Inverses

- Must be square
- $\det = 0 \Rightarrow$ No inverse
- $A^{-1}_{short} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
- $A^{-1}_{long} = (A|I) \to (I|A^{-1})$

Eigen Val, Vec

- Solve $\det(A-\lambda I)=0$
- for λ_n do:

$(A-\lambda_n I)=0$

$Q=(v_1 \cdots v_n)$

$Q^{-1}AQ = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$

- only keep cols from original where leading values exist in A

Dimensions, Basis

- Do Linear Dependent Elimination
- Basis is # of elements.

Determinant

- M_{ij} = matrix when row I and col j are removed
- $A_{ij} = -1^{(i+j)} \det(M_{ij})$ = cofactor
- $\operatorname{Det}(A) = \det(A^t)$
- $\operatorname{Det}(A)$ | A is triangular, determinant is just the product of the diagonal
- Row Operations
  - ↳ $\updownarrow = -1 * \det(A)$
  - ↳ $\cdot\alpha = \alpha \det(A)$
  - ↳ replace = No change.

Null, Row, Col

- Null = span(vectors from solve(A)) (R.E.F.)
- Nullity = dim(Null(A))
- Row space = span(Rows)
- Col space = span(cols)

reduce lin dep's

- Nullity = free variables
- Col dim = row dim = leading vars

Linear Transformations

- $T: \mathbb{R}^a \to \mathbb{R}^B$ $\boxed{1}$
- Check for Linearity:
  - ↳ $f(x_1 + x_2) = f(x_1) + f(x_2)$ | all $x_1, x_2 \in \mathbb{R}$
  - ↳ $f(cx) = c \cdot f(x)$ $\boxed{2}$ | $c \in \mathbb{R}$

Hermition
- $\bar{A}^t = A$

Similar
$B = Q^{-1}AQ$

Symmetric
$A^t = A$

Subspaces

- W is a subspace of $\mathbb{R}^n$ if
  - → $W \neq \emptyset$ (not empty)
  - → for $x, y \in W$ $x+y \in W$
  - → for $a \in \mathbb{C}$, $x \in W$, $ax \in W$

Expon: ↳ $A = \begin{pmatrix} \lambda & \\ & \lambda \end{pmatrix}$ ↓ $Q^{-1}\begin{pmatrix} e^{\lambda} & \\ & e \end{pmatrix}Q = e^A$.

Kernel, Range

- kernel = Null space (A)
- Range = Col space (A)
- Domain = first $\mathbb{R}^n$ in EQ

dim(Ker) + dim(Rng) = dim(dom)

Linear Dependence

- dependent if $\det(v_1 \cdots v_n) = 0$
- remove dependents: $\boxed{10}$ $\boxed{7}$

$A = (v_1\ v_2\ v_3 \dots) \sim$ Red. Echelon form $\boxed{8}$ $\boxed{9}$

invertible if $\det = 0$

$\lambda \longrightarrow 1/\lambda$ $\boxed{6}$

Coordinates

$\left(\begin{matrix}\text{Provided range}\end{matrix} \middle| \begin{matrix} x_1 \\ \vdots \\ x_n \end{matrix}\right) \sim \left(\begin{matrix}\text{echelon form}\end{matrix} \middle| \begin{matrix} c_1 \\ \vdots \\ c_n \end{matrix}\right)$ $[x]_v = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$

Matrix Representation

$\boxed{3}$ $\boxed{4}$ $\boxed{5}$

- for e_n in basis of $\mathbb{R}^n$ first in eq:

Stand. Represent. $= (T_{e_1} | T_{e_2} \cdots)$

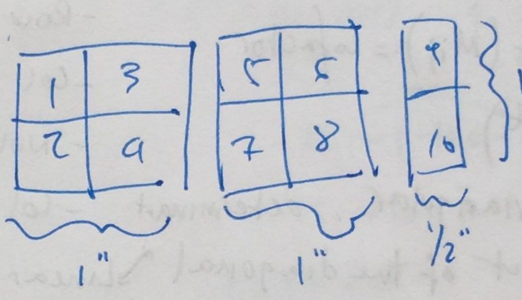
A note about Notecard Dimensions

Allowed: $\underbrace{2}_{\text{Sides}} \times \underbrace{4 \times 6}_{\text{NC}} \times \underbrace{2}_{\text{Qty}} = 96$

8.5 x 11 page $\underbrace{\text{(one side)}}_{\text{other side}}$ = 93.5

$96 - 93.5 = 2.5''^2$

eliminated in squares on opposite side



$= 2.5''^2$

Lyell Read 2019 Final Notecard