
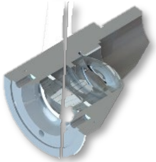



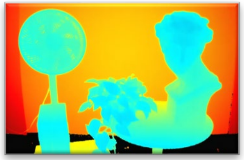


GAMES 204

Computational Imaging

Lecture 20: Temporal Encoding III： Indirect Time-of-flight Imaging

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香港中文大学（深圳）

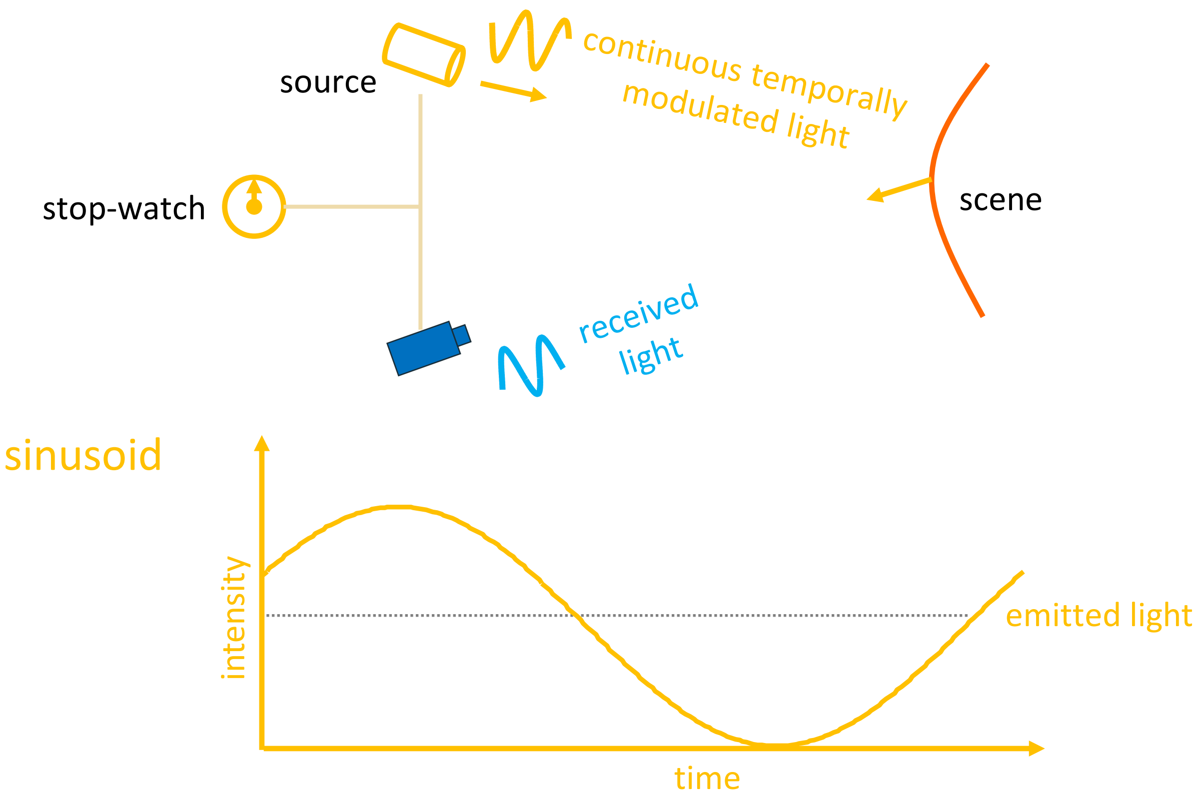
点昀技术（Point Spread Technology）

Today's **Topic**

- Continuous Wave ToF Imaging
- Imaging Model and Analysis of iToF
- Illumination
- Multipath Interference
- Phasor Imaging

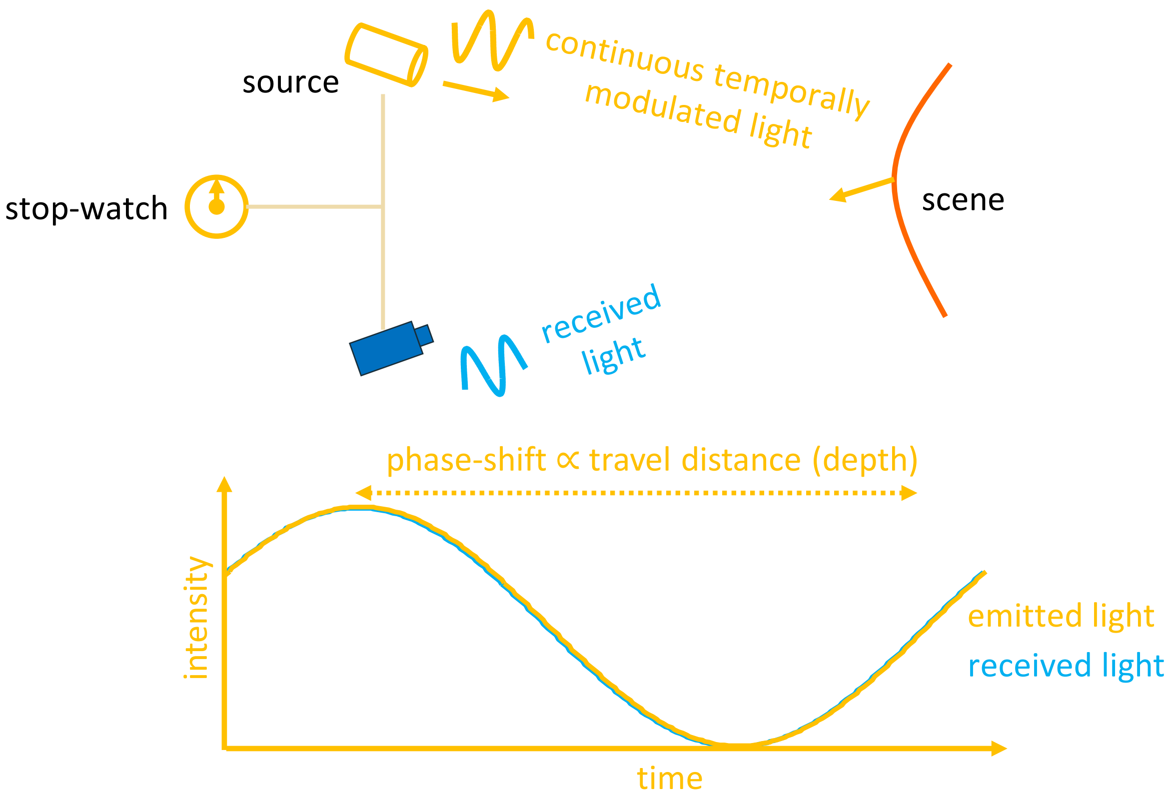
Continuous Wave ToF Imaging

Continuous Wave ToF Imaging



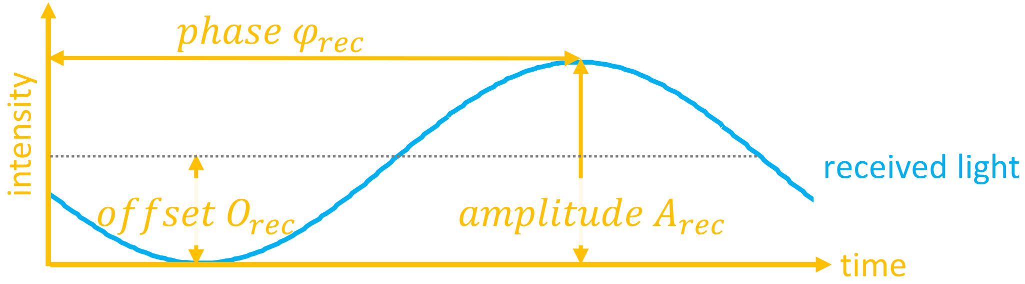
[Smith, 1980; Schwarte 1995; Lange, 2000]

Continuous Wave ToF Imaging



[Smith, 1980; Schwarte 1995; Lange, 2000]

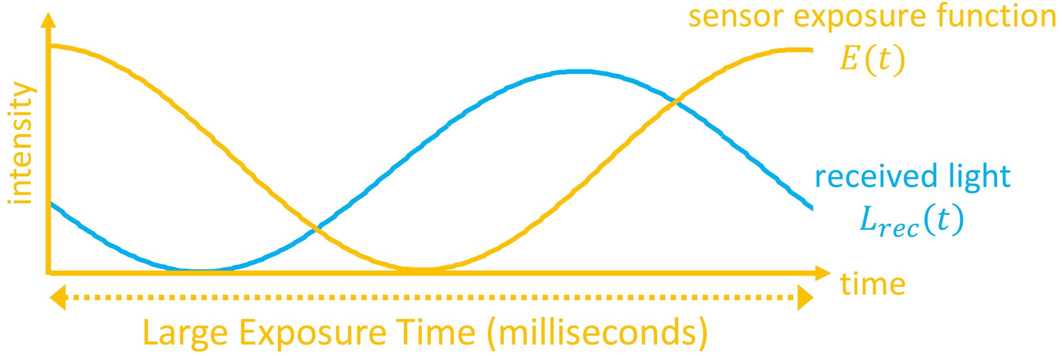
Measuring Phase-Shift



Three Unknowns

$$L_{rec}(t) = O_{rec} + A_{rec}\cos(\omega t - \phi_{rec})$$

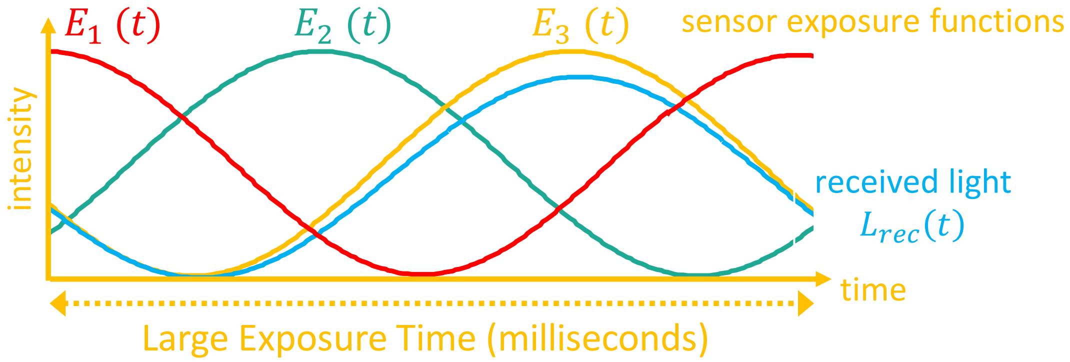
Measuring Phase-Shift: Correlation



Correlation: $I = \int E(t) \times L_{rec}(t)\, dt$

measured brightness — exposure function — received light

Measuring Phase-Shift: Correlation



High Signal-to-Noise Ratio

Real Time Capture

Correlation 1: $I_1 = \int E_1\ (t) \times L_{rec}(t)\ dt$

Correlation 2: $I_2 = \int E_2\ (t) \times L_{rec}(t)\ dt$

Correlation 3: $I_3 = \int E_3\ (t) \times L_{rec}(t)\ dt$

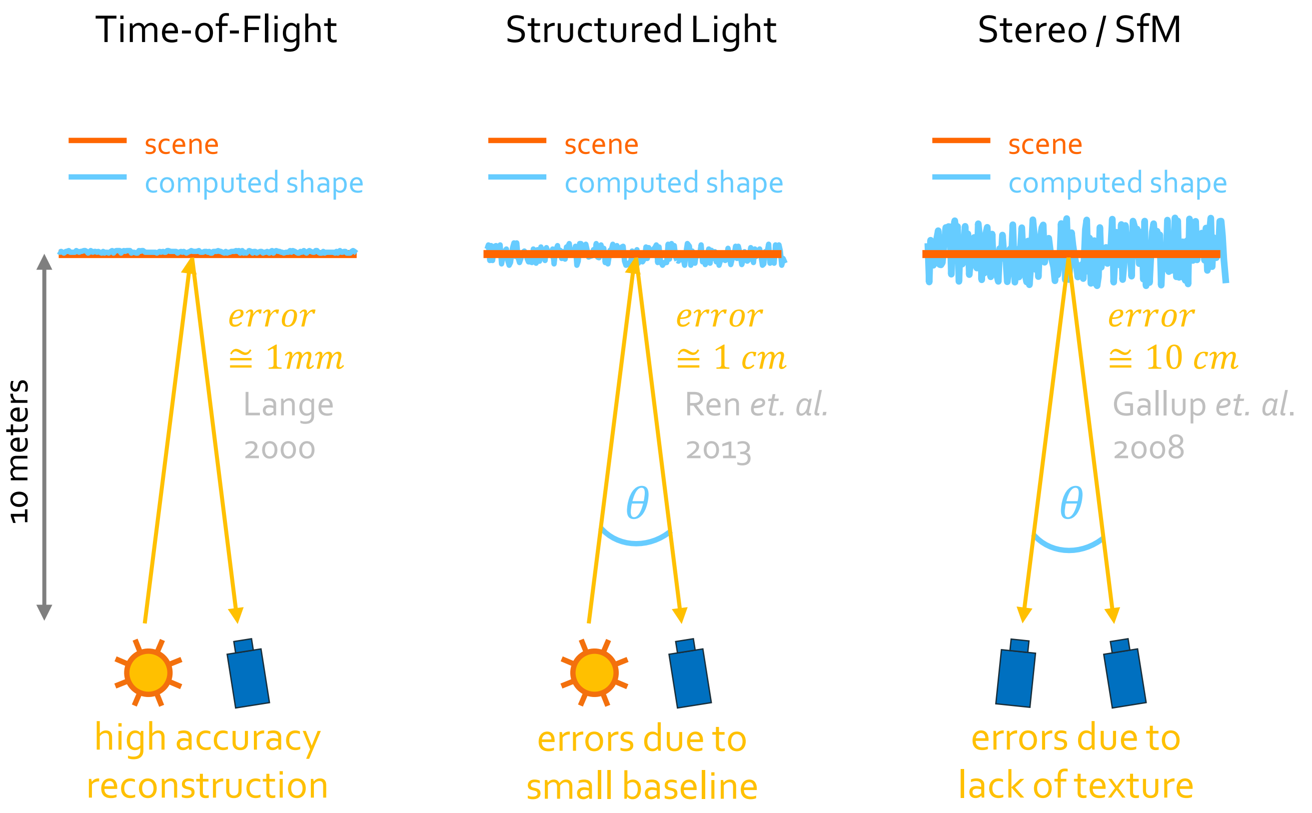
$depth$

$\boxed{phase\ \varphi_{rec}}$

$offset\ O_{rec}$

$amplitude\ A_{rec}$

Comparison of Depth Accuracy



Correlation-Based ToF Devices

Point Spread Technology
High-end ToF Camera Okulo P1

100/120fps RGBD

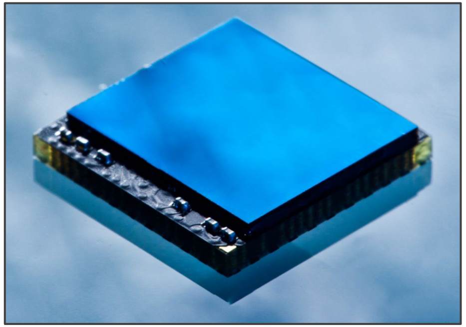
Microsoft Azure Kinect

30fps RGBD

Imaging Model and Analysis of iToF

Correlation Sensors

Also called “multi-bucket sensors”

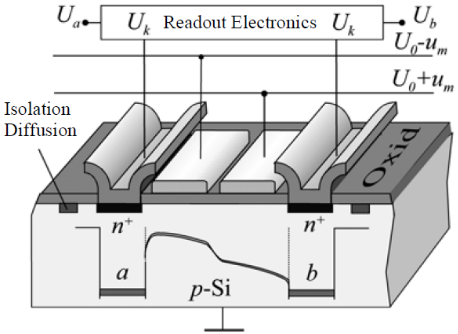


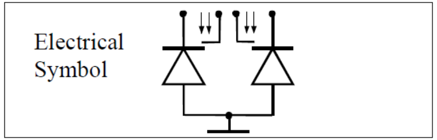
Correlation Sensor Chip
(e.g., Photonic Mixer Device)

Measure Temporal Correlation of Incident Light With Exposure Function

Low Cost, Compact, Real Time

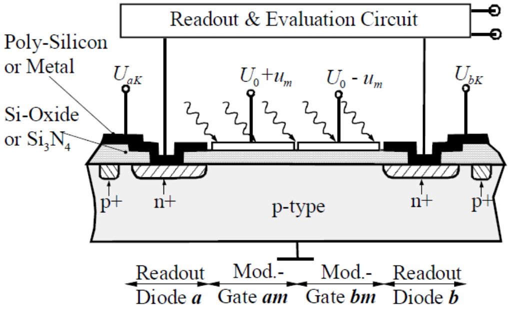
Chip layout

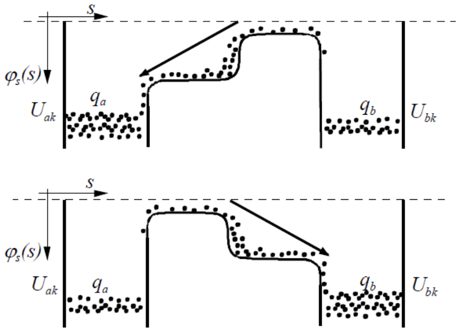




Electric symbol

Schematic view

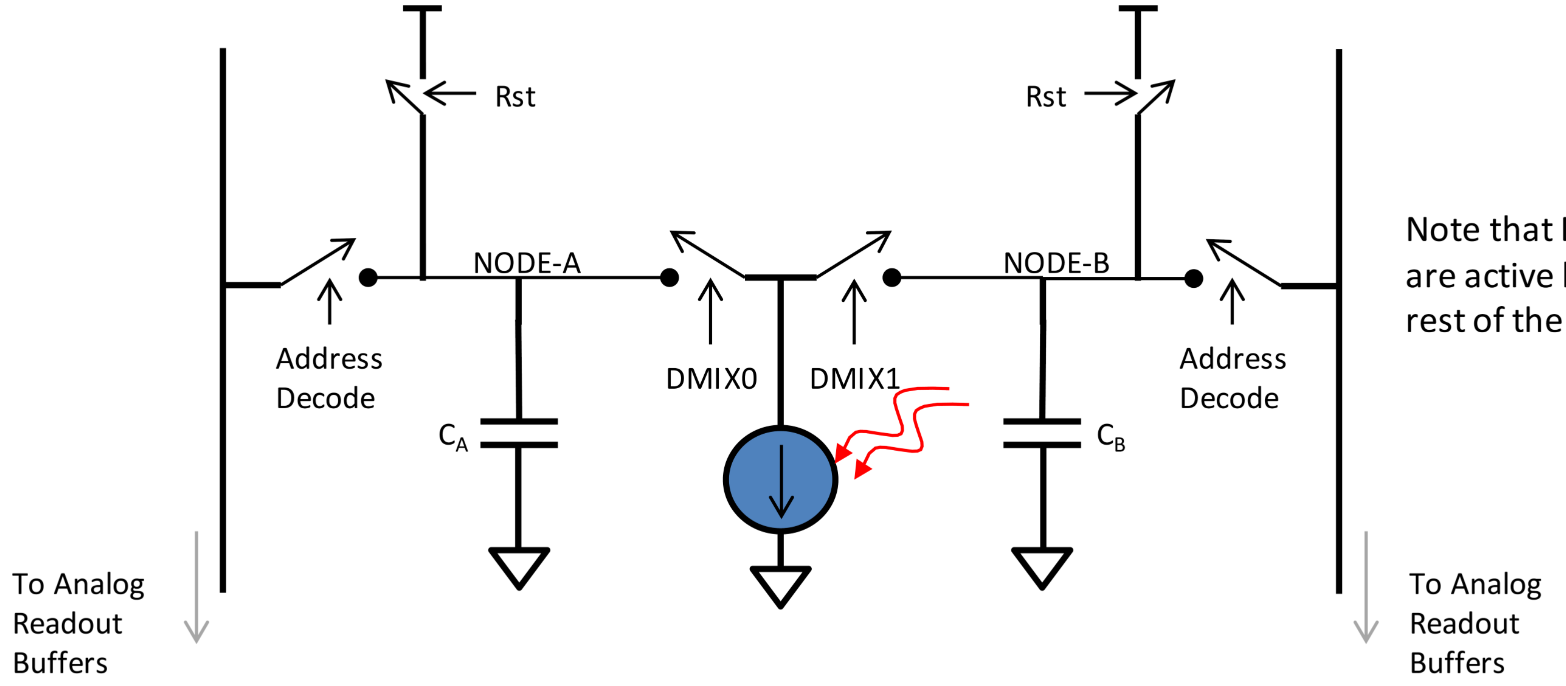




Fast on-chip modulation

Image from http://en.wikipedia.org/wiki/PMDTechnologies

Simplest Pixel Form



Note that DMIX signals are active low throughout rest of the document.

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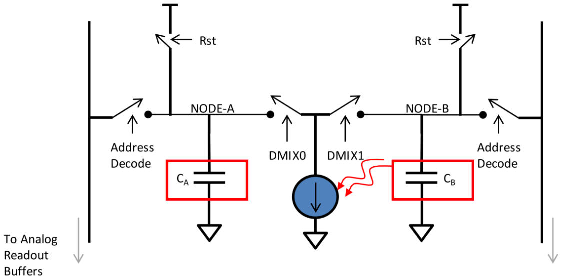
Ideal Pixel: Scene With No Ambient Light

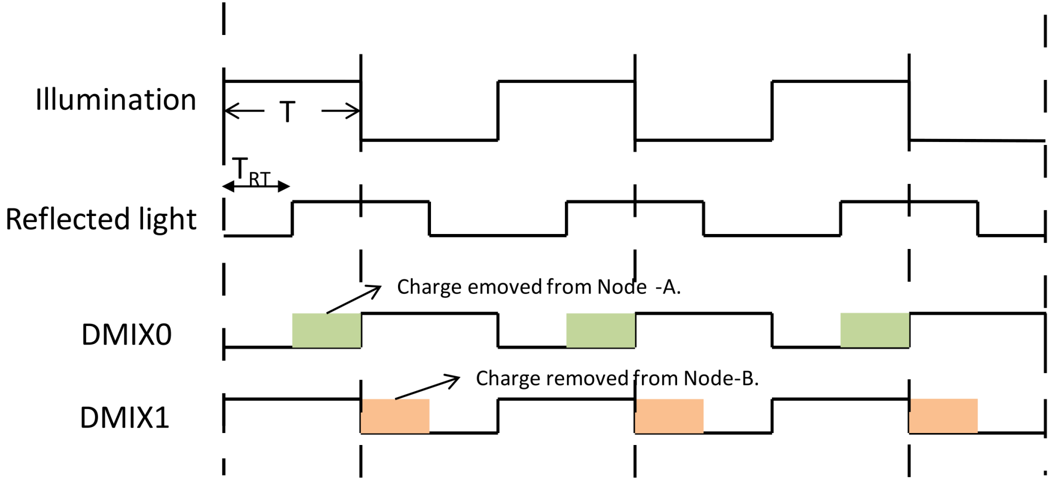
Assumption: No mismatch between C_A and C_B

A: Voltage on Node-A
B: Voltage on Node-B
V_R: Reference Voltage



Note that DMIX signals are active low throughout rest of the document.

$$A = V_R - (T - T_{RT}) \times K$$
$$B = V_R - T_{RT} \times K$$



$$Phase = \frac{T_{RT}}{T} = \frac{V_R - B}{2V_R - (A+B)}$$

$$range = \frac{T \times c}{2}$$

T_{RT} = Round trip time
T = Illumination pulse period

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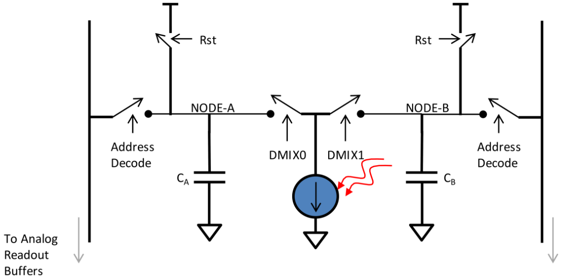
Practical Pixel: Offset, Scene with Ambient Light

A: Voltage on Node-A
B: Voltage on Node-B
V_R: Reference Voltage
V_{AX}: Ambient Light
$V_{OA} \neq V_{OB}$:Reset Voltage



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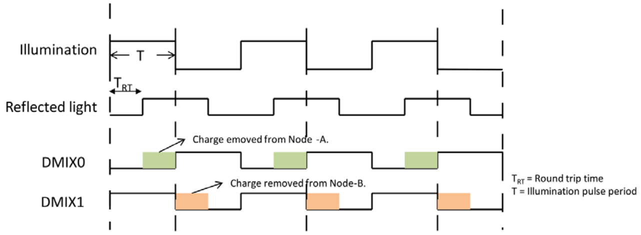

$$A = V_R - (T - T_{RT}) \times K$$
$$B = V_R - T_{RT} \times K$$

$$A = V_{AA} + V_{OA} + V_R - (T - T_{RT}) \times K$$
$$B = V_{AB} + V_{OB} + V_R - T_{RT} \times K$$

2 equations

but

4 unknowns:
$[(V_{AA} + V_{OA}), (V_{AB} + V_{OB}), K, T_{RT}]$

Additional Experiments

NO ILLUMINATION

$$A_o = V_{AA} + V_{OA} + V_R - (T - T_{RT}) \times K$$
$$B_o = V_{AB} + V_{OB} + V_R - T_{RT} \times K$$
$$A_{OFF} = V_{AA} + V_{OA} + V_R$$
$$B_{OFF} = V_{AB} + V_{OB} + V_R$$

4 equations, 4 unknowns!

$$Phase = \frac{B}{(A+B)}$$

$$A = -(T - T_{RT}) \times K$$
$$B = -T_{RT} \times K$$

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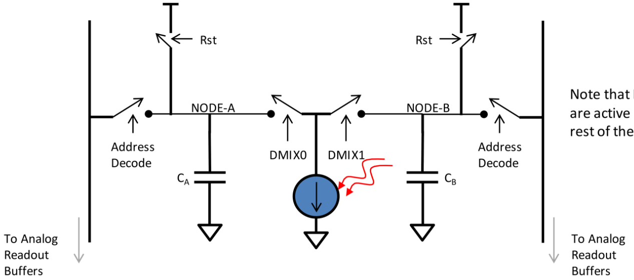
Practical Pixel: Gain Error

A: Voltage on Node-A
B: Voltage on Node-B
V_R: Reference Voltage
V_{AX}: Ambient Light
$V_{OA} \neq V_{OB}$:Reset Voltage
$G_A \neq G_B$: Gain Error



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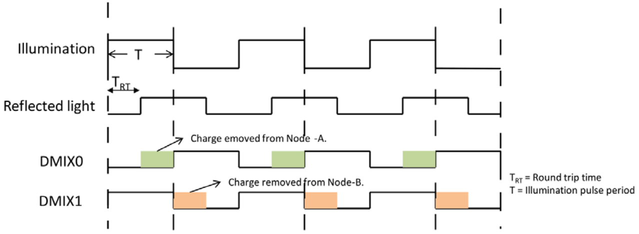

More Mearurements:4-Phase Method

$$A = -G_A (T - T_{RT}) \times K$$
$$B = -G_B T_{RT} \times K$$

4 unknowns:
$[G_A, G_B, K, T_{RT}]$

$$A_0 - B_0 = \text{Cons}_1 - (G_A + G_B) \times 2\cos(T_{RT}/T)$$
$$A_{180} - B_{180} = \text{Cons}_1 + (G_A + G_B) \times 2\cos(T_{RT}/T)$$
$$A_{90} - B_{90} = \text{Cons}_2 - (G_A + G_B) \times 2\sin(T_{RT}/T)$$
$$A_{270} - B_{270} = \text{Cons}_2 + (G_A + G_B) \times 2\sin(T_{RT}/T)$$

$$I = (A_0 - B_0) - (A_{180} - B_{180})$$
$$Q = (A_{90} - B_{90}) - (A_{270} - B_{270})$$

$$Phase = \tan^{-1} Q/I$$

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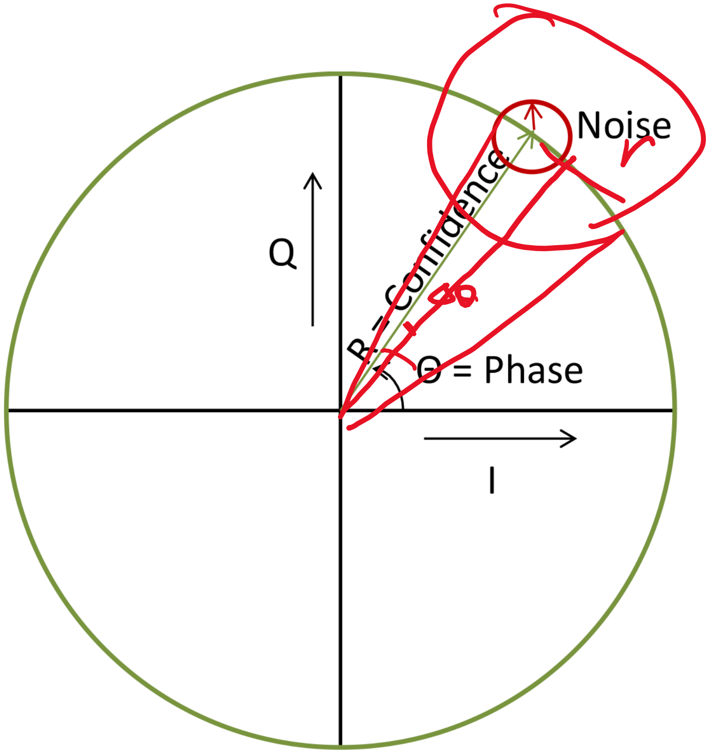
I and Q versus Phase



Noise ∝ Δθ

$\frac{r}{\text{confidence}}$ ∝ Δθ

$$Phase = \tan^{-1} Q/I$$

$$confidence = \sqrt{I^2 + Q^2}$$

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Non-Idealities in the Conversion of Photons to Voltage

Non-Idealities：

➢ 1. Fill factor

$$F_F = \frac{Active\ Pixel\ Area}{Total\ Pixel\ Area}$$

➢ 2. Responsivity (QE: quantum efficiency)

$$QE = \eta\ (\lambda) = \frac{n_e}{n_p}$$

Const

$$responsivity = \frac{i}{p} = \frac{\lambda q_e n_e}{hcn_p} = \boxed{\frac{\lambda q_e}{hc}} \underline{\eta\ (\lambda)}$$

$$v = \frac{q}{c} = \frac{responsivity \times p \times t_I}{c}$$

$$\sqrt{I^2+Q^2}$$

$$confidence \propto \frac{responsivity \times p_I \times F_F \times t_I}{c} \propto p_I \times tI$$

n_e: number of electrons generated
n_p: number of photons hitting the active pixel area

i: current generated
p: power incident on the active pixel area

h: the plank's constant
c: the speed of light in the medium
λ: wavelength of the light used
q_e: the charge of a single electron

q: the charge collected
t_I: the integration time
v: the voltage developed on each node of the pixel

p_I: the incident-modulated power on each pixel during integration time

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Non-Idealities in the Conversion of Photons to Voltage

Non-Idealities：

➢ 3. Demodulation Contrast

$$C_d = \frac{n_{ea} - n_{eb}}{n_{ea} + n_{eb}}$$

$C_d \downarrow$ then,
A-B ↓ then,
confidence ↓

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Noise Sources of iToF

- 1. Reset Noise

$$N_T = \sqrt{kTC}, \qquad nT = \sqrt{kTC}/qe,$$

- 2. Photon Shot Noise

$$n_s = \sqrt{n_e}$$

The Depth Noise/Resolution:

$$d_{noise} = dres = N_d \propto \frac{c\sqrt{n_e + nT^2}}{n_m fmCd} \propto \frac{c}{\sqrt{n_m} f_m C_d}$$

f_m: the modulation frequency
c: the speed of light in the medium
n_m: number of electrons generated resulting from the modulation signal light

Illumination

Illumination Power

$\sqrt{n_m}$, f_m , Cd

The Depth Noise/Resolution:

$$d_{noise} = dres = N_d \propto \frac{c\sqrt{n_e + nT^2}}{n_m\, fmCd}$$

To Reduce the Depth Noise: $n_m \uparrow$

QZ

$$p_E = \frac{4\pi \times p_I \times d^2 \times no.of\ pixels}{r \times aperture}$$

d: object distance
r: reflectance

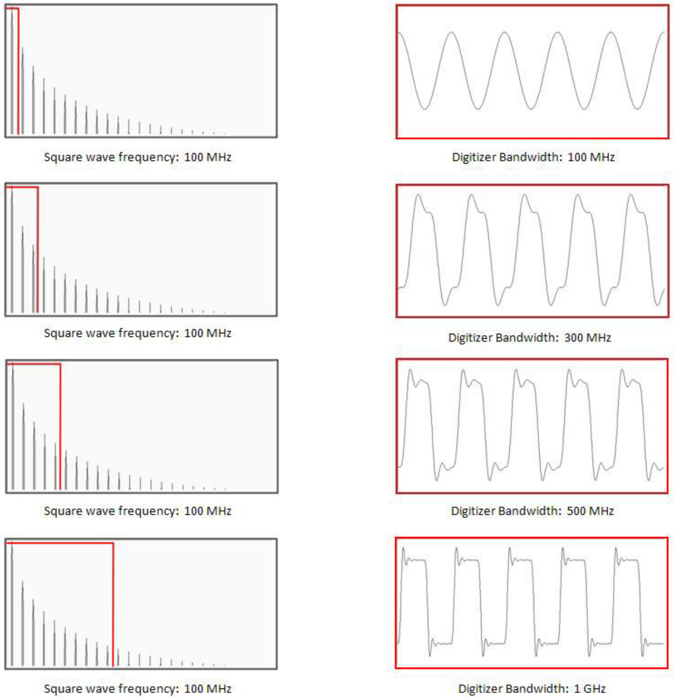
The average Illumination power:

$$p_{Eavg} = p_E \times \boxed{d_{TI}} \times fps$$

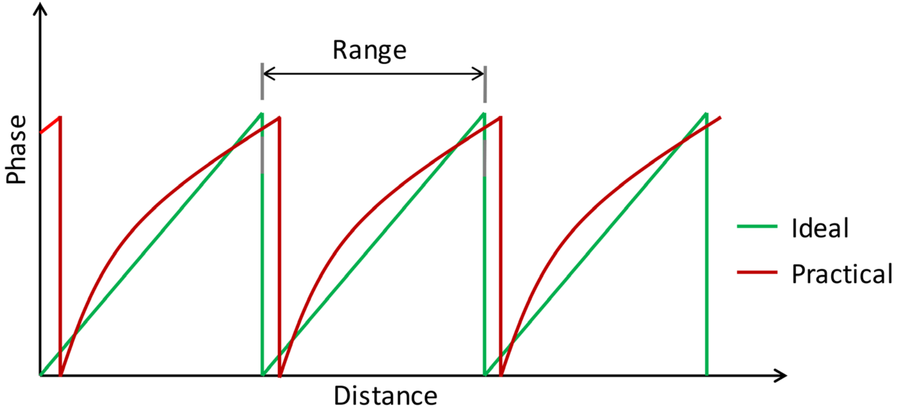
d_{TI}: integration duty cycle
fps: frame rate

Distance and Phase Relationship

However, the illumination CW and reference CW is far away from ideal.



From NI



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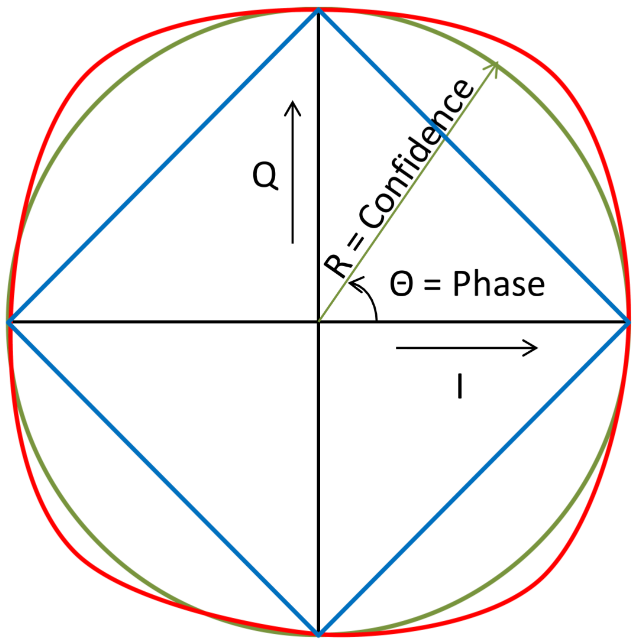
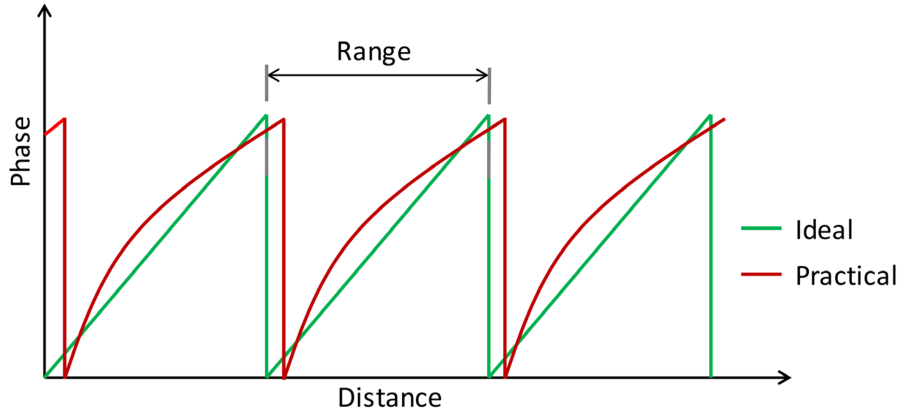
The Non-Linearity

However, the illumination CW and reference CW is far away from ideal.





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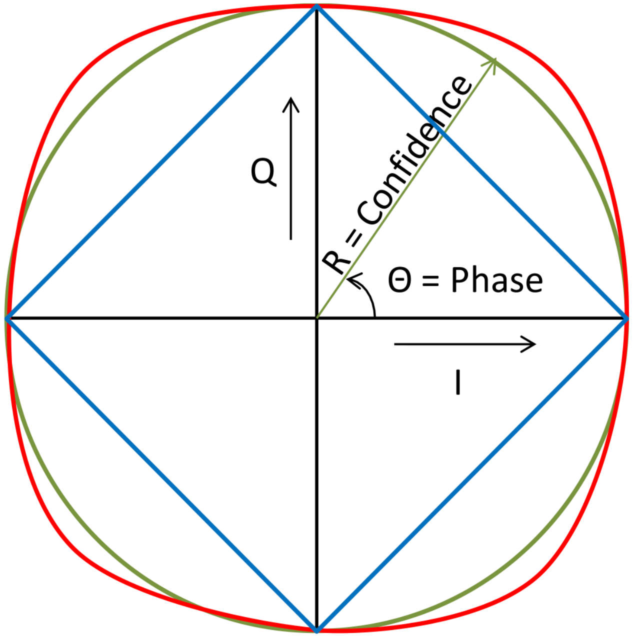
The Non-Linearity Correction



Solution: Build a Look UP Table (**LUT**) through

1. Phase offset sweeping
2. Calibrate use a movable object

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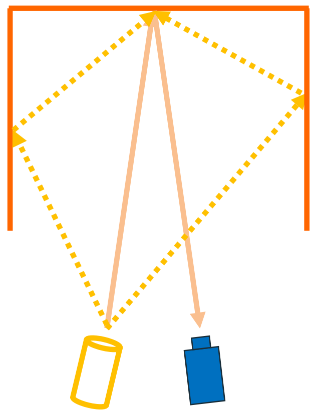
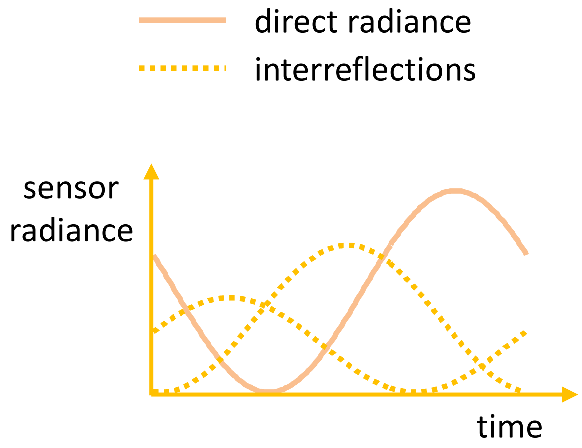
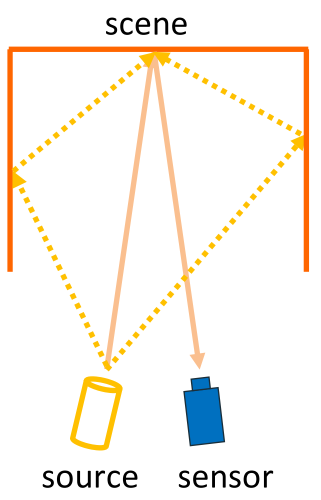
Multipath Interference

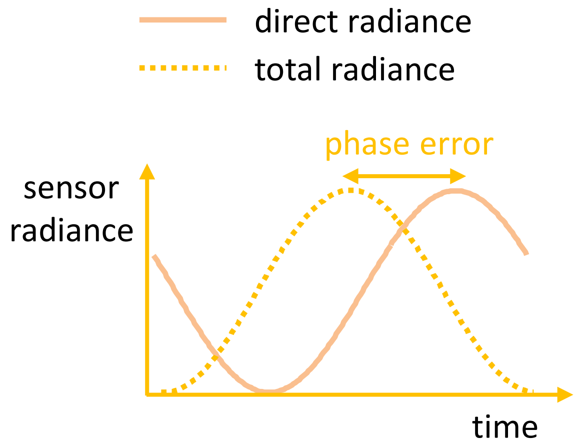
Interreflections and ToF Imaging





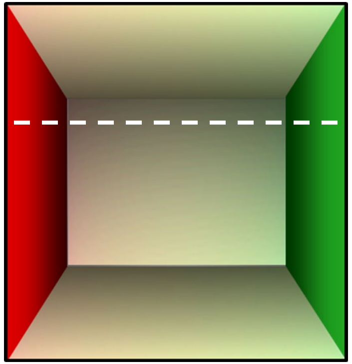
Interreflections and ToF Imaging



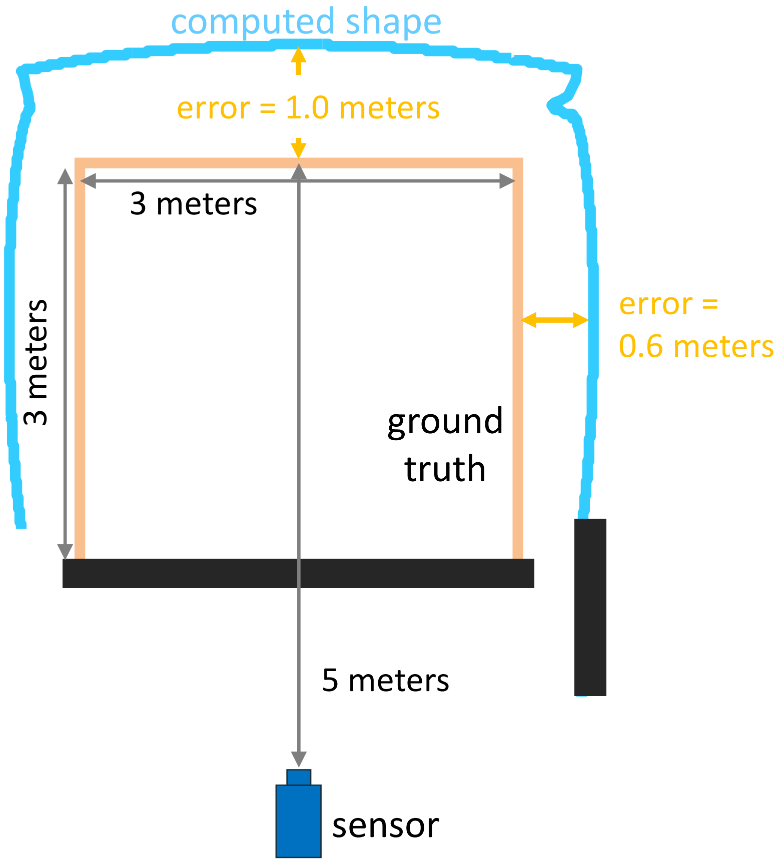


Interreflections Produce Incorrect Phase

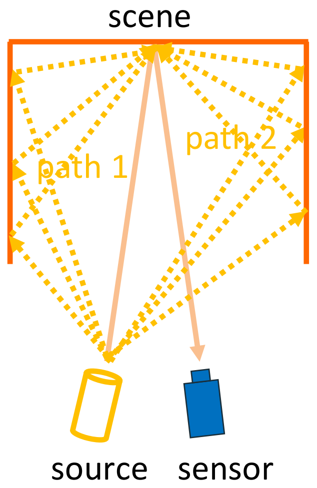
Errors in Shape Recovery

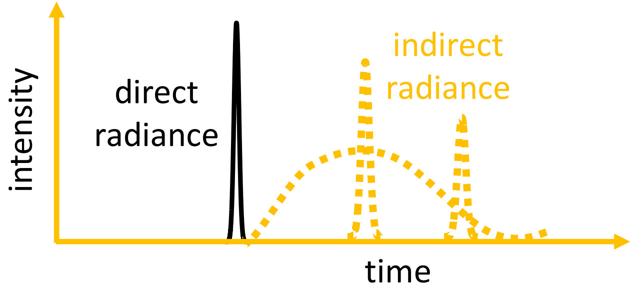


camera view



Multipath Interference





Sparse/Compressible Interreflections

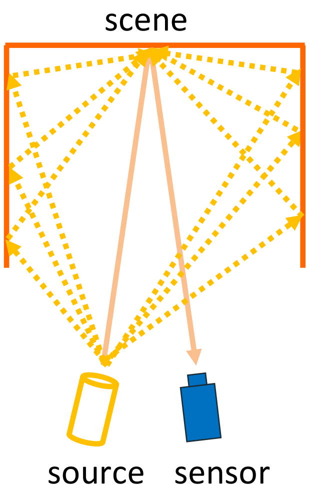
[Godbaz *et al.* 2008, Jimenez *et al.* 2012, Dorrington *et al.* 2011]

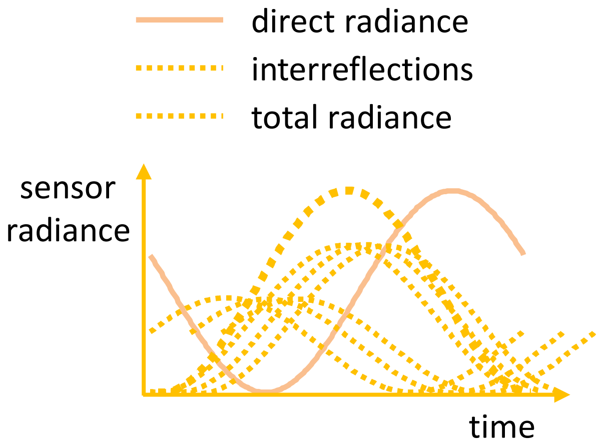
[Godbaz *et al.* 2012, Kadambi *et al.* 2013, Kirmani *et al.* 2013, Freedman *et al.* 2014]

Infinite Indirect Paths

2-3 Indirect Paths

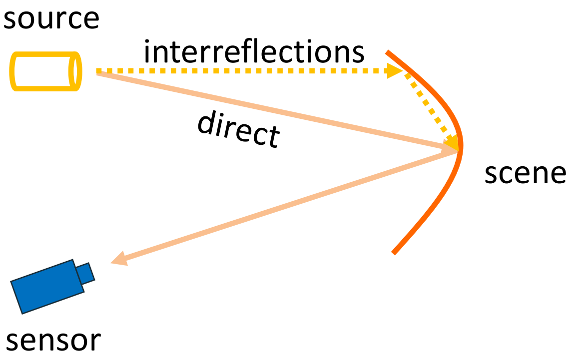
Multipath Interference

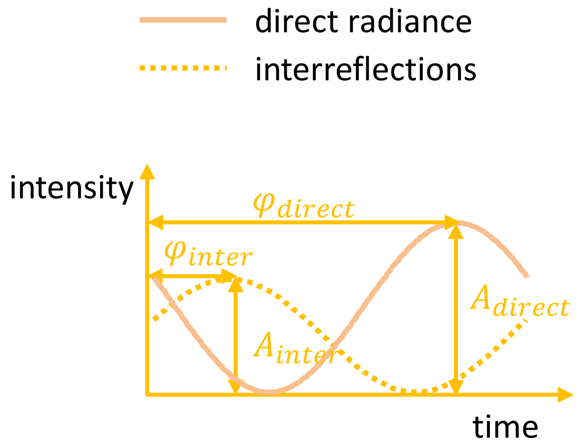




How To Separate Different Components?

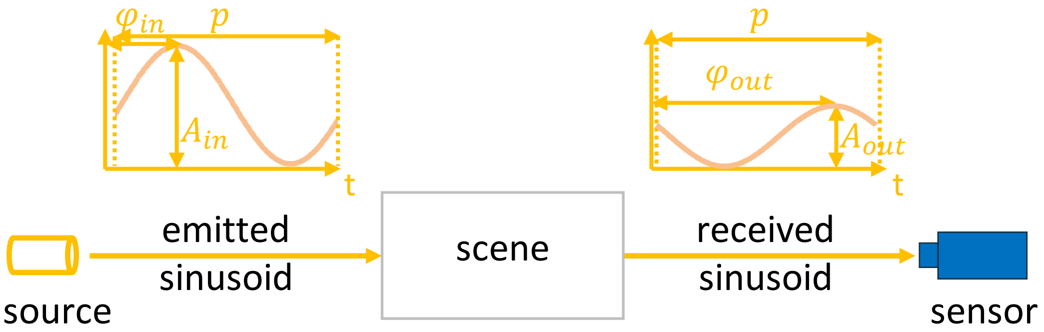
How To Represent Interreflections?





Different Phases   Different Amplitudes

Signal Processing View



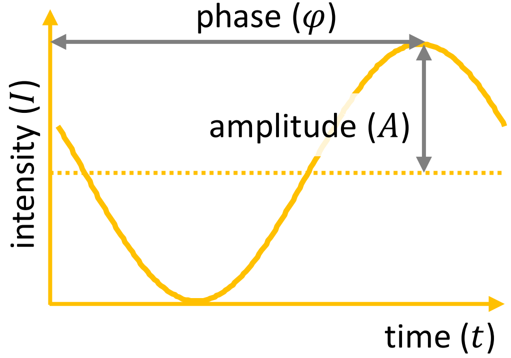
Scene Modulates Only the Phase and the Amplitude

Sinusoid Period (Frequency) Remains Same

Phasor Imaging

Phasor Representation of Sinusoids

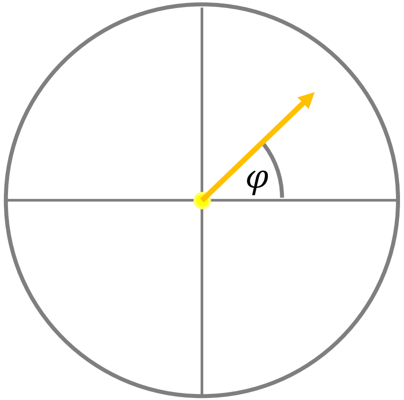
Sinusoid



modulation frequency

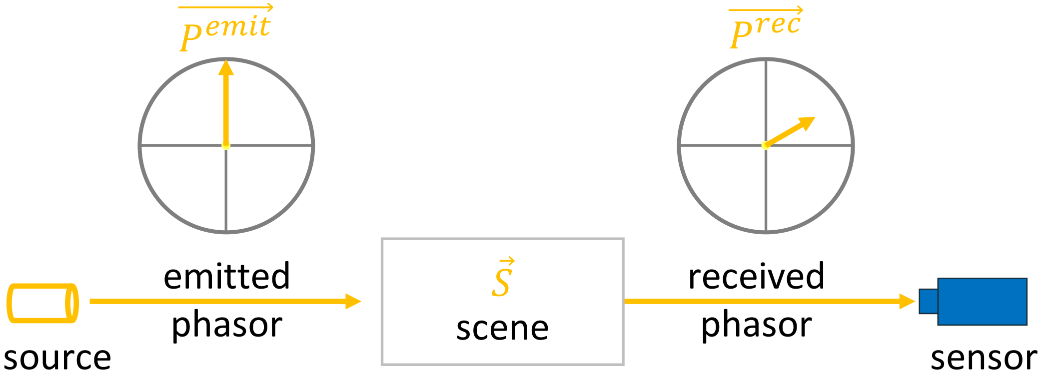
$$I = A\cos(\omega t - \varphi)$$

Phasor



$$\vec{I} = Ae^{-j\varphi}$$

Signal Processing View: Phasor

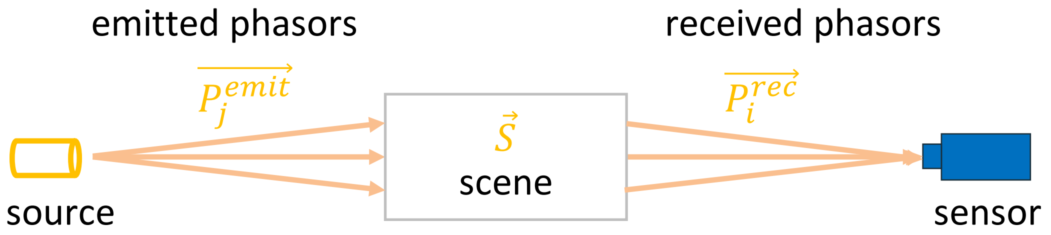


$$\overrightarrow{P^{rec}} = \vec{S} \times \overrightarrow{P^{emit}}$$

light transport coefficient (complex)

One Coefficient for Every Emitted-Received Light Ray Pair

Signal Processing View: Phasor



$$\overrightarrow{P_i^{rec}} = \sum_j \overrightarrow{S_{ij}} \times \overrightarrow{P_j^{emit}}$$

light transport coefficients (complex)

Linear Input-Output Relationship

Phasor Light Transport Matrix

light transport matrix (complex)

array of received intensities (complex)

$$\overrightarrow{[\mathbf{P}^{\mathbf{rec}}]} = \overrightarrow{[\mathbf{S}]} \times \overrightarrow{[\mathbf{P}^{\mathbf{emit}}]}$$

array of emitted intensities (complex)

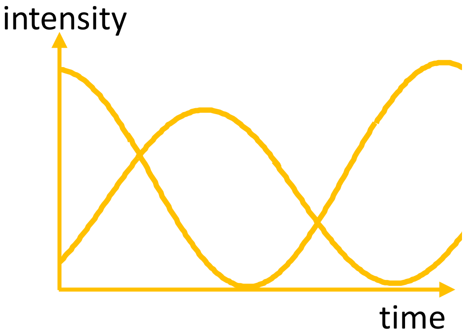
Phasor Light Transport Equation

Same Form As Conventional Light Transport

[O'Toole *et al.* 2014]

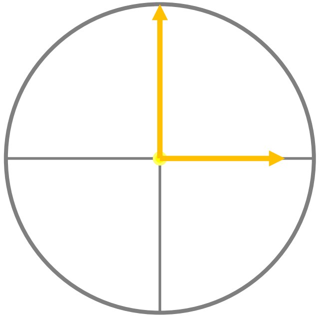
Phasor Light Transport: Reflection Propagation

Sinusoid

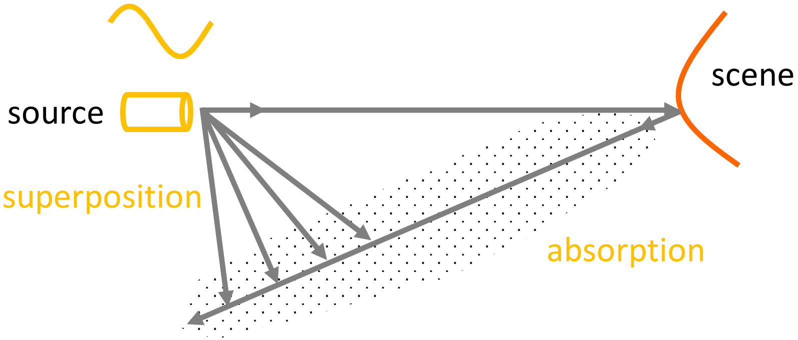


Only
Amplitude Phase
Changes

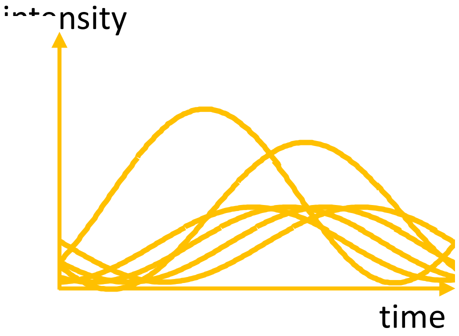
Phasor



Phasor Light Transport: Superposition

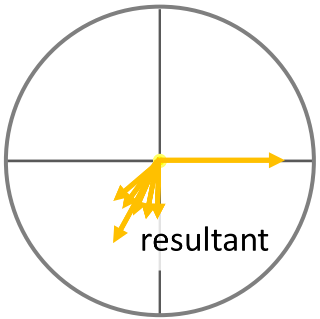


Sinusoid

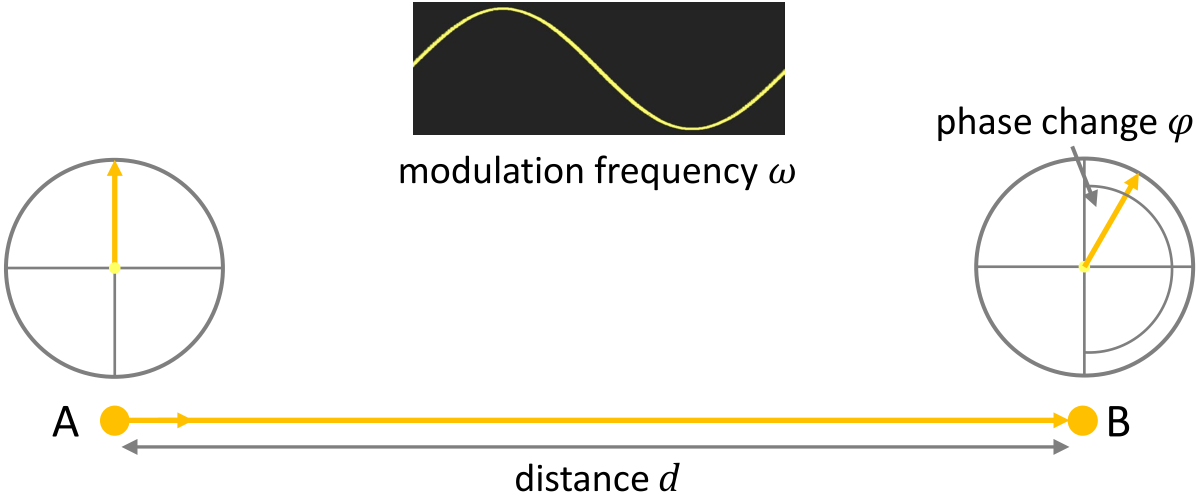


Both Phase and Amplitude Change

Phasor



Phase Change vs. Modulation Frequency

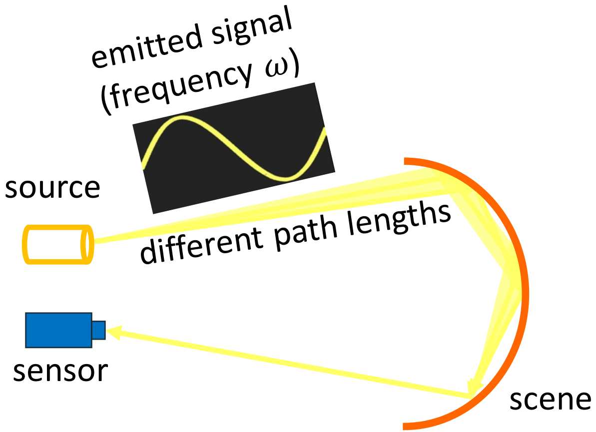


$$\varphi = \frac{\omega d}{c}$$

c ← speed of light

φ is proportional to both d and ω

Interreflections vs. Modulation Frequency



sensor radiance
(interreflections)

Increasing Modulation Frequency

Decreasing Resultant Amplitude

angular spread of phases

$\Delta\varphi \propto \omega$

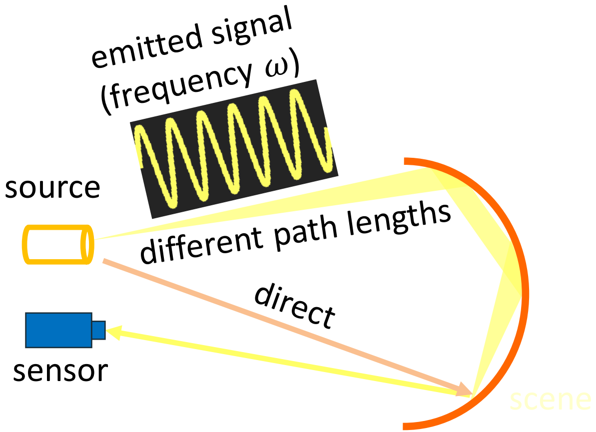
Local Smoothness of Light Transport

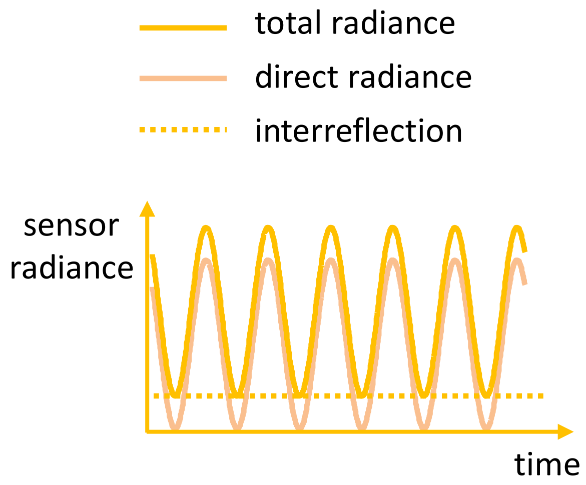
For High Temporal Frequency

[Nayar *et al.* 2006]

Interreflection Component is Constant

Interreflections vs. Modulation Frequency



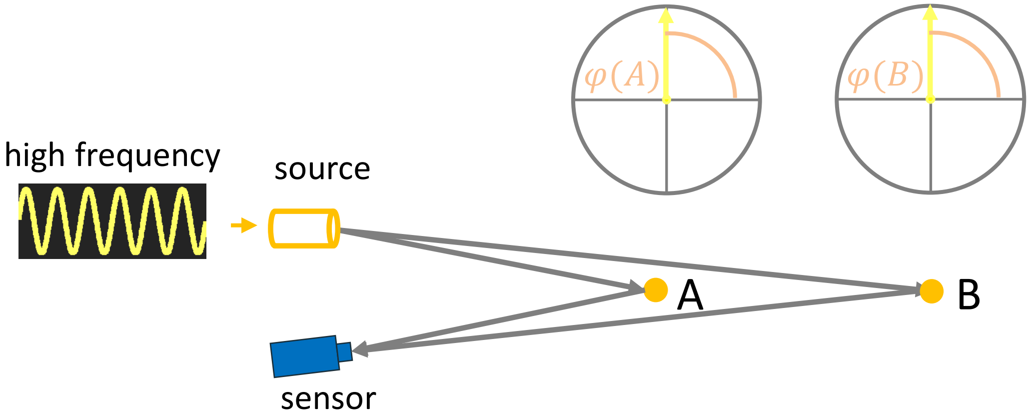


For High Temporal Frequency

Interreflection is Constant

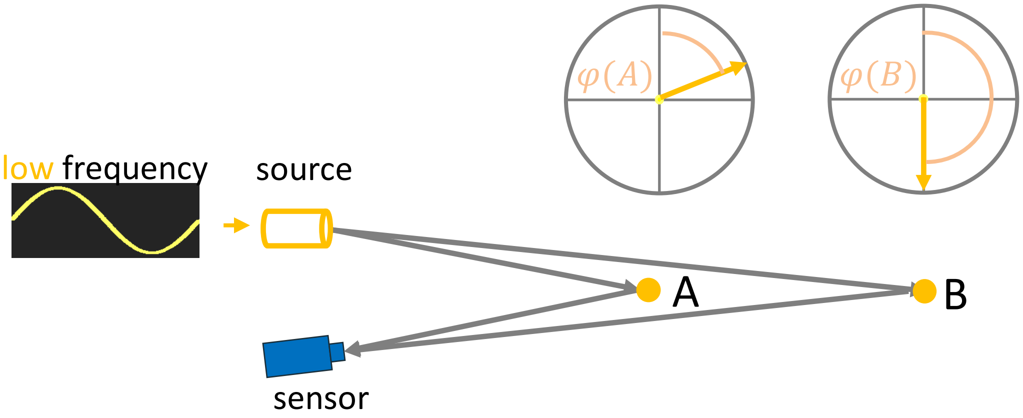
Interreflections Do Not Affect Phase

Phase Ambiguity



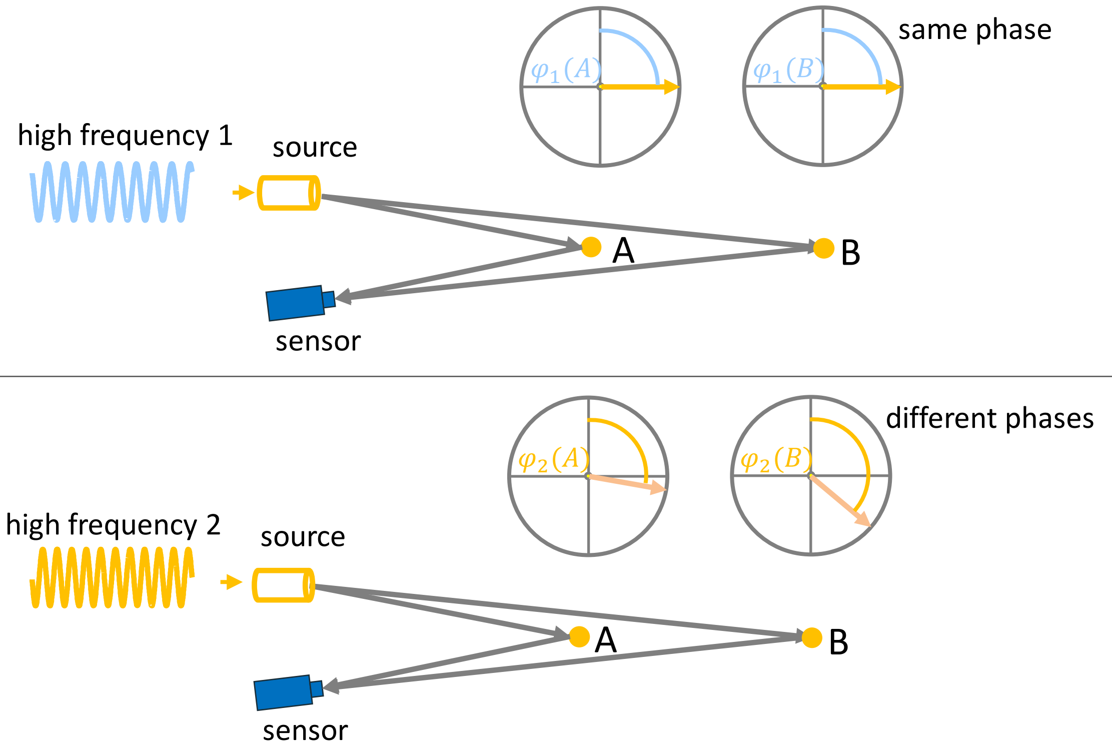
Different Scene Depths Have Same Phase

Phase Ambiguity



Unambiguous Depth Range: $R_{unambiguous} = \frac{1}{2\omega}$

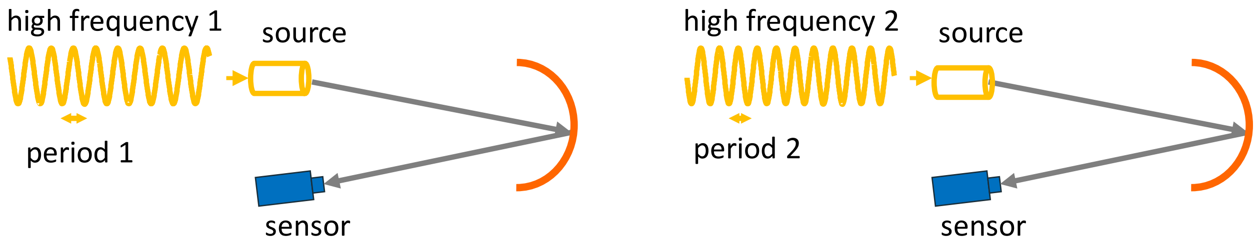
Disambiguating Phase



Compute Phases at Two High Frequencies

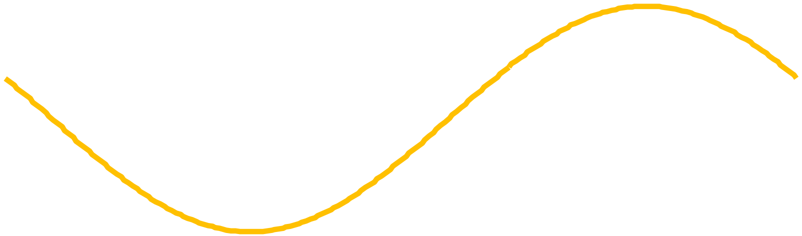
[Jongenelen *et al.* 2010, 2011]

Micro Time-of-Flight Imaging



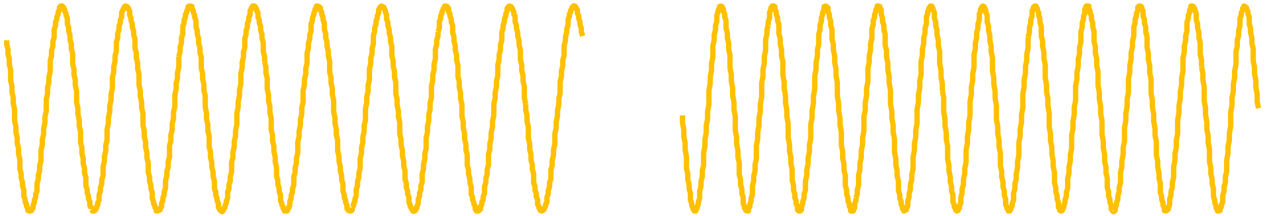
Modulation Signals With Micro (Small) Periods

Conventional vs. Micro ToF Imaging



Conventional ToF Shifting: One Low Frequency

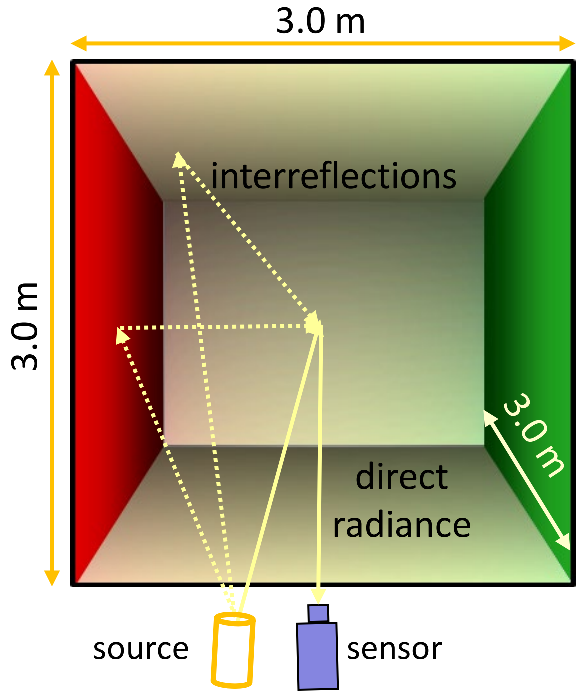
Three Measurements



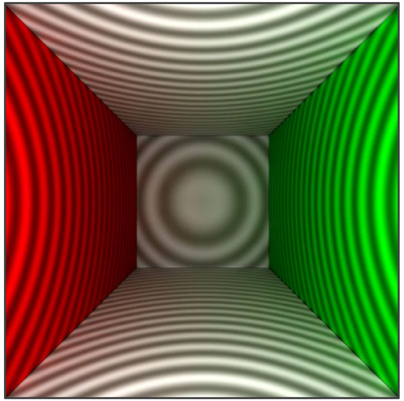
Micro ToF Shifting: Two High Frequencies

Four Measurements

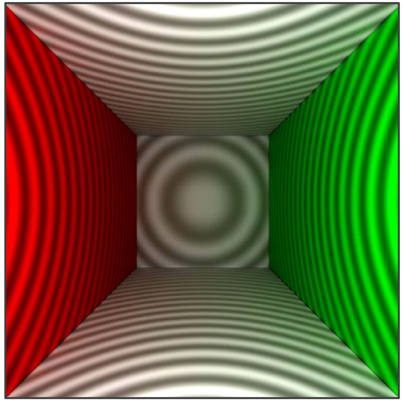
Simulations: Cornell Box



Cornell Box: Input Images

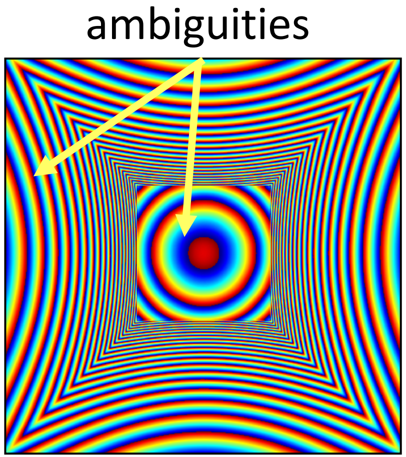


957 MHz.

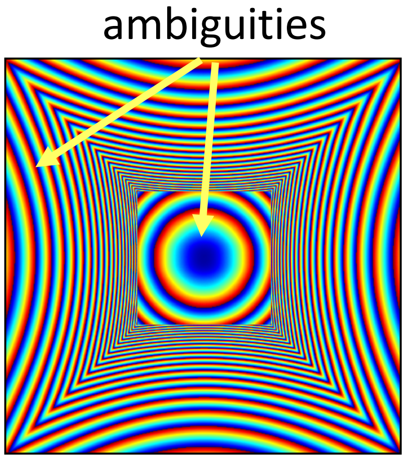


930 MHz.

Cornell Box: Phase Maps

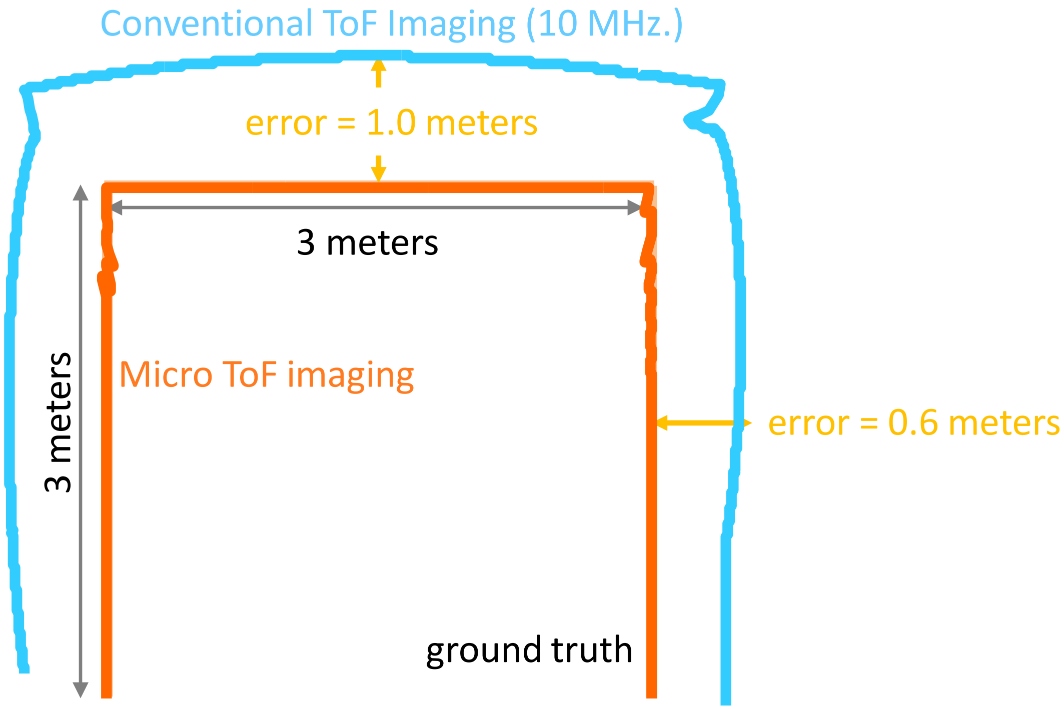


957 MHz.



930 MHz.

Cornell Box: Shape Comparison



Today's **Topic**

- Continuous Wave ToF Imaging
- Imaging Model and Analysis of iToF
- Illumination
- Multipath Interference
- Phasor Imaging

Thank You!

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