
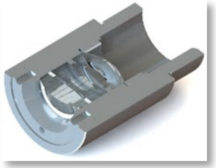



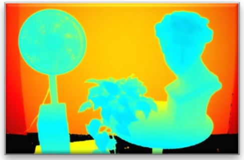


Computational Imaging

Lecture 19 Temporal Encoding II - Non-line-of-sight (NLOS) Imaging

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香港中文大学（深圳）

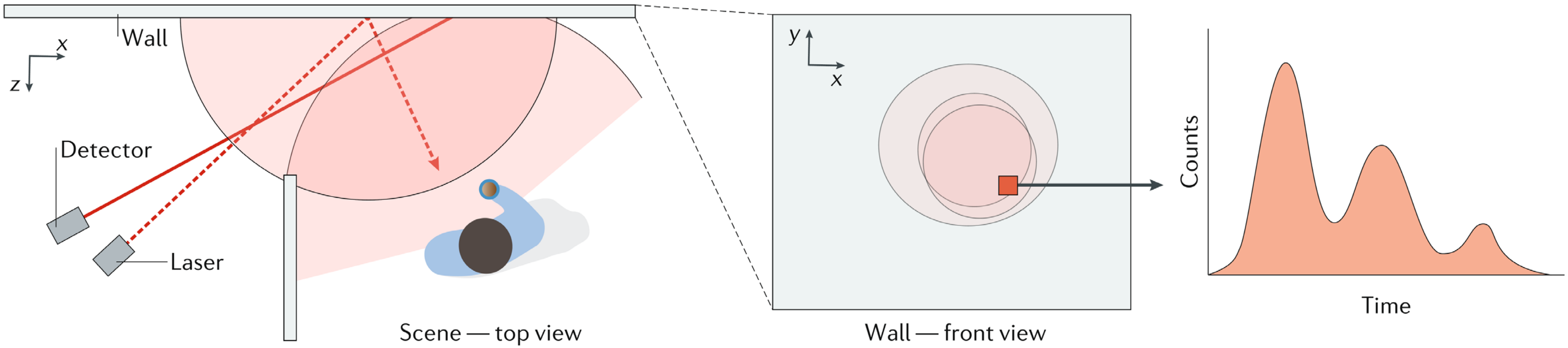
点昀技术（Point Spread Technology）

Today's **Topic**

- What's NLOS Imaging?
- Model of NLOS Imaging
  - NLOS Image Formation Model
- Confocal NLOS Imaging
- The f - $\mathbf{k}$ Migration Method
- Other Works

What's NLOS Imaging?

Layout of Time-resolved Non-line-of-sight Imaging



Daniele et al., Nature Review Physics, 2020

Challenges of NLOS Imaging

Only **a few** of the many recorded photons carry the information necessary to estimate hidden objects.

$$I_{direct} \propto 1/r^2$$

$$I_{scattered} \propto 1/r^{2*n}$$

Soluton

Ultra HDR or Gated Single Photon Dector

The inverse problem of estimating 3D hidden objects from intensity measurements alone is **ill-posed**.

Soluton

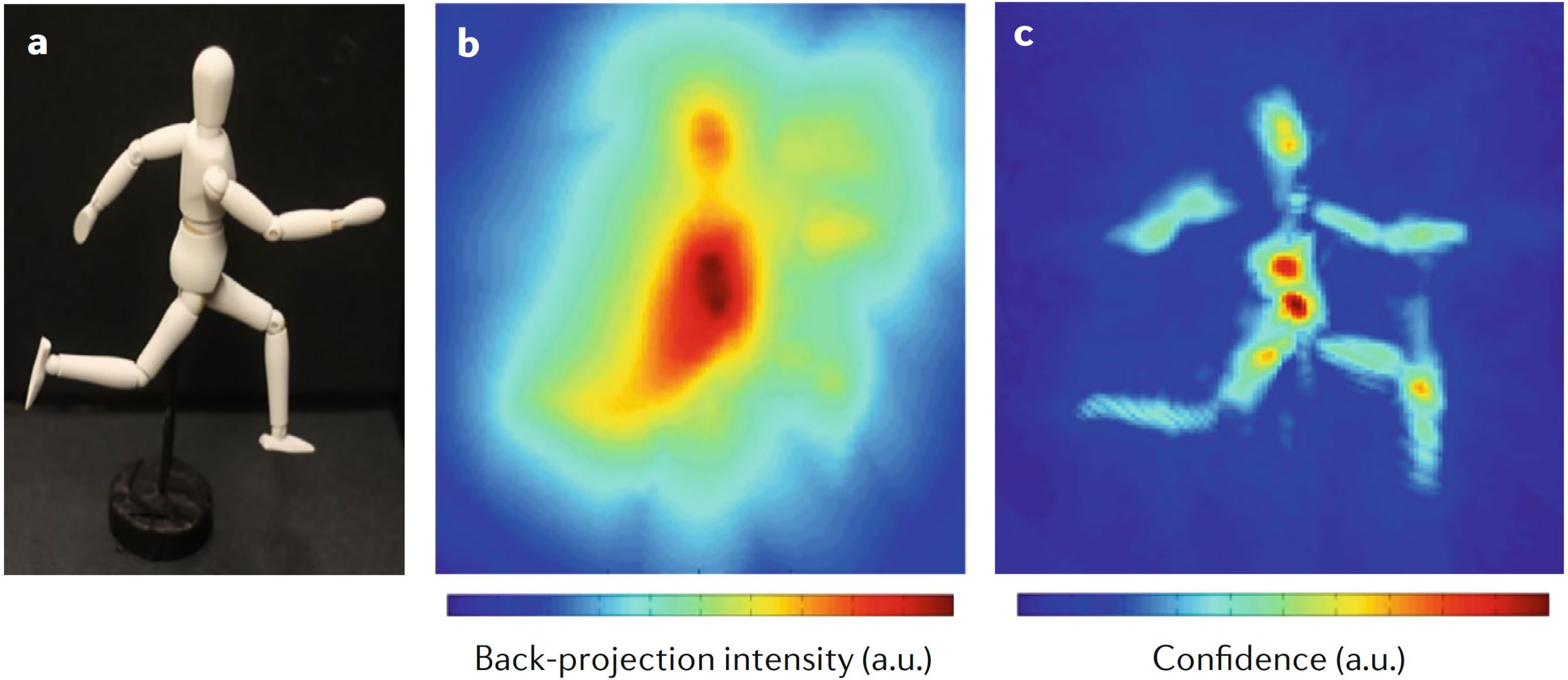
- Picosecond-accurate time-resolved measurement
- Mathematical priors on the imaged scenes
- Other unconventional approaches

The inverse problems associated with NLOS imaging are extremely **large**

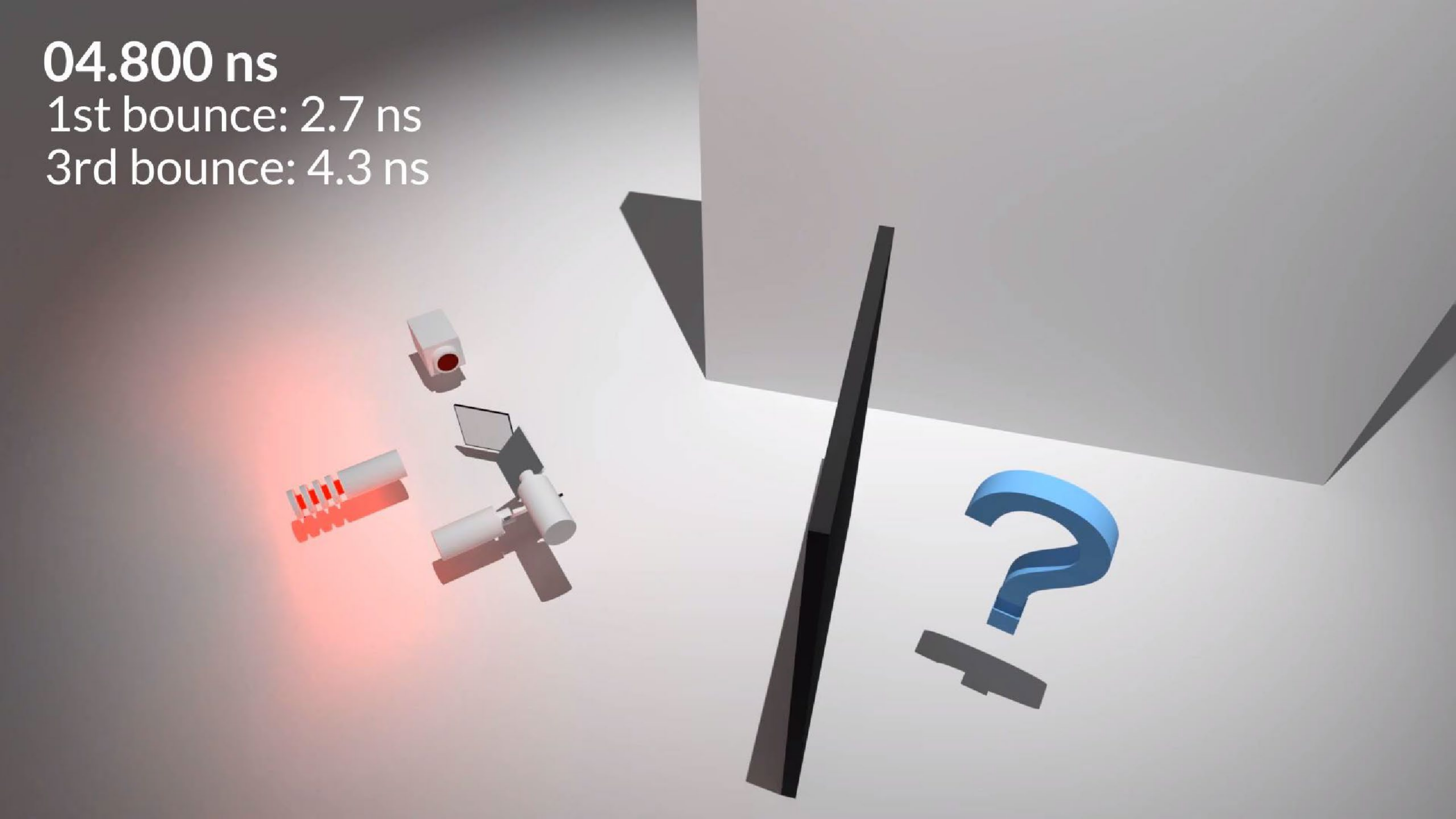
Soluton

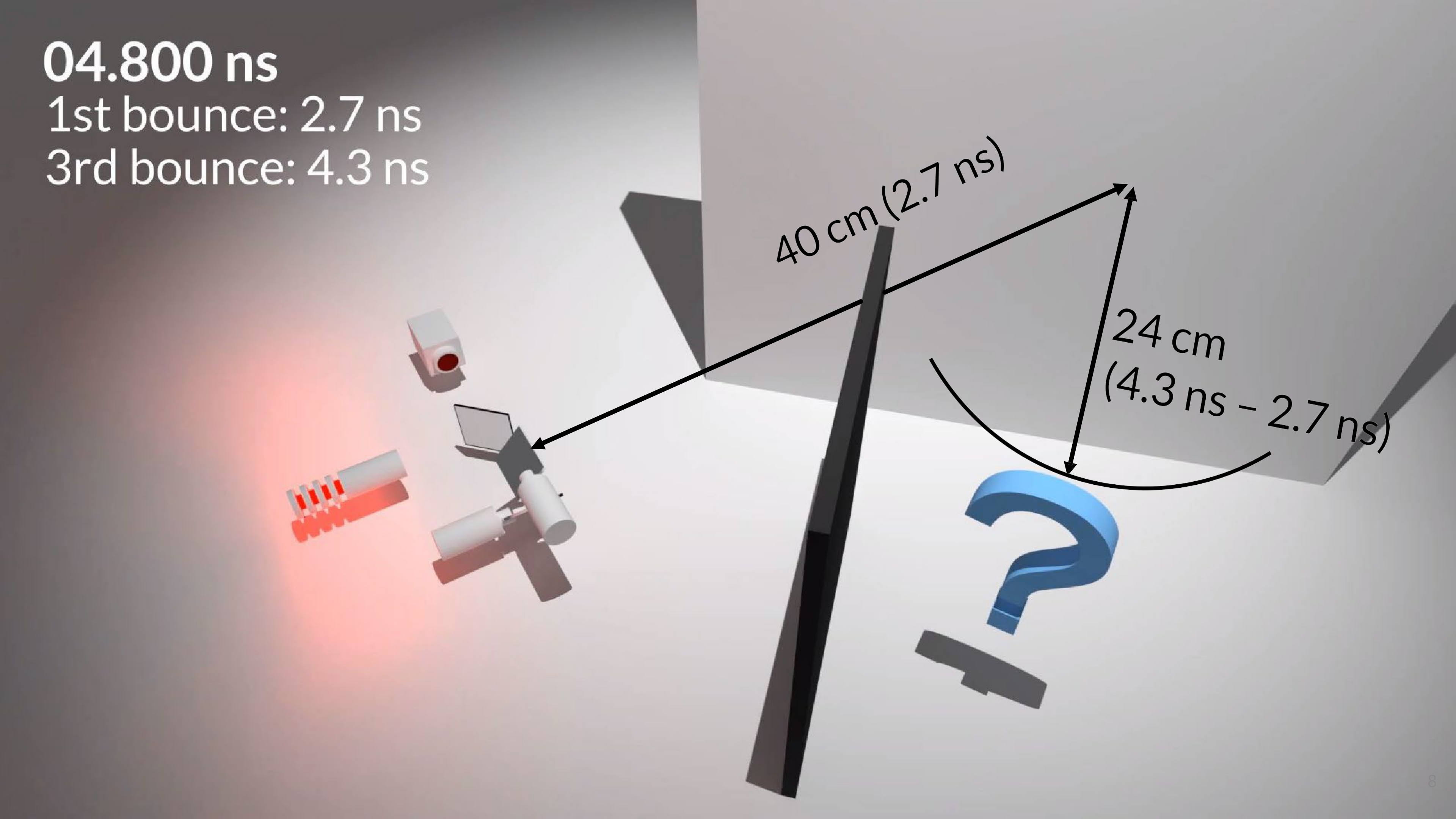
Efficient algorithms

First Trying of 'Looking Around Corners'

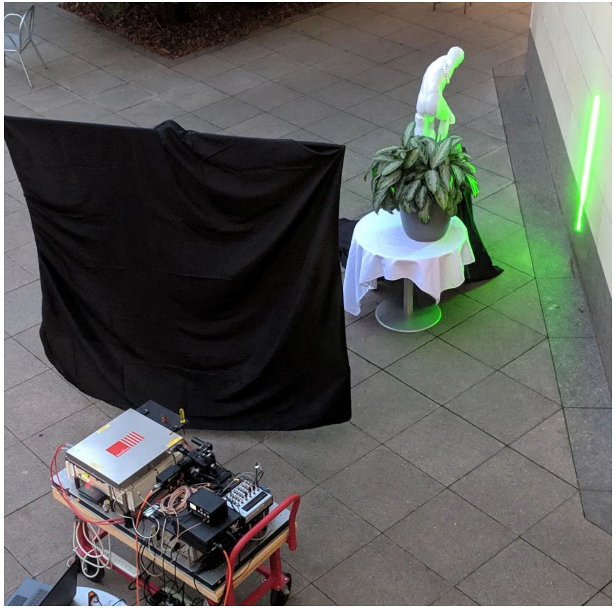


Velten, 12

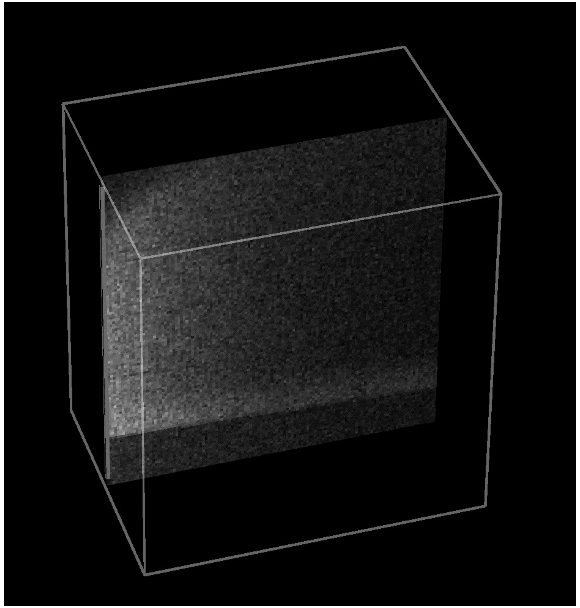

04.800 ns
1st bounce: 2.7 ns
3rd bounce: 4.3 ns


04.800 ns
1st bounce: 2.7 ns
3rd bounce: 4.3 ns
40 cm (2.7 ns)
24 cm
(4.3 ns – 2.7 ns)

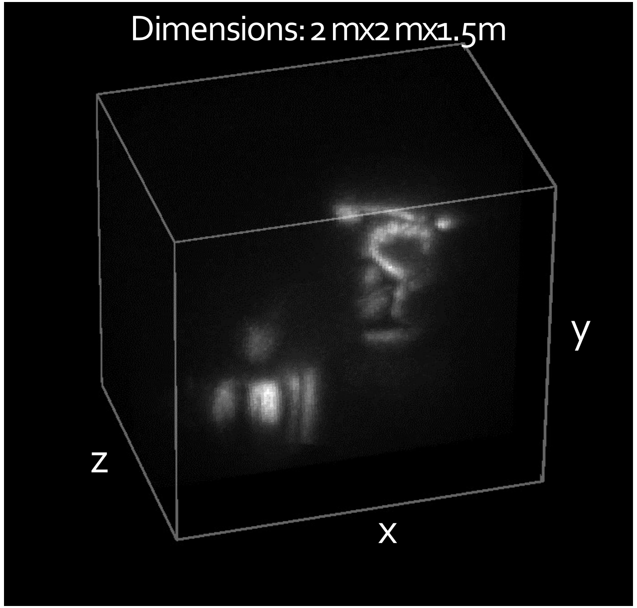
An Imaging Setup



scene



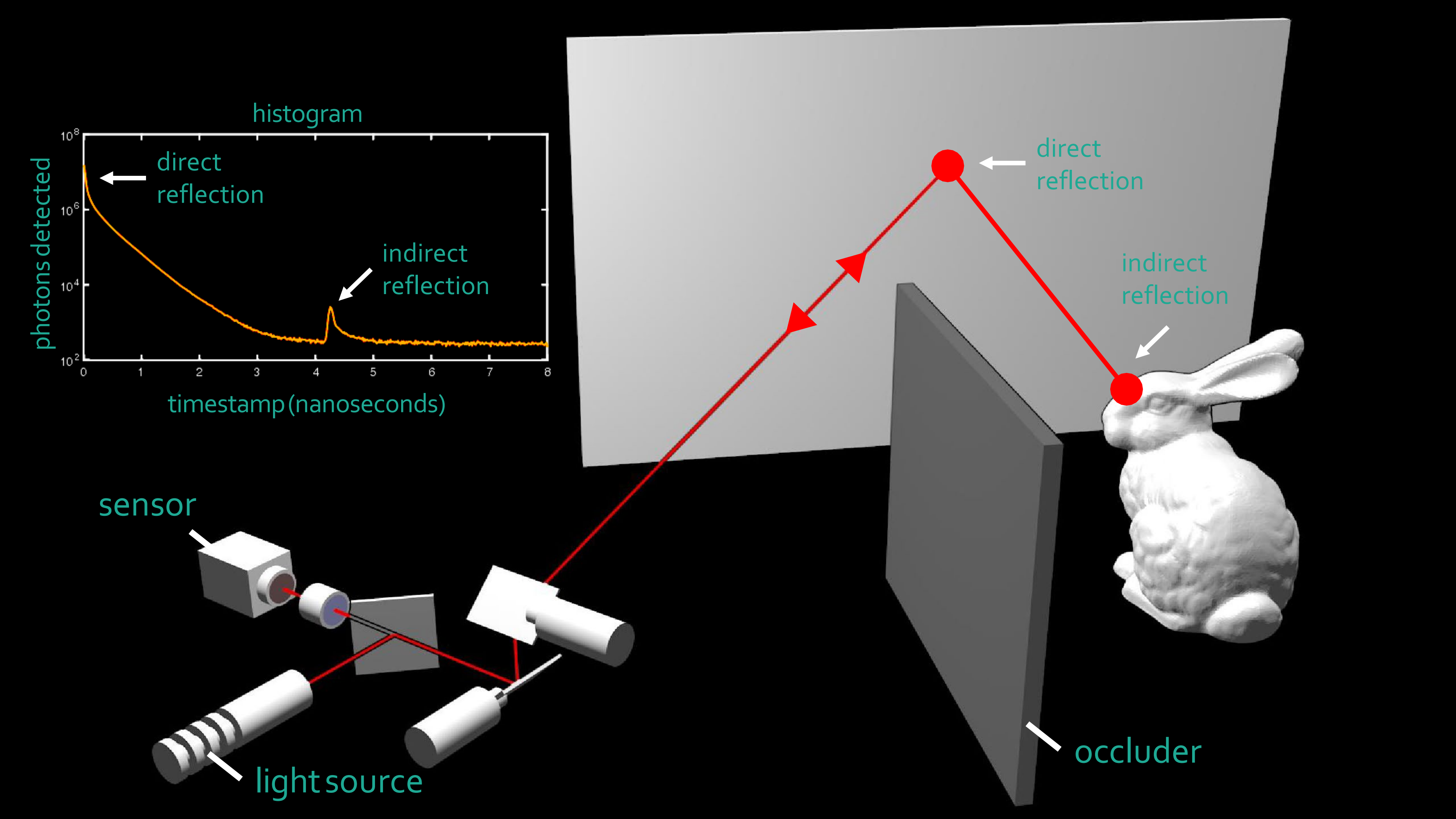
measurements

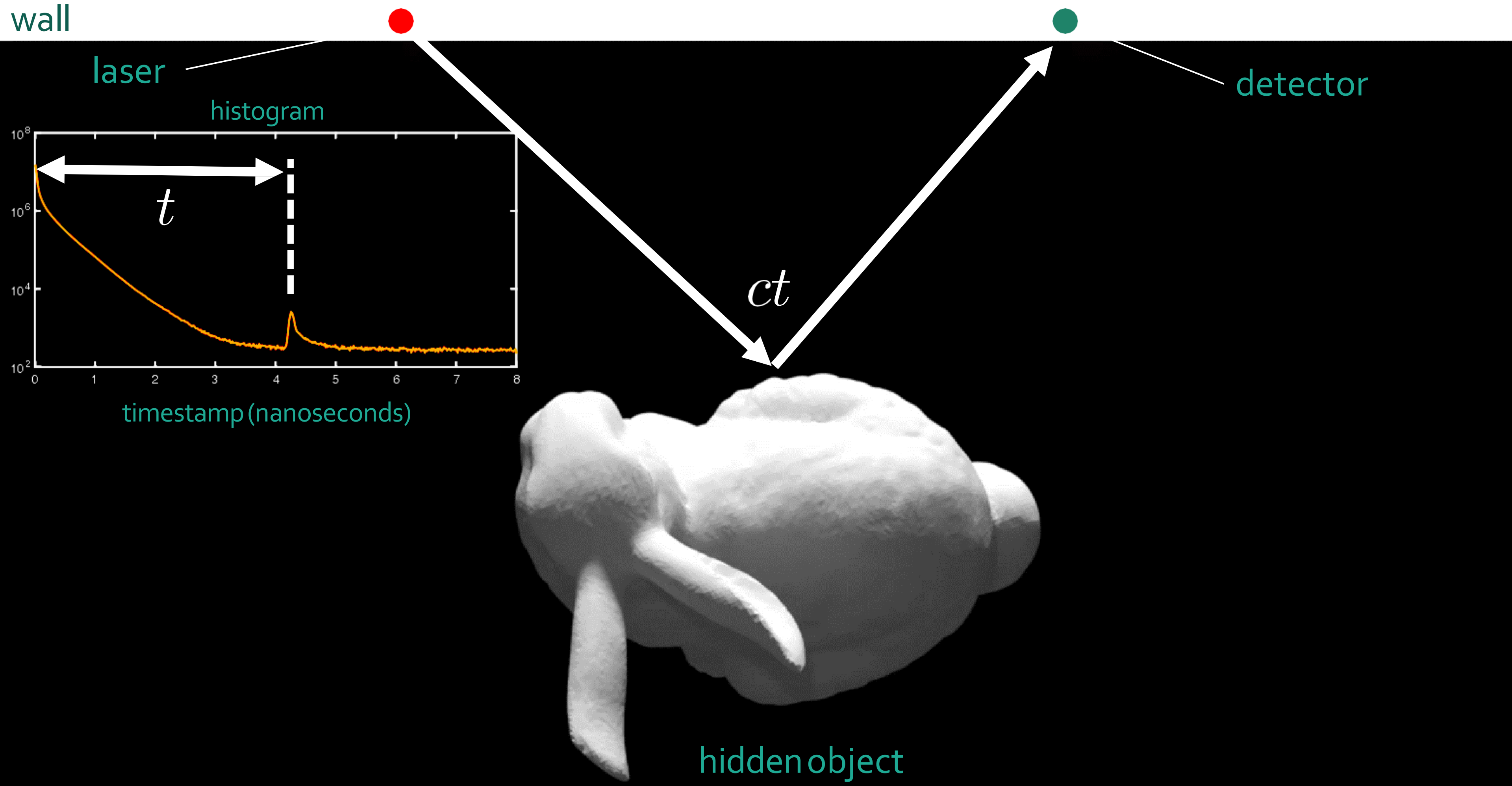


reconstruction

Lindell et al., SIGGRAPH 2019

Model of NLOS Imaging


histogram
photons detected
direct reflection
indirect reflection
timestamp (nanoseconds)
direct reflection
indirect reflection
sensor
light source
occluder



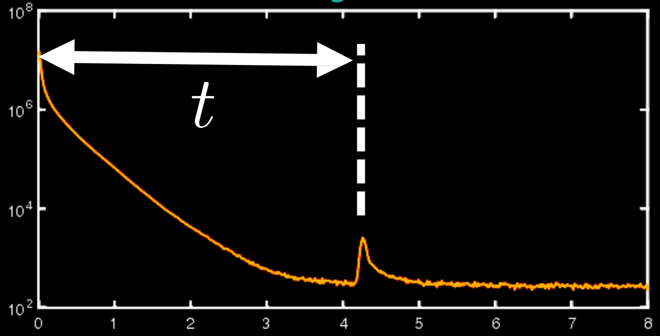
non-confocal sampling

wall

laser and detector focus on this point

ct

histogram

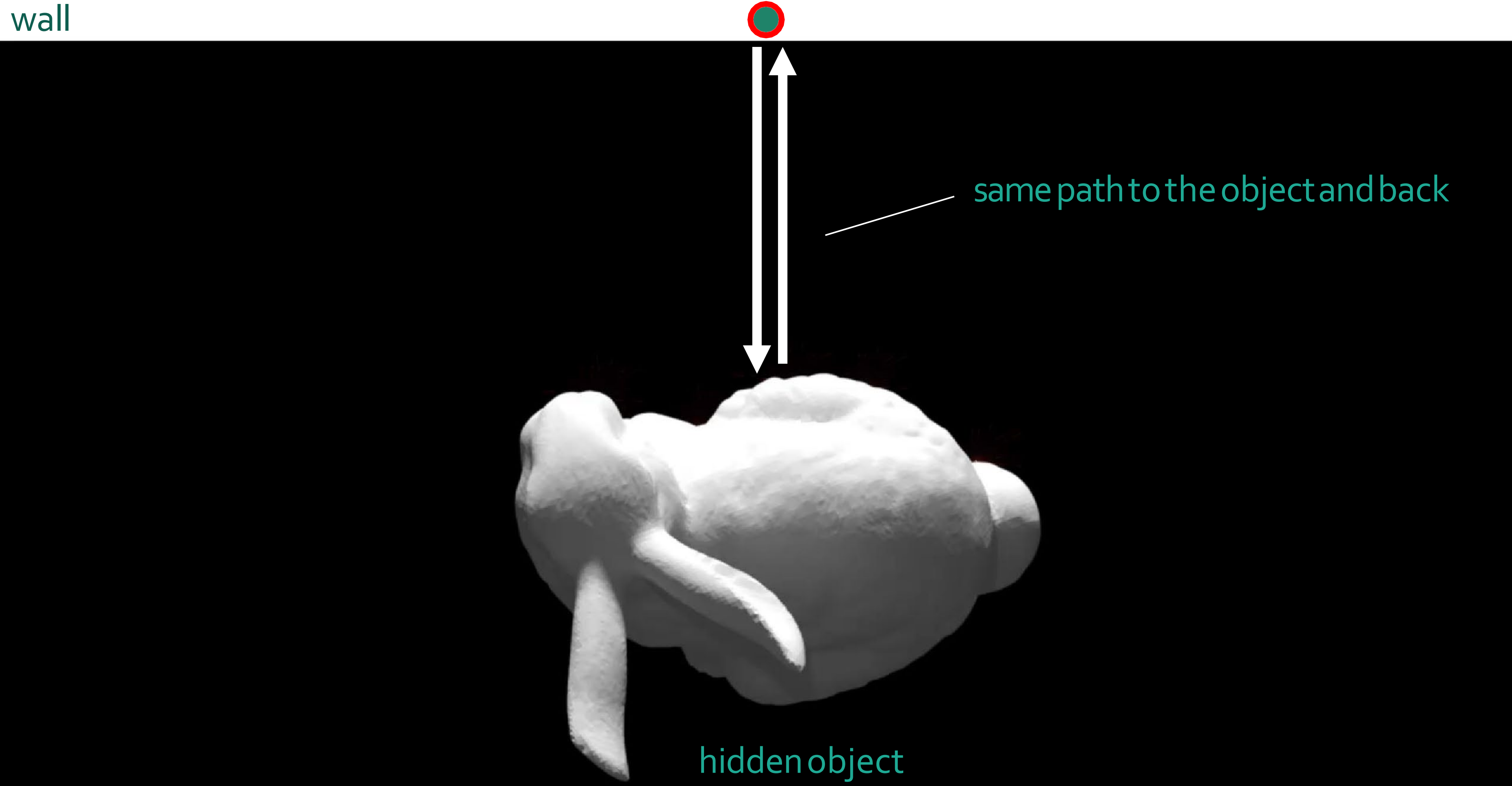


timestamp (nanoseconds)

hidden object

lasers and detectors illuminate and image **same** points

confocal sampling



confocal sampling

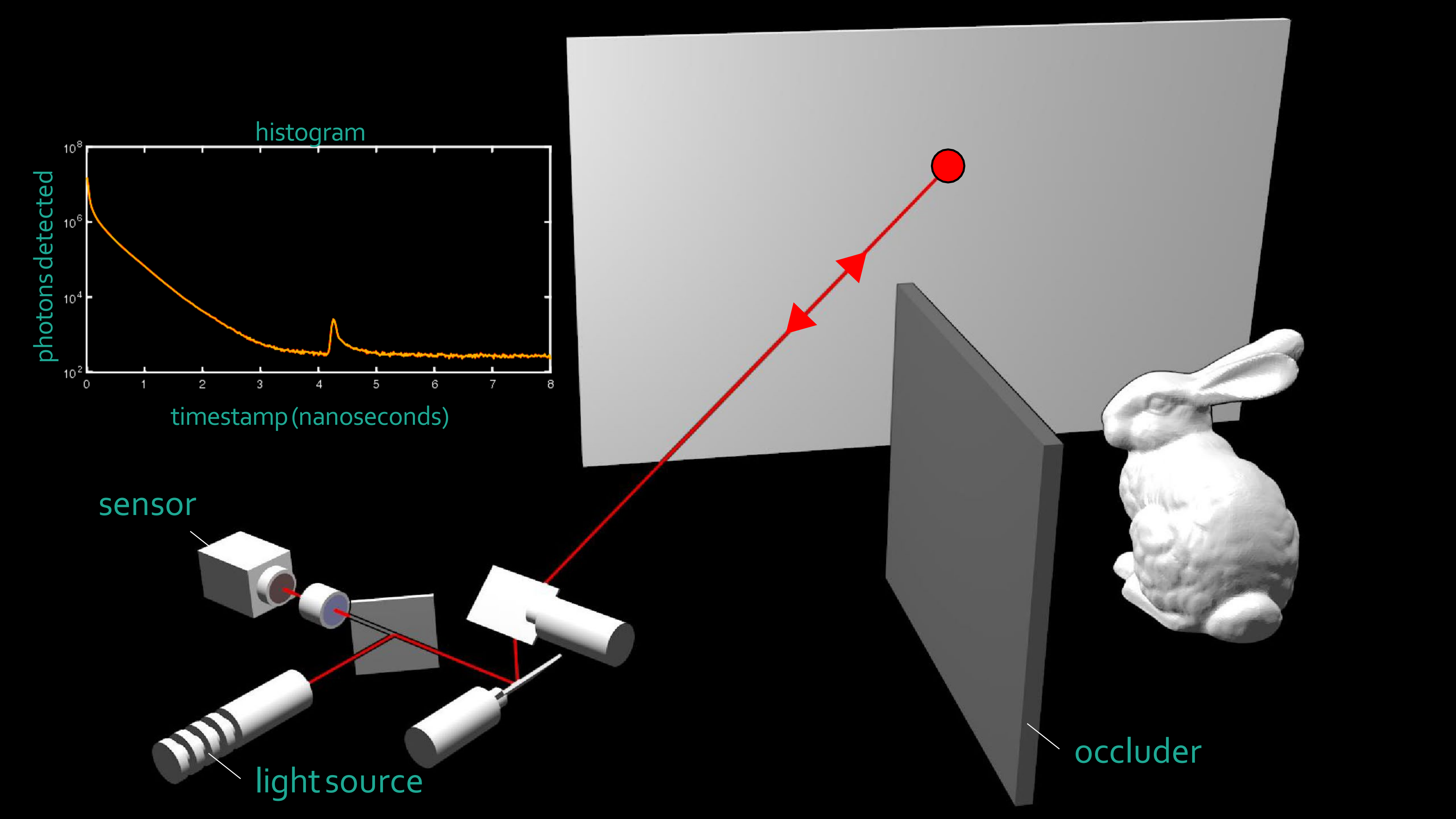
wall

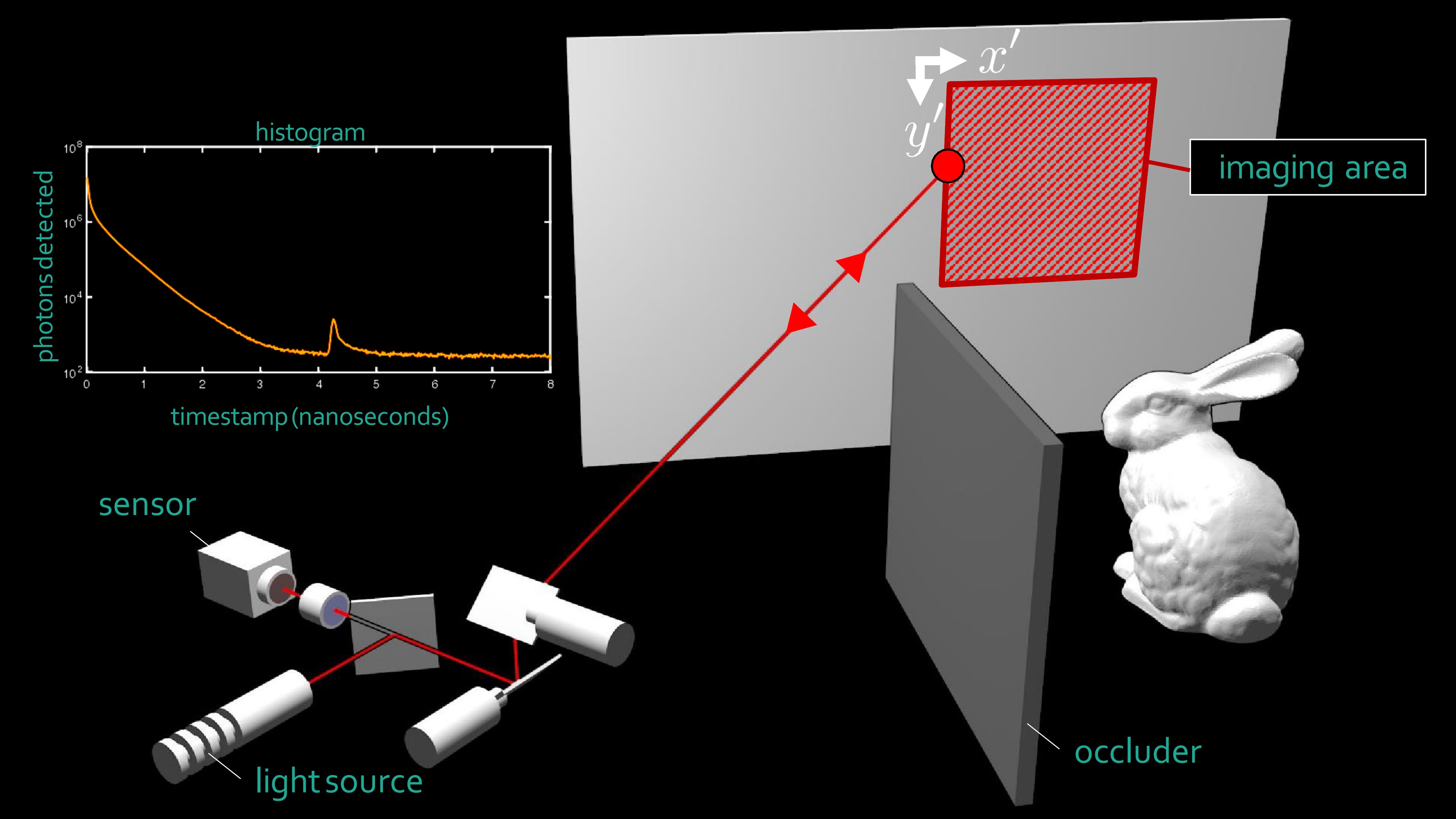
- Simplified NLOS mathematical model
- Enables efficient NLOS reconstruction

equivalent to one-way propagation at half-speed

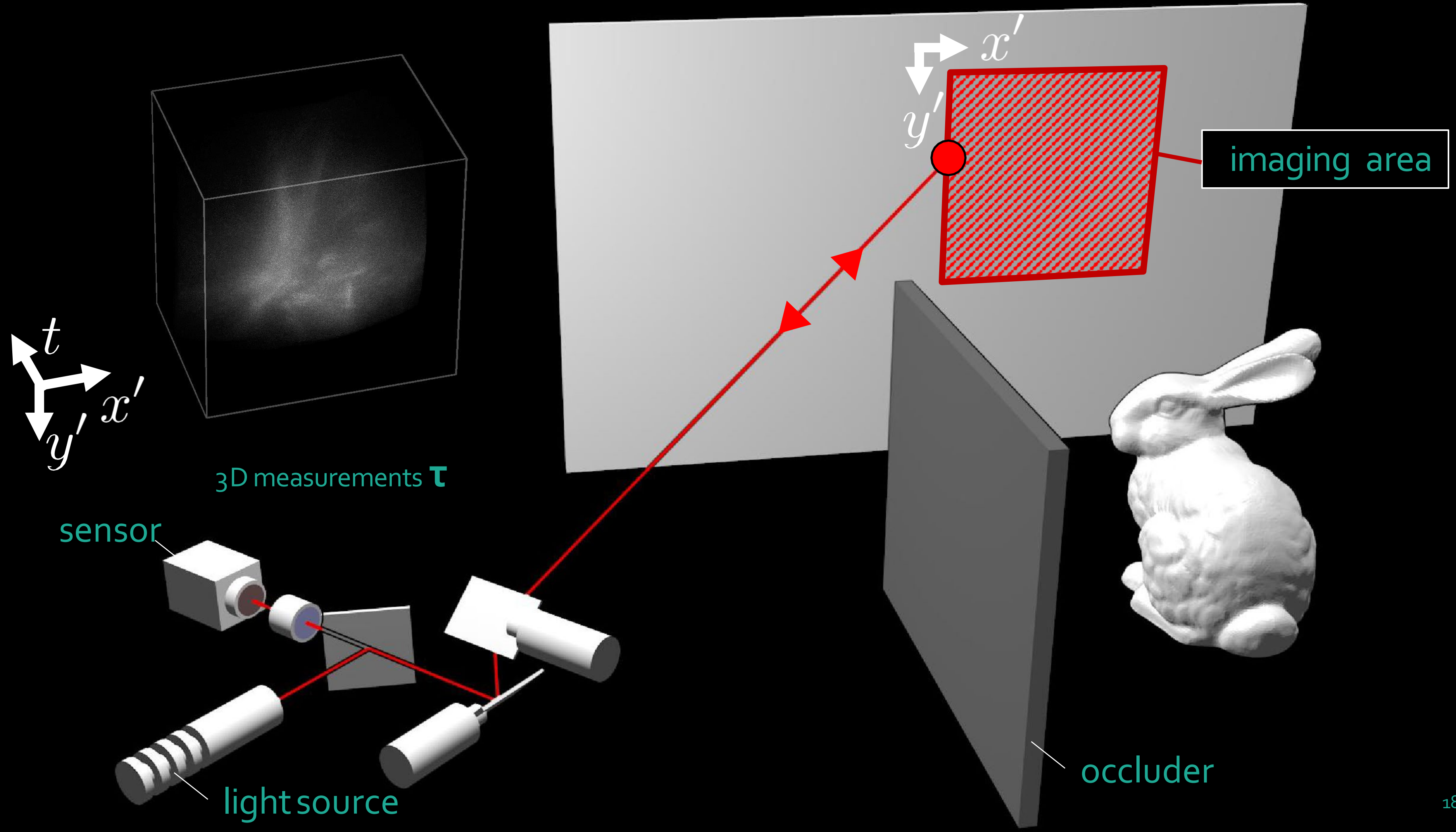
hidden object

confocal sampling


histogram
photons detected
timestamp (nanoseconds)
sensor
light source
occluder



histogram
photons detected
timestamp (nanoseconds)
imaging area
sensor
light source
occluder


x'
y'
imaging area
t
x'
y'
3D measurements $\boldsymbol{\tau}$
sensor
light source
occluder

NLOS Image Formation Model

$$\tau = A\rho$$

3D measurements $n^3 \times 1$ — transport matrix $n^3 \times n^3$ — unknown volume $n^3 \times 1$

Problem: **A** extremely large in practice

- for n=100, has 1 trillion elements
- for n=1000, sparse **A** needs petabytes of memory
- even matrix-free is computationally intractable

Computationally Intractable

Backprojection [Velten 12, Buttafava 15]

- Flops: $O(n^5)$
- Memory: $O(n^3)$
- Runtime: Approx. 10 min.

Iterative Inversion [Gupta 12, Wu 12, Heide 13]

- Flops: $O(n^5)$ per iter.
- Memory:
- Runtime: > 1 hour

Confocal NLOS Imaging

Confocal Scanning and Light-Cone Transform

$$\tilde{\tau} = a * \tilde{\rho}$$

measurements $n \times n \times n$

Blur kernel $n \times n \times n$

unknown volume $n \times n \times n$

3D Deconvolution (with Light-Cone Transform)
[O'Toole et al. 2018]

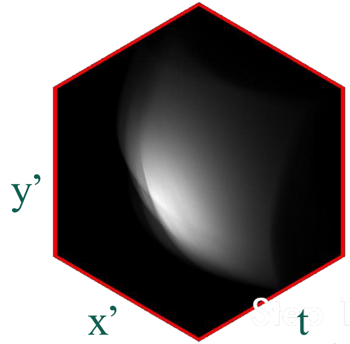
- Flops: $O(n^3 \log(n))$
- Memory: $O(n^3)$
- Runtime: < 1 second

Assumption:

- Isotropic scattering (only diffuse or retroreflective objects)

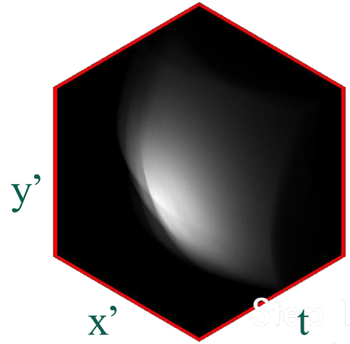
Confocal Scanning and Light-Cone Transform

measurements

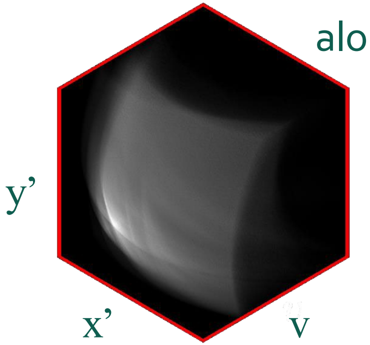


Confocal Scanning and Light-Cone Transform

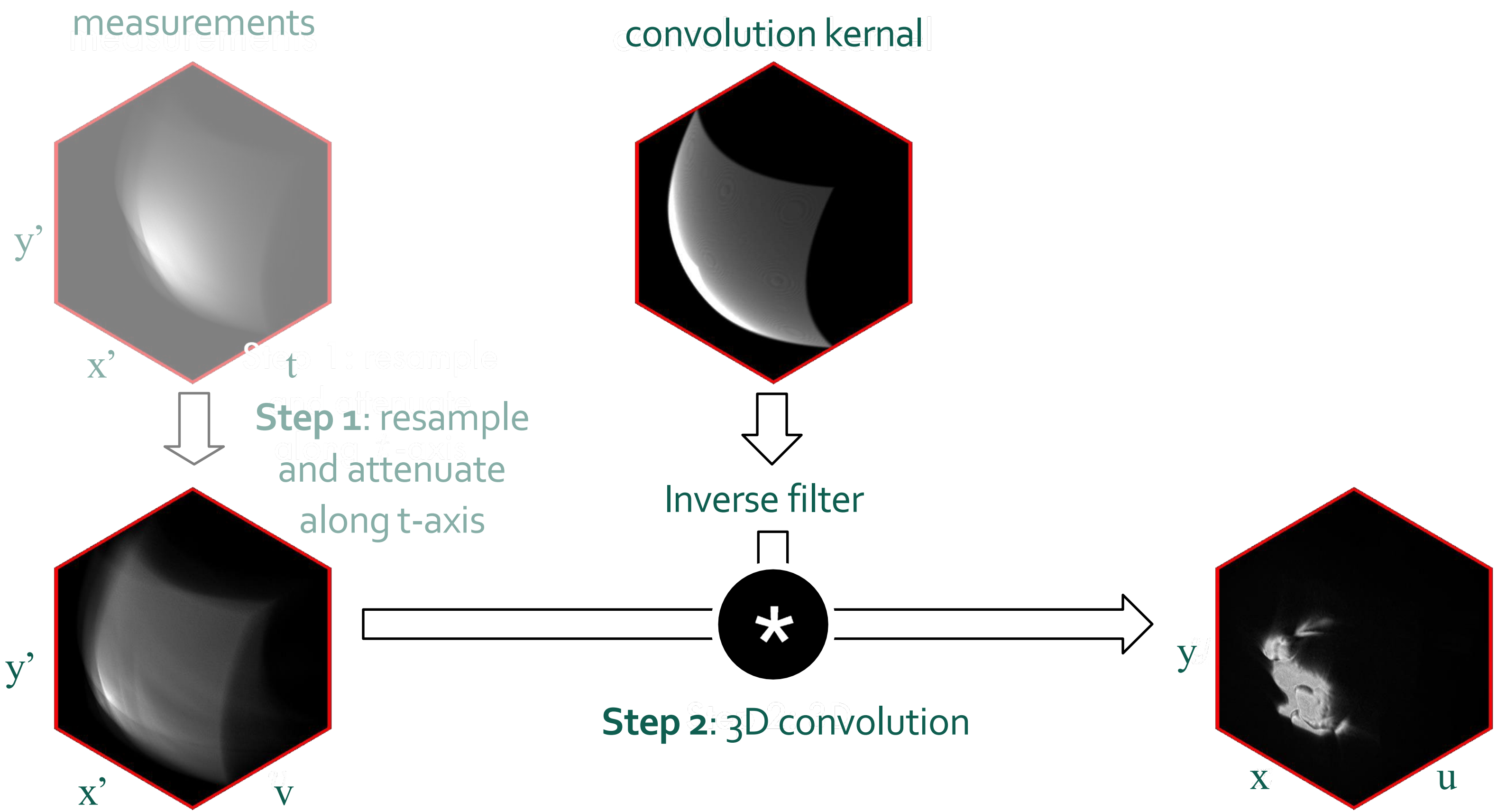
measurements



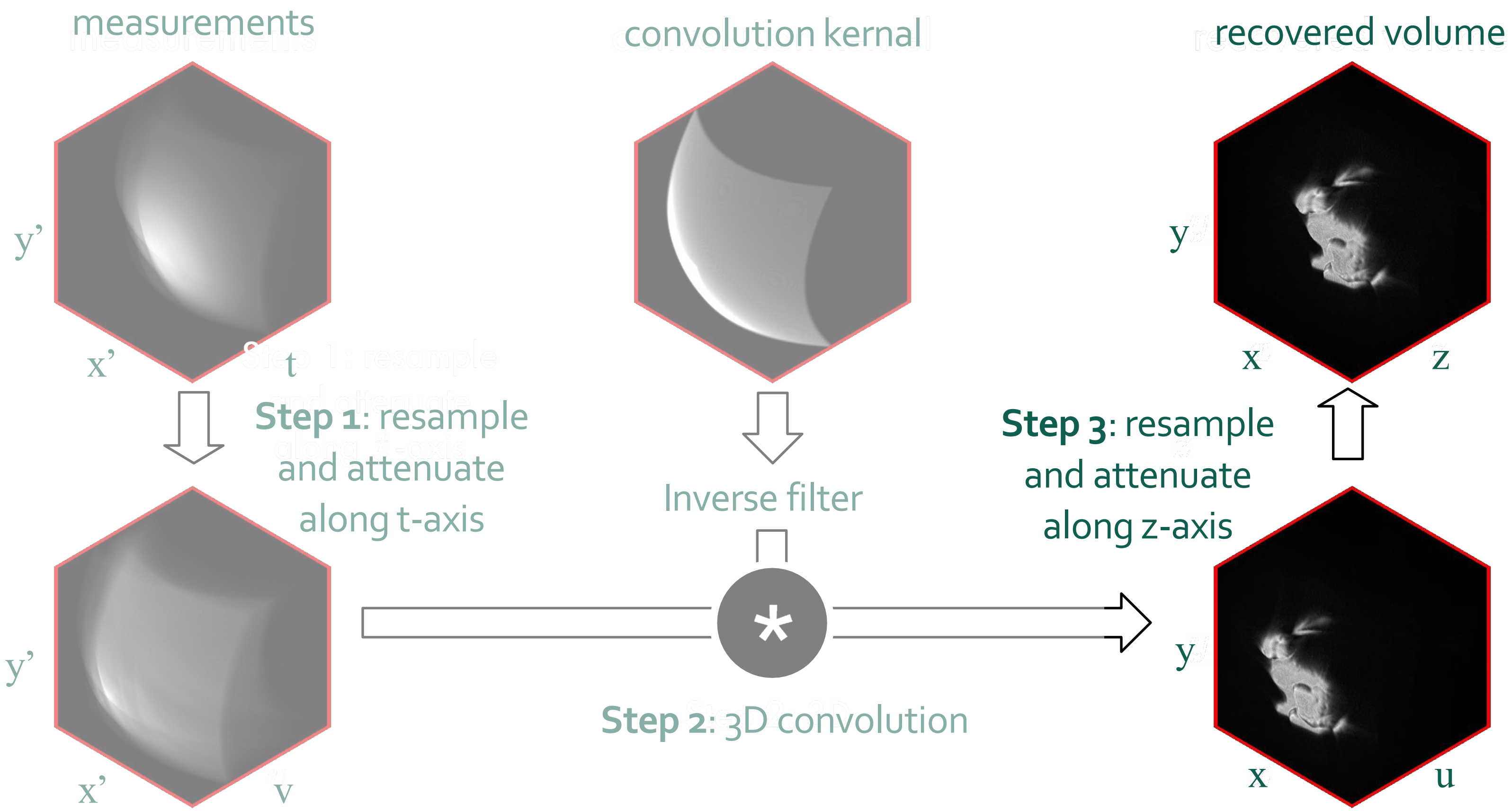
Step 1: resample and attenuate along t-axis



Confocal Scanning and Light-Cone Transform



Confocal Scanning and Light-Cone Transform

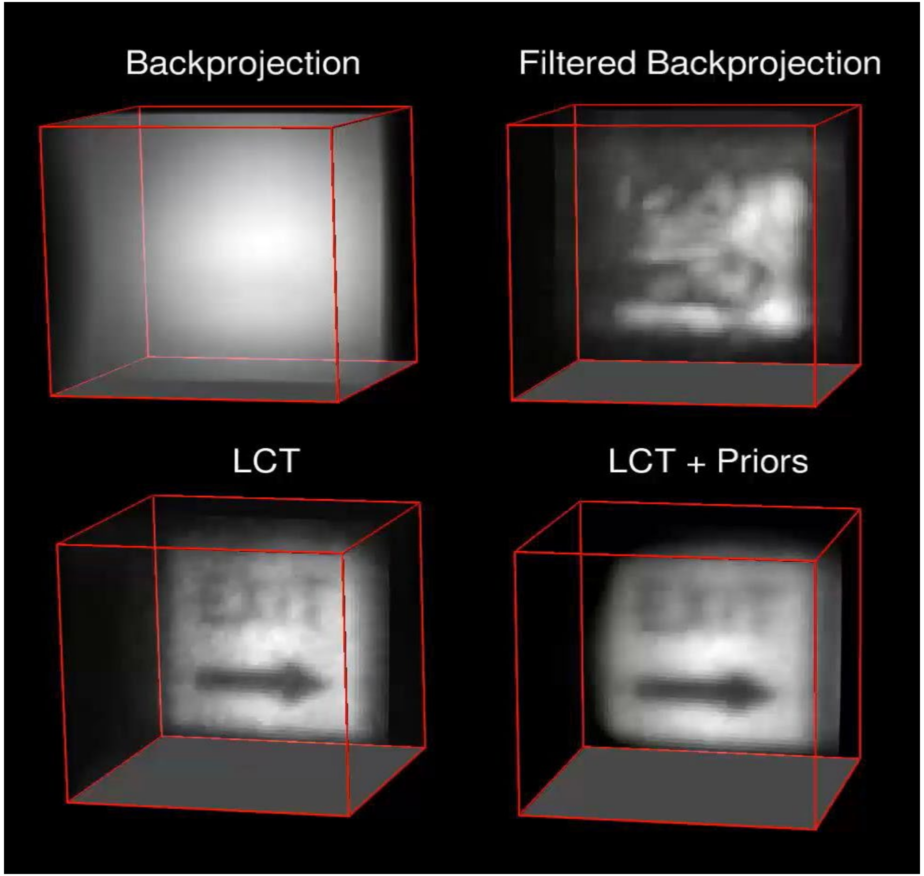


Experiments

Spatial resolution: 64x64

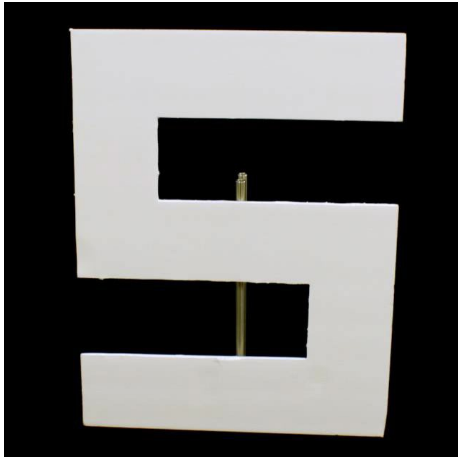
Exposure time (per sample): 0.1 sec

Retroreflective: Yes

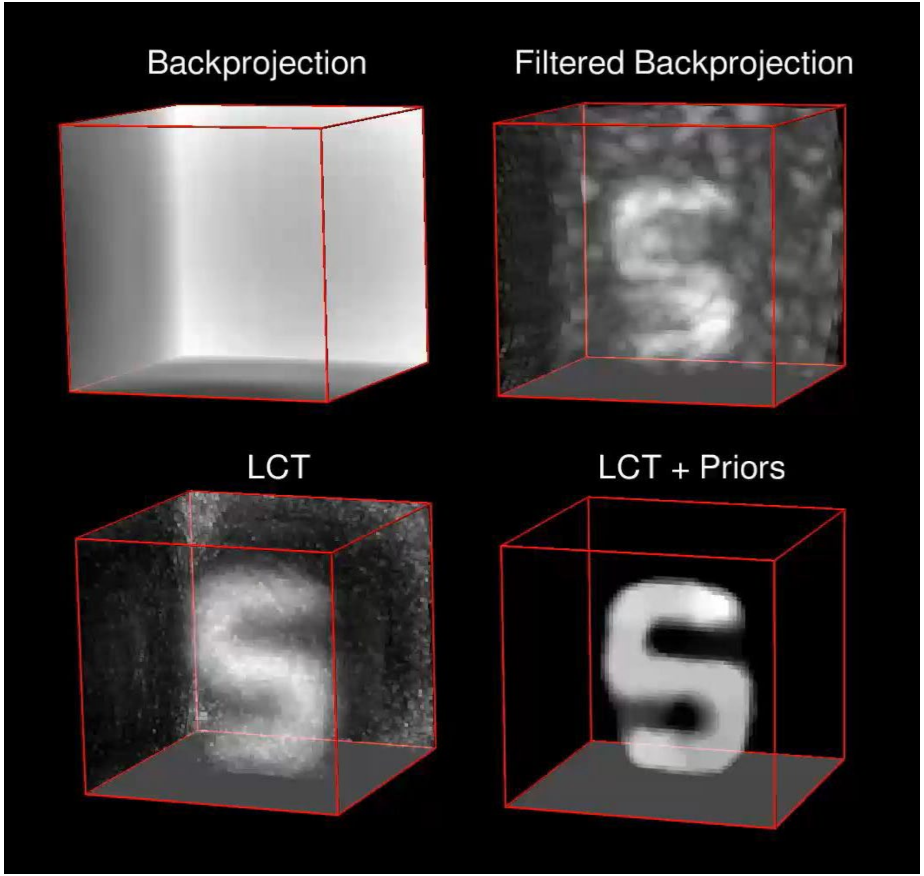


O'Toole et al., Nature 2018

Experiments



Spatial resolution: 64x64
Exposure time (per sample): 1 sec
Retroreflective: No



O'Toole et al., Nature 2018

The Frequency-Wavenumber（f - k）Migration Method

The Wave Equation

In free space, the wave equation constrains the propagation of Ψ in space and time according to

$$\nabla^2 \Psi - \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} = 0$$

Task: Migrate the boundary condition $z=0$ to another boundary condition $t=0$.

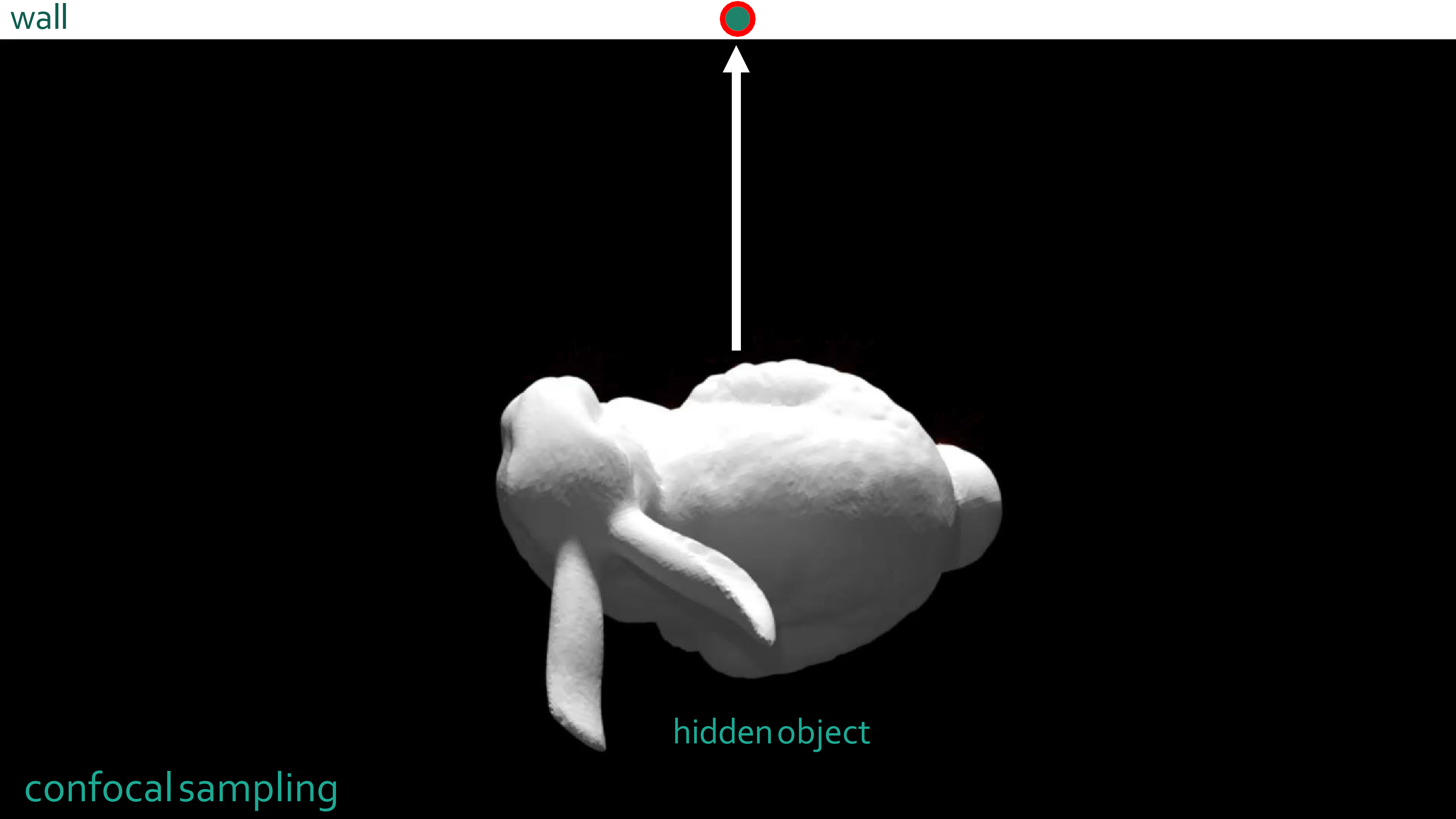
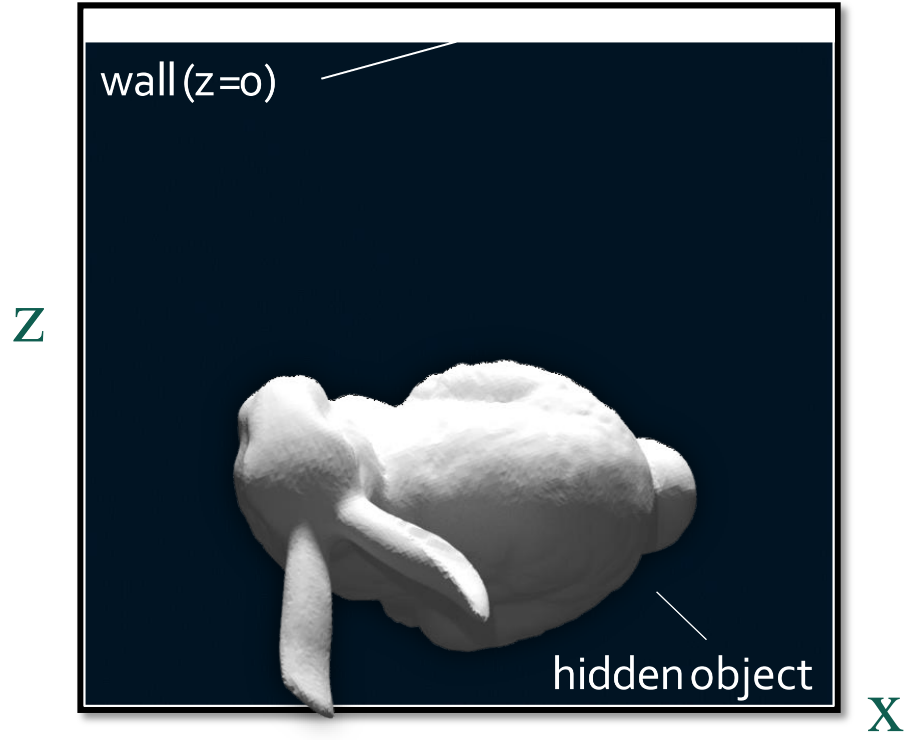

wall
hidden object
confocal sampling

Image Formation Model

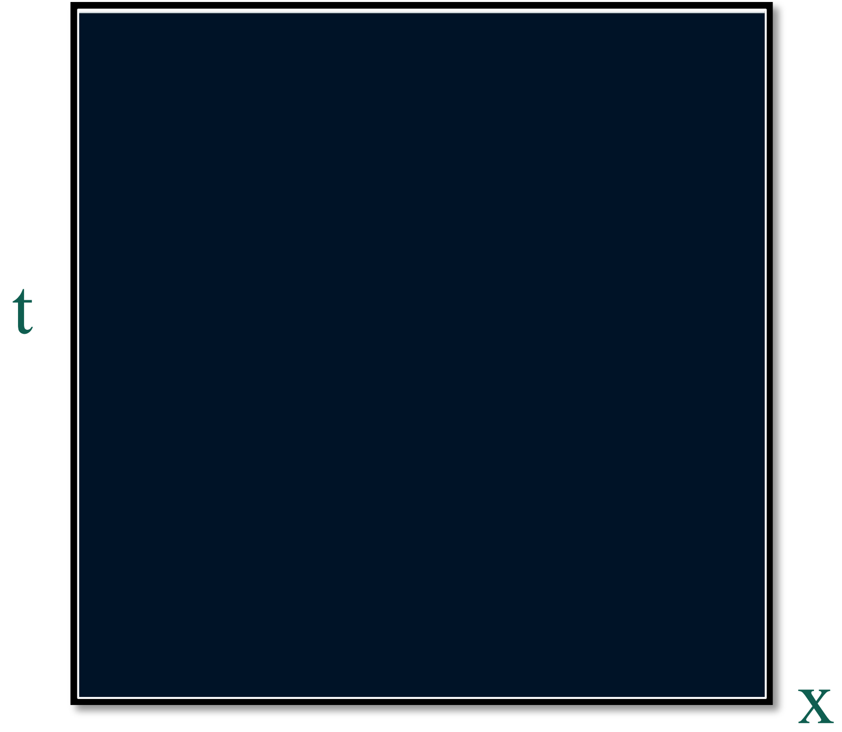
$$\Psi(x,z,t)$$

wavefield



$$\Psi(x,z=0,t)$$

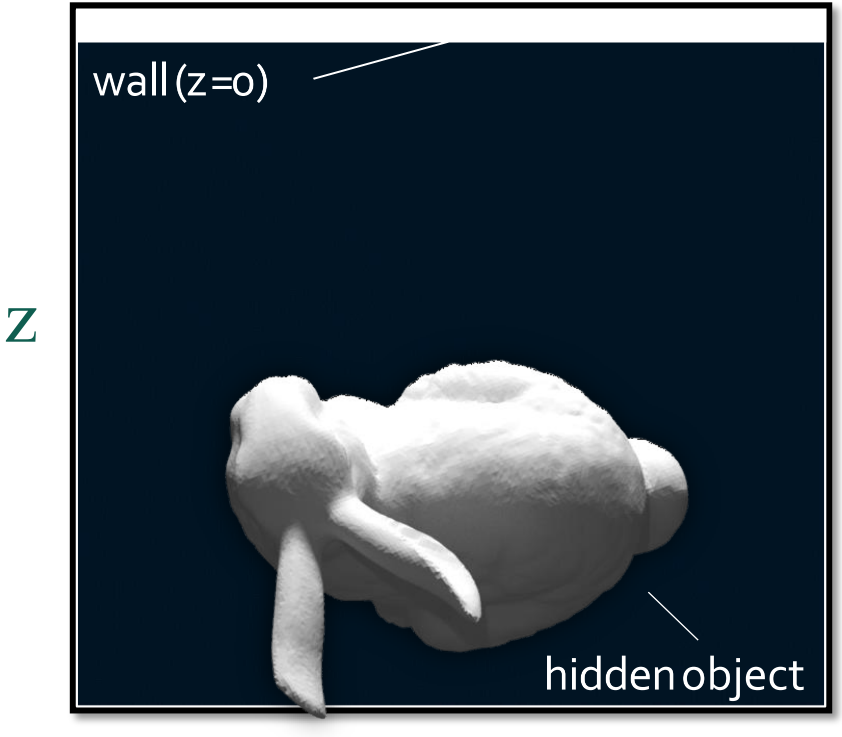
confocal measurements



General Time Reversal Solution

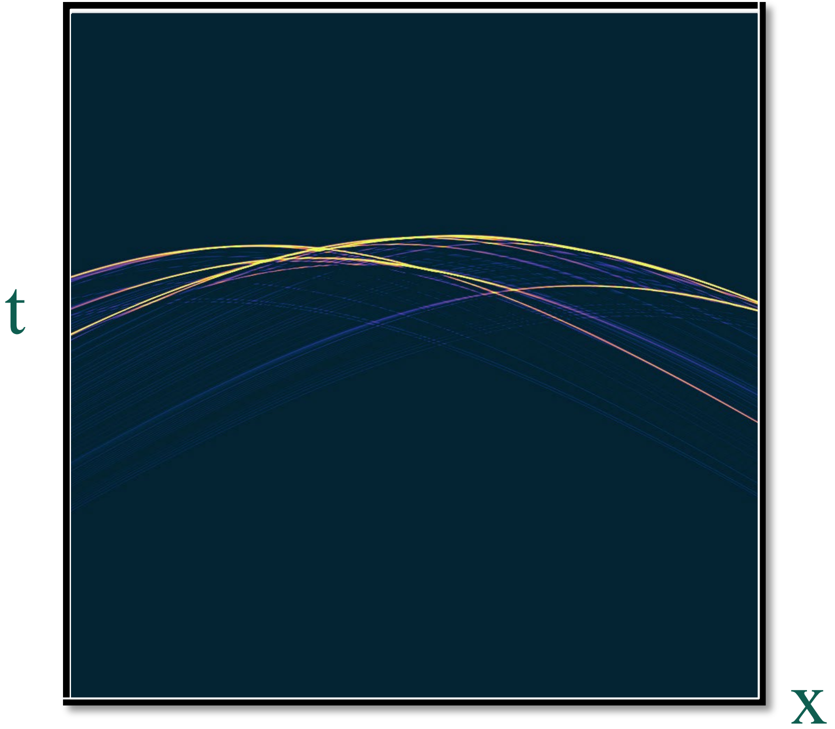
$$\Psi(x, z, t = 3.000ns)$$

wavefield



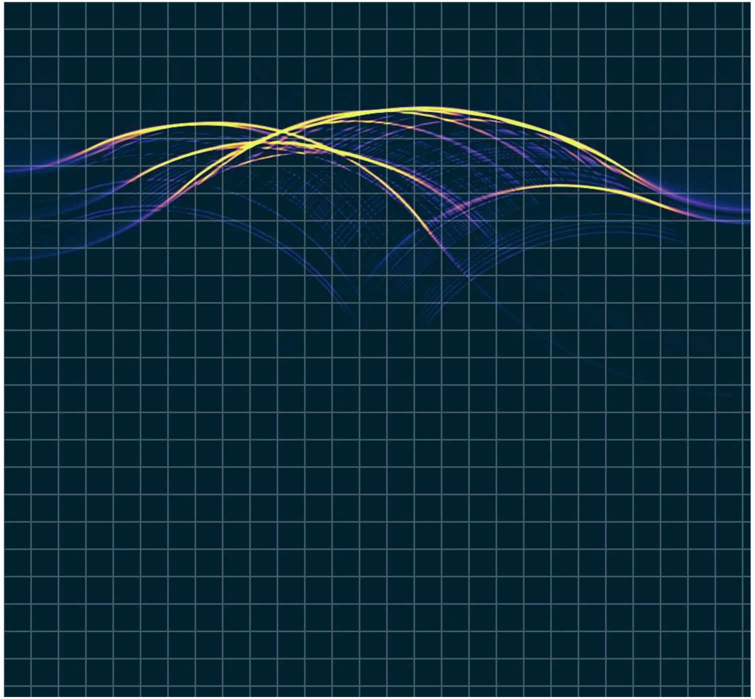
$$\Psi(x, z = 0, t)$$

confocal measurements



General Time Reversal Solution

finite-difference time-domain method



1. approximate wave equation with finite differences

$$\frac{\partial^2 \Psi}{\partial t^2} \approx \frac{\Psi_i^{n+1} - 2\Psi_i^n + \Psi_i^{n-1}}{(\Delta t)^2}$$

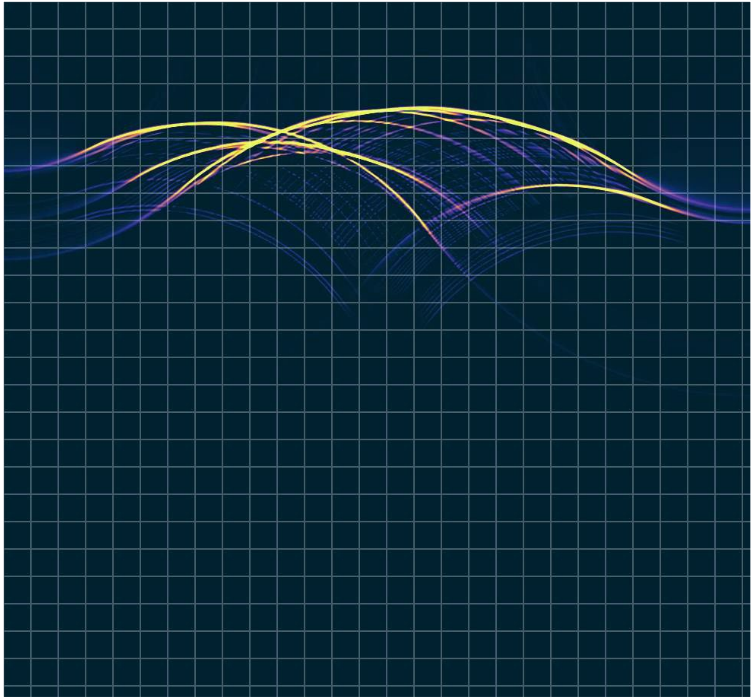
2. solve for previous time step

$$\Psi_i^{n-1} = f\left(\Psi^n, \Psi^{n+1}\right)$$

3. Repeatedly update **Ψ** at all grid cells

General Time Reversal Solution

finite-difference time-domain method



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2. solve for previous time step

$$\Psi_i^{n-1} = f\left(\Psi^{n}, \Psi^{n+1}\right)$$

3. Repeatedly update Ψ at all grid cells

Slow to get t=0 at high-resolution!

Frequency–wavenumber（f - k）Migration

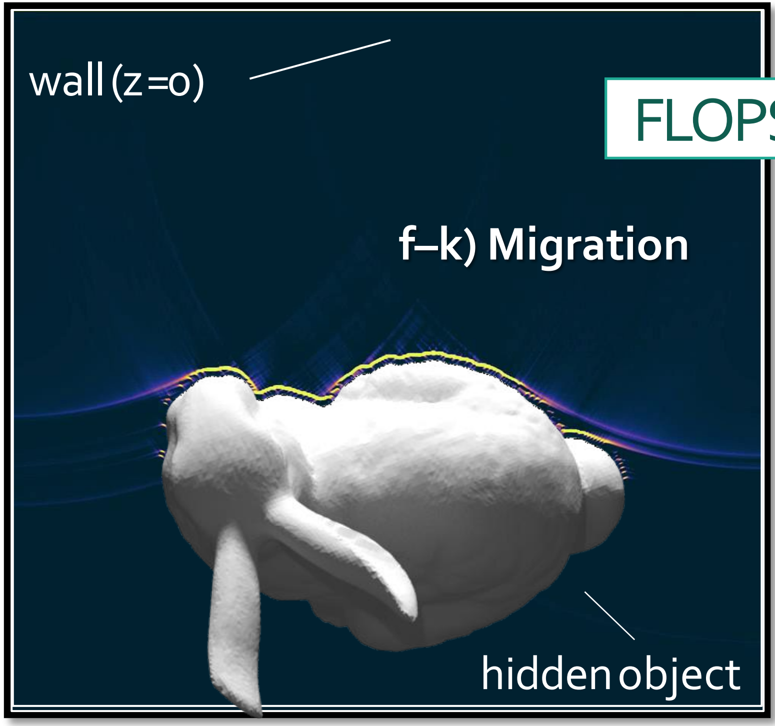
$\Psi(x, z, t)$

wavefield

$\Psi(x, z = 0, t)$

confocal measurements

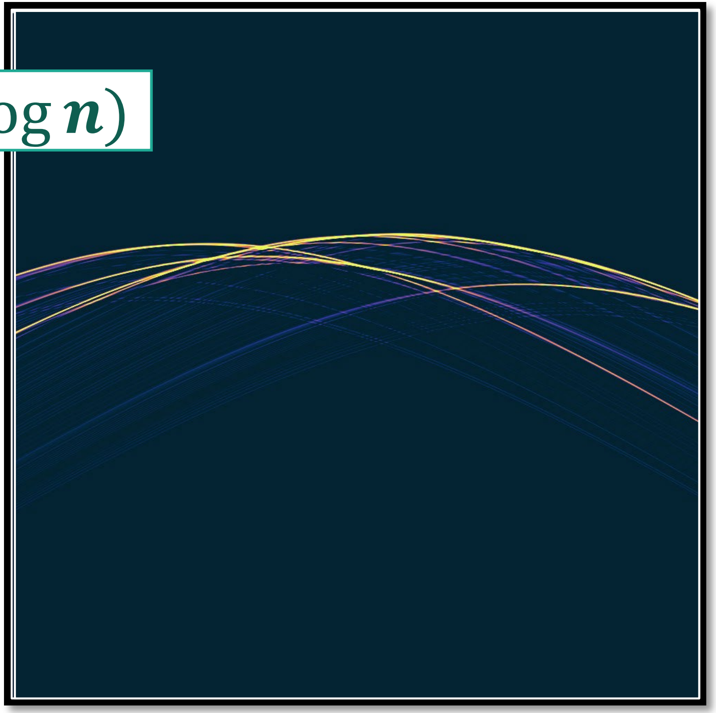
z



x

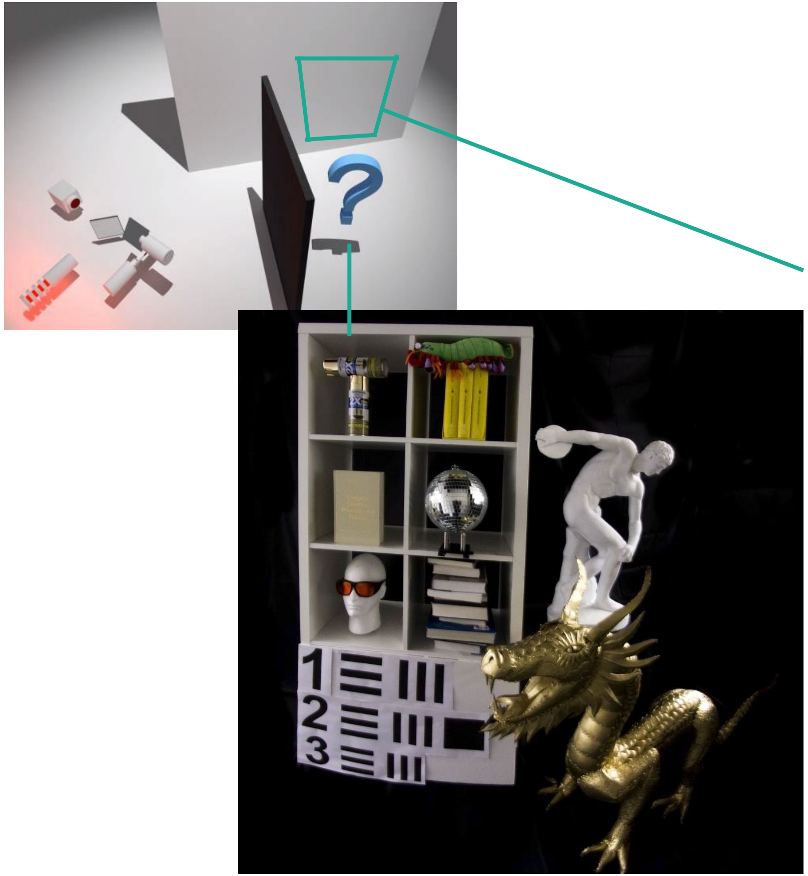
FLOPS: $O(n^3 \log n)$

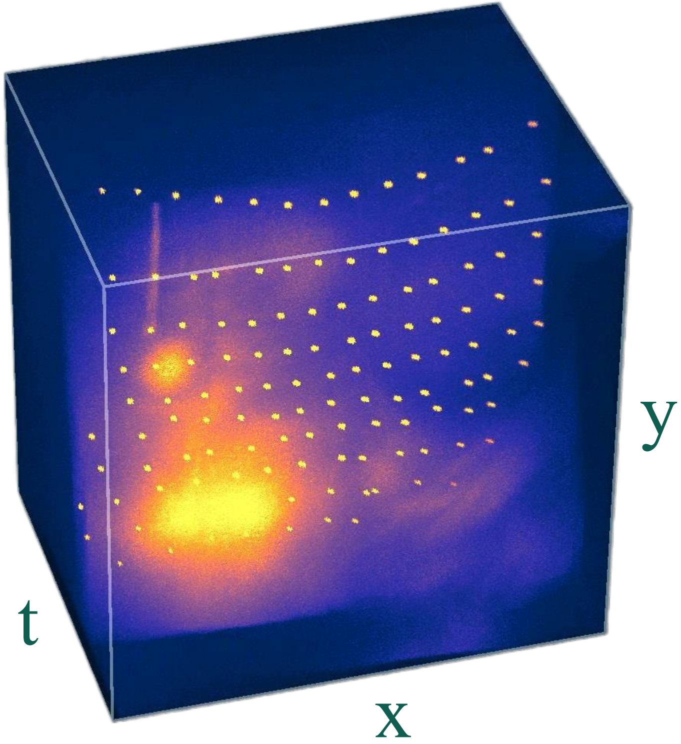
t



x

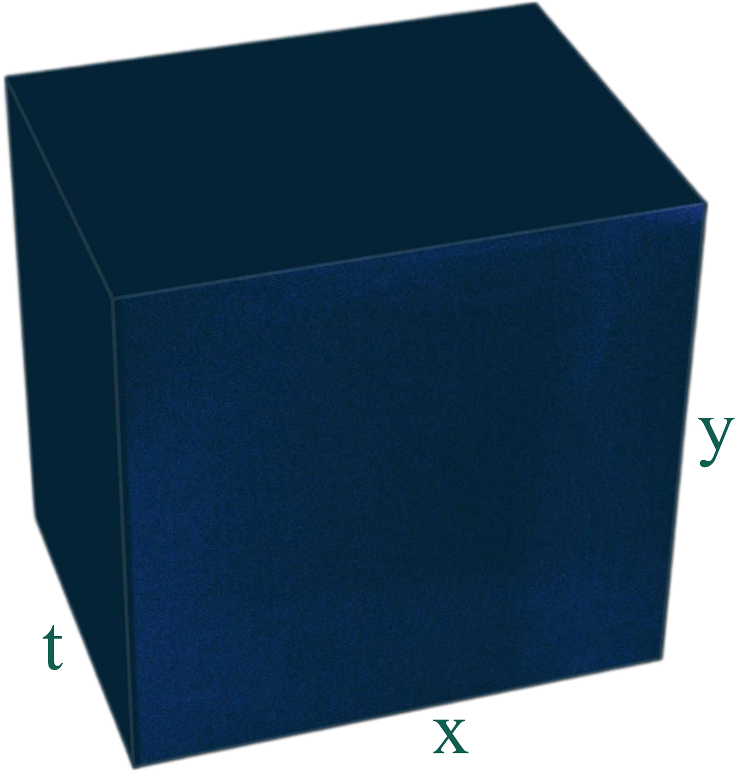
The f - k Migration: Measurements





Lindell et al., SIGGRAPH 2019

The f - k Migration: Measurements



Lindell et al., SIGGRAPH 2019

The f - k Migration

$\Psi(x, y, t)$

Measurements(z=0)

$\bar{\Phi}(k_x, k_y, f)$

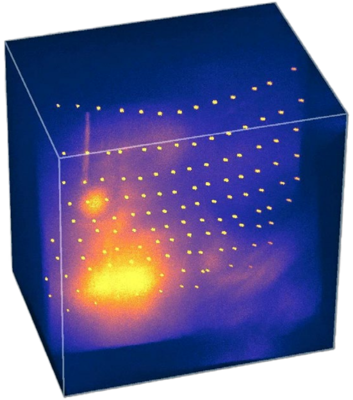
Spectrum

$\Phi(k_x, k_y, k_z)$

Interpolated Spectrum

$\Psi(x, y, z)$

Hidden Volume (t=0)



F

Resample

F^{-1}

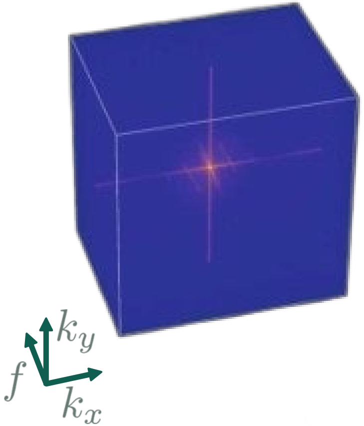
The f - k Migration

Express wavefield as function of measurement spectrum (plane wave decomposition)

$$\Psi(x, y, z, t) = \iiint \bar{\Phi}(k_x, k_y, f) e^{2\pi i (k_x x + k_y y + k_z z - f t)} dk_x dk_y df$$

wavefield

Fourier transform
Of measurements



Set $t=0$ to get migrated solution

$$\Psi(x, y, z, t=0) = \iiint \bar{\Phi}(k_x, k_y, f) e^{2\pi i (k_x x + k_y y + k_z z)} dk_x dk_y df$$

Almost an **inverse** Fourier Transform!

The f - k Migration

Set $t=0$ to get migrated solution

$$\Psi(x, y, z, t=0) = \iiint \bar{\Phi}(k_x, k_y, f) e^{2\pi i (k_x x + k_y y + k_z z)} dk_x dk_y df$$

Almost an **inverse** Fourier Transform!

Use dispersion relation[1] to perform substitution of variables

$$f = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$f \Rightarrow k_z$$

Weighted 1D interpolation, known as **Stolt** interpolation

$$\Phi(k_x, k_y, k_z) = \frac{v|k_z|}{\sqrt{k_x^2 + k_y^2 + k_z^2}} \bar{\Phi}\left(k_x, k_y, v\sqrt{k_x^2 + k_y^2 + k_z^2}\right)$$

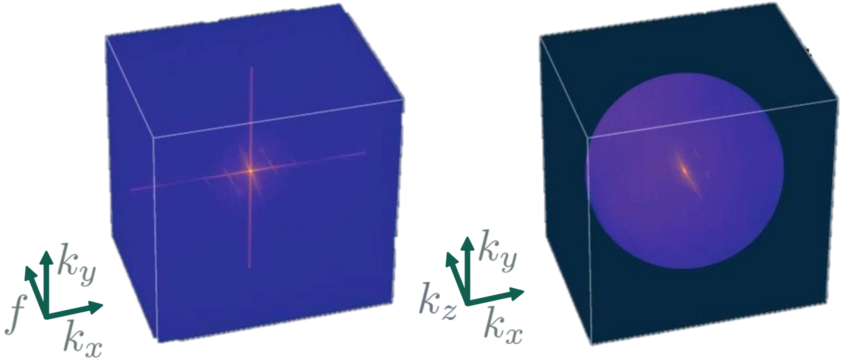
[1] Georgi, Howard. The physics of waves . Englewood Cliffs, NJ: Prentice Hall,

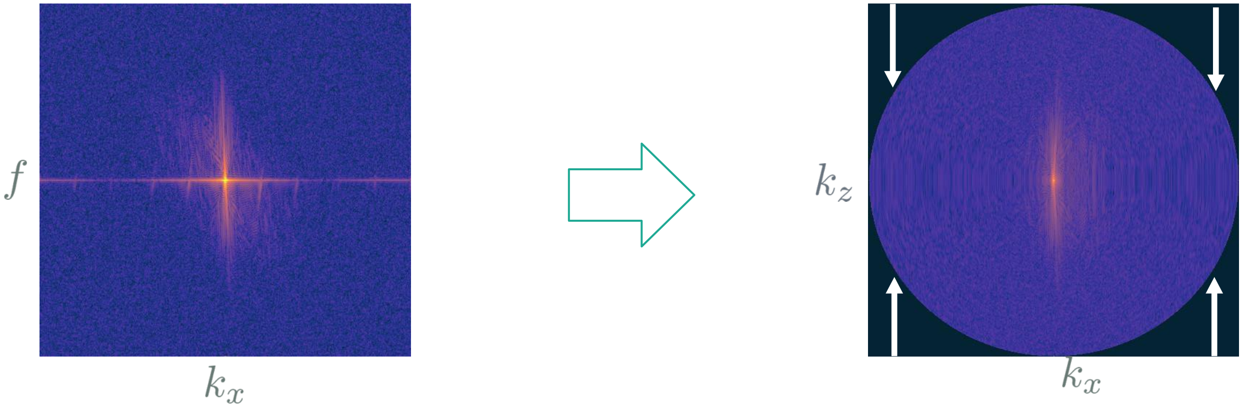
The f - k Migration

Use dispersion relation[1] to perform substitution of variables

$$f = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\boxed{f \Rightarrow k_z}$$



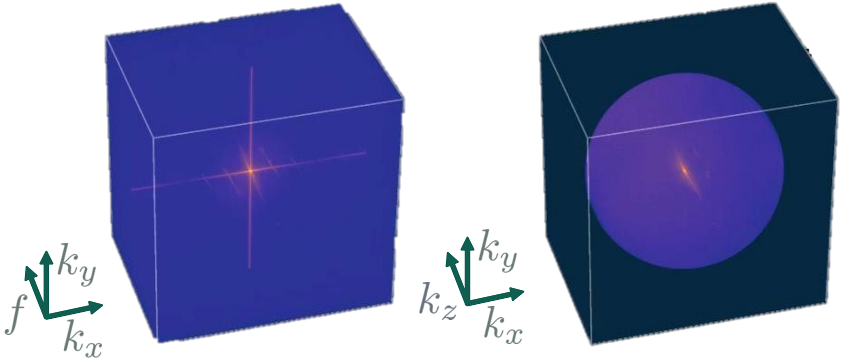


The f - k Migration

Use dispersion relation[1] to perform substitution of variables

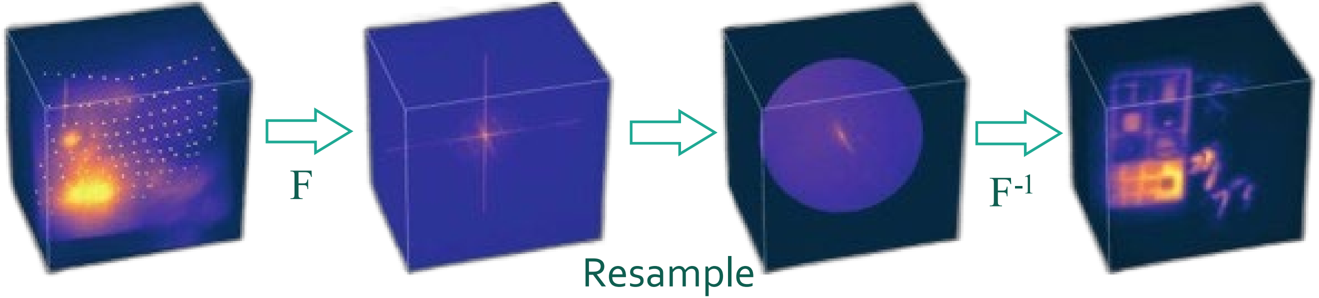
$$f = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\boxed{f \Rightarrow k_z}$$



The migrated solution is an inverse Fourier Transform!

$$\Psi(x, y, z, t = 0) = \iiint \Phi(k_x, k_y, k_z) e^{2\pi i (k_x x + k_y y + k_z z)} dk_x dk_y dk_z$$



The f - k Migration for NLOS Imaging

procedure FKMIG$(\tau(x, y, t))$

  // Pre-process data

  $\Psi(x, y, t) = t \cdot \sqrt{\tau(x, y, t)}$

  $\Psi(x, y, t) = pad\_volume(\Psi(x, y, t))$

  // Fast Fourier transform (Eqn. (4)**)**

  $\bar{\Phi}\left(k_x, k_y, f\right) = \mathcal{F}_{\{x,y,t\}}\left\{\Psi(x, y, t)\right\}$

  // Stolt interpolation (Eqn. (5)**)**

  $\Phi\left(k_x, k_y, k_z\right) = \frac{v|k_z|}{\sqrt{k_x^2+k_y^2+k_z^2}} \cdot resample\left(\bar{\Phi}(k_x, k_y, f)\right)$

  // Inverse Fast Fourier transform (Eqn. (3)**)**

  $\Psi(x, y, z) = \mathcal{F}^{-1}_{\{x,y,z\}}\left\{\Phi\left(k_x, k_y, k_z\right)\right\}$

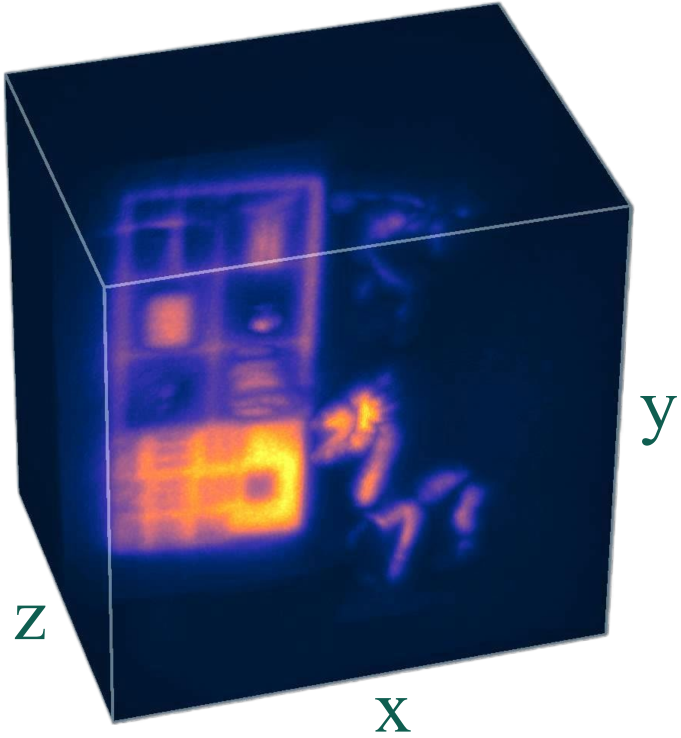
  // Post-process data

  $\Psi(x, y, z) = unpad\_volume(\Psi(x, y, z))$

  return $|\Psi(x, y, z)|^2$

end procedure

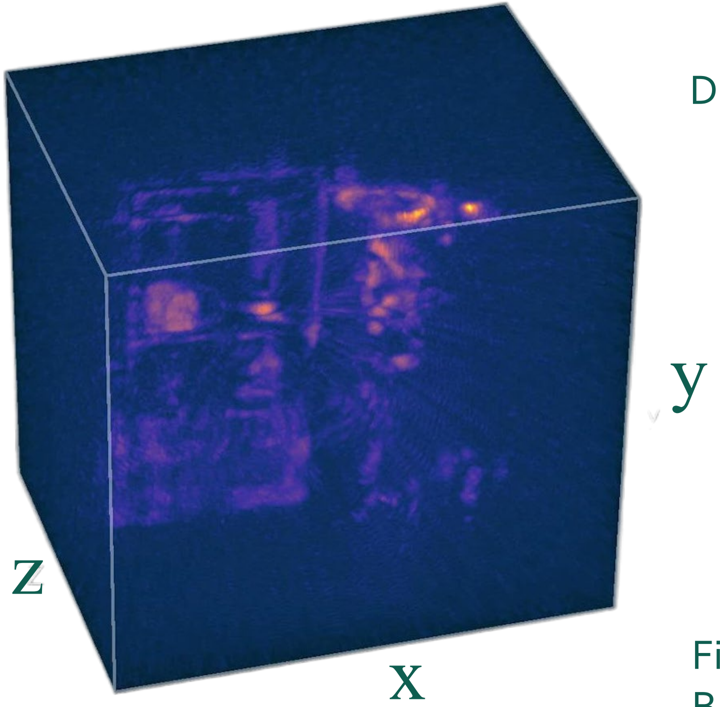
The f - k Migration



Dimensions: 2 x 2 m
Exposure: 180 min
Reconstruction time: ~90 sec (CPU)

Lindell et al., SIGGRAPH 2019

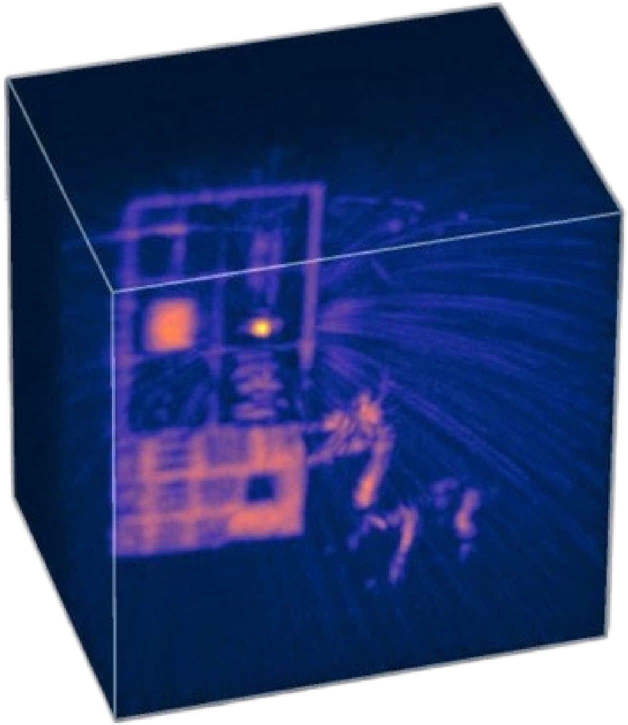
Reconstruction Comparison



Dimensions: 2 x 2 x 1.5m

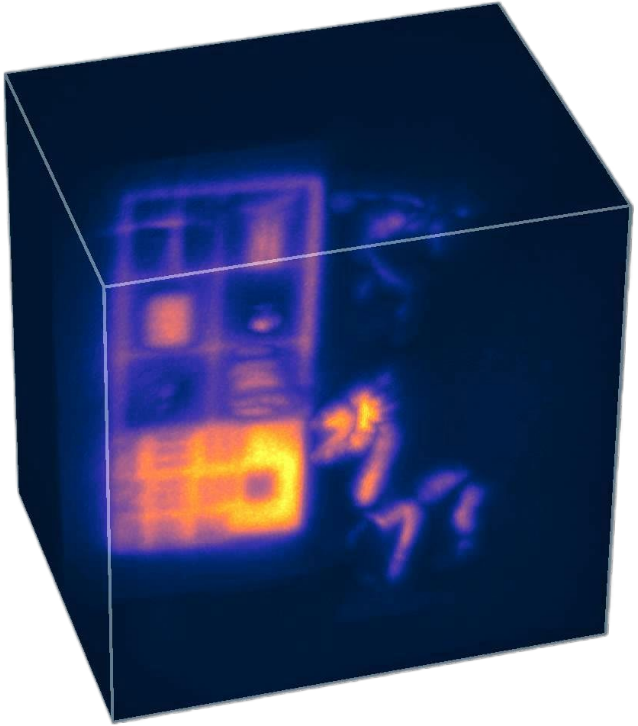
Filtered Backprojection

Reconstruction Comparison



Liu et al. 19 (293 s)

Virtual Wave Optics [Liu et al. 19 (Nature)]



f - k migration (94 s)

Other Works

Other NLOS Works

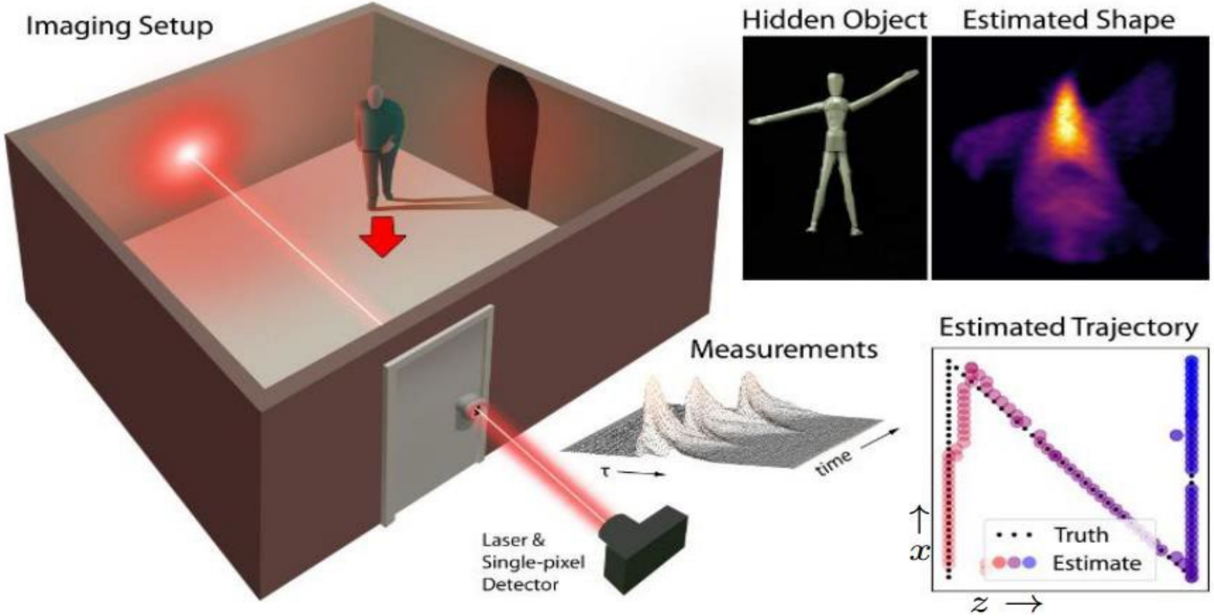
Directional Light-Cone Transform

hidden scene

Recovered surface

[Young et al., CVPR 2020]

Keyhole NLOS Imaging



[Metzler et al., IEEE TCI 2021]

Directional Light-Cone Transform

hidden scene

Recovered surface

[Young et al., CVPR 2020]

Today's **Topic**

- What's NLOS Imaging?
- Model of NLOS Imaging
  - NLOS Image Formation Model
- Confocal NLOS Imaging
- The f - $\mathbf{k}$ Migration Method
- Other Works

Thank You!

Qilin Sun（孙启霖）

香港中文大学（深圳）

点昀技术（Point Spread Technology）