Homework 4 · Lyell Read

279: 5.2.8(a), 5.2.9(a) | 290: 5.4.1 | 307: 5.6.6 | 314: 5.7.2

321: 5.8.3 |

5.2.8 (a)

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+2y+3z \\ 2y-3x+z \end{bmatrix}$$

Linearity Check: (1)

$$T\left(\begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}4\\5\\6\end{pmatrix}\right) = T\begin{pmatrix}1\\2\\3\end{pmatrix} + T\begin{pmatrix}4\\5\\6\end{pmatrix}$$

$$T\begin{pmatrix}1\\2\\3\end{pmatrix} \qquad T\begin{pmatrix}4\\5\\6\end{pmatrix}$$

$$\begin{bmatrix} 1+4+9 \\ 4-9+1 \end{bmatrix} \qquad \begin{bmatrix} 4+10+18 \\ 10-18+4 \end{bmatrix}$$

$$T\begin{pmatrix}5\\7\\9\end{pmatrix} \overset{\checkmark}{\underset{?}{=}} T\begin{pmatrix}1\\2\\3\end{pmatrix} + T\begin{pmatrix}4\\5\\6\end{pmatrix}$$

$$\begin{bmatrix} 5+14+27 \\ 14-27+5 \end{bmatrix} \qquad \begin{bmatrix} 5+14+27 \\ 14-27+5 \end{bmatrix}$$

Linearity Check (2)

$$T\left(2\begin{pmatrix}1\\1\\1\end{pmatrix}\right) = 2\left(T\begin{pmatrix}1\\1\\1\end{pmatrix}\right) : \longrightarrow$$

$$T\begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\begin{bmatrix} 2+4+6 \\ 4-6+2 \end{bmatrix} \overset{\checkmark}{\underset{?}{=}} 2\begin{bmatrix} 1+2+3 \\ 2-3+1 \end{bmatrix}$$

$\boxed{\text{↳ Linearity "confirmed"}}$

$$T(x) = Ax. \quad \boxed{A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & 1 \end{pmatrix}}$$

5.2.9 (a)

$$T\begin{pmatrix}x\\y\\z\end{pmatrix} = \cancel{\begin{bmatrix} x+2y+3z \\ 2y-3x+z \end{bmatrix}} = \begin{bmatrix} x+2y+3z+1 \\ 2y-3x+z \end{bmatrix}$$

Check linearity (1)

$$T\left(\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + \begin{pmatrix}x_4\\x_5\\x_6\end{pmatrix}\right) = T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + T\begin{pmatrix}x_4\\x_5\\x_6\end{pmatrix}$$

↳ probably is equal... don't need to solve

Check linearity (2)

$$T\left(c\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\right) = cT\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \longrightarrow$$

$$T\begin{pmatrix}cx_1\\cx_2\\cx_3\end{pmatrix} \qquad c \cdot T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$\begin{bmatrix} cx_1+2cx_2+3cx_3+1 \\ 2cx_2-3cx_1+cx_3 \end{bmatrix} \qquad c\begin{bmatrix} x_1+2x_2+3x_3+1 \\ 2x_2-3x_1+x_3 \end{bmatrix}$$

$$\neq \longrightarrow \begin{bmatrix} cx_1+2cx_2+3cx_3+c \\ 2cx_2-3cx_1+3x_3 \end{bmatrix}$$

$\boxed{\text{NOT LINEAR}}$

5.4.1:

find matrix for linear transformation that rotates every vector in $\mathbb{R}^2$ through $\theta = \pi/3$.

$$f\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x' \\ y' \end{pmatrix} : \mathbb{R}^2 \to \mathbb{R}^2.$$

$$= A\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

derived from I_2 rotated θ degrees...

$$\boxed{f\begin{pmatrix} x \\ y \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \cos\frac{\pi}{3}x & -\sin\pi/3\, y \\ \sin\frac{\pi}{3}x & +\cos\pi/3\, y \end{bmatrix}}$$

5.6.6:

$T: \mathbb{R}^4 \to \mathbb{R}^3$ DEFINED TO BE $Tx = \begin{bmatrix} 3 & 2 & 1 & 8 \\ 2 & 2 & -2 & 6 \\ 1 & 1 & -1 & 3 \end{bmatrix} \cdot \vec{x}$

im(T) = Range(T) = col. space of A

$$\to \begin{bmatrix} 3 & 2 & 1 & 8 \\ 1 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{1-3\cdot 2} \begin{bmatrix} 0 & -1 & 4 & -1 \\ 1 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

free free → EXISTS...

$$\boxed{im(T) = span\left\{ \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 8 \\ 6 \\ 3 \end{pmatrix} \right\}}$$

as
$$\begin{bmatrix} 3 & 2 & 1 & 8 \\ 2 & 2 & -2 & 6 \\ 1 & 1 & -1 & 3 \end{bmatrix} \begin{pmatrix} s \\ t \\ u \\ v \end{pmatrix} = s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + u\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} + v\begin{pmatrix} 8 \\ 6 \\ 3 \end{pmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 & 8 \\ 2 & 2 & -2 & 6 \\ 1 & 1 & -1 & 3 \end{bmatrix} \xrightarrow[3-2]{3\cdot 2} \begin{bmatrix} 3 & 2 & 1 & 8 \\ 2 & 2 & -2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[2\cdot 3]{2/2} \begin{bmatrix} 3 & 2 & 1 & 8 \\ 3 & 3 & -3 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to$$

$1-(2(-4)$

$$\begin{matrix} 2-1 \\ \to \end{matrix} \begin{bmatrix} 3 & 2 & 1 & 8 \\ 0 & 1 & -4 & 1 \end{bmatrix} \xrightarrow{1-2\cdot 2} \begin{bmatrix} 3 & 0 & 9 & 6 \\ 0 & 1 & -4 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 2 \\ & 1 & -4 & 1 \end{bmatrix} \to \Big\{$$

~~6x 3x 3 2x~~

NXT PG

$$\begin{cases} x_1 + 3x_3 + 2x_4 = 0 \\ x_2 - 4x_3 + x_4 = 0 \end{cases} \rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix} s, \begin{pmatrix} 0 \\ 1 \\ -4 \\ 1 \end{pmatrix} + \quad \ker(T)$$

$$\boxed{= \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -4 \\ 1 \end{pmatrix} \right\}}$$

5.7.2:

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$Im(T) = Range(T) \longrightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \rightarrow \Big\{$$

$$\boxed{R(T) = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \text{span}}$$

$$\ker(T) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \rightarrow \{x + y = 0 \quad x = -y$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0. \quad \boxed{\ker = \text{span}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

5.8.3

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \quad T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a+b \\ a-b \end{bmatrix}. \quad \text{consider} \begin{cases} B_1 = \{v_1, v_2\} = \left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right\} \\ B_2 = \left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\} \end{cases}$$

find $M_{B2,B1}$ of T w/rt B_1, B_2.

$$A \text{ for } T = \underset{2\times2}{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} \underset{2\times1}{\begin{pmatrix} a \\ b \end{pmatrix}}$$

See other side for corrected solution →

S = standard Basis

$$S \rightarrow B_1 = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$S \rightarrow B_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$B_1 \rightarrow B_2 = B_1 \rightarrow S \quad S \rightarrow B_2$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}}$$

Not sure about this, but I'll be asking.

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$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$B_1 = \left\{ \underset{v_1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} -1 \\ 1 \end{pmatrix}} \right\}$$

$$B_2 = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$

$$T(v_1) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}^{w}$$

$$T(v_2) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} = -w_1 + w_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\hookrightarrow \boxed{\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}}$$

(based on the solution you worked out after class 5/23)