MTH 341 Homework 1

Lyell Read

DUE 4/11/2019

(Pg, Prb): (46, 1.2.32), (46, 1.2.33), (46, 1.2.34) (91, 2.1.9)
(92, 2.1.14) (95, 2.1.35)

1.2.32

Find solution to

$$\begin{cases} 7x + 14y + 15z = 22 \\ 2x + 4y + 3z = 5 \\ 3x + 6y + 10z = 13 \end{cases}$$

Augmented matrix →

$$\begin{pmatrix} 7 & 14 & 15 & 22 \\ 2 & 4 & 3 & 5 \\ 3 & 6 & 10 & 13 \end{pmatrix} \xrightarrow{③ \cdot 2 \over 3} \begin{pmatrix} 7 & 14 & 15 & 22 \\ 2 & 4 & 3 = \frac{9}{3} & 5 = \frac{15}{3} \\ 2 & 4 & \frac{20}{3} & \frac{26}{3} \end{pmatrix}$$

$$\xrightarrow{③ - ②} \begin{pmatrix} 7 & 14 & 15 & 22 \\ 2 & 4 & 3 & 5 \\ 0 & 0 & \frac{11}{3} & \frac{11}{3} \end{pmatrix} \xrightarrow{③ \cdot \frac{3}{11}} \begin{pmatrix} 7 & 14 & 15 & 22 \\ 2 & 4 & 3 & 5 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{② - 3 \cdot ③} \begin{pmatrix} 7 & 14 & 15 & 22 \\ 2 & 4 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{② / 2} \begin{pmatrix} 7 & 14 & 15 & 22 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{① - 7 \cdot ②} \begin{matrix} x & y & z & = \\ \end{matrix} \begin{pmatrix} 0 & 0 & 15 & 15 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} \rightarrow z = 1 \\ \rightarrow x + 2y = 1 \\ \rightarrow z = 1 \end{matrix}$$

$z = 1$

$x = 1 - 2y$ → $\boxed{[z = 1,\ x = 1 - 2t,\ y = t]}$

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1.2.33

Find Solution to

$$\begin{cases} 3x - y + 4z = 6 \\ y + 8z = 0 \\ -2x + y = -4 \end{cases}$$

augmented matrix →

$$\begin{matrix} x & y & z & = \\ \end{matrix} \begin{pmatrix} 3 & -1 & 4 & 6 \\ 0 & 1 & 8 & 0 \\ -2 & 1 & 0 & -4 \end{pmatrix} \xrightarrow{① + ③} \begin{pmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & 8 & 0 \\ -2 & 1 & 0 & -4 \end{pmatrix}$$

$$\xrightarrow{③ + 2①} \begin{pmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & 8 & 0 \\ 0 & 1 & 8 & 0 \end{pmatrix} \xrightarrow{③ - ②} \begin{pmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & -8 & -4 \end{pmatrix} \xrightarrow{③ \cdot -\frac{1}{2}} \begin{matrix} x & y & z & = \\ \end{matrix} \begin{pmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 3 & 2 \end{pmatrix}$$

$x + 4z = 2 \rightarrow x = 2 - 4z$

$y + 8z = 0 \rightarrow y = -8z$

$z = t$ → $\boxed{[x = 2 - 4t \quad y = -8t \quad z = t]}$

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1.2.34

Find the solution to:

$$\begin{cases} 9x - 2y + 4z = -17 \\ 13x - 3y + 6z = -25 \\ -2x + 0y - 1z = 3 \end{cases} \xrightarrow{\text{aug. mtrx}} \begin{pmatrix} 9 & -2 & 4 & -17 \\ 13 & -3 & 6 & -25 \\ -2 & 0 & -1 & 3 \end{pmatrix} \xrightarrow{③/-2} \begin{pmatrix} 9 & -2 & 4 & -17 \\ 13 & -3 & 6 & -25 \\ 1 & 0 & \frac{1}{2} & -\frac{3}{2} \end{pmatrix}$$

$$\xrightarrow{\text{swp } ③①} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 13 & -3 & 6 & -25 \\ 9 & -2 & 4 & -17 \end{pmatrix} \xrightarrow{②-13\cdot①} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & -3 & -\frac{1}{2} & -\frac{63}{2} \\ 9 & -2 & 4 & -17 \end{pmatrix} \xrightarrow{②/-3} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & \frac{1}{6} & \frac{63}{6} \\ 9 & -2 & 4 & -17 \end{pmatrix}$$

$$\xrightarrow{③-9\cdot①} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & \frac{1}{6} & \frac{63}{6} \\ 0 & -2 & -\frac{1}{2} & -\frac{7}{2} \end{pmatrix} \xrightarrow{③+2\cdot②} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & \frac{1}{6} & \frac{63}{6} = \frac{21}{2} \\ 0 & 0 & -\frac{1}{6} & \frac{35}{2} \end{pmatrix} \xrightarrow{②+③} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & 0 & 28 \\ 0 & 0 & -\frac{1}{6} & \frac{35}{2} \end{pmatrix}$$

$$\xrightarrow{③\cdot -3} \begin{pmatrix} 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & 0 & 28 \\ 0 & 0 & -\frac{1}{2} & \frac{105}{2} \end{pmatrix} \xrightarrow[③\cdot -2]{①+③} \begin{pmatrix} 1 & 0 & 0 & \frac{102}{2} \\ 0 & 1 & 0 & 28 \\ 0 & 0 & 1 & -105 \end{pmatrix} \rightarrow \boxed{\begin{array}{l} x = 51 \\ y = 28 \\ z = -105 \end{array}}$$

(not book answers, but from the check I did, they work...)

2.1.9

a] $-3A$ $\quad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 7 \end{bmatrix}$ $\quad \boxed{-3A = \begin{bmatrix} -3 & -6 & -9 \\ -6 & -3 & -21 \end{bmatrix}}$

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b] $3B - A$ $\quad A = \uparrow \quad B = \begin{bmatrix} 3 & -1 & 2 \\ -3 & 2 & 1 \end{bmatrix}$ $\quad 3B = \begin{bmatrix} 9 & -3 & 6 \\ -9 & 6 & 3 \end{bmatrix}$ $\quad -A = \boxed{\begin{bmatrix} 8 & -5 & 3 \\ -11 & 5 & -4 \end{bmatrix}}$

c] AC $\quad \boxed{N/P}$: $\text{cols}(A) \neq \text{Rows}(B)$

d] CB $\quad C = \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}}$ $\quad B = \underset{2\times 3}{\begin{bmatrix} 3 & -1 & 2 \\ -3 & 2 & 1 \end{bmatrix}}$ $\quad CB = \begin{bmatrix} 3-6_{11} & 4-1_{12} & 2+2_{13} \\ 9-3_{21} & 2-3_{22} & 6+1_{23} \end{bmatrix} = \boxed{\begin{bmatrix} -3 & 3 & 4 \\ -6 & -1 & 7 \end{bmatrix}}$

e] $A_{2\times 3}$ $\quad E_{2\times 1}$ $\quad \boxed{N/P}$ $\quad \text{cols}(A) \neq \text{Rows}(E)$

f] $E_{2\times 1}$ $\quad A_{2\times 3}$ $\quad \boxed{N/P}$ $\quad \text{cols}(E) \neq \text{Rows}(A)$

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2.1.14

Find k such that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & k \end{bmatrix}$ $AB = BA$

$AB = \begin{bmatrix} 7 & 2+2k \\ 15 & 6+4k \end{bmatrix}$ $BA = \begin{bmatrix} 7 & 10 \\ 3+3k & 6+4k \end{bmatrix}$ (11, 12, 21, 22)

$2+2k=10 \quad 2k=8 \quad k=4$

↳ $15 = 3+3k \quad 12 = 3k$ $\boxed{k=4}$

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2.1.35

Find A^{-1} if possible

$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ $A^{-1}_{test} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$ $\Longrightarrow$ TEST with $AB = I = BA$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= B$ ↳ $B = A^{-1}_{test}$

$A \quad A^{-1}_{test} = B$

$\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & \emptyset \\ \emptyset & 7 \end{bmatrix} \rightarrow c(AB) \mid c = \frac{1}{7} = AcB$

⇓

DIV BY 7 → $\boxed{A^{-1} = \begin{bmatrix} \frac{3}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}}$

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