MTH341 NOTES 5/30/2019

- for an $n \times n$ matrix, it is diagonalizable iff you can find n linearly independent eigenvalues
  - ↳ for $\{\lambda_1 \cdots \lambda_k\}$ which are distinct eigenvalues of A, $x_1 \ldots x_k$ are the corresponding vectors. $x_1 \cdots x_k$ are linearly independent.
  - ↳ for $A_{n \times n}$, n distinct E.V's → A is diag.
- for $A_{n\times n}$ complex matrix's characteristic equation $\det(A - \lambda I) = 0$

$$= (\lambda - \lambda_1)^{a_1} (\lambda - \lambda_2)^{a_2} \cdots (\lambda - \lambda_n)^{a_n}$$

| algebraic multiplicity $\equiv a_i$
$\operatorname{algmult}_A(\lambda_i) = a_i$

- to find Eigen Vectors, solve

$$(A - \lambda I)\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

| geometric multiplicity is the dim of null space of $A - \lambda I$ == # E.vec (lin indep) found

- for any $A_{n\times n}$ complex matrix, for each eigenvalue of A:

$$\operatorname{alg\,mult}_A(\lambda) \geq \operatorname{geog\,mult}_A(\lambda)$$

$$1 \longrightarrow 1$$

$$2 \longrightarrow (1|2)$$

- $A_{n\times n}$ only diagonalizable iff $\operatorname{alg\,mult}_A(\lambda) = \operatorname{geog\,mult}_A(\lambda)$!

Ex: $A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix}$

1] Eigen Values

$$0 = \det(A - \lambda I)$$

$$= \det\begin{pmatrix} 3-\lambda & -1 & -2 \\ 2 & -\lambda & -2 \\ 2 & -1 & -1-\lambda \end{pmatrix}$$

col + row ops

$$\begin{pmatrix} 1-\lambda & -1 & -2 \\ 0 & -\lambda & -2 \\ 0 & 0 & 1-\lambda \end{pmatrix}$$

— is upper triangular, so

$$\det() = -\lambda(1-\lambda)^2$$

$$\lambda_1 = 0 \quad \lambda_2, \lambda_3 = 1$$

alg mult $(0) = 1$ → geog mult $(0) = 1$

alg mult $(1) = 2$

$\lambda = 1 \quad (A - I)x = 0 = \begin{pmatrix} 2 & -1 & -2 \\ 2 & -1 & -2 \\ 2 & -1 & -2 \end{pmatrix}$

$$\downarrow$$

$$\begin{pmatrix} 2 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Null Null

geog mult $(1) = 2$
2 free variables

↳ A is diagonalizable

- All E-values of a hermitian matrix are real!
  - $A^* = \bar{A}^t = A$
  - ↳ Also is diag.