MTH 341 NOTES 5/23/2019

Complex Numbers

- $i = \sqrt{-1}$ $\quad i^2 = 1$
- $a + bi$
- $z = a+bi$ $\quad$ conjugate $\bar{z} = a - bi$
- $\overline{zw} = \bar{z} \cdot \bar{w}$
  $\overline{z+w} = \bar{z} + \bar{w}$

Solving Polynomials

- $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$
- if $b^2 - 4ac > 0$
  ↳ 2 real solutions
- if $b^2 - 4ac = 0$
  ↳ 1 solution (rep 2x)
- if $b^2 - 4ac < 0$
  ↳ 2 unreal solutions
  ↳ if $a, b, c \in \mathbb{R}$
  ↳ roots are conjugates

Fundamental theorem Of Algebra

- A polynomial equation of degree $n \geq 1$ has at least 1 complex (incl $\mathbb{R}$) root.
  ↳ it has n complex roots

Applications to Eigen*

- $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$$0 = \det\left(\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right)$$

$$-1 = (-\lambda+1)^2 = (1-\lambda)^2$$

$$1 - \lambda = \pm i$$

$$\hookrightarrow \begin{cases} \lambda_1 = 1+i \\ \lambda_2 = 1-i \end{cases}$$

$$(A - (i+1)I)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad \begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\longrightarrow \begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \longrightarrow \begin{cases} x_2 = s \\ x_1 = is \end{cases} \rightarrow s\begin{pmatrix} 1 \\ i \end{pmatrix}$$

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for $\lambda = 1 - i$: $(A - (1-i)I)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$$\hookrightarrow \begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix} \rightarrow \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix} \rightarrow \begin{cases} x_2 = s \\ x_1 = -is \end{cases} \rightarrow s\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\longrightarrow Q = \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix} \qquad Q^{-1}AQ = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$\begin{pmatrix} 1+i & 0 \\ 0 & 1-i \end{pmatrix} =$$

more Quick Notes

- ~~If you get~~ [illegible] || Just kidding, that's "coincidence"
- for 3x3's pull a factor out of a row is a viable strategy.

Hermition Matrix

- $\bar{A}^T = A$
  ↳ conjugate + transpose

Ex: $A = \begin{pmatrix} 2 & 1-i \\ 1+i & 1 \end{pmatrix}$

$$\bar{A} = \begin{pmatrix} 2 & 1+i \\ 1-i & 1 \end{pmatrix}$$

$$\bar{A}^T = \begin{pmatrix} 2 & 1-i \\ 1+i & 1 \end{pmatrix}$$

Symmetric Matrix

- $A^T = A$
- symmetric $\subseteq$ hermition

Hermition matrix Eigenvalues are all real

Symmetric Matrix is diagonizable

Some Interesting Phenomena

- consider $A_{n \times n} = (a_{ij})$
- the trace of A is the sum of all diagonal entries
  ↳ denoted tr A.

Phenomena, Cont'd

$$A = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$$

$$\operatorname{tr} A = 4+1 = 5$$
$$\det(A) = 4+2 = 6$$

$$\lambda_{1A} = 2 \qquad \lambda_1 + \lambda_2 = 5$$
$$\lambda_{2A} = 3 \qquad \lambda_1 \cdot \lambda_2 = 6$$

$$\lambda_1 + \lambda_2 = \operatorname{tr} A$$
$$\lambda_1 \cdot \lambda_2 = \det(A)$$

Homework Problem Solution 5.8.3

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \qquad B_1 = \left\{ \overset{v_1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}, \overset{v_2}{\begin{pmatrix} -1 \\ 1 \end{pmatrix}} \right\} \qquad B_2 = \left\{ \underset{w_1}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}, \underset{w_2}{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \right\}$$

$$T(v_1) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = w_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$T(v_2) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} = -w_1 + w_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$