MTH341 NOTES FROM 4/25/2019

CH 10 subspaces

- $\mathbb{R}^2 = \{(x,y) \mid x,y \in \mathbb{R}\}$
  $= \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid x,y \in \mathbb{R} \right\}$
- $\mathbb{R}^n = \left\{ x_1 \cdots x_n \mid \begin{matrix} x_i \in \mathbb{R} \\ i = 1 \ldots n \end{matrix} \right\}$

Linear Combination

- for $\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ L.C. $= \begin{pmatrix} cv_1 \\ \vdots \\ cv_n \end{pmatrix}$

Subspaces

- A subset V of $\mathbb{R}^n$ is a subspace if whenever $v_1, v_2 \in V$, then $cv_1 + cv_2$ is in V.

- Ex with zero vector

Claim: $\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \in \mathbb{R}^n$

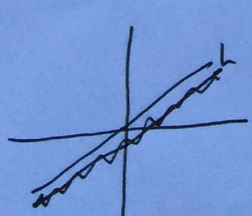
set

↳ $\{0\}$ ~~$\in \mathbb{R}^2$~~ is a subspace of $\mathbb{R}^n$

$V_1 = 0 \quad V_2 = 0$

$C_1 V_1 + C_2 V_2 = 0 \in \{0\}$ ✓

Ex: a line through $(0,0)$ in $\mathbb{R}^2$



for L: $ax + by = 0$

Claim: L is a subspace of $\mathbb{R}^2$

$L = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid ax + by = 0 \right\}$

NEXT COL

- Let $V_1 \in L \quad V_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$
  $V_2 \in L \quad V_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

linear combination

$C_1 V_1 + C_2 V_2 = \begin{pmatrix} C_1 x_1 + C_2 x_2 \\ C_1 y_1 + C_2 y_2 \end{pmatrix}$

Is this in L?

$= a(C_1 x_1 + C_2 x_2) + b(C_1 y_1 + C_2 y_2)$

$= C_1 \underbrace{(ax_1 + by_1)}_{0} + C_2 \underbrace{(ax_2 + by_2)}_{0}$

$= 0 \in L$ ✓

Ex2: Let A be an $m \times n$ matrix consider the set of all solutions to the homogenious system $AX = 0$

$A_{m \times n} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \quad \{X \in \mathbb{R}^n \mid AX = 0\}$

consider 2 solutions X_1, X_2

Claim: subspace of $\mathbb{R}^n$

$A(C_1 x_1 + C_2 x_2) = 0$

Null Space

- $\mathrm{Null}(A) = \left\{ \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n \mid A \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = 0 \right\}$

Span

- $\mathrm{Span}(S) = \left\{ x : x = \sum_{i=1}^{n} c_i v_i \mid v_i \in S, n < \infty \right\}$

In Summary

NEXT PG

Ex $A = (1, -1, 3)$

$$\text{Null}(A) = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; x_1 - x_2 + 3x_3 = 0 \right\}$$

~~Null (A) = {~~

let: $x_2 = s$, $x_3 = t$ || they are free variables

$$\text{Null}(A) = \left\{ \begin{pmatrix} s-3t \\ s \\ t \end{pmatrix} \;\middle|\; s, t \in \mathbb{R} \right\} = \left\{ s \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \;\middle|\; s, t \in \mathbb{R} \right\}$$

$$= \text{SPAN}\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right)$$