_{1n}A_{1n}$$

- One can take the determinant with regard to any row/col
- When choosing where to take the determinant, seek the place that will ~~generate~~ include the most 0's
- $\mathrm{Det}(A^T) = \mathrm{Det}(A)$
- Triangular Matricies

↳ $\begin{pmatrix} a_{11} & & * \\ & a_{22} & \\ 0 & & \ddots & a_{nn} \end{pmatrix}$ = Upper triangular

↳ $\begin{pmatrix} a_{11} & & 0 \\ & a_{22} & \\ * & & \ddots & a_{nn} \end{pmatrix}$ = lower triangular

$$\mathrm{Det}(\text{triangular}) = a_{11} \cdot a_{22} \cdot \cdots \cdot a_{nn}$$

- If A has a row or column of 0's, the determinant is 0
- If A has two identical rows, or cols then $\mathrm{Det}(A) = 0$
- If you switch two rows in a matrix:

$A \xrightarrow{(i)(j)} \tilde{A}$ → $\det(\tilde{A}) = -\det(A)$

- Constant Multiple

$A \xrightarrow{\alpha (i)} \tilde{A}$ → $\det \tilde{A} = \alpha \det(A)$

- Replace:

$A \xrightarrow{\text{replace } (i) \text{ by } (j) + \alpha (i)} \tilde{A}$, then $\det(A) = \det(\tilde{A})$