MTH 341 QUIZ Review 4/23/2019

- Determinant of $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} \cdot A_{11} + \dots + a_{13} \cdot A_{13}$ || A_{ij} = cofactor (see below)

- Cofactor $A_{ij} = -1^{(i+j)} \cdot \det(M_{ij})$ || M_{ij} is the minor (see below)

- Minor M_{ij} = Matrix formed by deleting row i, col j from $A_{m \times n}$. (original)

- Fancy Rules related to Determinants
  - → If there exists a row or col of "0"'s, then $\det(A) = 0$
  - → If there exist two same rows or cols, then $\det(A) = 0$
  - → Switching two rows causes sign change on $\det(A)$
  - → If $A \xrightarrow{① \cdot \alpha} \tilde{A} \rightarrow \det(\tilde{A}) = \alpha \det(\tilde{A})$ (constant multiple travels thru)
  - → Replacing rows (elementary add w/ multiple) $\det(A) = \det(\tilde{A})$

- Inverses. Only if $ad - bc \neq 0$ (for 2x2).

- Finding Inv's with Gaussian Elimination

$$\underbrace{\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \xrightarrow{\frac{1}{2}①} \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & 2 \end{pmatrix}} \cdots \xrightarrow{f} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

|| If this solves to I, then there is an inverse.

$$E_n = \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}$$

|| transform identity as you would target matrix

- Inverse Algorithm: Consider $(A \mid I)_{m \times 2n} \rightarrow E_n (A \mid I)_{m \times 2n} = (E_n A \mid E_n I)$

$\rightarrow E_k \cdots E_1 (A \mid I) = (I \mid A^{-1})$. If solution ∴ (see above)