Homework 3 / Lyell Read

4.10.11

Dependent? $\begin{pmatrix}1\\3\\-1\\1\end{pmatrix}\begin{pmatrix}1\\4\\-1\\1\end{pmatrix}\begin{pmatrix}1\\4\\0\\1\end{pmatrix}\begin{pmatrix}1\\10\\2\\1\end{pmatrix} \longrightarrow \begin{pmatrix}1&1&1&1\\3&4&4&10\\-1&-1&0&2\\1&1&1&1\end{pmatrix}$ Det = 0 because two identical lines.

↳ $\boxed{\text{Linearly Dependent.}}$

$\begin{pmatrix}1\\10\\2\\1\end{pmatrix} = -6 + 4 + 3$. Redundant.

↳ $\begin{pmatrix}1&1&1\\3&4&4\\-1&-1&0\\1&1&1\end{pmatrix}$ which is NOT! lin dep b/c:

$\begin{pmatrix}1&1&1\\3&4&4\\-1&-1&0\\1&1&1\end{pmatrix}$ ③ + ④ $\rightarrow \begin{pmatrix}1&1&1\\3&4&4\\0&\boxed{0&-1}\\1&1&1\end{pmatrix}$ impossible.

$$\boxed{\text{Linear indep set with same span} : \left\{\begin{pmatrix}1\\3\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\4\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\4\\0\\1\end{pmatrix}\right\}}$$

4.10.32

find basis, dim for: $\left\{\begin{pmatrix}2\\1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\-1\\-1\end{pmatrix}, \begin{pmatrix}5\\2\\3\\3\end{pmatrix}, \begin{pmatrix}-1\\1\\-2\\-2\end{pmatrix}\right\}$

$\begin{pmatrix}2&-1&5&-1\\1&0&2&1\\1&-1&3&-2\\1&-1&3&-2\end{pmatrix} \longrightarrow \begin{pmatrix}1&0&2&1\\0&1&-1&3\end{pmatrix} \rightarrow \begin{pmatrix}2\\1\\1\\1\end{pmatrix} + 3\begin{pmatrix}-1\\0\\-1\\-1\end{pmatrix} = \begin{pmatrix}-1\\1\\-2\\-2\end{pmatrix}.$

$\begin{pmatrix}2&-1&5\\1&0&2\\1&-1&3\\1&-1&3\end{pmatrix}$ ⑤ - ③ $\longrightarrow \begin{pmatrix}1&0&2\\0&1&-1\\0&0&0\\0&0&0\end{pmatrix} \rightarrow 2\begin{pmatrix}2\\1\\1\\1\end{pmatrix} - \begin{pmatrix}-1\\0\\-1\\-1\end{pmatrix} = \begin{pmatrix}5\\2\\3\\3\end{pmatrix}$

↳ Basis = $\left\{\begin{pmatrix}2\\1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\-1\\-1\end{pmatrix}\right\}$, dim = 2.

4.10.33

find dim, basis for $H = \text{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 5 \\ -5 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \\ -2 \end{pmatrix} \right\}$

$$\begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -2 & 5 & 2 \\ -1 & 2 & -5 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -2 & 5 & 2 \\ \cancel{1} & \cancel{-2} & \cancel{5} & \cancel{2} \end{pmatrix}$$

↳ means lin dep, but in solving b/c det = 0.

$$\begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -2 & 5 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

↳ impossible.

Therefore, Basis: $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 5 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ -2 \end{pmatrix} \right\}$

Dim = 4

10.4.47

is $S = \left\{ \begin{matrix} 4u + v - 5w \\ 12u + 6v - 6w \\ 4u + 4v + 4w \end{matrix} \;\middle|\; u, v, w \in \mathbb{R} \right\}$ subspace of $\mathbb{R}^3$

$= \text{span}\left(\begin{pmatrix} 4 \\ 21 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}, \begin{pmatrix} -5 \\ -6 \\ 4 \end{pmatrix} \right)$

↳ $\det \begin{pmatrix} 4 & 1 & -5 \\ 21 & 6 & -6 \\ 4 & 4 & 4 \end{pmatrix} \neq 0 \rightarrow$ linearly independent. (calc on review).

3 linearly independent vectors <u>do form a basis</u> for $\mathbb{R}^3$,

and 1] $0 \in$ Span S

2] closed under addition

3] closed under mult

$\boxed{\begin{matrix} \text{Dim} = 3 \\ \text{Yes} \end{matrix}}$

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$$\begin{pmatrix} 1 & 0 & 3 \\ 3 & 1 & 10 \\ 1 & 1 & 4 \\ 1 & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 4 \\ 1 & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & \cancel{0} & \cancel{0} \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{array}{ccc} \Delta & \Delta & \times \end{array}$$

$\boxed{\text{Rank} = 2}$

2 leading variables

1 free variable.

Basis for Rowspace:

$$\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Basis for Col space: ~~$\begin{pmatrix} 3 \\ 10 \\ 4 \\ 2 \end{pmatrix}$~~

~~(free var, taken from initial)~~

$$\begin{pmatrix} 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$

4.10.68

$$\begin{pmatrix} 1 & 0 & 3 & 0 \\ 3 & 1 & 10 & 0 \\ -1 & 1 & -2 & 1 \\ 1 & -1 & 2 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 3 & 1 & 10 & 0 \\ 1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{array}{cccc} L & L & F & F_{\text{or } L} \end{array}$$

$\boxed{\text{Rank} = \text{leading} = 3 \text{ if } -1 \text{e last col, else} = 2}$

Col Basis: $\begin{pmatrix} 1 \\ 3 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix}$ row basis $\begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

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