



中国科学技术大学
University of Science and Technology of China



GAMES 102在线课程

几何建模与处理基础

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GAMES 102在线课程：几何建模与处理基础

曲面简化

挑战：大规模网格数据

- 存储
- 传输
- 处理
- 渲染



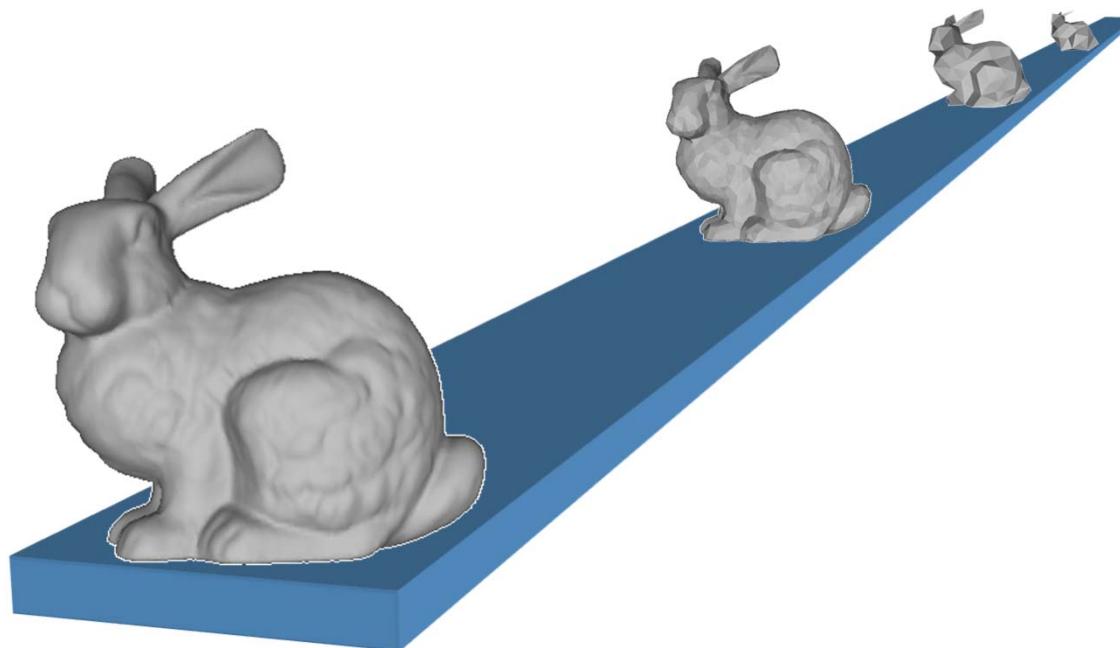
*1087716 faces
543652 vertices*

*1765388 faces
882954 vertices*

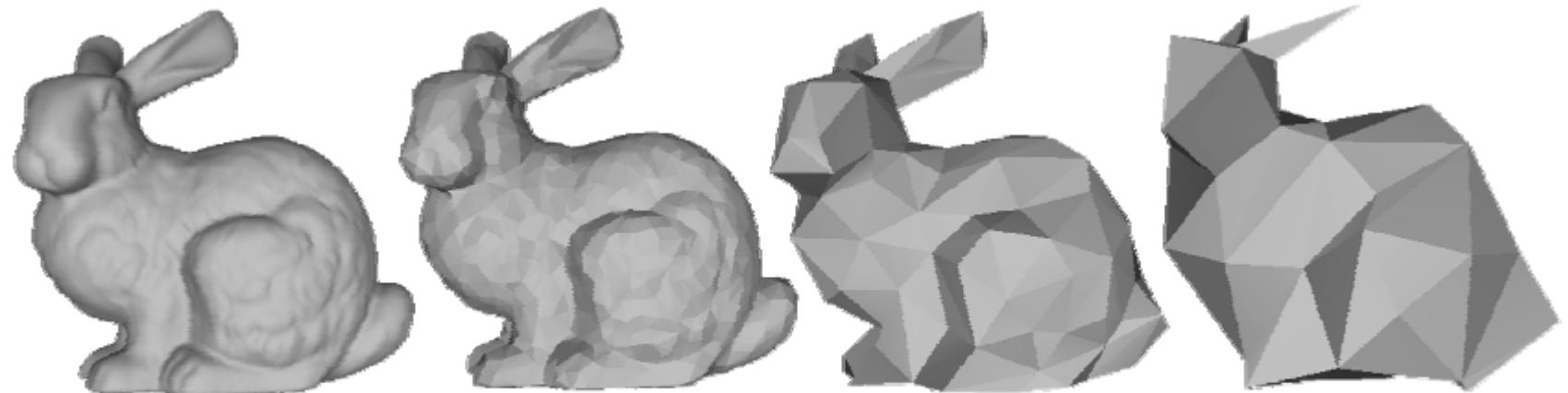


挑战：大规模网格数据

- 冗余数据：信息熵
- 在不损失视觉效果的情况下减少数据量
- Level of details (LOD)



Simplification Examples

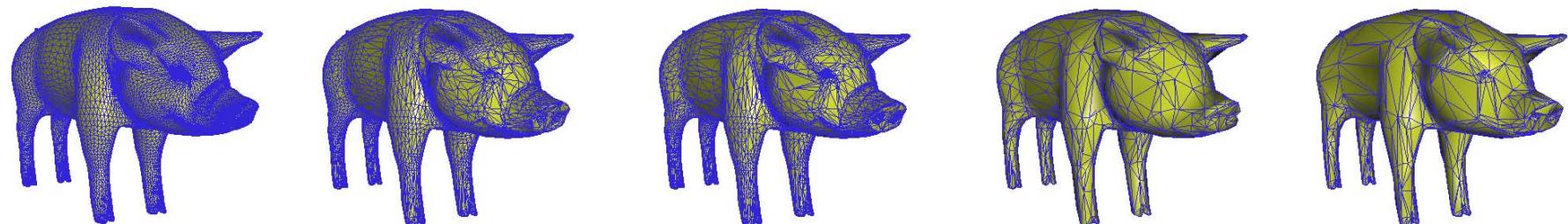


69,451 polys

2,502 polys

251 polys

76 polys



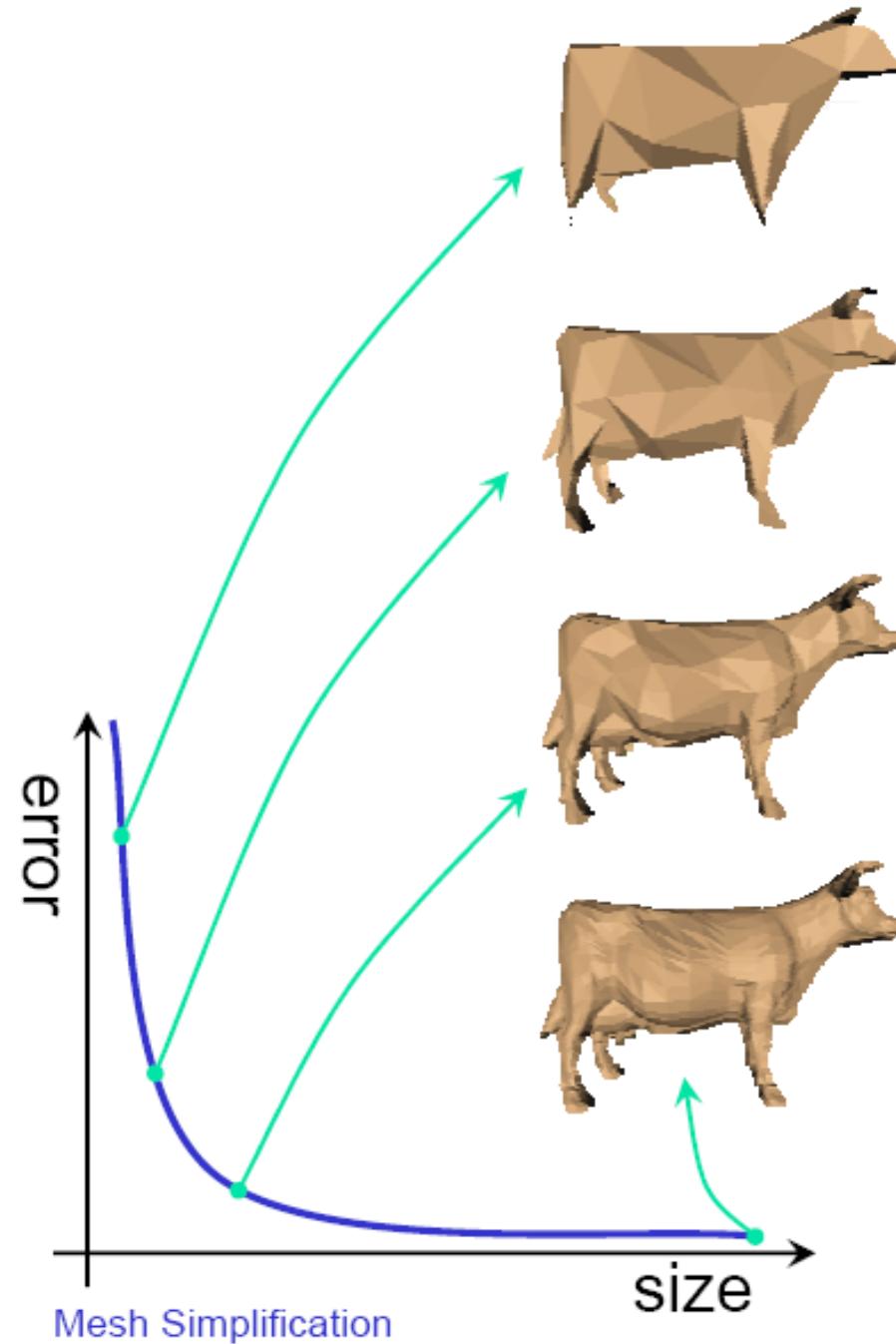
Simplification Applications



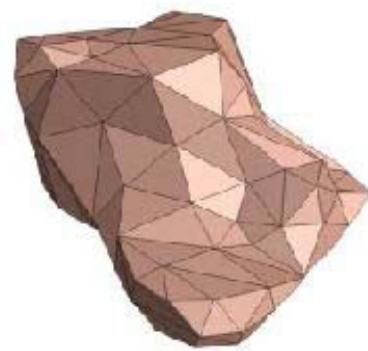
- Level-of-detail modeling
 - Generate a family of models for the same object with different polygon counts
 - Select the appropriate model based on estimates of the object's projected size
- Simulation proxies
 - Run the simulation on a simplified model
 - Interpolate results across a more complicated model to be used for rendering

Tradeoff

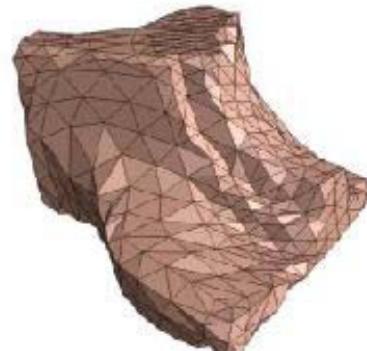
- Size
- Error



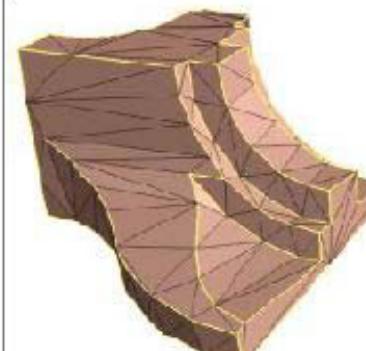
Quality



92 faces



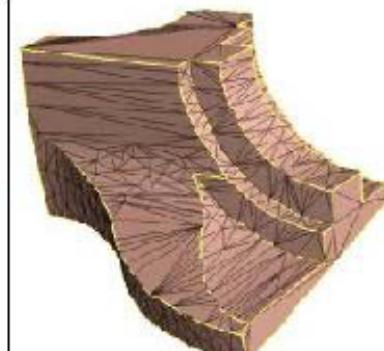
1,070 faces



PM (200 faces)



Mⁿ (12,946 faces)



PM (1,000 faces)

Performance Requirements

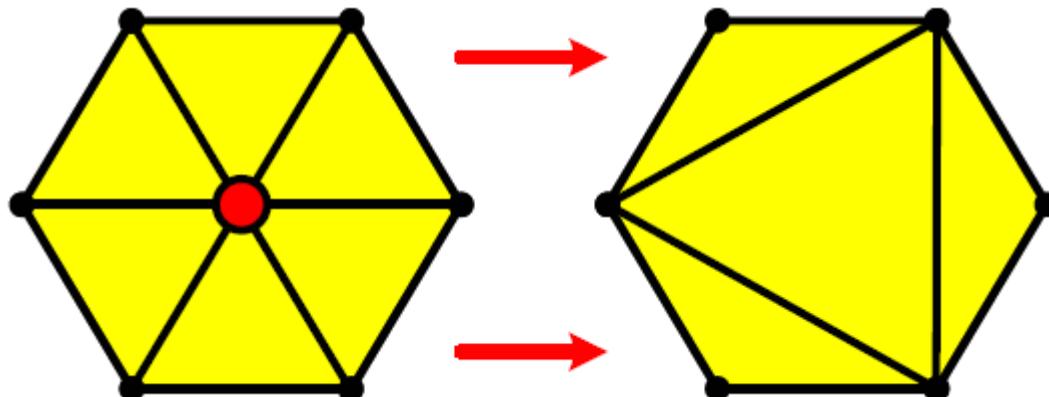
- Offline
 - Generate model at given level(s) of detail
 - Focus on quality
- Real-time
 - Generate model at given level(s) of detail
 - Focus on speed
 - Requires preprocessing
 - Time/space/quality tradeoff

简化方法

- 几何对象
 - 顶点
 - 边
 - 面
- 简化度量
 - 几何
 - 视觉：纹理、材质、法向...

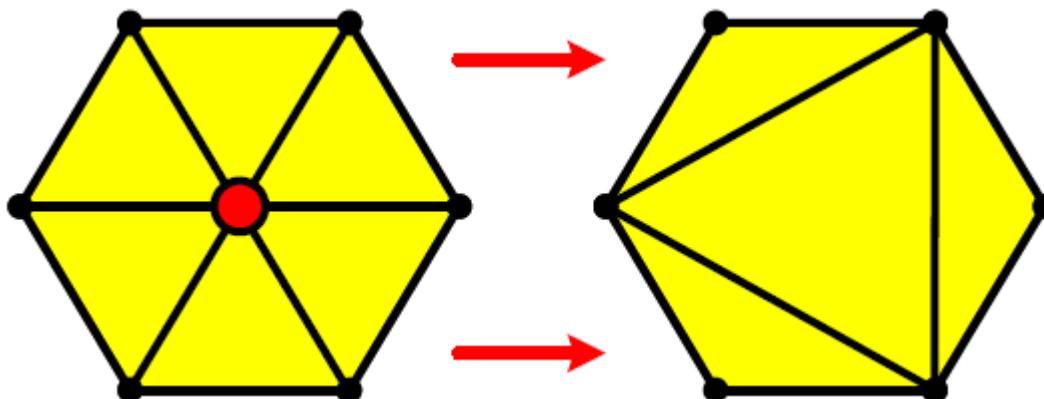
Methodology

- Sequence of local operations
 - Involve near neighbors - only small patch affected in each operation
 - Each operation introduces error
 - Find and apply operation which introduces the least error



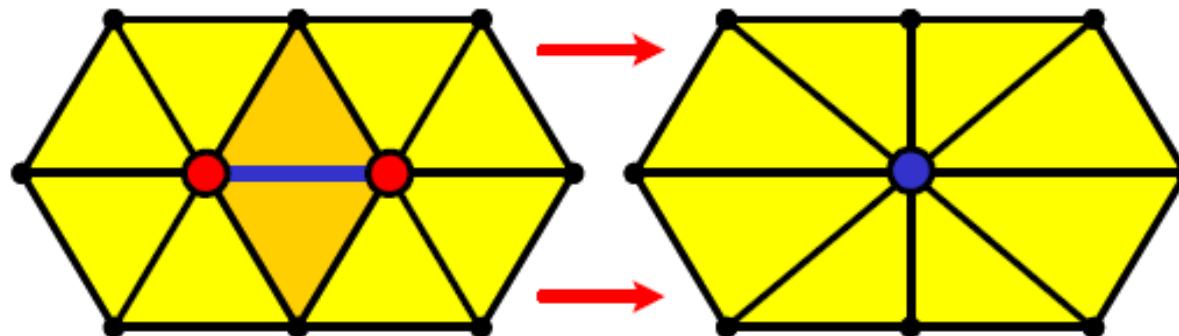
Simplification Operations (1)

- Decimation
 - Vertex removal:
 - $v \leftarrow v-1$
 - $f \leftarrow f-2$
 - Remaining vertices - subset of original vertex set



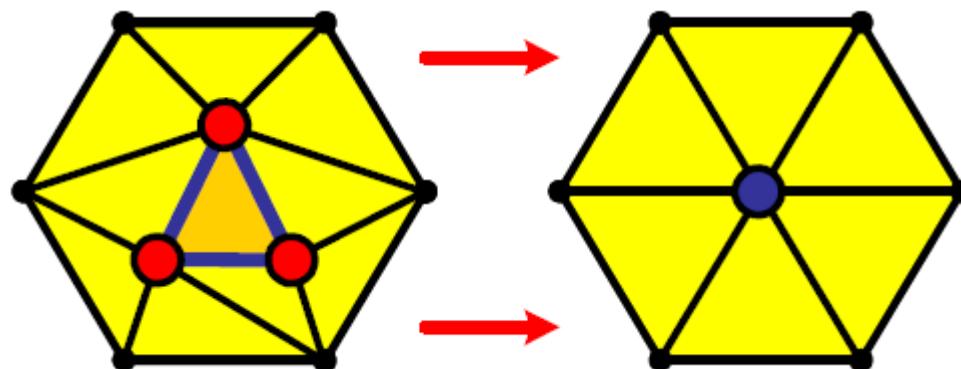
Simplification Operations (2)

- Decimation
 - Edge collapse
 - $v \leftarrow v-1$
 - $f \leftarrow f-2$
 - Vertices may move



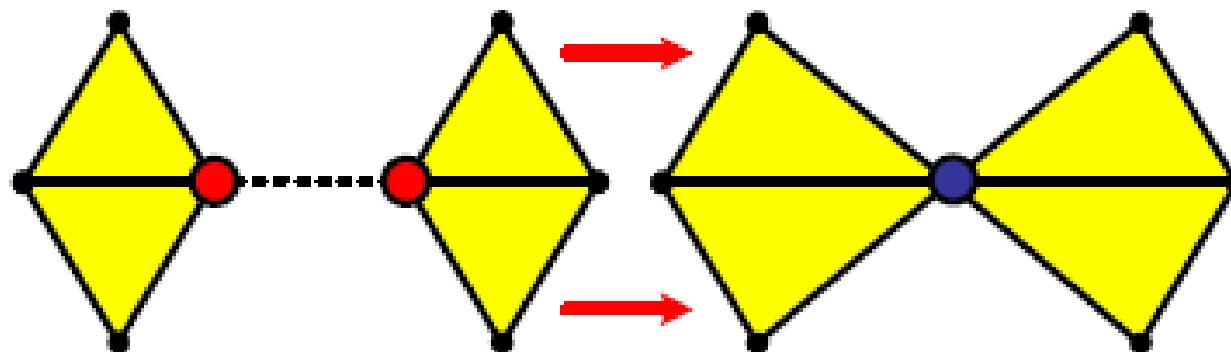
Simplification Operations (3)

- Decimation
 - Triangle collapse
 - $v \leftarrow v-2$
 - $f \leftarrow f-4$
 - Vertices may move



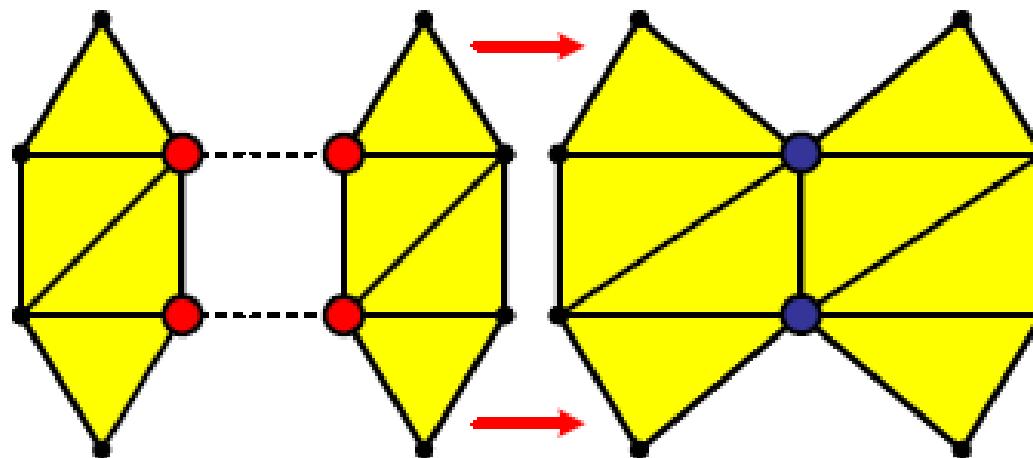
Simplification Operations (4)

- Contraction
 - Pair contraction (cluster of two vertices)
- Vertices may move



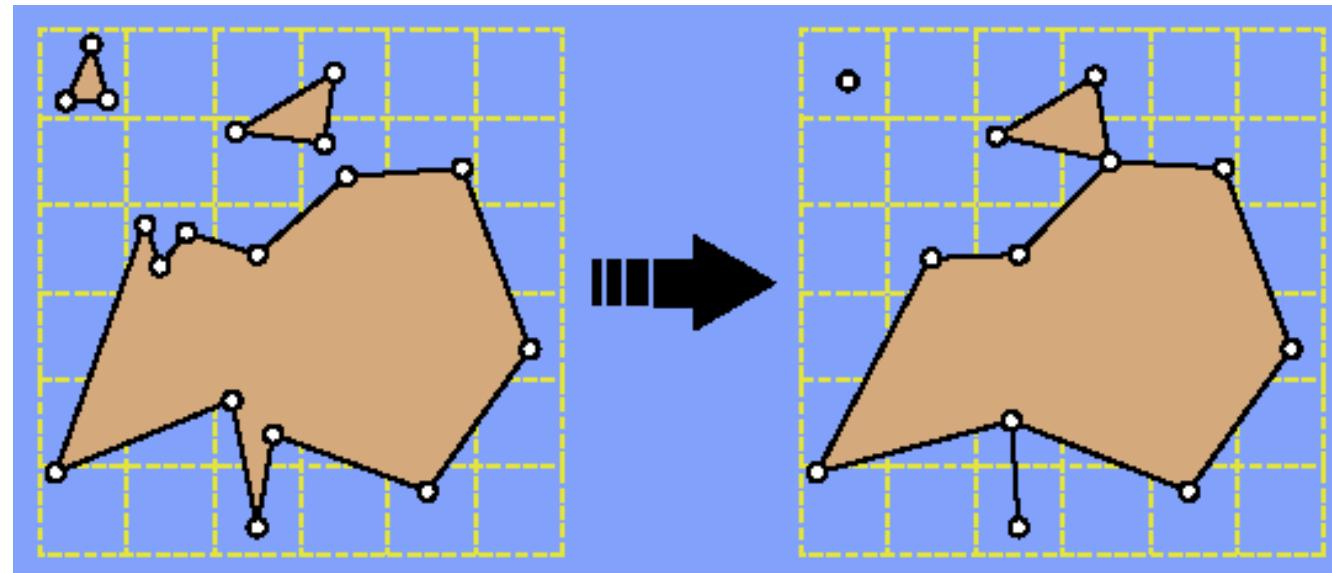
Simplification Operations (5)

- Contraction
 - Cluster contraction (set of vertices)
- Vertices may move



Simplification Operations: Vertex Clustering

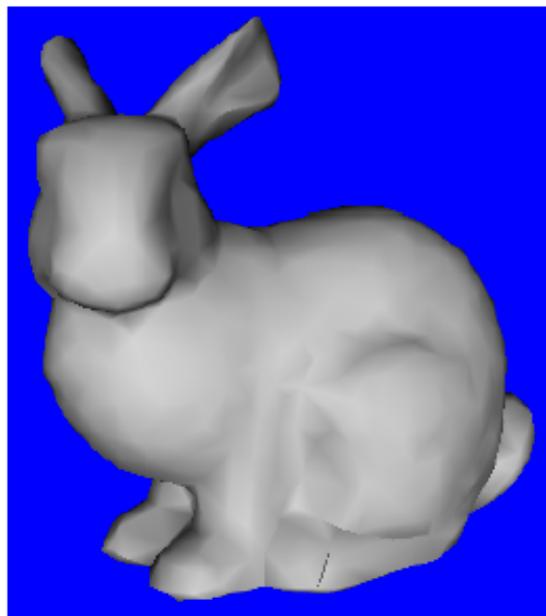
- Merge all vertices within the same cell



Error Control

- Local error: Compare new patch with previous iteration
 - Fast
 - Accumulates error
 - Memory-less
- Global error: Compare new patch with original mesh
 - Slow
 - Better quality control
 - Can be used as termination condition
 - Must remember the original mesh throughout the algorithm

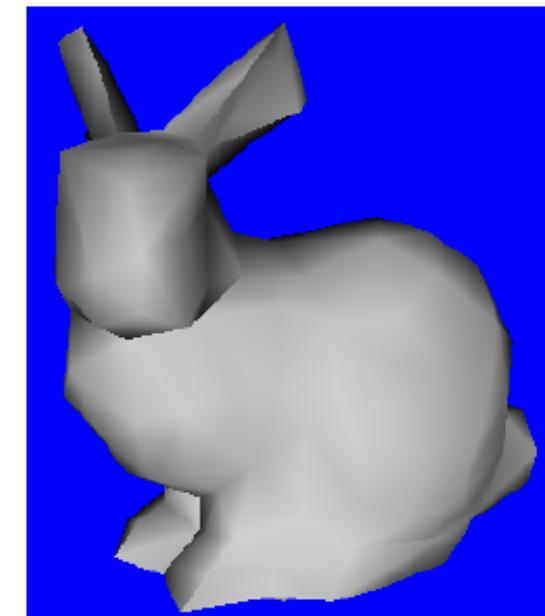
Local vs. Global Error



2000 faces



488 faces



488 faces

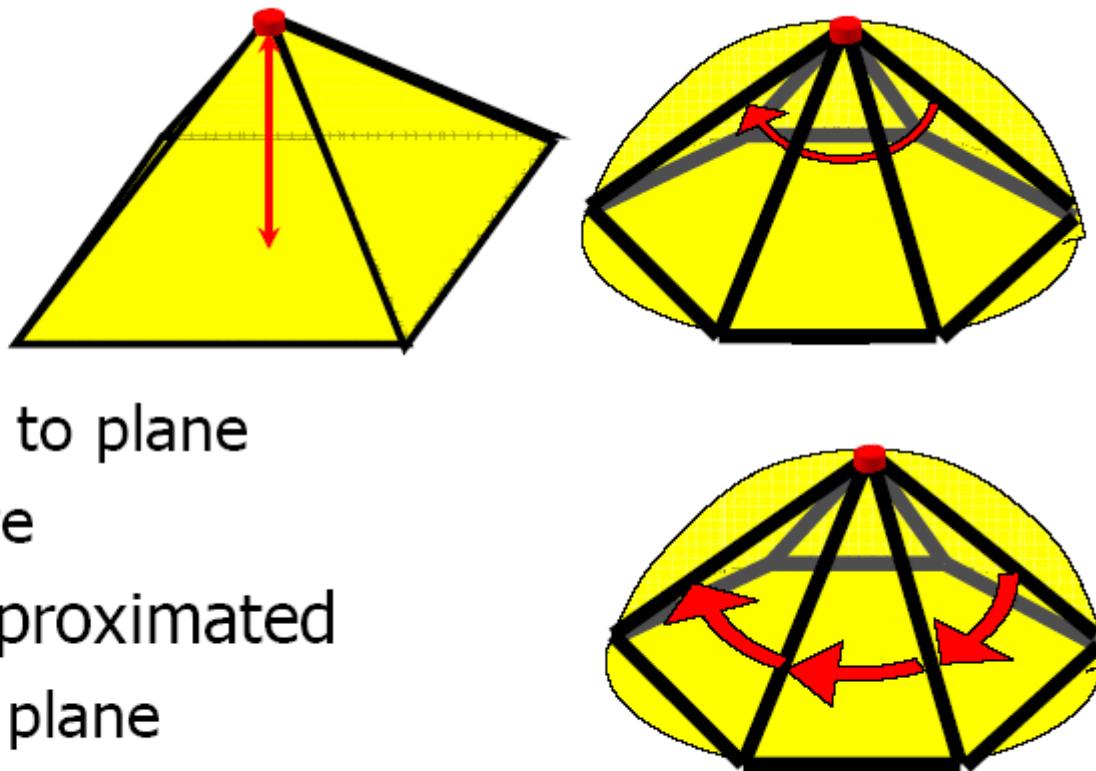
1. Local Simplification Strategies

The Basic Algorithm

- Repeat
 - Select the element with minimal error
 - Perform simplification operation (remove/contract)
 - Update error (local/global)
- Until mesh size / quality is achieved

Simplification Error Metrics

- Measures
 - Distance to plane
 - Curvature
- Usually approximated
 - Average plane
 - Discrete curvature

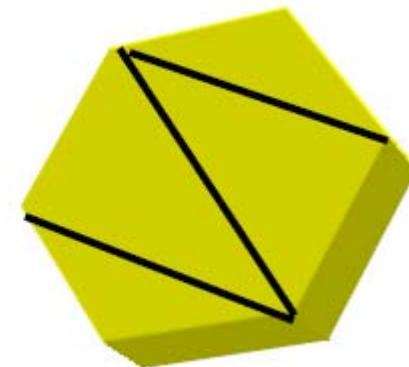
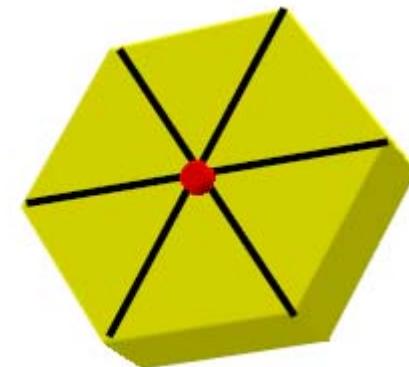


Implementation Details

- Vertices/Edges/Faces data structure
 - Easy access from each element to neighboring elements
- Use priority queue (e.g. heap)
 - Fast access to element with minimal error
 - Fast update

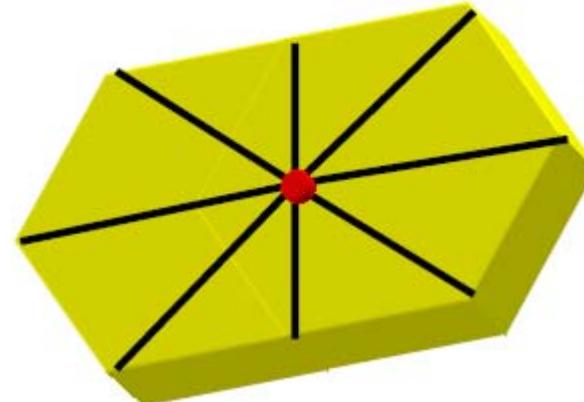
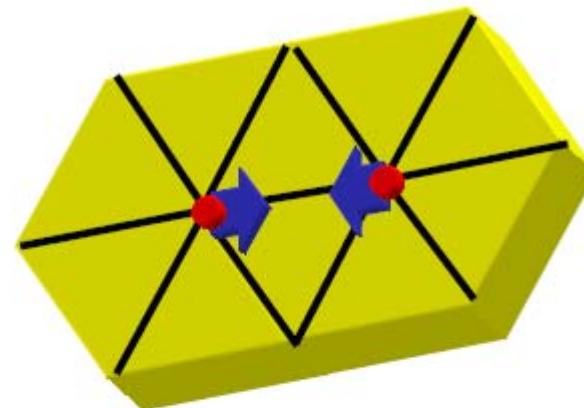
1.1 Vertex Removal [Schroeder et al 92]

- Simplification operation:
Vertex removal
- Error metric: Distance to
average plane
- May preserve mesh
features (creases)



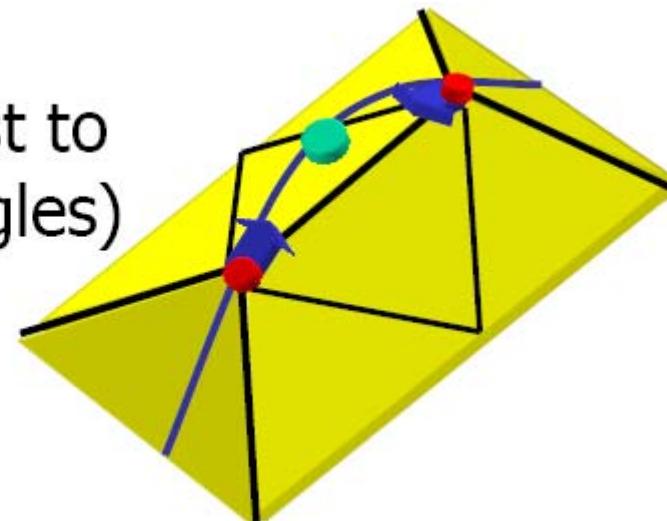
1.2 Edge Collapse [Hoppe et al 93]

- Simplification operation:
Pair contraction
- Error metric:
distance, pseudo-global
- Simplifies also topology

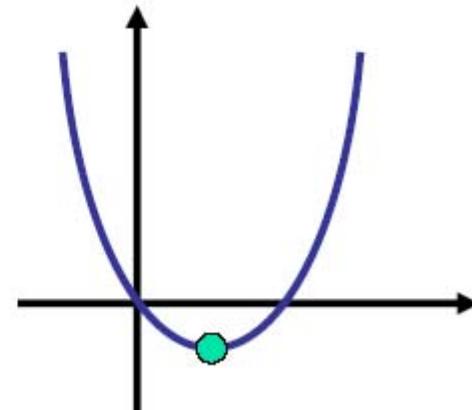


Distance Metric: Quadrics

- Choose point closest to set of planes (triangles)



- Sum of squared distances to set of planes is quadratic \Rightarrow has a minimum



The Quadric Error Metric (QEM)

[Garland & Heckbert 1997]

- Given a plane, we can define a **quadric** Q

$$Q = (\mathbf{A}, \mathbf{b}, c) = (\mathbf{n}\mathbf{n}^T, d\mathbf{n}, d^2)$$

measuring squared distance to the plane as

$$Q(\mathbf{v}) = \mathbf{v}^T \mathbf{A} \mathbf{v} + 2\mathbf{b}^T \mathbf{v} + c$$

$$Q(\mathbf{v}) = [x \ y \ z] \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + 2[ad \ bd \ cd] \begin{bmatrix} x \\ y \\ z \end{bmatrix} + d^2$$

The Quadric Error Metric

- Sum of quadrics represents set of planes

$$\sum_i (\mathbf{n}_i^T \mathbf{v} + d_i)^2 = \sum_i Q_i(\mathbf{v}) = \left(\sum_i Q_i \right)(\mathbf{v})$$

- Each vertex has an associated quadric

- $\text{Error}(v_i) = Q_i(v_i)$
- Sum quadrics when contracting $(v_i, v_j) \rightarrow v'$
- Cost of contraction is $Q(v')$

$$Q = Q_i + Q_j = (\mathbf{A}_i + \mathbf{A}_j, \mathbf{b}_i + \mathbf{b}_j, c_i + c_j)$$

The Quadric Error Metric

- Sum of endpoint quadrics determines \mathbf{v}'
 - Fixed placement: select v_1 or v_2
 - Optimal placement: choose \mathbf{v}' minimizing $Q(\mathbf{v}')$
$$\nabla Q(\mathbf{v}') = 0 \Rightarrow \mathbf{v}' = -\mathbf{A}^{-1}\mathbf{b}$$
 - Fixed placement is faster but lower quality
 - But it also gives smaller progressive meshes
 - Fallback to fixed placement if \mathbf{A} is non-invertible

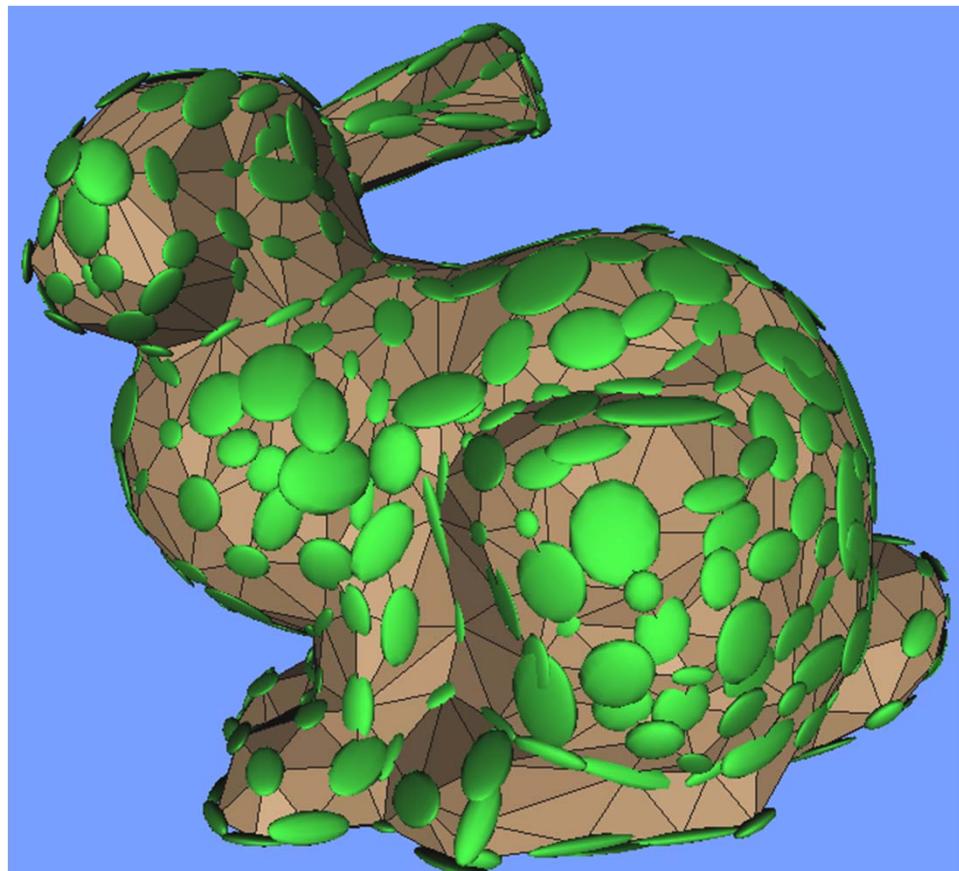
Contracting Two Vertices

- **Goal:** Given edge $e = (v_1, v_2)$, find contracted
 $v = (x, y, z)$ that minimizes $\Delta(v)$:
 $\partial\Delta/\partial x = \partial\Delta/\partial y = \partial\Delta/\partial z = 0$
- Solve system of linear *normal equations*:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

If no solution - select the edge midpoint

Visualizing Quadratics



- Quadric isosurfaces
 - Are ellipsoids (maybe degenerate)
 - Centered around vertices
 - Characterize shape
 - Stretch in least-curved directions

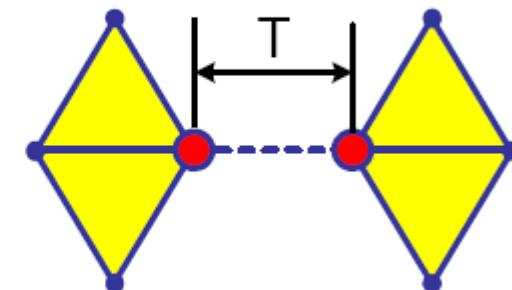
Selecting Valid Pairs for Contraction

- Edges:

$$\{(v_1, v_2) : (v_1 v_2) \text{ is in the mesh}\}$$

- Close vertices:

$$\{(v_1, v_2) : ||v_1 - v_2|| < T\}$$

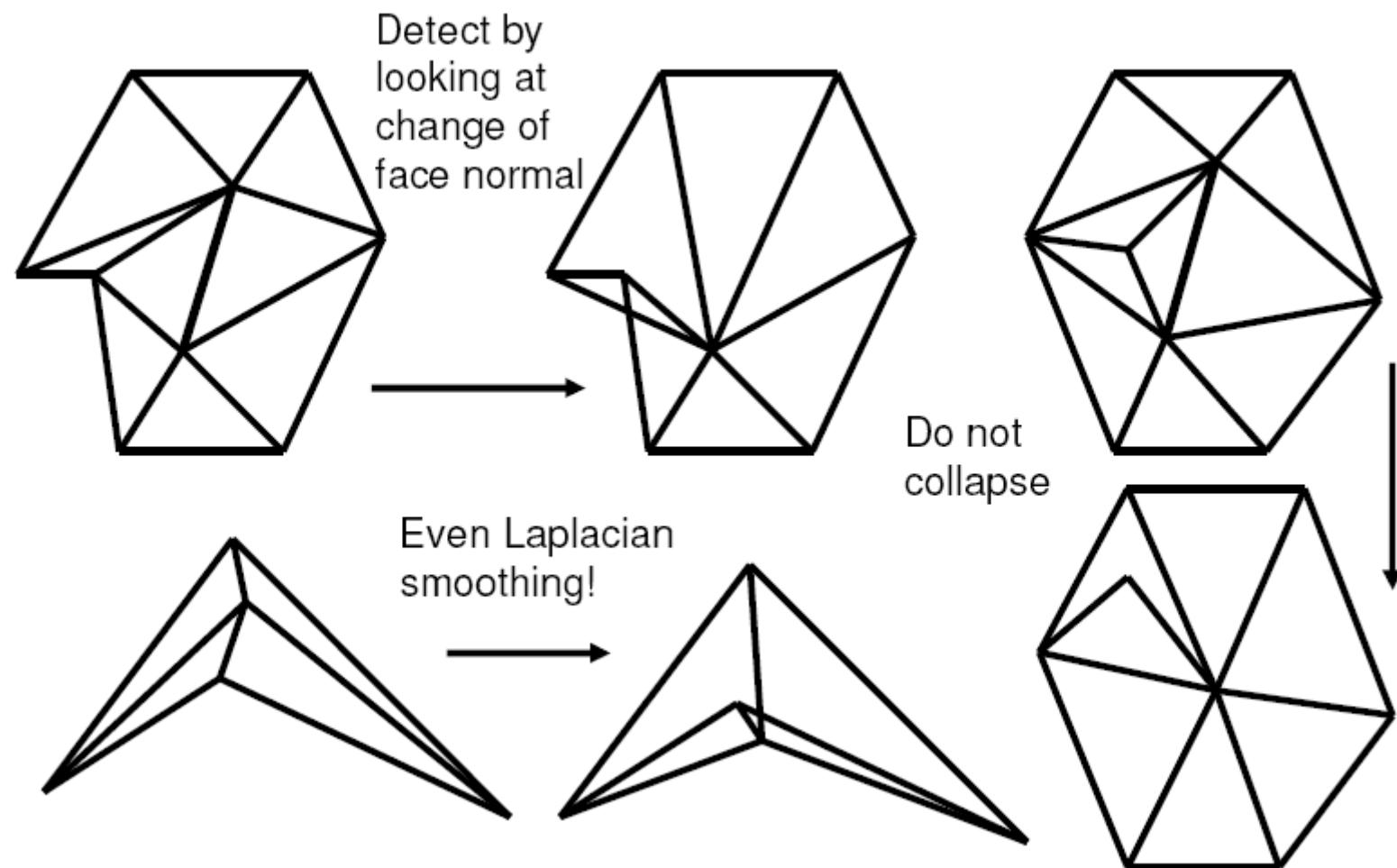


- Threshold T is input parameter

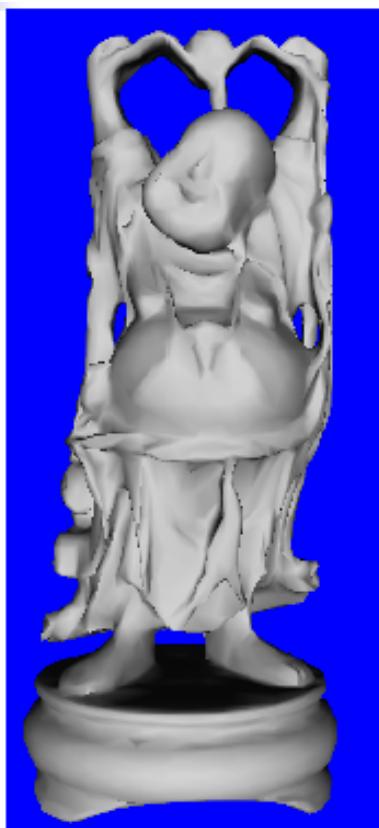
Algorithm

- Compute Q_v for all the mesh vertices
- Identify all valid pairs
- Compute for each valid pair (v_1, v_2) the contracted vertex v and its error $\Delta(v)$
- Store all valid pairs in a priority queue (according to $\Delta(v)$)
- While reduction goal not met
 - Contract edge (v_1, v_2) with the smallest error to v
 - Update the priority queue with new valid pairs

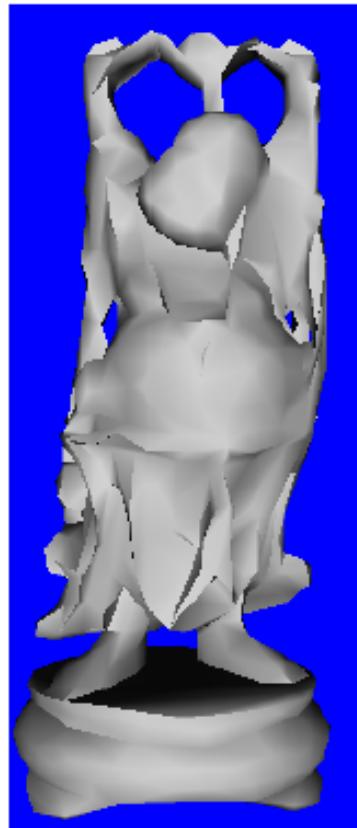
Artifacts by Edge Collapse



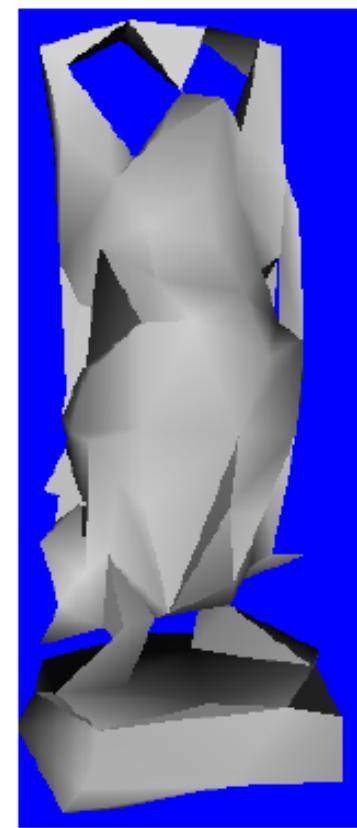
Examples



Original - 12,000



2,000 faces



298 faces (140 vertices)

Pros and Cons

- Pros
 - Error is bounded
 - Allows topology simplification
 - High quality result
 - Quite efficient
- Cons
 - Difficulties along boundaries
 - Difficulties with coplanar planes
 - Introduces new vertices not present in the original mesh

1.3 Appearance-based metrics

- Generalization required to handle appearance properties
 - color
 - texture
 - normals
 - etc.
- Treat each vertex as a 6-vector $[x,y,z,r,g,b]$
 - Assume this 6D space is Euclidean
 - Of course, color space is only roughly Euclidean
 - Scale xyz space to unit cube for consistency

Generalized Quadric Metric

	Vertex	Dimension
Color	[x y z r g b]	6x6 quadrics
Texture	[x y z s t]	5x5 quadrics
Normal	[x y z u v w]	6x6 quadrics
Color+Normal	[x y z r g b u v w]	9x9 quadrics

$$Q(\mathbf{v}) = \mathbf{v}^T \mathbf{A} \mathbf{v} + 2\mathbf{b}^T \mathbf{v} + c$$

Example

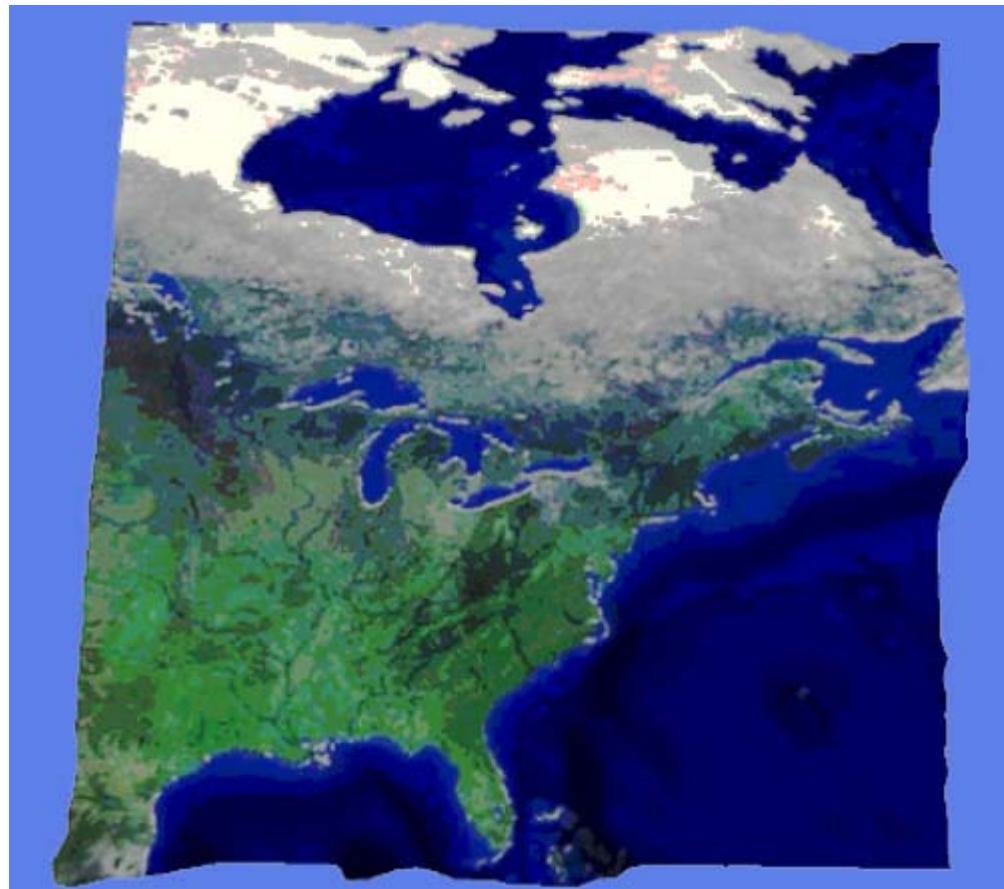


50761 triangles

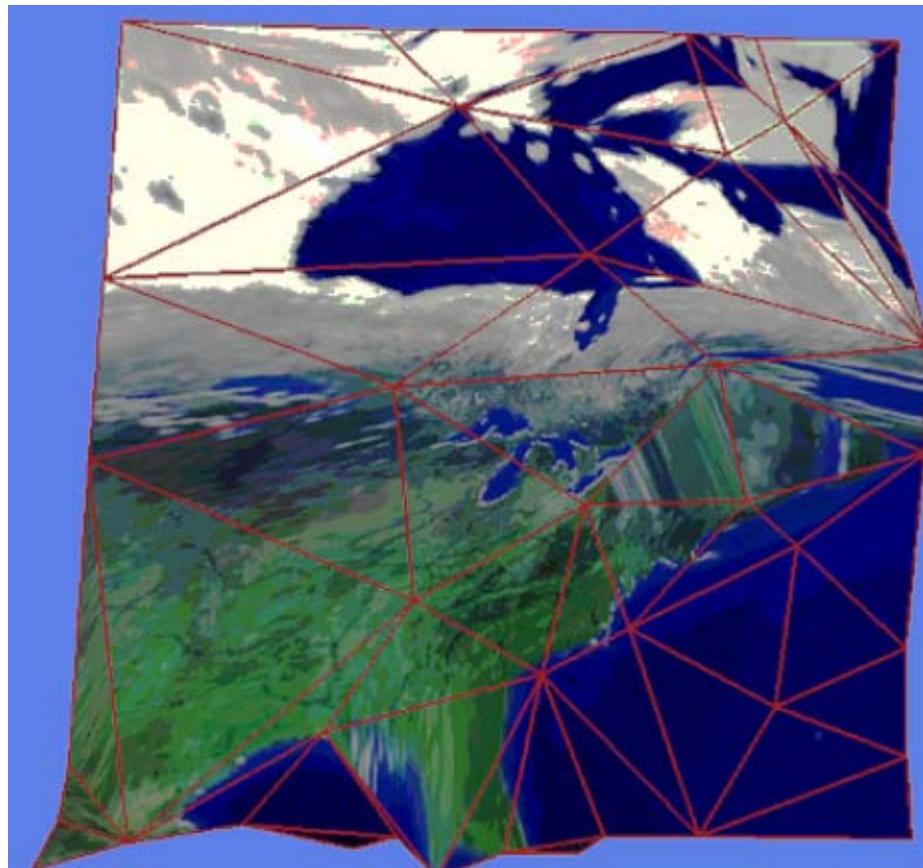


1500 triangles

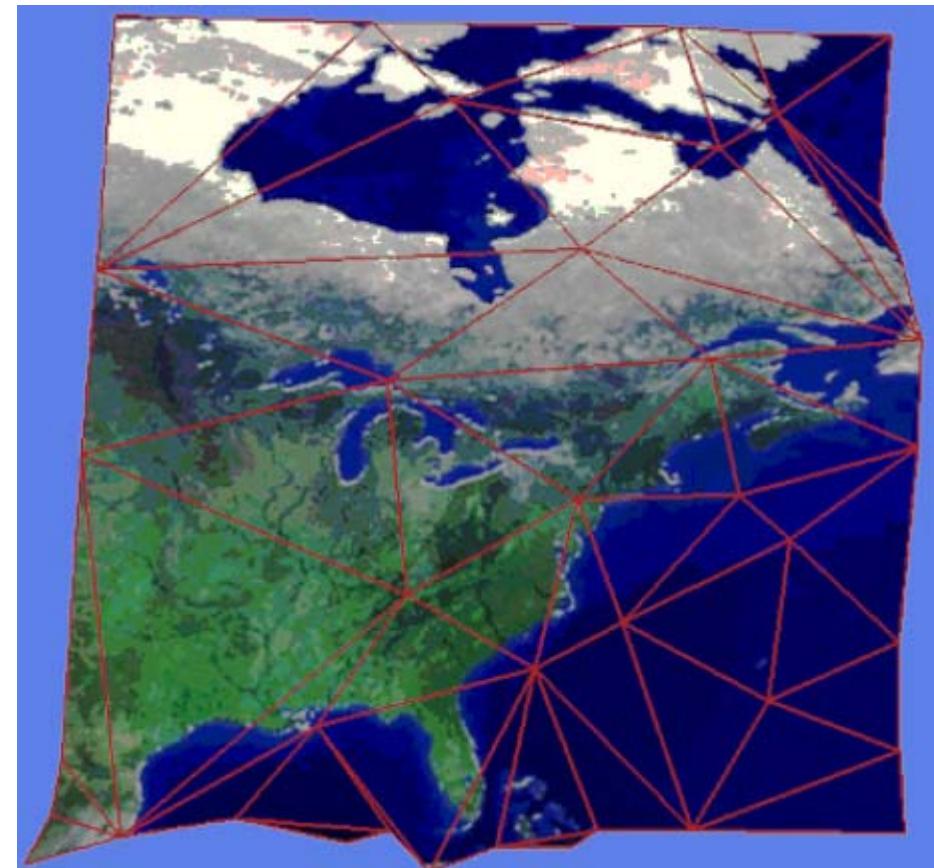
A Sample Textured Surface



Comparison



Simplifying geometry only



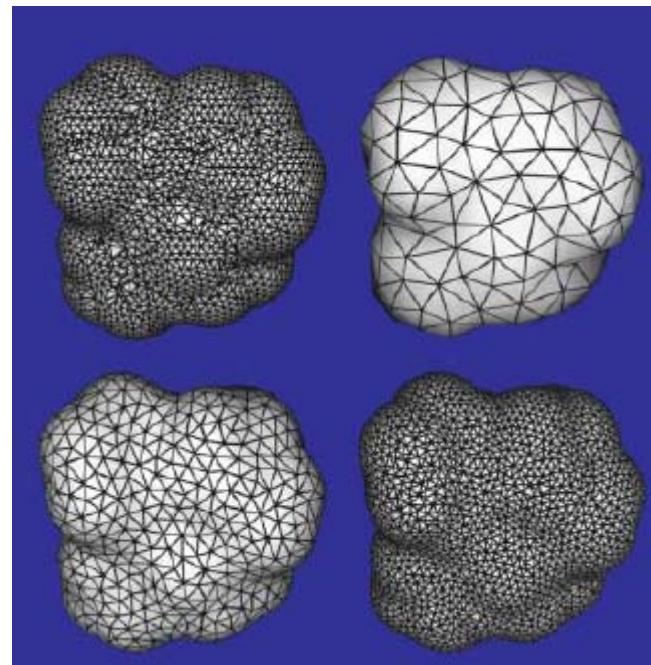
Simplifying geometry + texture coordinates

2. Global Simplification Strategies

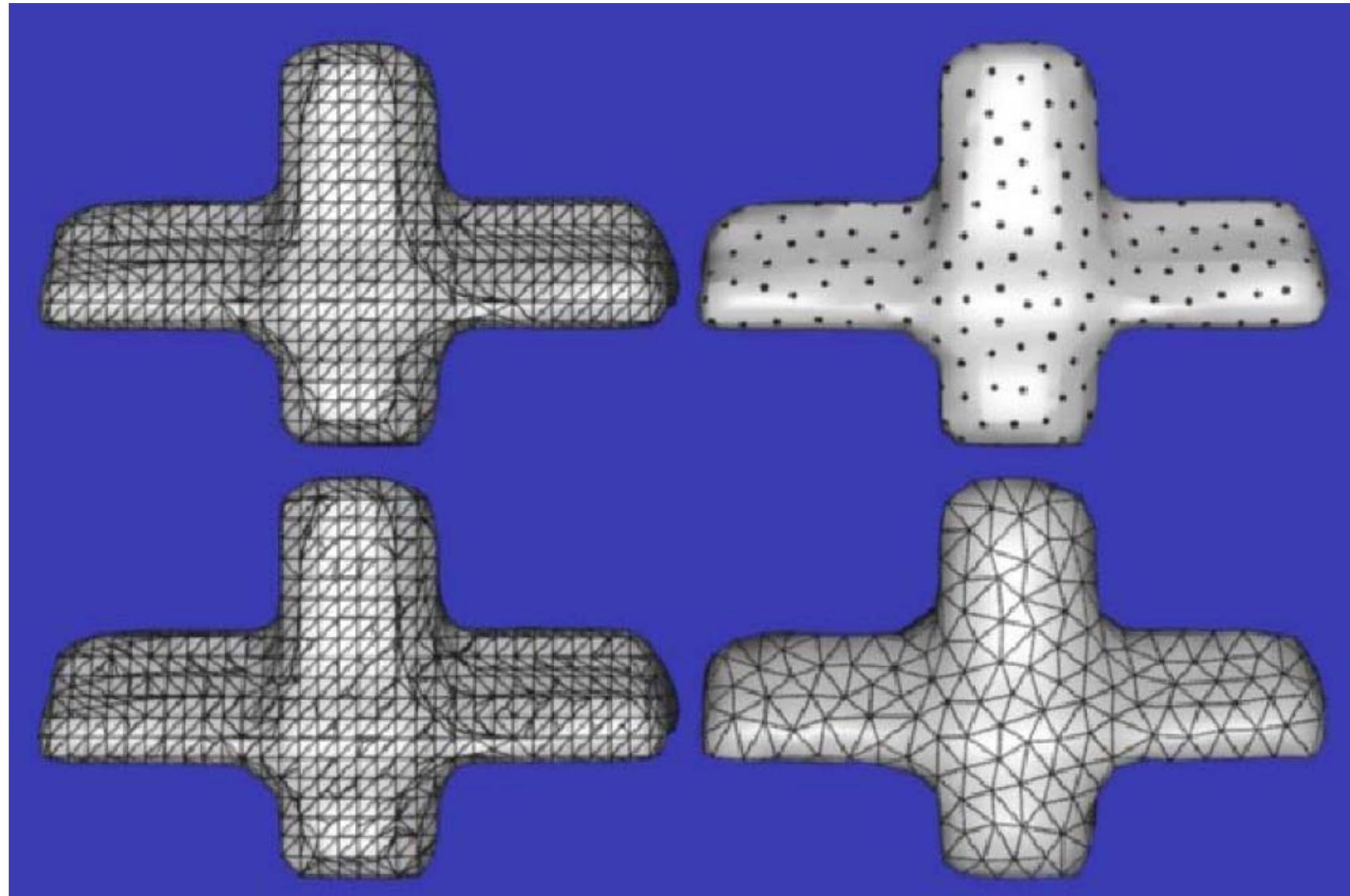
Resampling

2.1 Mesh Re-Tiling [Turk 92]

- Re-tiling attempts to simplify as well as improve meshing by introducing new “uniformly spaced” vertices

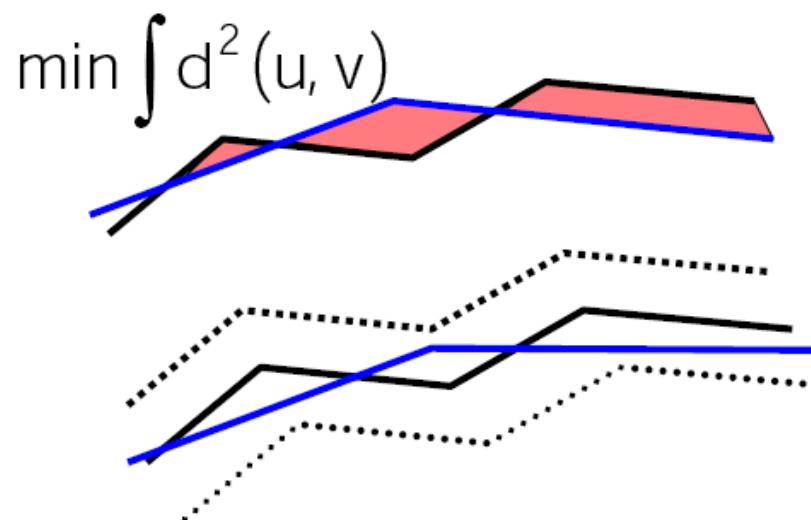


Re-Tiling Example



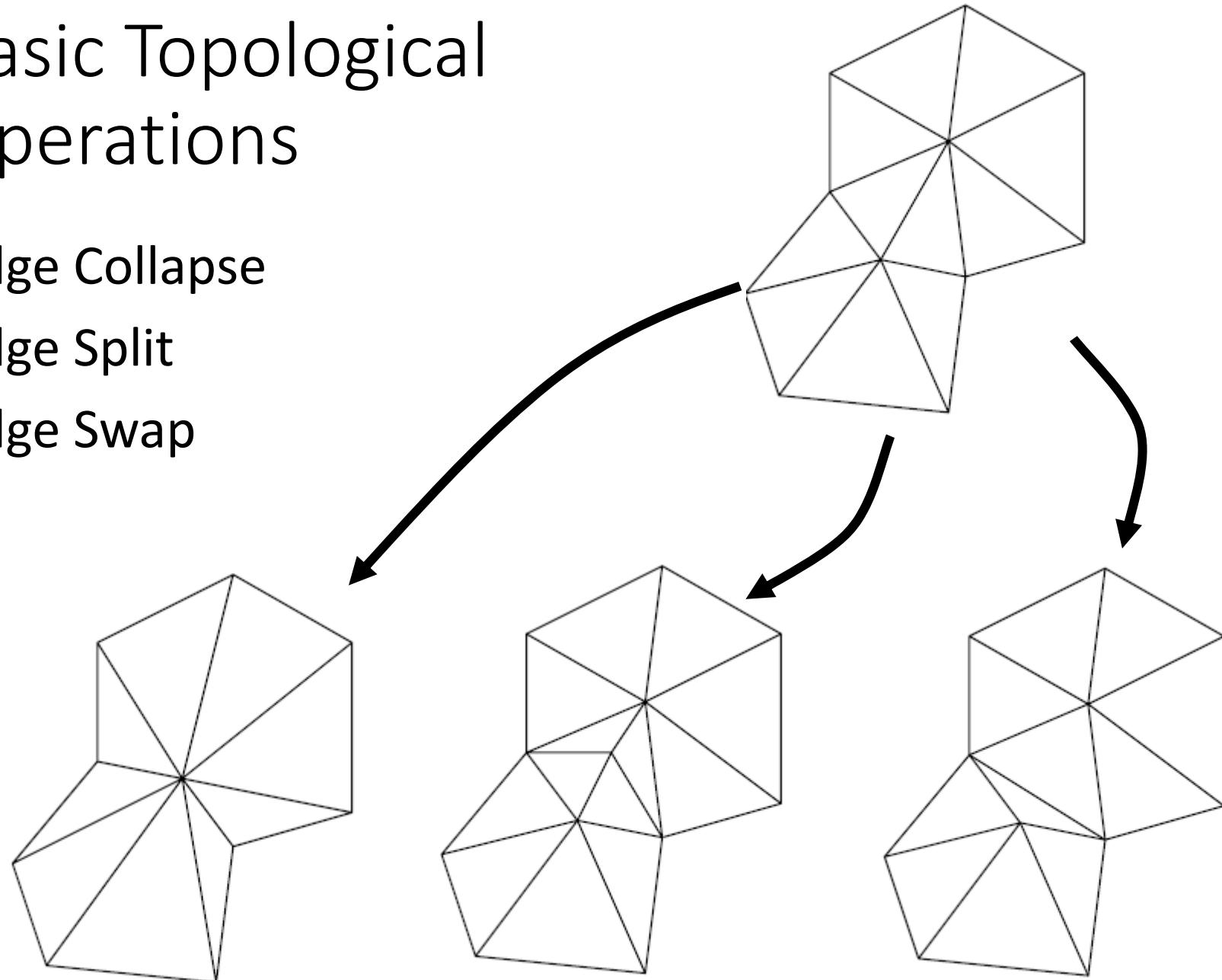
2.2 Mesh Optimization [Hoppe et al 93]

- Frames simplification as an optimization problem
 - Minimizes some **energy function**
 - Make simple changes to the topology of the mesh
 - Evaluate the energy before and after the change
 - Accept any change that reduces the energy

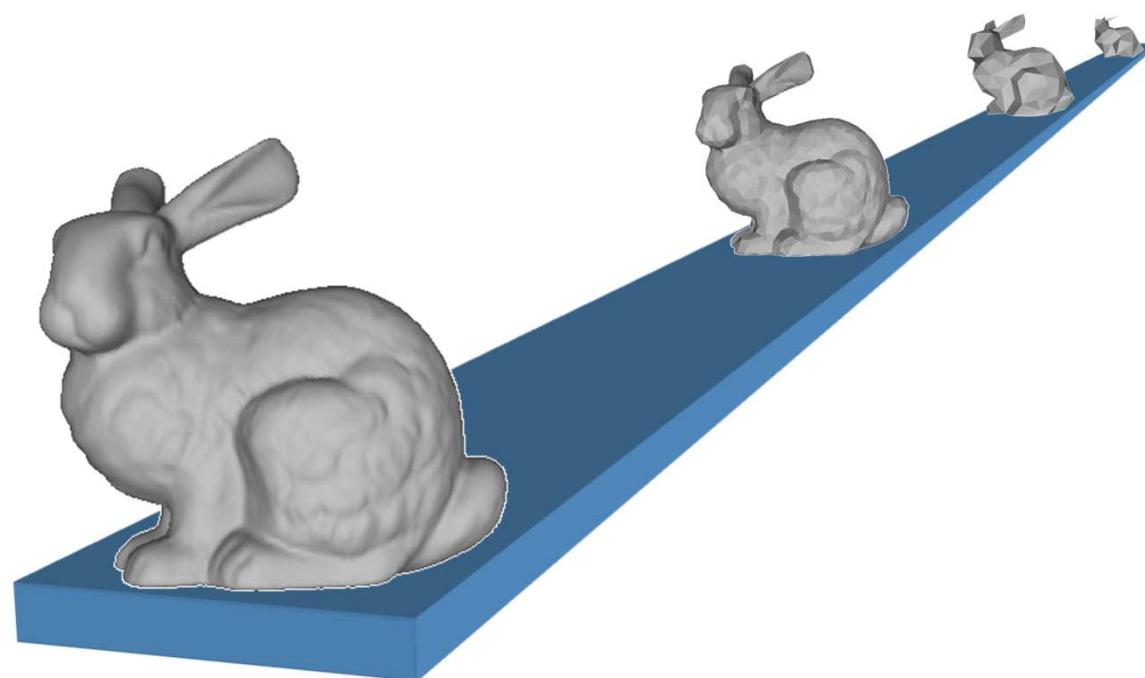


Basic Topological Operations

- Edge Collapse
- Edge Split
- Edge Swap



Level of Detail (LOD)



Multiresolution Representation

Multiresolution Representation of M

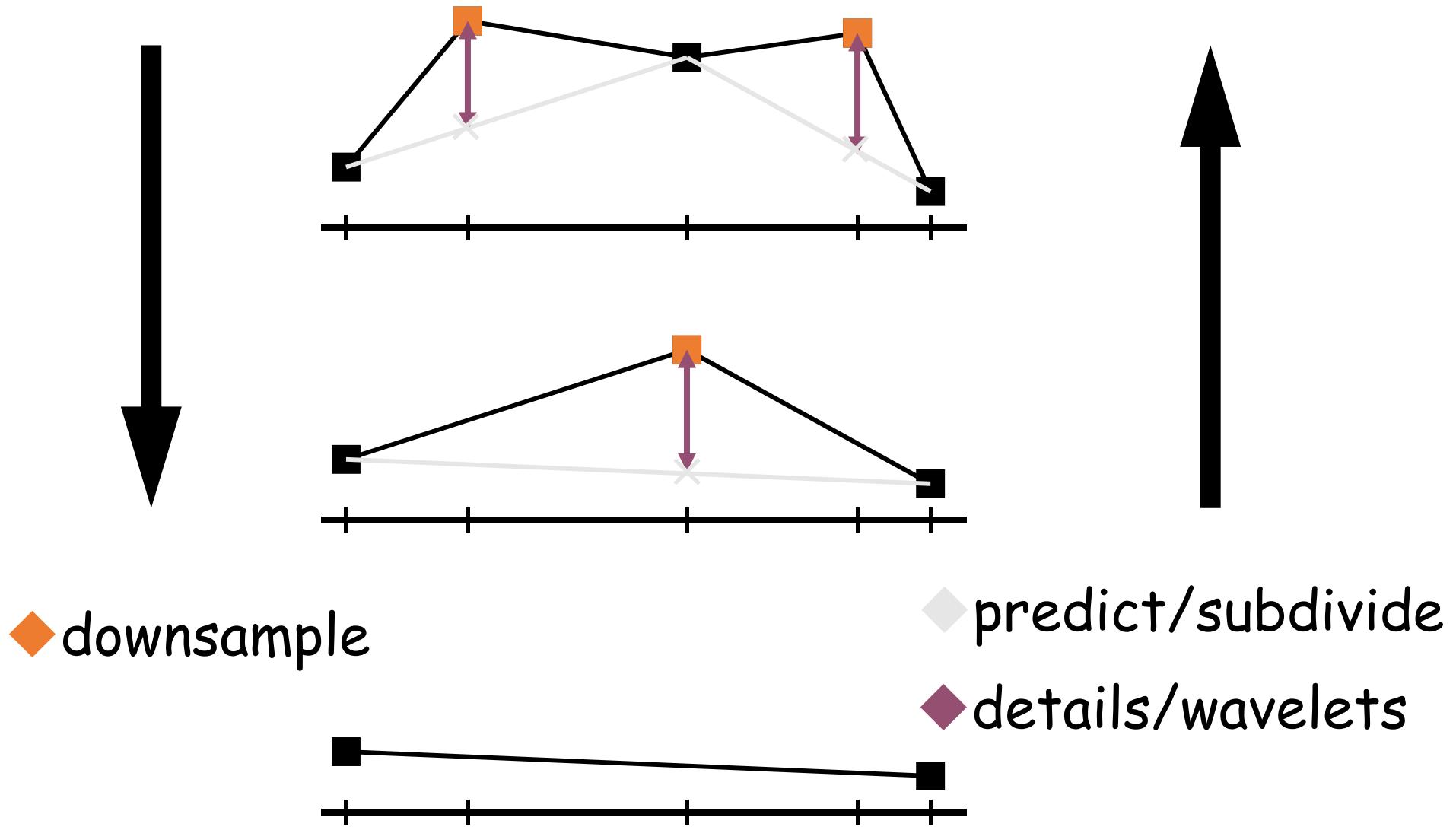
=

Base mesh M^0

+

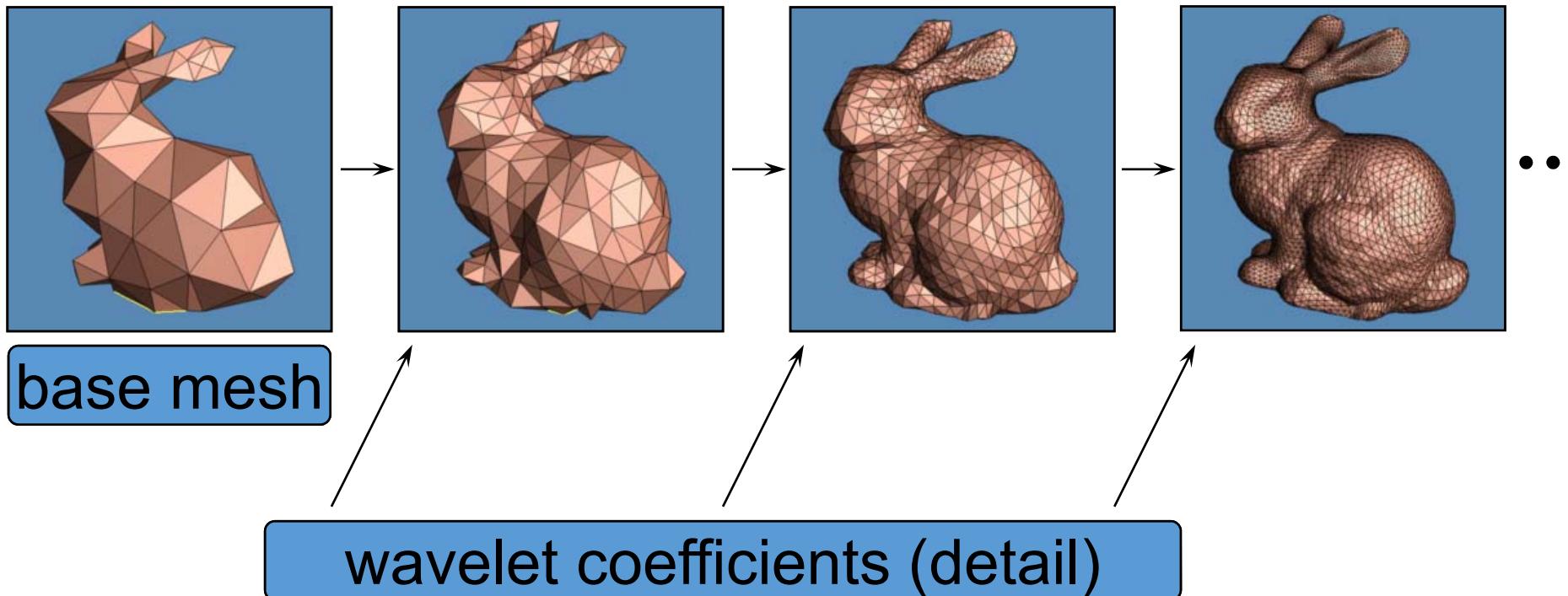
A sequence of refinements M^1

Multiresolution



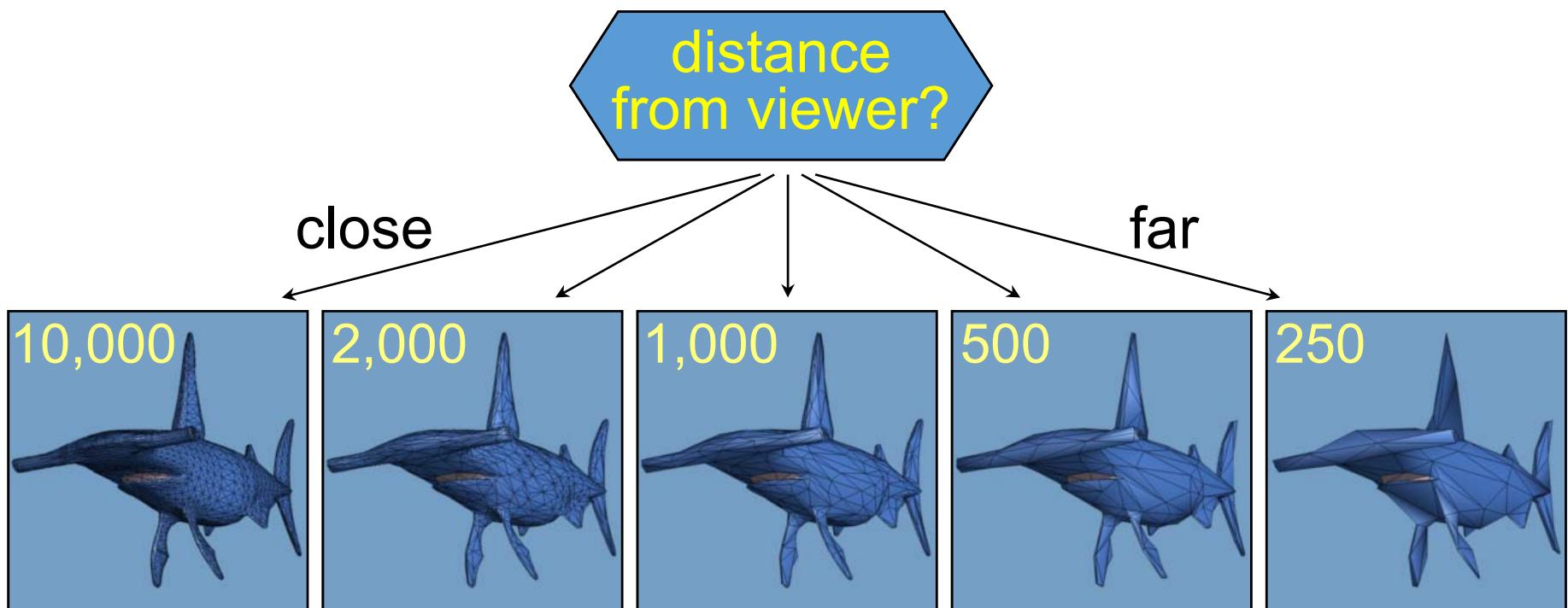
Multiresolution Analysis

[Lounsberry-etal93] [Eck-etal95] [Certain-etal96]



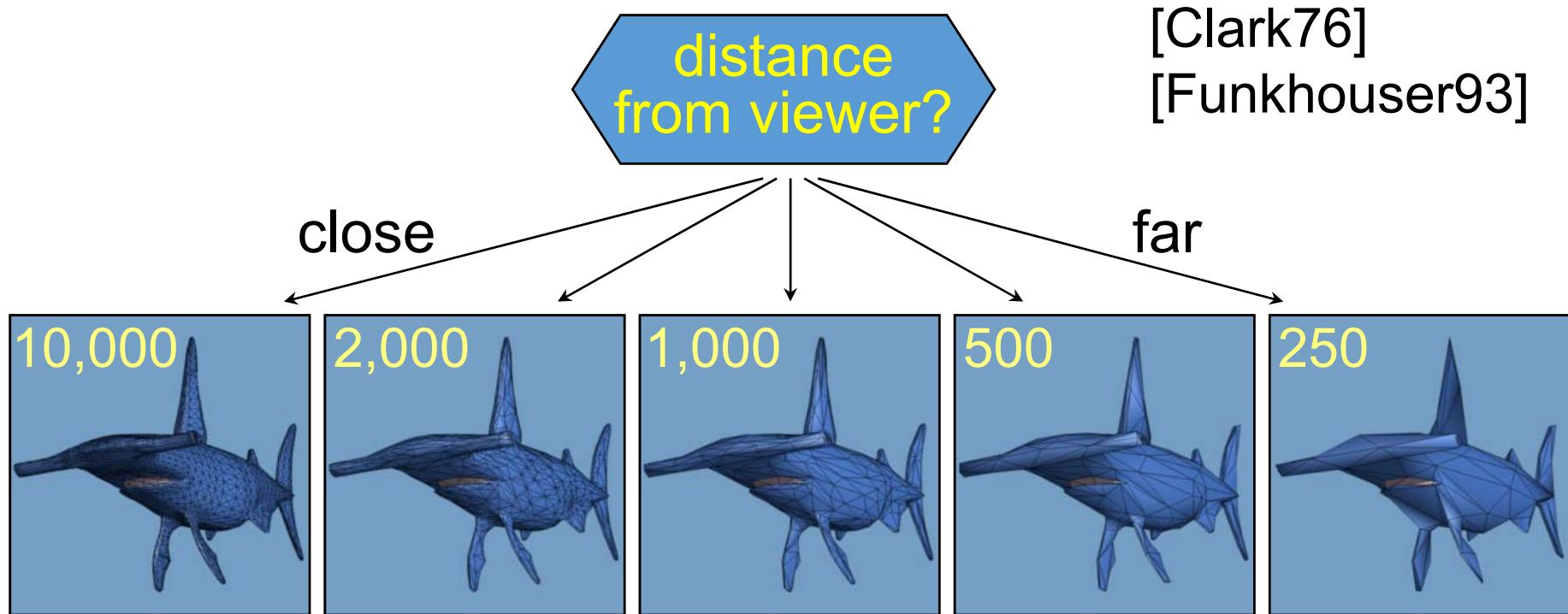
1. Discrete LOD

- “Pop” discontinuity



Concern: transitions may “pop”

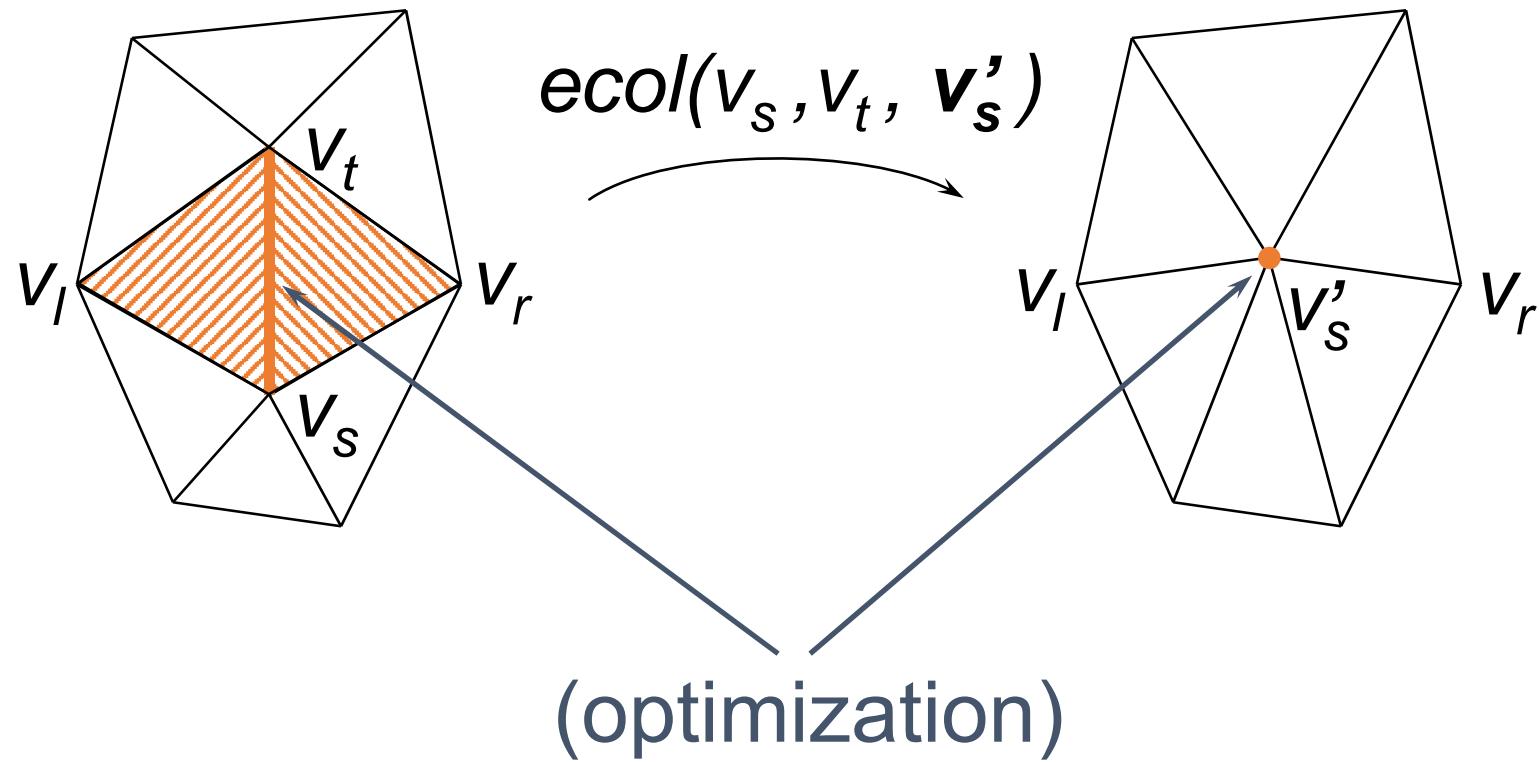
2. Continuous LOD



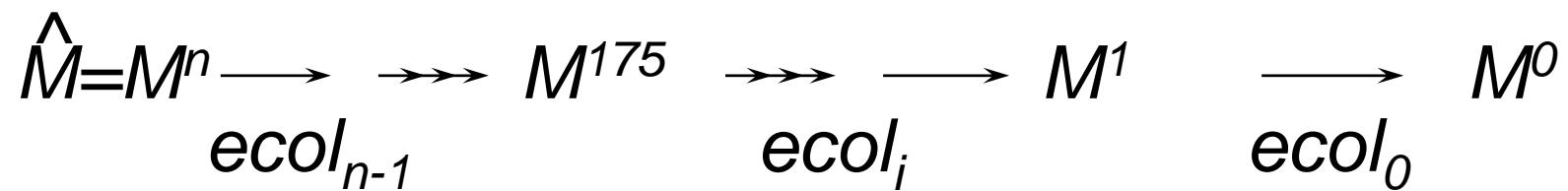
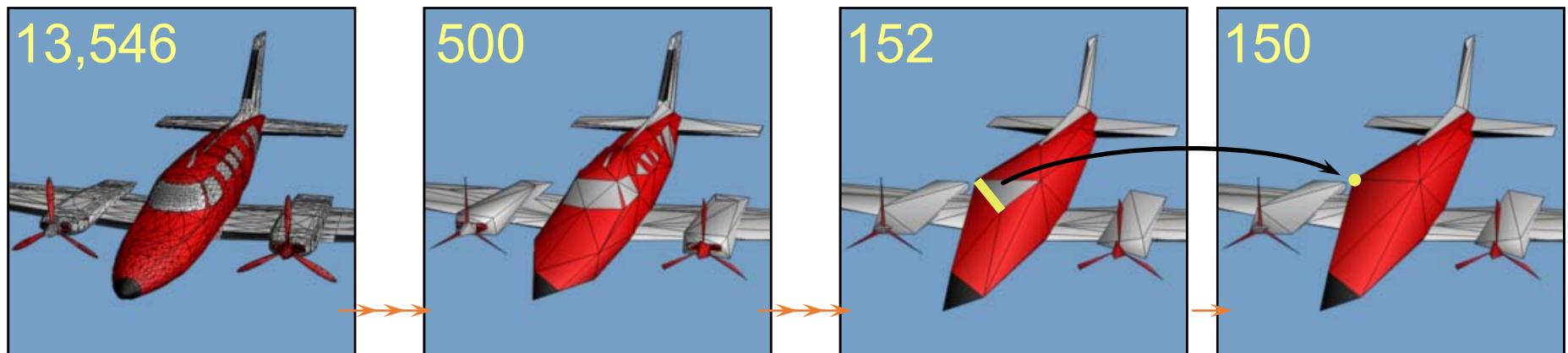
Concern: transitions may “pop”
→ would like smooth LOD

Mesh Simplification Procedure

- Idea: recode sequence of edge collapses

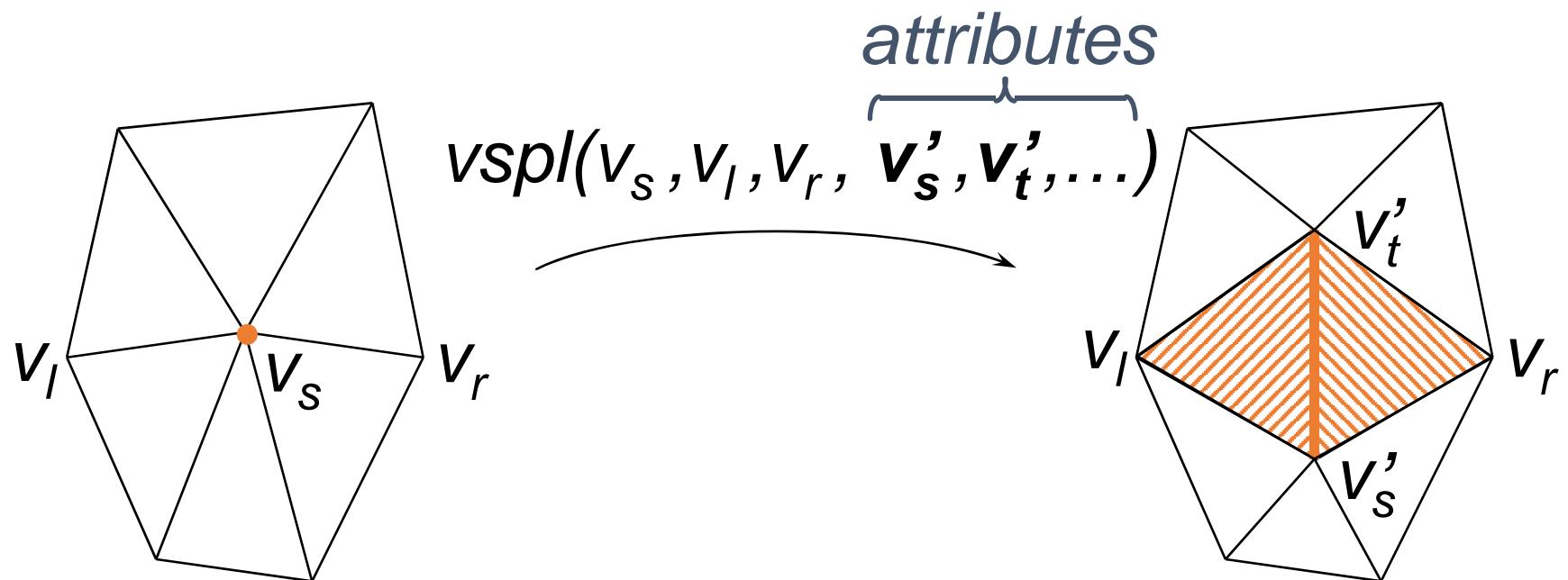


Simplification Process

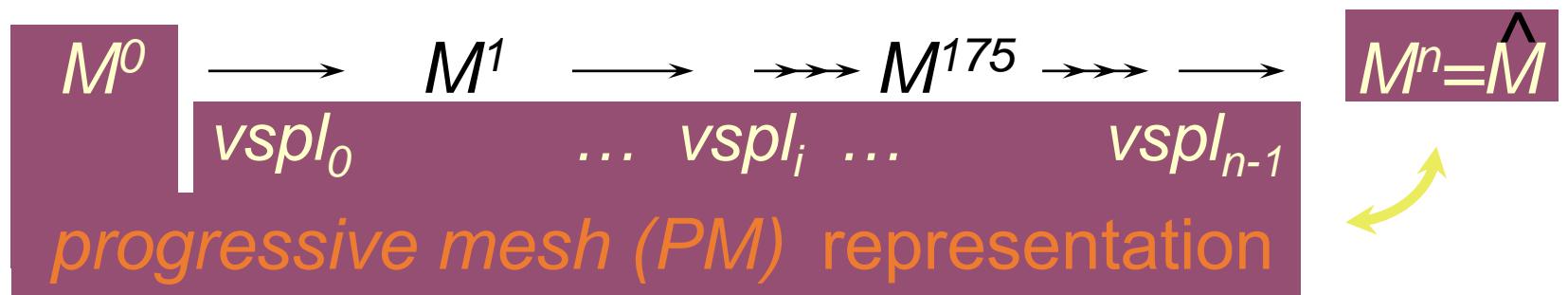
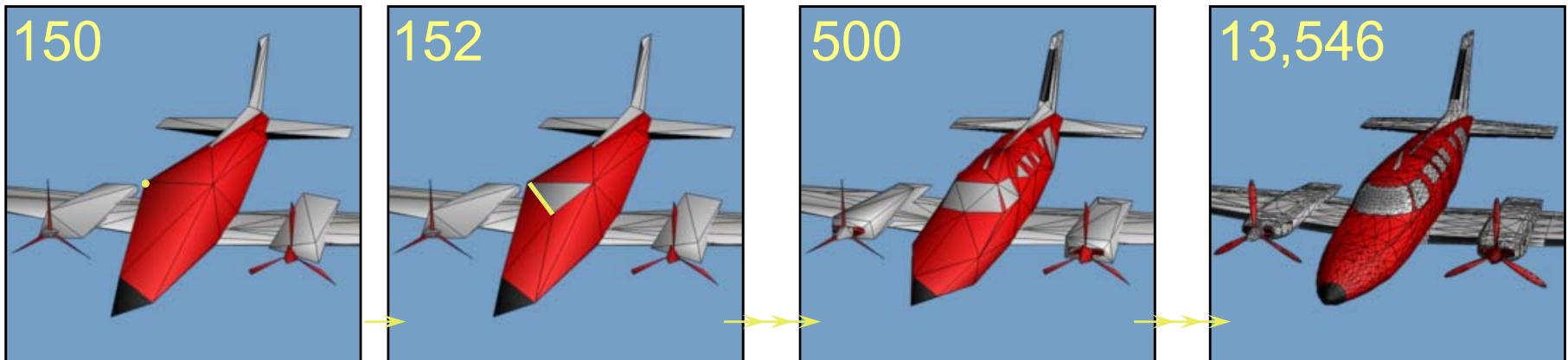


Invertible!

Vertex split transformation:

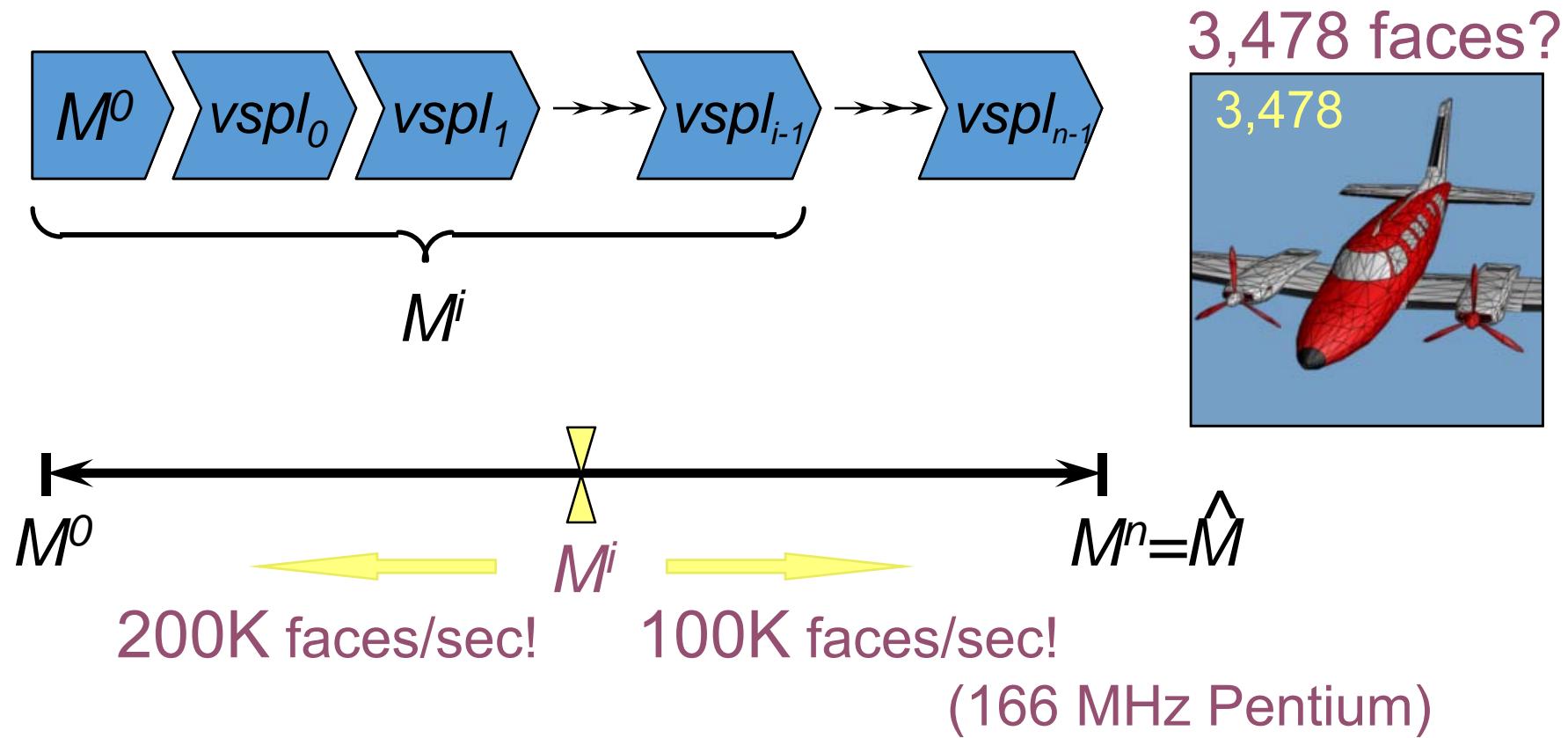


Reconstruction Process



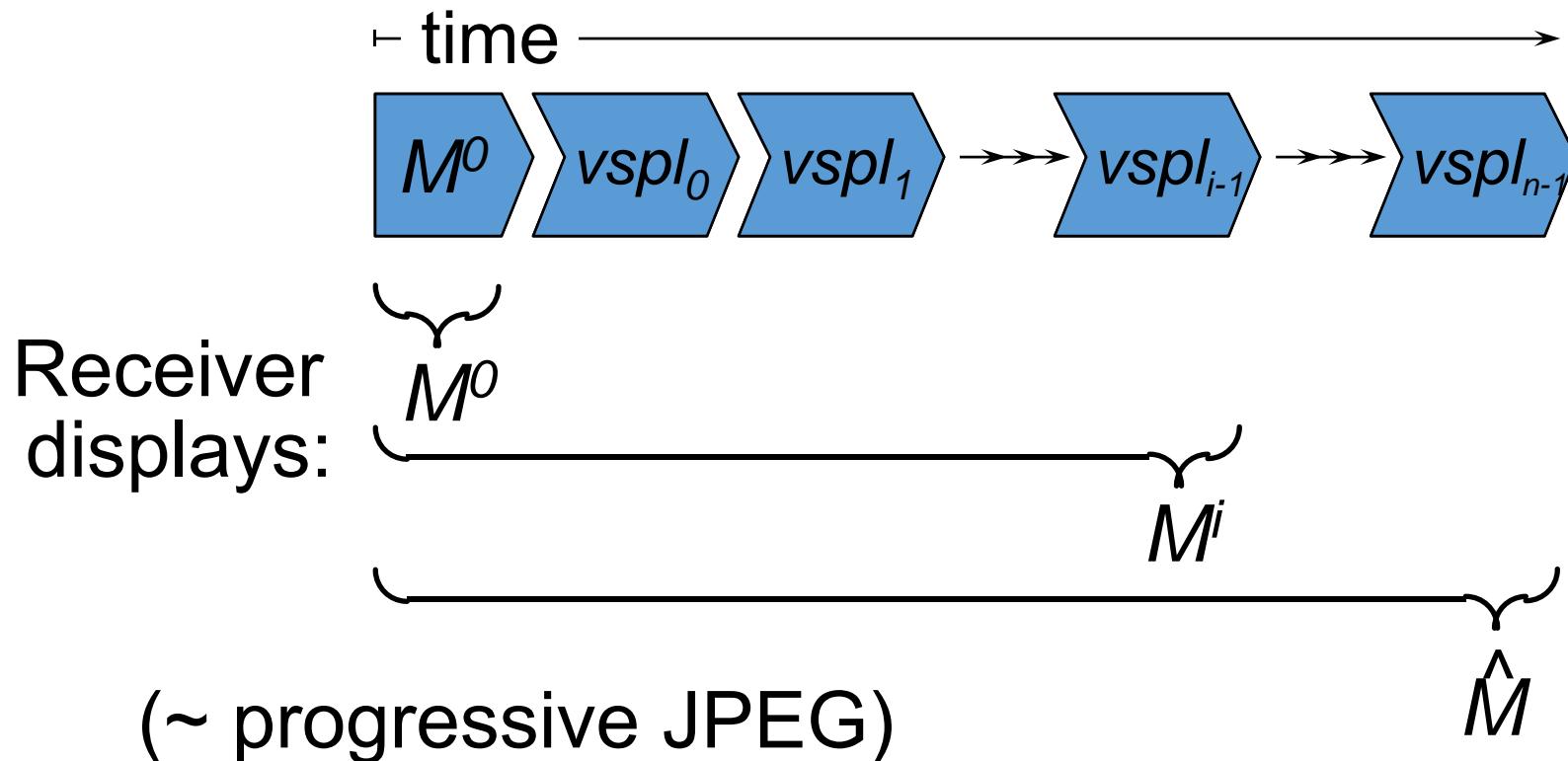
Continuous-resolution LOD

From PM, extract M^i of any desired complexity.



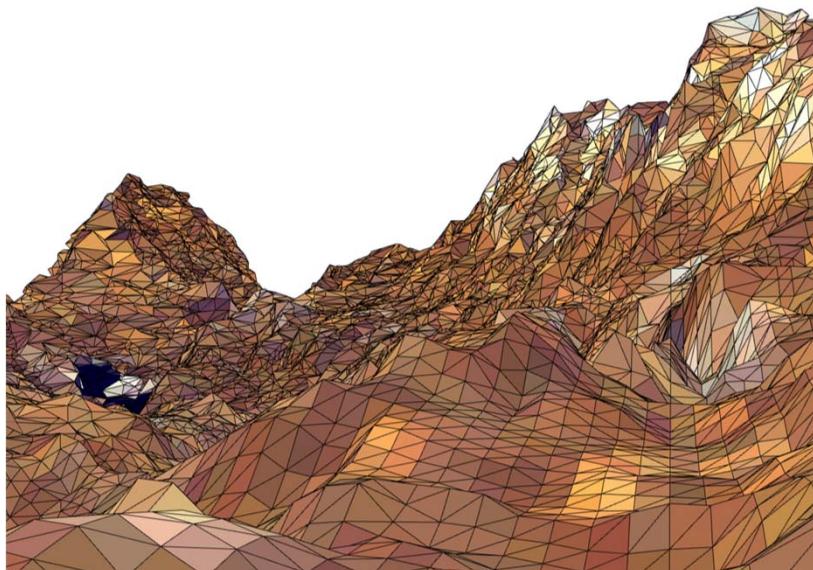
Progressive Transmission

Transmit records progressively:

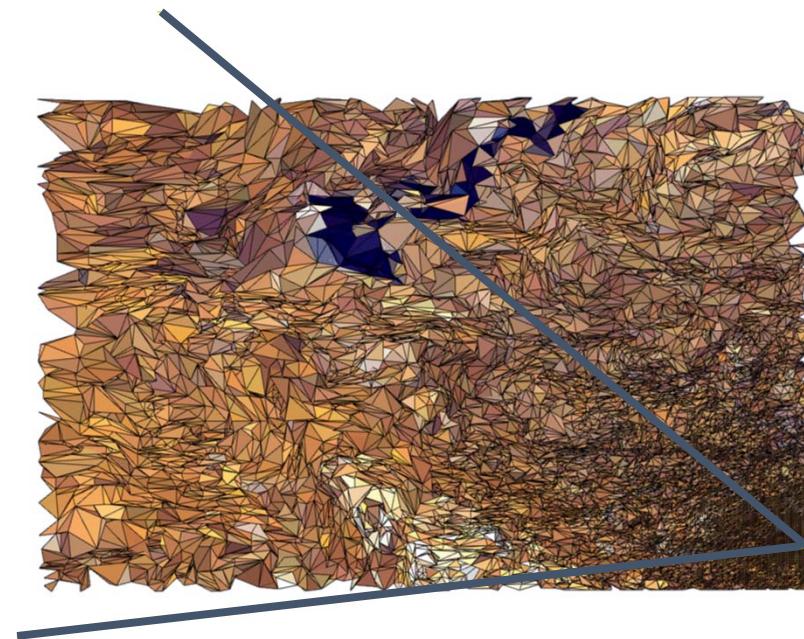


3. View-Dependent LOD

- Show nearby portions of object at higher resolution than distant portions

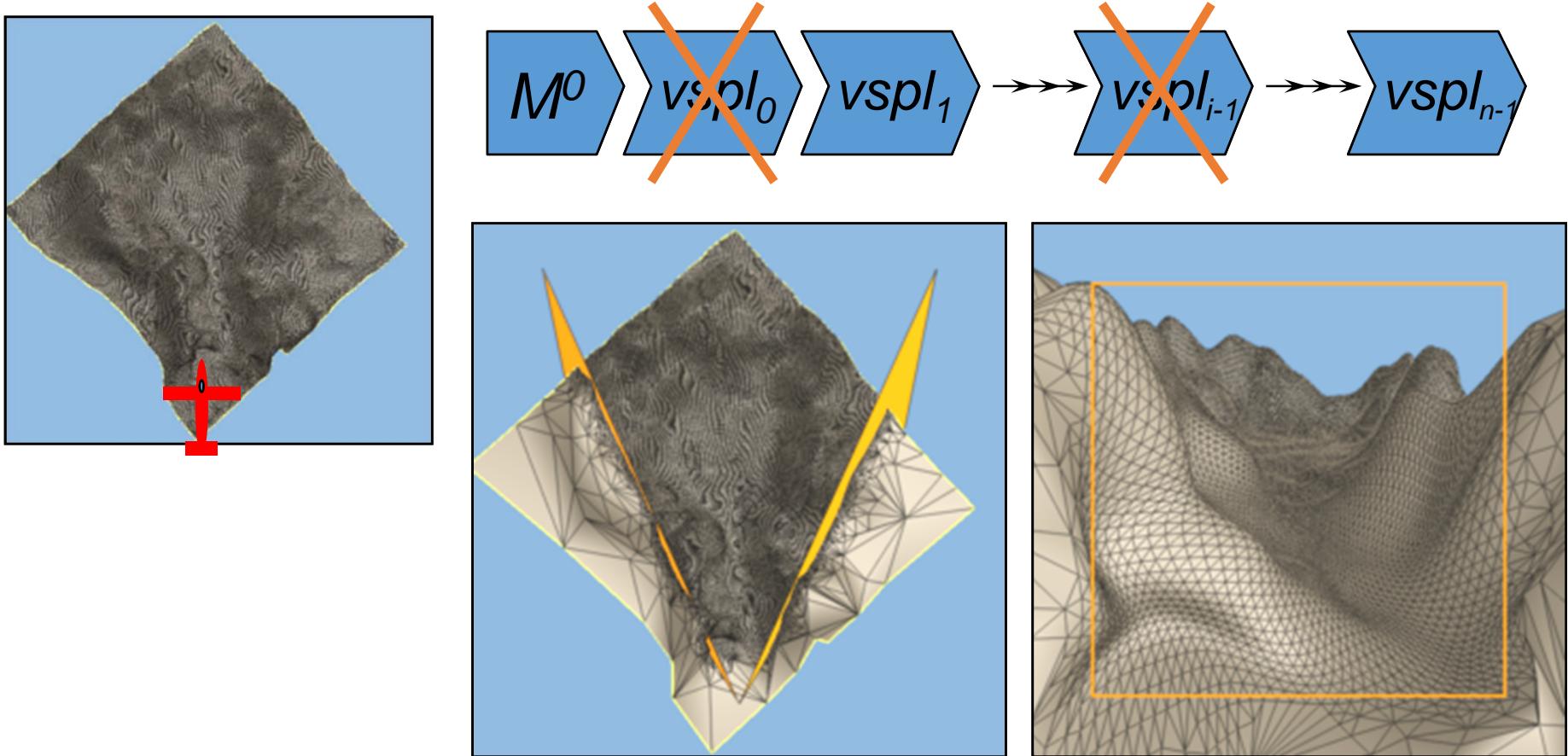


View from eyepoint



Birds-eye view

Selective Refinement



[SIGGRAPH 96]: incremental update not possible

Challenges: Unreal 5 demo



Challenges: Digital Cities



Challenges

- 纹理的简化
 - 熵、保特征...
- 数据组织与调度
 - 虚拟纹理
 - 虚拟几何
- 计算模式
 - Client/Server (CS)
 - Browser/Server (BS)
 - Cloud-Edge-Client computing: 云、边、端
- ...

Resources

- Internet, Papers, Siggraph courses
- VDSlib - <http://vdslib.virginia.edu>
 - A public-domain view-dependent simplification and rendering package/library
- Luebke's work on view-dependent simplification:
 - <http://www.cs.virginia.edu/~luebke/simplification.html>
- Hoppe's work on progressive meshes:
 - <http://www.research.microsoft.com/~hhoppe>
- Garland's work on quadric error metrics:
 - <http://www.uiuc.edu/~garland>
 - <http://www.cs.cmu.edu/afs/cs/user/garland/www/multires/survey.html>
- The Multi-Tesselation (MT) homepage:
 - <http://www.disi.unige.it/person/MagilloP/MT>

作业9

- 任务：
 - 实现网格简化的QEM方法
 - Garland and Heckbert. Surface Simplification Using Quadric Error Metrics. Siggraph 1997.
 - <http://mgarland.org/files/papers/quadrics.pdf>
 - 目的
 - 学习网格的拓扑关系（点、边、面）更新的维护
 - 交互界面（可选）
 - 通过操纵网格顶点个数 $n \leq N$ （输入网格顶点数），可以生成简化网格（减少 n ），也可以回退到原始网格（增加 n ）
 - 思考：如何记录简化的操作？
 - Deadline: **2021年1月2日晚**



中国科学技术大学
University of Science and Technology of China

谢谢！