MTH 341 NOTES 4/9/2019

Chapter 3

- Elementary Matrix: any matrix that results from applying an elementary row operation to an Identity Matrix.

- EX: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{3 \text{ ①}} \underbrace{\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Elementary matrix}} = E_1$

Ex: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[\text{② - 5①}]{\text{rep ② by}} \begin{pmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$E_1 A = \begin{pmatrix} 3a & 3b & 3c \\ d & e & f \\ g & h & i \end{pmatrix}$ $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

same as

$\underbrace{A \xrightarrow{3 \text{ ①}}}_{\text{same as done to elementary matrix}}$

↳ This can be done over and over again $(E_1 E_2 E_3 \dots A)$ to solve sets of sim Eq's

- Performing ERO's:
  - → perform operation on $I \to E_n$
  - → multiply $E_n A$

- Leading Entry of a matrix row
  - → the first nonzero element in a row, starting from the left

- Echelon form (NON REDUCED)

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- If leading Entry of row i is in position k and $(i+1)$ is not a zero-row, then its leading entry is $(k+j) \mid j \geq 1$

- All zero-rows are at the bottom of the Matrix

- All non-zero rows have leading Entry $=1$.

Ex's (all in EF)

$$\begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & * & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 1 & * \end{pmatrix}$$

- REDUCED Echelon form
  - R is in Reduced Echelon Form if
    - → (a) R is in echelon form
    - → (b) Each leading entry is the only nonzero entry in its column.

Ex: (all in REF)

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & * & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{pmatrix}$$