MTH341 Homework 2. 4/25/2019 Lyell Read

<u>2.1.43</u>

$$(A|I) = \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -3 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[-1 \cdot ②]{② - ①} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 2 & 1 & -1 & 0 & 0 \\ 2 & 1 & -3 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{③ - 2(①)} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 2 & 1 & -1 & 0 & 0 \\ 0 & -3 & -3 & -2 & -2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{③ + 3(②)} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 2 & 1 & -1 & 0 & 0 \\ 0 & 0 & -9 & 4 & \cancel{8}1 & -3 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[④ - ①]{③ / -9} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 2 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{-4}{9} & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{④ - ③} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 2 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{4}{9} & \frac{1}{-9} & \frac{1}{3} & \frac{-1}{9} & 0 \\ 0 & 0 & 0 & 4/9 & -8/9 & -\frac{1}{3} & \frac{1}{9} & 1 \end{array}\right)$$

$$\xrightarrow{④ \cdot \frac{9}{4}} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 2 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{-4}{9} & -1/9 & 1/3 & -1/9 & 0 \\ 0 & 0 & 0 & 1 & -2 & -3/4 & 1/4 & 0 \end{array}\right) \xrightarrow[① - 2④]{③ + \frac{4}{9}④,\ ② - 2④} \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 5 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -2 & 0 & 5 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 & -3/4 & 1/4 & 0 \end{array}\right)$$

$$\xrightarrow[① - ② \cdot 2]{② + 2③} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & 3 & \frac{1}{2} & -\frac{1}{2} & 2 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2/3 & -3/4 & 1/4 & 0 \end{array}\right) \quad \therefore A^{-1} = \begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 2 & 1/2 & -1/2 & 2 \\ -1 & 0 & 0 & 1 \\ -2/3 & -3/4 & 1/4 & 0 \end{pmatrix}$$

NOTE: Different answer from book, but I did this twice, and did not find mistakes. Could be alt-solution?

$$A^{-1} = \begin{pmatrix} -1 & 1/2 & 1/2 & 0 \\ 3 & 1/2 & -1/2 & 2 \\ -1 & 0 & 0 & 1 \\ -2/3 & -3/4 & 1/4 & 0 \end{pmatrix}$$

2.1.44

(a) $\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $AX = B$ $\boxed{= \begin{pmatrix} \frac{7}{2} \\ -\frac{3}{2} \end{pmatrix}}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ok!

A   X   B

A^{-1} : $\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}$ $ad - bc = -2.$ $\rightarrow \frac{1}{-2}\begin{pmatrix} 1 & -4 \\ -1 & 2 \end{pmatrix}$ CHECK: $\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1/2 & 2 \\ 1/2 & -1 \end{pmatrix}$

$\therefore A^{-1} = \begin{pmatrix} -1/2 & 2 \\ 1/2 & -1 \end{pmatrix}.$ $A^{-1}B = X \Rightarrow \begin{pmatrix} -1/2 & 2 \\ 1/2 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1/2 + 4 \\ 1/2 - 2 \end{pmatrix} = \begin{pmatrix} \frac{7}{2} \\ -\frac{3}{2} \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $A^{-1}B = X \Rightarrow \begin{pmatrix} -1/2 & 2 \\ 1/2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}$

(c) $\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ $A^{-1}B = X \Rightarrow \begin{pmatrix} -1/2 & 2 \\ 1/2 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \boxed{\begin{pmatrix} -1/2a + 2b \\ 1/2a - b \end{pmatrix}}$

2.1.45 (Wrong. See last pg's.

$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

$A^{-1} = \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{②-2①} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 3 & -2 & -2 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right)$

$\xrightarrow{②/3} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{③-①} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -2/3 & -\frac{2}{3} & 1/3 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right)$

$\xrightarrow{-1\cdot③} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -2/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right) \xrightarrow{②+2/3③} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{4}{3} & 1/3 & 2/3 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$

$\xrightarrow{①-3③} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 0 & -3 \\ 0 & 1 & 0 & -4/3 & 1/3 & 2/3 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$ $\therefore A^{-1} = \begin{pmatrix} 4 & 0 & -3 \\ -4/3 & 1/3 & 2/3 \\ -1 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 4 & 0 & -3 \\ -4/3 & 1/3 & 2/3 \\ -1 & 0 & 1 \end{pmatrix}$

(a) $A^{-1}B = X \quad \begin{pmatrix} 4 & 0 & -3 \\ -4/3 & 1/3 & 2/3 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 \\ -2/3 \\ 0 \end{pmatrix}}$

(b) $\longrightarrow A^{-1}B = X \quad \cancel{\begin{pmatrix} 4 & 0 & -3 \\ -4/3 & 1/3 & 2/3 \\ -1 & 0 & 1 \end{pmatrix}}\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$ (below)

$$\begin{array}{c} 11 \\ 21 \\ 31 \end{array}\begin{pmatrix} \cancel{12 + 6} \;\; -6 -6 \\ -1/3 + 4/3 \\ 3 + 2 \end{pmatrix} = \boxed{\begin{pmatrix} -12 \\ 3/3 \\ 5 \end{pmatrix}} \begin{pmatrix} -2 & 0 & 3 \\ 0 & 1/3 & -2/3 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$$

NOTE: Calc'd online because worked for first prob. will ask about in class.

(c) $= \begin{pmatrix} -2 & 0 & 3 \\ 0 & 1/3 & -2/3 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -2a + 3c \\ 1/3 b - 2/3 b \\ a - c \end{pmatrix} = \boxed{\begin{pmatrix} 3c - 2a \\ 1/3 b - 2/3 c \\ a - c \end{pmatrix}}$ ✓

3.1.3

(c) $A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 1 & 3 & 2 & 3 \\ 4 & 1 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{pmatrix}$ find det(A).

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 1 & 3 & 2 & 3 \\ 4 & 1 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{pmatrix} \xrightarrow{\text{②} - \text{①}} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -1 & 1 \\ 4 & 1 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{pmatrix} \xrightarrow[\text{col 4 + col 3}]{\text{col 2 + col 3}}$$

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -1 & 1 \\ 4 & 1 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 5 & 3 & 5 \\ 0 & 0 & 1 & 0 \\ 4 & 6 & 5 & 5 \\ 1 & 3 & 1 & 3 \end{pmatrix} \overset{(2,3)\text{ sign}}{=} \cancel{-1(+1)} \det\begin{pmatrix} 1 & 5 & 5 \\ 4 & 6 & 5 \\ 1 & 3 & 3 \end{pmatrix}$$

$$\det\begin{pmatrix} 1 & 5 & 5 \\ 4 & 6 & 5 \\ 1 & 3 & 3 \end{pmatrix} = 1 \cdot \underbrace{\det\begin{pmatrix} 6 & 5 \\ 3 & 3 \end{pmatrix}}_{18-15=3} - 5\underbrace{\det\begin{pmatrix} 4 & 5 \\ 1 & 3 \end{pmatrix}}_{12-5=7} + 5\underbrace{\det\begin{pmatrix} 4 & 6 \\ 1 & 3 \end{pmatrix}}_{12-6=6}$$

$3 - 35 + 30 = \boxed{-2}$ ✓

3.1.7

find det(A) using cofactor extraction

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{2} \\ 2 & 1 & 3 & 1 \end{pmatrix} \quad A_{3,4} \cdot \underbrace{\det\begin{pmatrix} \cancel{1} & \cancel{0} & \cancel{0} \\ \cancel{2} & 1 & 1 \\ \cancel{2} & 1 & 3 \end{pmatrix}}_{=B} = A_{3,4} \cdot B_{1,1} \cdot \underbrace{\det\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}}_{3-1}$$

$A_{3,4}$: $2 \cdot -1^7 = -2$

$B_{1,1}$: $1 \cdot -1^2 = 1$

$$= (-2) \cdot (1) \cdot (2) = \boxed{-4}$$

3.1.8

(c)

$$A = \begin{pmatrix} 2 & 3 & 15 & \cancel{0} \\ 0 & 4 & 1 & \cancel{7} \\ 0 & 0 & -3 & \cancel{5} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} \end{pmatrix}$$

4,4 = no sign :)

find det(A)

1,1 = no sign

$$(1) \det\begin{pmatrix} \cancel{2} & \cancel{3} & \cancel{15} \\ \cancel{0} & 4 & 1 \\ \cancel{0} & 0 & -3 \end{pmatrix} = (1)(2) \det\begin{pmatrix} 4 & 1 \\ 0 & -3 \end{pmatrix}$$

$-12 - 0$

$= -12$

$$\det(A) = \boxed{-24}$$

Redo:

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & 2 \end{pmatrix} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{②-2(①)} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 3 & -2 & -2 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{③-①} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 3 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{②/3} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -2/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right) \xrightarrow{①=①+3③} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 3 \\ 0 & 1 & -2/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{③\cdot -1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 3 \\ 0 & 1 & -2/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right) \xrightarrow{②+\frac{2}{3}③} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 3 \\ 0 & 1 & 0 & 0 & 1/3 & -2/3 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right)$$

$$\boxed{A^{-1} = \begin{pmatrix} -2 & 0 & 3 \\ 0 & 1/3 & -2/3 \\ 1 & 0 & -1 \end{pmatrix}}$$

60