MTH391 Homework 5 - 6/6/19

Lyell Read

7.2.1; 7.2.2, 7.2.4, 7.2.9; 7.2.11, 7.2.12.

7.2.1 $A = \begin{pmatrix} 5 & -18 & -32 \\ 0 & 5 & 4 \\ 2 & -5 & -11 \end{pmatrix}$ find $\begin{cases} \text{eigenvalues} \\ \text{eigenvectors} \end{cases}$, diagonalizeable?

<u>eigenvalues</u> : solve $0 = \det(A - I\lambda)$

$$0 = \det \begin{pmatrix} 5-\lambda & -18 & -32 \\ 0 & 5-\lambda & 4 \\ 2 & -5 & -11-\lambda \end{pmatrix}$$

$(5-\lambda) \det \begin{pmatrix} 5-\lambda & 4 \\ -5 & -11-\lambda \end{pmatrix} \cancel{0} + 2 \det \begin{pmatrix} -18 & -32 \\ 5-\lambda & 4 \end{pmatrix}$

Note: substituting x for λ for ease of solving :)

$(5-\lambda)(-11-\lambda) - (-20)$

$-55 - 5x + 11x + x^2 + 20$

$-35 - 5x + 11x + x^2$

$(5-x)(-35 + 6x + x^2)$

$-175 + 30x + 5x^2 + 35x - 6x^2 - x^3$

$-175 + 65x - x^2 - x^3$

oops

$-72 - (-160 + 32x)$

$-72 + 160 - 32x$

$2(88 - 32x)$

$176 - 64x$

$$-175 + 65x - x^2 - x^3 + 176 - 64x = -x^3 - x^2 + x + 1$$

$$-x^3 - x^2 + x + 1 = -1(x^3 + x^2) + (x+1) = -x^2(x+1) + x + 1$$

$$= (-x^2 + 1)(x+1) = -(x^2 - 1)(x+1) = -(x+1)(x-1)(x+1) = \begin{matrix} -(x-1) \\ (x+1)^2 \end{matrix}$$

↳ <u>Eigen Values</u> $\boxed{1, -1, -1.}$

Not sufficient vectors to form $\mathbb{R}^3$ basis (-1 dup 2). $\boxed{\text{Not diagonalizeable}}$

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7.2.2. $A = \begin{pmatrix} -13 & -28 & 28 \\ 4 & 9 & -8 \\ -4 & -8 & +9 \end{pmatrix}$ find $\left\{ \begin{array}{l} \text{eigenvalues} \\ \text{eigenvectors} \end{array} \right.$ and diagonalizeability

eigenvalues: solve det$(A - IX) = 0$ || note: $\lambda = x$ for simplicity of solve.

$$0 = \det \begin{pmatrix} -13-\lambda & -28 & 28 \\ 4 & 9-\lambda & -8 \\ -4 & -8 & 9-\lambda \end{pmatrix} \rightarrow \begin{pmatrix} -13-\lambda & -28 & 0 \\ 4 & 9-\lambda & -8-9-\lambda \\ -4 & -8 & 9-\lambda-8 \end{pmatrix} \rightarrow \begin{array}{l} -17-\lambda \\ 1-\lambda \end{array}$$

$$\rightarrow \begin{pmatrix} -13-\lambda & -28 & 0 \\ 4 & 9-\lambda & 9-\lambda-8 \\ -4 & -8 & 9-\lambda-8 \end{pmatrix} \xrightarrow{②-③} \begin{pmatrix} -13-\lambda & -28 & 0 \\ 8 & 9-\lambda+8 & 0 \\ -4 & -8 & 9-\lambda-8 \end{pmatrix}$$

$$0 = 1-x \det \begin{pmatrix} -13-x & -28 \\ 8 & 1-x \end{pmatrix} = (1-x) \cdot \left[(-13-x)(1-x) - (-224) \right]$$

$$0 = (1-x) \cdot \left[-13 + 13x - x + x^2 + 224 \right] = (1-x) \cdot \left[x^2 + 12x + 211 \right]$$

$$0 = x^2 + 12x + 211 - x^3 - 12x^2 - 211x = -x^3 - 11x^2 - 199x + 211$$

online solver $\rightarrow \underbrace{-x^3 + 5x^2 - 7x + 3}$ ← arithmetic error? :(

$-x^3 + 5x^2 - 7x + 3$ solved using graph: $x = 1$ m2, $x = 3$

↳ eigenvalues $\boxed{\lambda = 1, 1, 3}$

eigenvectors: solve $(A - \lambda_i I) = 0$

$\lambda = 1$

$$A = \begin{pmatrix} -14 & -28 & 28 \\ 4 & 8 & -8 \\ -4 & -8 & 8 \end{pmatrix} \rightarrow \begin{pmatrix} -14 & -28 & 28 \\ 4 & 8 & -8 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -14 & -28 & 28 \\ 14 & 28 & -28 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -14 & -28 & 28 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} -1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{array}{c} \begin{pmatrix} 1 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{array}{ccc} & \Delta & \Delta \\ & F & F \end{array} \end{array} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

~~$v_1 + 2v_2 - 2v_3 = 0$~~

~~$v_1 + 2v_2 = 2v_3$~~

~~$v_1 = 2v_3 - 2v_2$~~

$v_1 = t$

$$\rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$v_1 = 2v_3 - 2v_2$
$v_2 = s$
$v_3 = t$

$$\rightarrow s\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

eigenvector $\lambda = 3$

$$A_3 = \begin{pmatrix} -16 & -28 & 28 \\ 4 & 6 & -8 \\ -4 & -8 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & 7 & -7 \\ 4 & 6 & -8 \\ -4 & -8 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & 7 & -7 \\ 0 & -1 & -1 \\ \cancel{0} & \cancel{1} & \cancel{1} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & \frac{7}{4} & -\frac{7}{4} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} \frac{4}{7} & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{4}{7} & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \frac{7}{4} & -\frac{7}{4} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & \cancel{0}\frac{7}{4} & -\frac{7}{4} \\ 0 & \cancel{1} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$v_1 \Rightarrow 0 = v_1 - \frac{7}{4}v_3$
$v_2 \Rightarrow 0 = v_2$
$v_3 \Rightarrow v_3 = s$

$$0 = v_1 + \frac{7}{4}v_2 - \frac{7}{4}v_3 \rightarrow \begin{bmatrix} 7 \\ -2 \\ 2 \end{bmatrix}$$

Diagonalizability - $\boxed{\text{Yes}}$

$$\boxed{Q = \begin{bmatrix} -2 & 2 & 7 \\ 1 & 0 & -2 \\ 0 & 1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}}$$

7.2.4 $A = \begin{pmatrix} 1 & 90 & 0 \\ 0 & -2 & 0 \\ 3 & 89 & -2 \end{pmatrix}$ find $\begin{cases} \text{e.val} \\ \text{e.vec} \end{cases}$, Diagonalizeability

$$0 = \det(A - I\lambda) = \det\begin{pmatrix} 1-\lambda & 90 & 0 \\ 0 & -2-\lambda & 0 \\ 3 & 89 & -2-\lambda \end{pmatrix}$$

$$= (-2-\lambda)\begin{pmatrix} 1-\lambda & 90 \\ 0 & -2-\lambda \end{pmatrix} \rightarrow (-2-\lambda+2\lambda+\lambda^2) + \cancel{0}$$

$$= (-2-\lambda)(-2+\lambda+\lambda^2) = 0 = 4 - \cancel{2\lambda} + 2\lambda^2 + \cancel{2\lambda} + \lambda^2 - \lambda^3$$

$$= \cancel{\lambda^3 \; 3\lambda^2 - 4} = -\lambda^3 + 4\lambda^2 + 4\lambda - 4$$ arithmetic err.

$$-\lambda^2(\lambda + 1) \quad 4(\lambda - 1) = -(\lambda^2 - 4)(\lambda - 1)$$

$\boxed{\cancel{\lambda = 2, 2, 1}}$ $\lambda = -2, 2, 1$

eigenvectors

$$\lambda=-2: \begin{pmatrix} 3 & 90 & 0 \\ 0 & 0 & 0 \\ 3 & 89 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & 90 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 30 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} 0 \\ 0 \\ v_3 = s \end{matrix} \quad s\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \boxed{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

$$\lambda = 1 \begin{pmatrix} 2 & 90 & 0 \\ 0 & -1 & 0 \\ 3 & 89 & -1 \end{pmatrix} \begin{pmatrix} 2 & 90 & 0 \\ 0 & 1 & 0 \\ 3 & 89 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & -1 \end{pmatrix} \begin{pmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 6 & 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} v_1 \rightarrow \\ v_2 = 0 \\ v_3 = s \end{matrix} \quad 0 = 6v_1 - 2v_3 \quad 6v_1 = 2v_3 \quad \frac{1}{3}s = v_1$$

$$\begin{pmatrix} 0 & 90 & 0 \\ 0 & -3 & 0 \\ 3 & 89 & -3 \end{pmatrix} \cdot -30 \Rightarrow \begin{pmatrix} 0 & 90 & 0 \\ 0 & 90 & 0 \\ 3 & 89 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & -1 & -3 \end{pmatrix} \rightarrow$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ v_2 = 0 \\ v_3 = s \end{matrix} \quad 0 = v_1 + v_3 \quad v_1 = v_3 \quad \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = s\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \boxed{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}$$

→ $\boxed{\text{undiagonalizeable}}$ b/c matricies arent basis of $\mathbb{R}^3$!

7.2.9 $A = \begin{pmatrix} 15 & -24 & 7 \\ -6 & 5 & -1 \\ -58 & 76 & -20 \end{pmatrix}$ find $\begin{cases} \text{eigenvalues} \\ \text{eigenvectors} \end{cases}$, determine diagon.

eigenvectors solve for $(A - I\lambda)\det = 0$

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$$0 = \det \begin{pmatrix} 15-\lambda & -24 & 7 \\ -6 & 5-\lambda & -1 \\ -58 & 76 & -20-\lambda \end{pmatrix} \to 0 = -\lambda^3 - 13\lambda - 34$$

Note: I used online symolab calculator to speed this process up...

$$-\lambda^3 - 13\lambda - 34$$

$$= -(x+2)(x^2 - 2x + 17)$$

$$\boxed{\lambda = -2}$$

$a=1$, $b=-2$, $c=17$

$$\frac{2 \pm \sqrt{4 - 4\cdot 17}}{2} = \frac{\cancel{2} \pm \cancel{2}\sqrt{-16}}{\cancel{2}} = 1 \pm \sqrt{-16} = \boxed{1 \pm 4i = \lambda}$$

eigenvectors: solve $(A - \lambda I) = 0$

$$\lambda = -2 \quad \begin{pmatrix} 17 & -24 & 7 \\ -6 & 7 & -1 \\ -58 & 76 & -18 \end{pmatrix} \underset{\text{symolab calc online}}{\sim} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{matrix} 0 = v_1 - v_3 \\ 0 = v_2 - v_3 \\ v_3 = s \end{matrix} \quad \boxed{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}$$

$$\lambda = 1 + 4i \quad \begin{pmatrix} 14-4i & -24 & 7 \\ -6 & 4-4i & -1 \\ -58 & 76 & -21-4i \end{pmatrix} \underset{\text{symolab calc online}}{\sim} \begin{pmatrix} 1 & 0 & \frac{1}{10} + i\frac{1}{5} \\ 0 & 1 & -\frac{1}{5} + i\frac{1}{10} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{Bmatrix} x + \left(\frac{1}{10} + i\frac{1}{5}\right) z = 0 \\ y + \left(-\frac{1}{5} + i\frac{1}{10}\right) z = 0 \end{Bmatrix} \longrightarrow \begin{Bmatrix} x = -\left(\frac{1}{10} + i\frac{1}{5}\right) z \\ y = -\left(-\frac{1}{5} + i\frac{1}{10}\right) z \end{Bmatrix}$$

$z = 10$

$$\eta = \begin{pmatrix} -(1+2i) \\ -(-2+i) \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -1-2i \\ 2-i \\ 10 \end{pmatrix} = \lambda_{1+4i} \; ; \; \begin{pmatrix} -1+2i \\ 2+i \\ 10 \end{pmatrix} = \lambda_{1-4i}}$$

$$Q = \begin{pmatrix} 1 & -1-2i & -1+2i \\ 1 & 2-i & 2+i \\ 1 & 10 & 10 \end{pmatrix} \; ; \; D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1+4i & 0 \\ 0 & 0 & 1-4i \end{pmatrix}$$

7.2.12 $A = \begin{pmatrix} -11 & -12 & 4 \\ 8 & 17 & -4 \\ -4 & 28 & -3 \end{pmatrix}$ find $\begin{cases} \text{E.val} \\ \text{E.vec} \end{cases}$ and diagonalizability

eigenvalues : solve $0 = \det(A - \lambda I)$

$$0 = \det \begin{pmatrix} -11-\lambda & -12 & 4 \\ 8 & 17-\lambda & -4 \\ -4 & 28 & -3-\lambda \end{pmatrix} \xrightarrow[\text{calculator used}]{\text{symolab online}} = \underbrace{-\lambda^3 + 3\lambda^2 - 19\lambda + 17}$$

$-\lambda^3 + 3\lambda^2 - 19\lambda + 17$ $(x-1)$ is a factor.

$$\begin{array}{c|cccc} 1 & -1 & 3 & -19 & 17 \\ & & -1 & 2 & -17 \\ \hline & -1 & 2 & -17 & \emptyset \end{array}$$

$(x-1)(-x^2+2x-17)$

$\boxed{\lambda = 1}$

$a = -1$
$b = 2$
$c = -17$

$$\frac{-2 \pm \sqrt{4 + 4 \cdot -17}}{-2}$$

$$\frac{-2 \pm \sqrt{4-68}}{-2} = \cancel{\frac{-2 \pm 2\sqrt{-16}}{-2}} \quad \frac{-1 \pm \sqrt{-16}}{\cancel{-2}}$$

$4i$

$$\boxed{\begin{matrix} 1 + 4i \\ \lambda \\ 1 - 4i \end{matrix}}$$

eigenvectors (all solved using symolab calculator)

$$\lambda = 1 \longrightarrow \begin{pmatrix} 1 & 0 & -1/6 \\ 0 & 1 & -1/6 \\ 0 & 0 & 0 \end{pmatrix} z \text{ is free} \quad \eta = \begin{pmatrix} 0 = x - 1/6\, z \\ 0 = y - 1/6\, z \\ z \end{pmatrix} z = 6$$

$$\begin{pmatrix} x = 1/6\, z \\ y = 1/6\, z \\ z \end{pmatrix} \to \begin{pmatrix} x = 1 \\ y = 1 \\ 6 \end{pmatrix} \longrightarrow \boxed{\begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix} = EV_{\lambda=1}}$$

$$\lambda = 1 \pm 4i \longrightarrow \begin{pmatrix} 1 & 0 & -\frac{3}{25} + i\frac{4}{25} \\ 0 & 1 & -\frac{4}{25} - i\frac{3}{25} \\ 0 & 0 & 0 \end{pmatrix} z \text{ free} \quad \eta = \begin{pmatrix} x = \left(\frac{3}{25} - i\frac{4}{25}\right) z \\ y = \left(\frac{4}{25} + i\frac{3}{25}\right) z \\ z \end{pmatrix} z = 25$$

solve for pos:

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$$\eta = \begin{pmatrix} x = \left(\frac{3}{25} - i\frac{4}{25}\right)z \\ y = \left(\frac{4}{25} + i\frac{3}{25}\right)z \\ z \end{pmatrix} \Bigg|_{z=25} \longrightarrow \boxed{\begin{pmatrix} 3-4i \\ 4+3i \\ 25 \end{pmatrix} \quad \begin{pmatrix} 3+4i \\ 4-3i \\ 25 \end{pmatrix} \\ \lambda = 1+4i \qquad \lambda = 1-4i}$$

$$Q = \begin{pmatrix} 1 & 3-4i & 3+4i \\ 1 & 4+3i & 4-3i \\ 6 & 25 & 25 \end{pmatrix} ; \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+4i & 0 \\ 0 & 0 & 1-4i \end{pmatrix}$$

7.2.12 $A = \begin{pmatrix} 14 & -12 & 5 \\ -6 & 2 & -1 \\ -69 & 51 & -21 \end{pmatrix}$ find $\begin{cases} \text{eigenvec's} \\ \text{eigenval's} \end{cases}$ and diagonalizability

NOTE: Some parts solved on symbolab online calculator.

<u>eigenvalues</u>

$$\det\begin{pmatrix} 14-\lambda & -12 & 5 \\ -6 & 2-\lambda & -1 \\ -69 & 51 & -21-\lambda \end{pmatrix} = 0 \longrightarrow -\lambda^3 - 5\lambda^2 - 16\lambda - 30 \quad , \; x+3 \text{ is fac.}$$

$$\begin{array}{r|rrrr} & -1 & -5 & -16 & -30 \\ -3 & & 3 & 6 & 30 \\ \hline & -1 & -2 & -10 & \cancel{0} \end{array} \longrightarrow (x+3)(-x^2-2x-10)$$

$\downarrow$

$\lambda = -3$

$a=-1$
$b=-2$
$c=-10$ $\longrightarrow$ $\dfrac{2 \pm \sqrt{4-40}}{-2}$

$$\frac{2 \pm \sqrt{-36}}{-2} \longrightarrow \frac{2 \pm 6i}{-2} \longrightarrow \boxed{-1 \pm 3i = \lambda}$$

<u>eigenvectors:</u>

$$\lambda = -3 : A \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}_{z \text{ free}} \qquad \eta \begin{pmatrix} 0 = x+z \\ 0 = y+z \\ z \end{pmatrix} z=1$$

$$\begin{pmatrix} x=-1 \\ y=-1 \\ 1 \end{pmatrix} \longrightarrow \boxed{\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \lambda_3^v}$$

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$$\lambda = \begin{cases} -1+3i \longrightarrow \\ -1-3i \;\; -x \end{cases} \quad A \longrightarrow \begin{pmatrix} 1 & 0 & \frac{1}{6} + i\frac{1}{6} \\ 0 & 1 & -\frac{1}{6} + i\frac{1}{6} \\ 0 & 0 & 0 \end{pmatrix} \quad \eta \begin{pmatrix} 0 = x + \left(\frac{1}{6} + i\frac{1}{6}\right) z \\ 0 = y + \left(-\frac{1}{6} + i\frac{1}{6}\right) z \\ z \end{pmatrix}$$

$$\eta = \begin{pmatrix} x = \left(-\frac{1}{6} - i\frac{1}{6}\right) z \\ y = \left(+\frac{1}{6} - i\frac{1}{6}\right) z \\ z \end{pmatrix} z = 6 \longrightarrow \boxed{\begin{array}{cc} \begin{pmatrix} -1-i \\ 1-i \\ 6 \end{pmatrix} & \begin{pmatrix} -1+i \\ 1+i \\ 6 \end{pmatrix} \\ \lambda = -1+3i & \lambda = -1-3i \end{array}}$$