MTH 341 NOTES 4/16/2019

Finding Inverses

- conduct elementary row operations to bring $A \longrightarrow I$.
- Record all those ERO's as $E_1 \cdots E_n$

Ex $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$\xrightarrow{\frac{1}{2}①} \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & 2 \end{pmatrix}$ $E_1 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$

...

$\xrightarrow{\cdots} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- Then, $E_n \cdots E_4 E_3 E_2 E_1 A = I$

  $\hookrightarrow E_n \cdots E_4 E_3 E_2 E_1 = A^{-1}$

Inverse Finding Algorithm

- assume $E_k \cdots E_1 A = I$
- consider $(A|I)_{n \times 2n}$
- now $E_1(A|I) = (E_1A \ E_1I)$

  $= (E_1A | E_1)$

$E_k \cdots E_1 (A|I)$

$= (E_k \cdots E_1 A | E_k \cdots E_1 I)$

$= (I, A^{-1})$

Reminder about Inverses

- only invertible if $ad-bc \neq 0$
- inverse typically is $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$