MTH331 4/2/2019

CH 1

- a matrix is a rectangular array of numbers or variables
- Example: $A = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}_{2\times 2}$ A is a 2x2
- $A = (\mathring{A}_{ij}) = \begin{pmatrix}\mathring{A}_{11} & \mathring{A}_{12}\\ \mathring{A}_{21} & \mathring{A}_{22}\end{pmatrix}_{2\times 2}$ $\mathring{A}_{ij} \simeq i^{th}$ row, j^{th} col

NOTE: Å is used in place of "a" here.

- a real number is a 1x1 matrix, i.e. $(8)_{1\times 1}$
- A vector is a 3x1 matrix: $v = \underbrace{\begin{pmatrix}0\\ 1\\ 2\end{pmatrix}_{3\times 1}}_{\text{column vector}}$

→ can also be row vector ⤵

$v = (0\ 1\ 2)_{1\times 3}$

- Matrix Addition

→ If two matricies A and B have same size then their sum is A+B.

→ Then: $(A+B)_{ij} = (A)_{ij} + (B)_{ij}$

→ Also: if $A = (a_{ij})_{m\times n}$

$B = (b_{ij})_{m\times n}$

Then: $A+B = (a_{ij}+b_{ij})_{m\times n}$

→ EX: $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} + \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}2 & 2\\ 3 & 5\end{pmatrix}$

Matrix Difference

- $A-B = (a_{ij} - b_{ij})_{m\times n}$
- Ex: $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} - \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}0 & 2\\ 3 & 3\end{pmatrix}$

Scalar Multiplication

- multiplying the whole matrix as a constant.
- Ex: $3A \mid A = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$

$3A = \begin{pmatrix}3 & 6\\ 9 & 12\end{pmatrix}$

Zero Matrix:

- simple: $(0)_{m\times n}$. this represents a matrix whose entries are zero.

- Also note: $A_{m\times n} + \overset{\text{Zero Matrix}}{O_{m\times n}} = A_{m\times n}$

Equality of Matricies

- A = B if:

→ ① A, B have same size

→ ② $a_{ij} = b_{ij}$ for all i, j.

Multiplication (Product)

- Define A_{mk} and B_{kn} (rows of B = cols of A)
- Product $(AB)_{ij} = a_{i1}b_{1j} + a_{i2}+b_{2j} \cdots$
- EX:

$A = \begin{pmatrix} & \cdots & \\ a_{i1} & a_{i2} \cdots & a_{ik} \\ & \cdots & \end{pmatrix} \quad B = \begin{pmatrix} & B_{1j} & \\ \cdots & B_{2j} & \cdots \\ & \vdots & \\ & B_{kj} & \end{pmatrix}$

- EX 2:

$A = \begin{pmatrix}1 & 2 & 3\\ -1 & 0 & 4\end{pmatrix}_{2\times 3} \quad B = \begin{pmatrix}-1 & 0\\ 4 & 2\\ 1 & 3\end{pmatrix}_{3\times 2}$ sizes

OK!

$AB = \begin{pmatrix}\text{first row tms first col} & \cdots \\ \vdots & \text{last row last col}\end{pmatrix} = \begin{pmatrix}10 & 13\\ 5 & 12\end{pmatrix}$

- matrix product is <u>not</u> <u>commutative</u>

Ex: $A = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} \quad B = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$

$AB = \begin{pmatrix}(1\times1)+(2\cdot0) & (1\cdot0)+(2\cdot0)\\ (3\cdot1)+(4\cdot0) & (3\cdot0)+(4\cdot0)\end{pmatrix} = \begin{pmatrix}1 & 0\\ 3 & 0\end{pmatrix}$

$BA = \begin{pmatrix}(1\cdot1)+(0\cdot3) & (1\cdot2)+(0\cdot4)\\ (0\cdot1)+(0\cdot3) & (0\cdot2)+(0\cdot4)\end{pmatrix} = \begin{pmatrix}1 & 2\\ 0 & 0\end{pmatrix}$

$AB \neq BA$ ∴ not commutative

- EX: $A = \begin{pmatrix}2\\ 3\end{pmatrix} \quad B = (1\ 2)$

$AB = \begin{pmatrix}2 & 4\\ 3 & 6\end{pmatrix}_{2\times 2} \quad BA = \begin{pmatrix}8\end{pmatrix}_{1\times 1}$

Rotation Matrix

- $\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$

- usage: rotating point $(x,y) \to (x',y')$

$$\begin{pmatrix}x' \\ y'\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix}$$

Multiplication

- note: $ABC \Rightarrow A(BC) = (AB)C$

Double Rotation Matrix

- $\begin{pmatrix}\cos\phi\cos\theta - \sin\phi\sin\theta & -\cos\phi\sin\theta - \sin\phi\cos\theta \\ \sin\phi\cos\theta + \cos\phi\sin\theta & -\sin\phi\sin\theta + \cos\phi\cos\theta\end{pmatrix}$

$$= \begin{pmatrix}\cos(\phi+\theta) & -\sin(\phi+\theta) \\ \sin(\phi+\theta) & \cos(\phi+\theta)\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix}$$

Properties of Matricies

① If AB and AC are defined, Then $A(B+C) = AB + AC$

② If AB is defined, and c is a scalar, then $A(cB) = (cA)B = c(AB)$

Transposition of Matricies

- transpose of A is defined as A^t
- first row of A becomes first col of A^t

... ...

- Ex: $A = \begin{pmatrix}1 & 2 \\ 3 & 4 \\ 5 & 6\end{pmatrix}_{3\times 2}$ $A^t \begin{pmatrix}1 & 3 & 5 \\ 2 & 4 & 6\end{pmatrix}_{2\times 3}$

- If $A_{m\times n}$ then $A^t_{n\times m}$ (sizes swap)

- If $A = (a_{ij})$ then $A^t = (a_{ji})$
- If AB is defined:
  - $\rightarrow (AB)^t = \cancel{A^t} B^t A^t$

NOTE: $(AB)^t \neq A^t B^t$

Square Matrix

- is a matrix with the same number of rows as columns
- $\rightsquigarrow A_{n\times n}$

Identity Matrix

- the identity Matrix I_n has ones on the diagonal and 0's elsewhere
- $I_3 = \begin{pmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{pmatrix}_{3\times 3}$
- $I_n = (a_{ij})$ where $a_{ii} = 1$ and a_{ij} (where $i \neq j$) $= 0$
- Ex: AI is A, IA is A

$$\begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}\begin{pmatrix}a & b \\ c & d\end{pmatrix} = \begin{pmatrix}a & b \\ c & d\end{pmatrix}$$

- You can sort of treat I as 1

Inverses

- If $BA = I = AB$ then B is inverse of A and A is inverse of B
  - ↳ $B = A^{-1}$, $A = B^{-1}$

Also:

- $AB = I \rightarrow c(AB) = cI \rightarrow c(AB) = c$