Lyell C Read

CHAPTER 11.* REVIEW

10/4/2018

REVIEW PR'S
59, 81, 86!
THESE ARE CHALLENGES

11.1 VECTORS IN THE PLANE

- Vector is any segment that has both magnitude ($|\vec{v}|$) and direction (θ). Denoted as $\vec{v} = \langle \ldots, \ldots, \ldots \rangle$
- Displacement vectors: A displacement vector from P to Q is indicated by $\overrightarrow{PQ}$
- Position vector: Any vector that has O (0,0,0) as it's tail end ("starts at O")
- Unit vector: vector $\vec{v}$ with magnitude $|\vec{v}| = 1$
  → can be calculated by $\frac{\vec{v}}{|\vec{v}|}$ = unit vector in dir. of $\vec{v}$
- Vector Magnitude: $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ assuming $\vec{v} = \langle v_1, v_2, v_3 \rangle$
- $\mathbb{R}$: $\mathbb{R}^1$: Number line. $\mathbb{R}^2$: Plane (typically xy). $\mathbb{R}^3$: 3D space (x, y, z)
- Vector properties:

Define vectors $\vec{v} = \langle v_1, v_2, v_3 \rangle$
$\vec{u} = \langle u_1, u_2, u_3 \rangle$

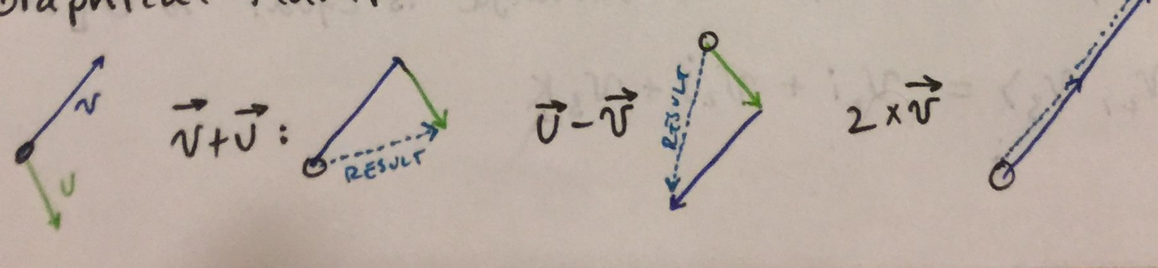
$$n^* \vec{v} = \langle n v_1, n v_2, n v_3 \rangle$$

$$\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$$

$$\vec{v} - \vec{u} = \vec{v} + (-\vec{u})$$

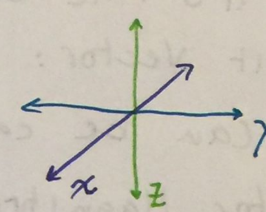
$$-\vec{v} = \langle -v_1, -v_2, -v_3 \rangle$$

- Graphical Manipulation:



11.2 VECTORS IN THREE DIMENSIONS

- 3 principal unit vectors: $i\langle 1,0,0\rangle$, $j\langle 0,1,0\rangle$, $k\langle 0,0,1\rangle$
- Equation for a circle, centered at (a,b) with radius r:
  $\rightarrow r^2 = (x-a)^2 + (y-b)^2$
- Equation for a sphere centered at (a,b,c) with radius r
  $\rightarrow r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$
- Ball: all the points on, or inside of a sphere. Equation for a ball centered at (a,b,c) with radius r:
  $\rightarrow (x-a)^2 + (y-b)^2 + (z-c)^2 \leq r^2$
- x, y, z axis placement "standard":



- sets and "constraints":

$\mathbb{R}^2$

$\{(x,y): x=a\}$ = line $x=a$

$\{(x,y): x=a, y=b\}$ = point (a,b)

$\{(x,y): x=a \text{ or } y=b\}$ = lines intersecting at (a,b)

$\mathbb{R}^3$

$\{(x,y,z): x=a\}$ = plane $x=a$

$\{(x,y,z): x=a, y=b\}$ = intersect. 2 planes AKA a line

$\{(x,y,z): x=a, y=b, z=c\}$: point

$\{(x,y,z): x=a \text{ or } y=b\}$: Two planes that meet at a line

- "Circle Magic": for a circle with $r=1$, center at O:
  $\rightarrow$ all points on circle are $(x = r\cos\theta,\ y = r\sin\theta)$
  $\rightarrow$ arc length from $(1,0)$ to an angle is equal to θ $\left[\begin{matrix} r=1 \\ \downarrow \\ s=r\theta \end{matrix}\right]$
- $\vec{v} = \langle v_1, v_2, v_3\rangle = v_1 i + v_2 j + v_3 k$

Lyell C Read CHs 11.3, 11.4, 11.5 Review 10/2/2018

FOR QUIZ ON 10/3/2018

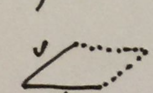
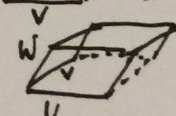
$\boxed{11.3}$ DOT PRODUCTS

- u dot v = $|u||v|\cos\theta$ | θ = between u, v
- u dot v = $u_1v_1 + u_2v_2 + v_3u_3$ if $u=\langle u_1, u_2, u_3\rangle$, $v=\langle v_1, v_2, v_3\rangle$
- $\cos\theta$ = u dot v $/ |u||v|$
- Projection: $\text{proj}_u v = |v|\cos\theta\left(\frac{u}{|u|}\right) = \frac{u\cdot v}{|u|}\left(\frac{u}{|u|}\right) = \left(\frac{u\cdot v}{u\cdot u}\right)u$
- Scalar: $\text{scal}_u v = |v|\cos\theta = \frac{u\cdot v}{|u|}$
- dot product of $\perp$ vectors is always 0 $\cos(90)=0$!
- $\perp$ and // components to a force (grav). vec $\sin\theta$ = //
  vec $\cos\theta = \perp$

$\boxed{11.4}$ CROSS PRODUCTS

- Matrix Determinants: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ for 2X2
- Matrix 3X3 Determinant:

$$\begin{vmatrix} a & -b & c \\ -d & e & f \\ g & -h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$ ~~then~~ use above 2X2

- cross Product: $u \times v = |u||v|\sin\theta$ between u, v
  - perpendicular to both u, v. Use Right Hand Rule.
- Area of a parallelogram  $= |u||v|\sin\theta$
- Area of a parallelopeipet  $u \times v \cdot w$
- Torque $\tau = |F||L|\sin\theta = |F \times L|$

• cross Product w/ Matricies $u = \langle u_1, u_2, u_3 \rangle \times v = \langle v_1, v_2, v_3 \rangle$

$$\begin{vmatrix} i & j & k \\ v_1 & v_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \longrightarrow \text{Determinant} \rightarrow ix \; jy \; kz \rightarrow \langle \rangle$$

• Area of $\triangle$ $\frac{1}{2}|u \times v| = \frac{1}{2}|u||v|\sin\theta$ or use Matricies

$\boxed{11.5}$ **VECTOR DEFINED FUNCTIONS**

• line through point, in direction $r(t) = \text{point} + t\,\text{dir.}$

• line through u, v $v - u = \text{dir}$ $r(t) = u + t\,\text{dir}$

• line dir from eq (remove everything but what is mult by t)

11.6 CALCULUS ON VECTOR DEFINED FUNCTIONS

- Derivative of $r(t) = \langle x(t), y(t), z(t) \rangle = r'(t) = \langle x'(t), y'(t), z'(t) \rangle$
- Integrals of $r(t) = \langle x(t), y(t), z(t) \rangle =$

  $\rightarrow \int r(t)\,dt = \left\langle \int x(t), \int y(t), \int z(t) \right\rangle dt + \langle C_1, C_2, C_3 \rangle$

  $\rightarrow$ same as $\left\langle \int x(t)\,dt + C, \int y(t)\,dt + C, \int z(t)\,dt + C \right\rangle$
- Bounded integrals: same as integrals, just with bounds and without C. Remember: $\int_a^b r(t) \overset{\text{ANTI}}{=} R(b) - R(a)$
- Tangent vector at a point is the derivative evaluated at that point.
- Unit Tangent Vector $\frac{r'(t)}{|r'(t)|} = U_{Tt}$
- $r(t)$ from $r'(t)$: calc ANTIDERIVATIVE then make sure that the point given works (and ± a for any values $[\langle \text{value} \ldots, \ldots, \ldots \rangle]$ that need adjusting).
- Tangent Line Equation: for $r(t)$ at point a

  $\rightarrow T(t) = \text{tangent line} = \underline{r(a)} + t^*(\underline{r'(a)})$ "Point on curve" "Direction"
- Points where $r(t)$ and $r'(t)$ are orthogonal:

  $\rightarrow$ where $r(t) \cdot r'(t) = 0 \therefore r_1(t) r_1'(t) + r_2(t) r_2'(t) \ldots = 0$
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11.7 MOTION IN SPACE

- Position, velocity, speed, acceleration:
  - → Position: $r(t)$
  - → velocity: $v(t) = r'(t)$
  - → speed: $|v(t)|$ = speed
  - → acceleration: $a(t) = v'(t) = r''(t)$

- Dealing with initial points...
  - → if given $v(t) = \langle a, b, c \rangle$ $\quad r(t) = \langle \int a, \int b, \int c \rangle$

    $= \langle A + c, B + c, C + c \rangle$

    Substitute the time value for the points (usually 0) in for t, and change C for each value so that the equation = point....

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11.8 LENGTH AND ARC LENGTH, PARAMETRIZATION

- Length along a curve $(r(t))$ $\quad L = \int_a^b |r'(t)|\, dt$
- Parametrized with arc length if $s(t) = t$
- Parametrize with arc length for $r(t)$

$$s(\tau) = \int_0^\tau |r'(t)|\, dt$$

→ then solve for $S(t) = \ldots xyz\ t \ldots 123$

→ sub s for $S(t)$

→ solve for $t = \ldots$

→ substitute ... in for t in original equation.