# MTH341 NOTES FROM 5/28/2019

## Recall

- for eigenvalues $\lambda_1, \lambda_2 \dots \lambda_n$ of matrix $A_{n\times n}$, then

$$\det(A) = \lambda_1 \cdot \lambda_2 \cdot \dots \cdot \lambda_n$$
$$tr(A) = \lambda_1 + \lambda_2 + \dots + \lambda_n$$

## Invertability

- A is invertible if $\det(A) = 0$.
  - ↳ if and only if at least one eigenvalue is equal to 0

- other wise put:

  if A is invertible:

$$\Leftrightarrow A\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

has nontrivial solutions

$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ is eigenvector of A corresp. to eigenvalue of 0

## Similarity

- If A and B are similar

$$\left( B = Q^{-1} A Q \mid Q \text{ exists, fulfills} \right)$$

then A, B have same eigenvalues

## Transpose-ality

- A and $A^T$ have the same eigenval's

## Invertability (cont'd)

- If A has e.v's $\lambda_1 \dots \lambda_n$ then $A^{-1}$ has e.v.'s $\frac{1}{\lambda_1} \dots \frac{1}{\lambda_n}$.

## Even More Propositions

- If A is diagonalizeable and has eigen values $\lambda_1 \dots \lambda_n$ then $A^k$ has E.V.'s : $\lambda_1^k \dots \lambda_n^k$

## Powers

- $A^k = (QDQ^{-1})^k$

$$= QD\cancel{Q^{-1}}\cancel{Q}D\cancel{Q^{-1}} \dots \cancel{Q}DQ^{-1}$$

$$= QD^kQ^{-1}$$

- $e^A$ : let $g(x) = \sum_{i=0}^{\infty} c_i x^i$

$$g(A) = \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

if $A = Q\begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{pmatrix}Q^{-1}$

↳ $e^A = Q\begin{pmatrix} e^{\lambda_1} & & \\ & e^{\lambda_2} & \\ & & \ddots \end{pmatrix}Q^{-1}$

Yeet!

- Example solve

$A = \begin{pmatrix} 2 & -3 \\ 2 & -5 \end{pmatrix}$ E.Va = 1, −4

E.Ve = $\begin{pmatrix} 3 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$Q = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

$$Q^{-1}AQ = \begin{pmatrix} 1 & 0 \\ 0 & -4 \end{pmatrix}$$

$$A = Q\begin{pmatrix} 1 & 0 \\ 0 & -4 \end{pmatrix}Q^{-1}$$

$$e^A = Q\begin{pmatrix} e & 0 \\ 0 & e^{-4} \end{pmatrix}Q^{-1}$$

$$\rightsquigarrow \frac{1}{5}\begin{pmatrix} 6e - e^{-4} & -3e + 3e^{-4} \\ 2e - 2e^{-4} & -e + 6e^{-4} \end{pmatrix}$$