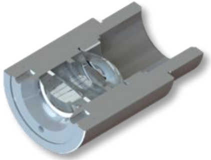
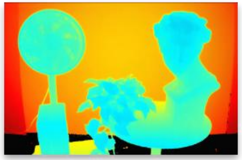








GAMES 204

Computational Imaging

Lecture 08: Imaging Toolbox： Lens&Abberations

Qilin Sun（孙启霖）

香港中文大学（深圳）

点昀技术（Point Spread Technology）

Today's Topic

- Paraxial optics.
- Ray Transfer Matrix
- Aberrations and compound lenses.

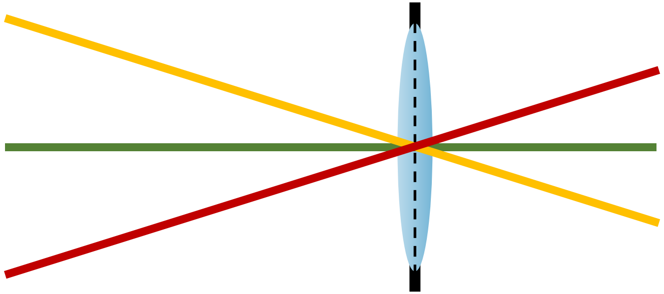
About Homework

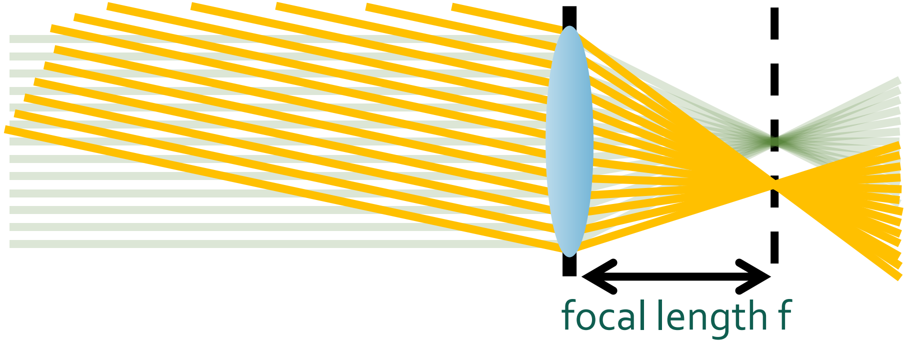
Rating： Code (60%), Results (20%), Report and Description (20%)

Paraxial Optics

Thin Lens Model

Simplification of geometric optics for well-designed lenses.





- Where do the thin lens properties come from?
- What determines the focal length of a thin lens?

$$\frac{1}{S'}+\frac{1}{S}=\frac{1}{f}$$

$$m=\frac{S'-f}{f}$$

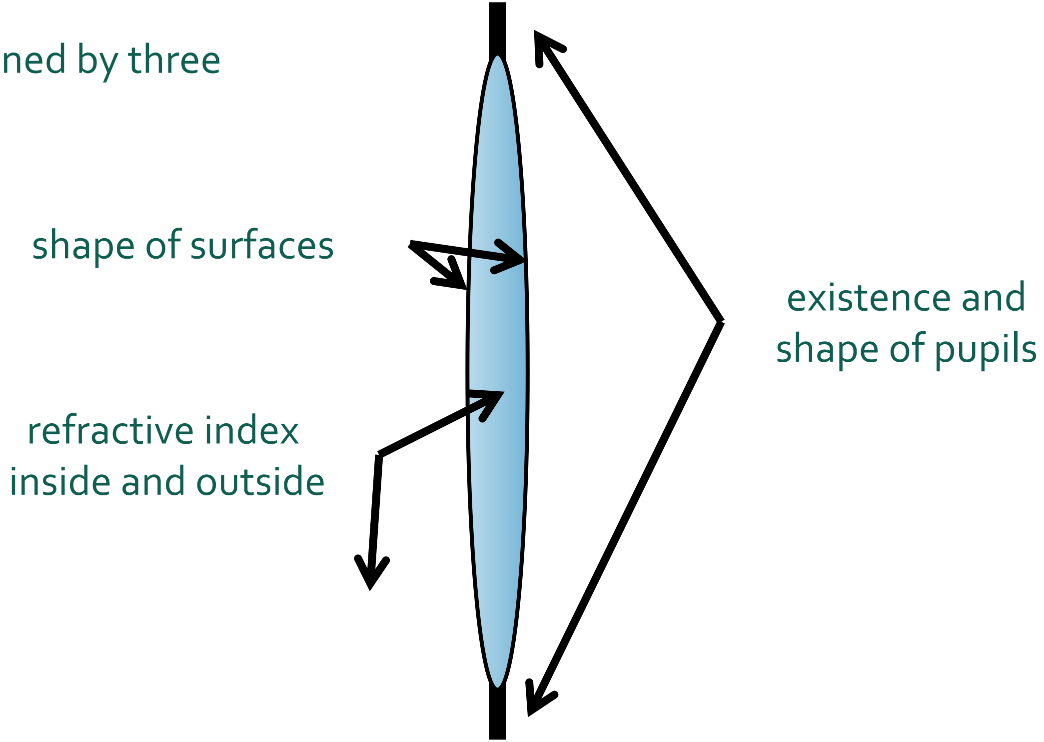
Two assumptions:

1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.

Real lenses

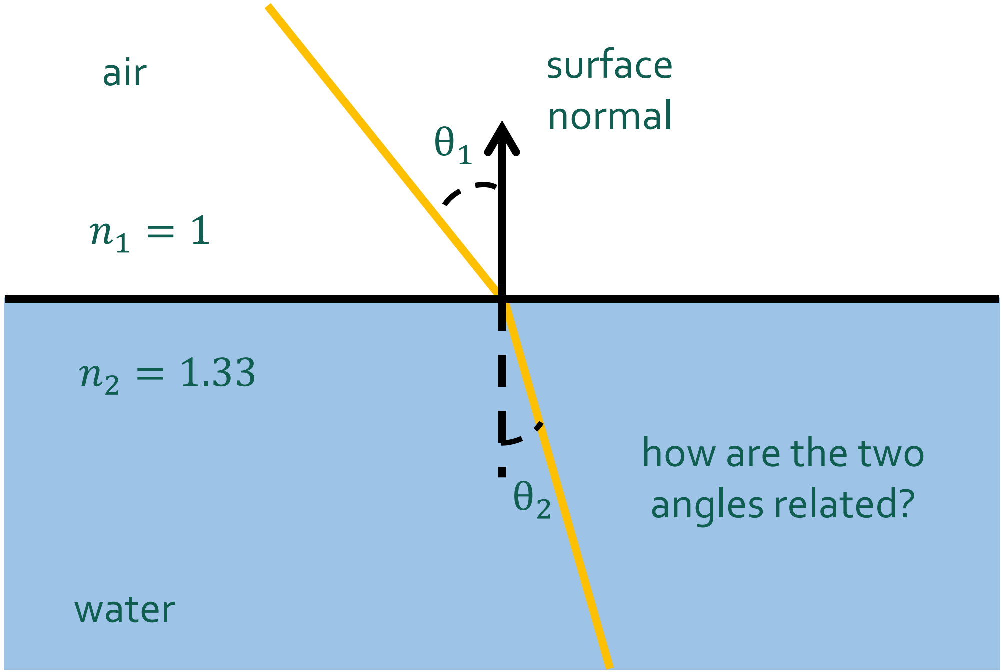
Simple case: a system with an individual lens element

The lens' behavior is determined by three characteristics:



Refraction

Refraction is the bending of rays of light when they cross optical interfaces (i.e., surfaces where the index of refraction changes).



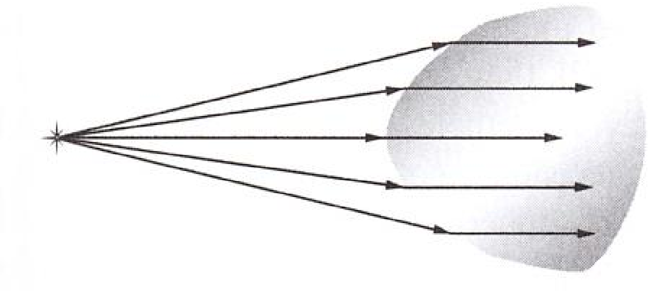
Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

How do we prove Snell's law?

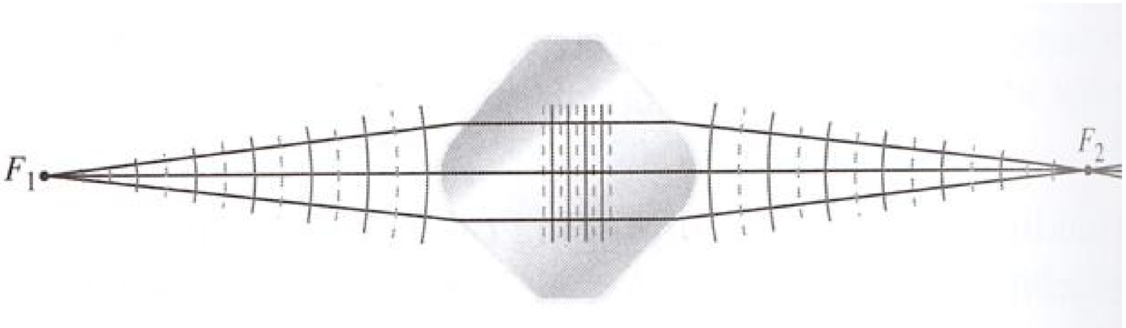
Refraction at Interfaces of Complicated Shapes

What shape should an interface have to make parallel rays converge to a point?



Single hyperbolic interface:
point to parallel rays

Double hyperbolic interface:
point to point rays



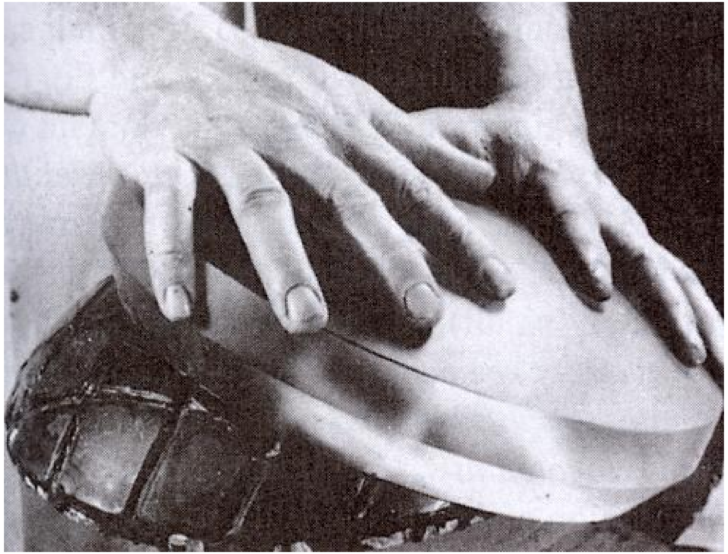
(Note: conics have different reflective and refractive properties.)

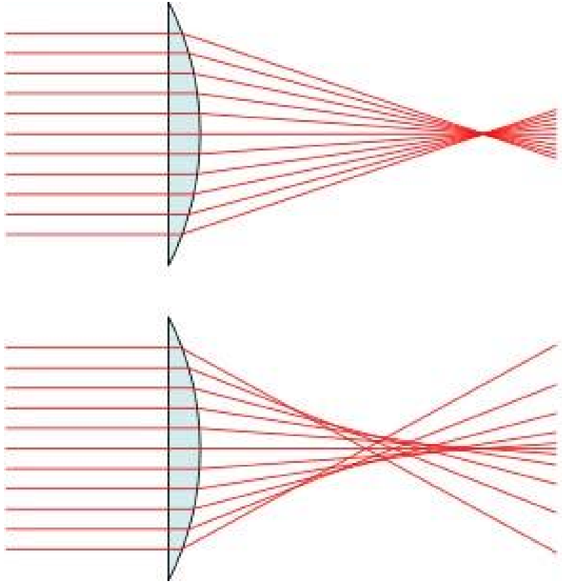
Therefore, lenses should also have hyperbolic shapes.

Spherical lenses

In practice, lenses are often made to have spherical interfaces for ease of fabrication.

- Two roughly fitting curved surfaces ground together will eventually become spherical.





Spherical lenses don't bring parallel rays to a point.

- This is called ***spherical aberration***.
- Approximately axial (i.e., paraxial) rays behave better.

Paraxial Approximation (a.k.a. first-order optics)

Assume angles are small. Then:

$$\sin\theta \simeq \theta \qquad \cos\theta \simeq 1 \qquad \tan\theta \simeq \theta$$

Where do these approximations come from?

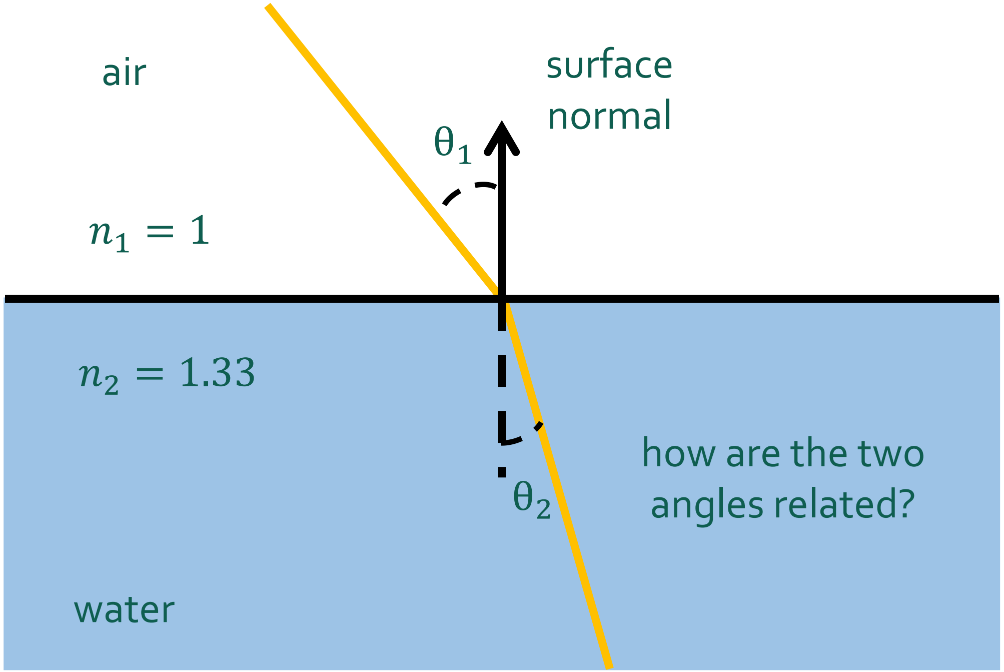
- First-order expansions of sin and cos functions.

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

Refraction

Refraction is the bending of rays of light when they cross optical interfaces (i.e., surfaces where the index of refraction changes).



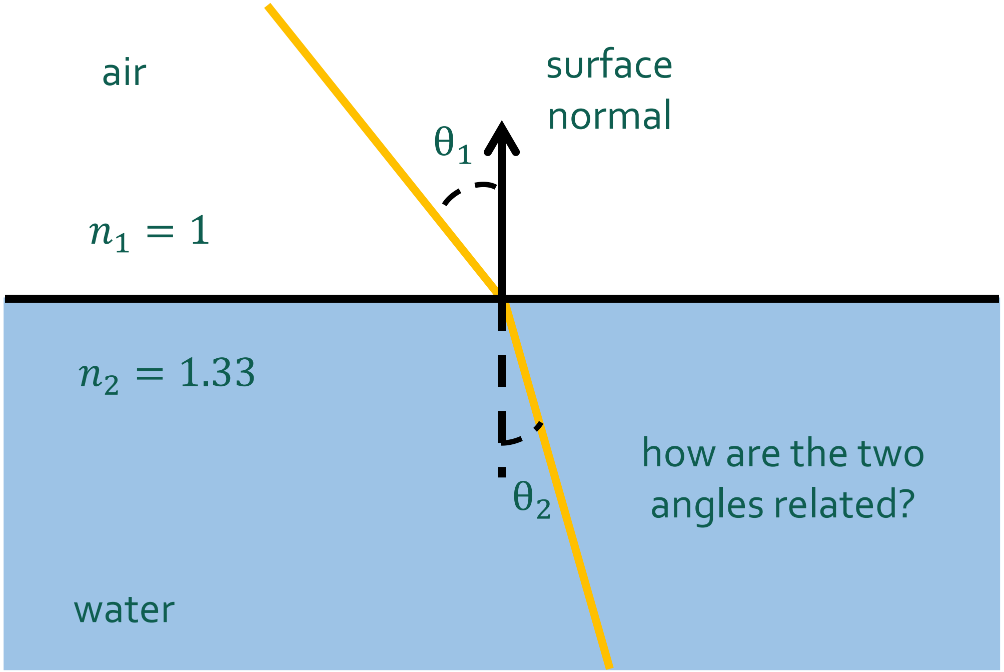
Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

How is Snell's law simplified under paraxial approximation?

Refraction

Refraction is the bending of rays of light when they cross optical interfaces (i.e., surfaces where the index of refraction changes).



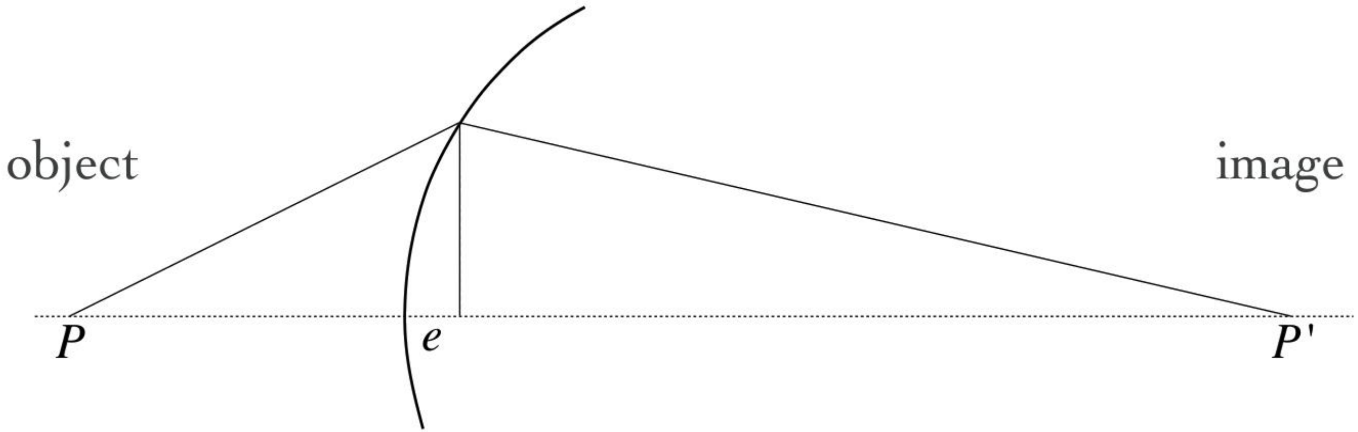
Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

How is Snell's law simplified under paraxial approximation?

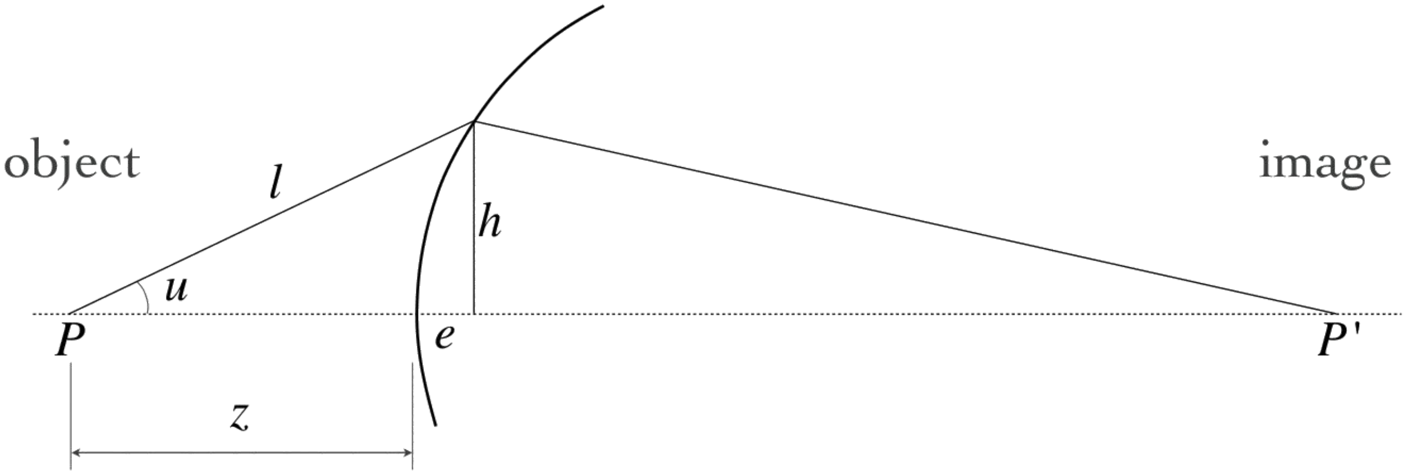
$$n_1\theta_1 = n_2\theta_2$$

Paraxial Focusing



spherical lens surface with radius r

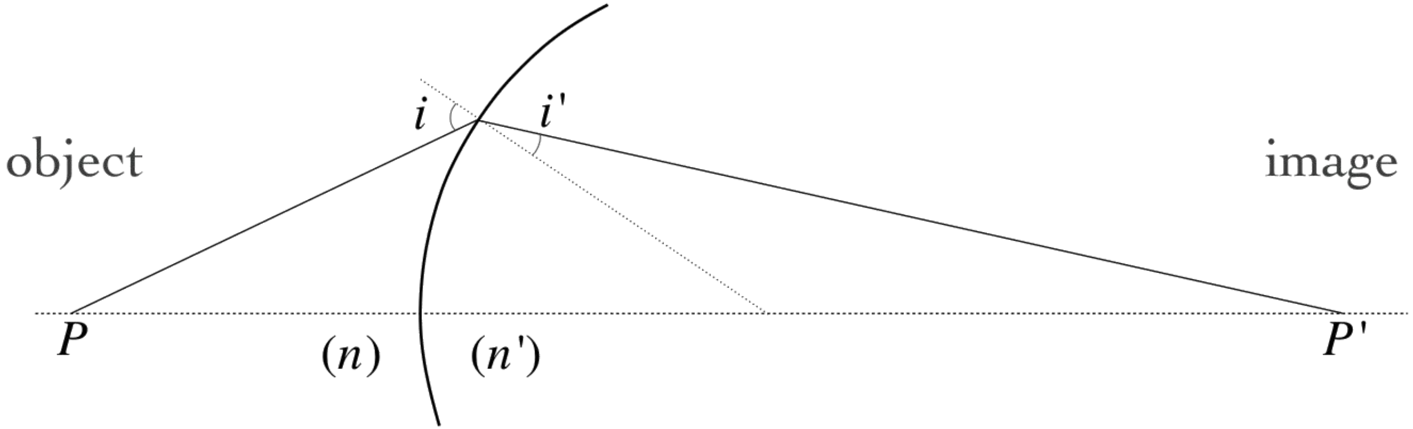
Paraxial Focusing



- assume $e \approx 0$
- assume $\sin u = h/l \approx u$ (for u in radians)
- assume $\cos u \approx z/l \approx 1$

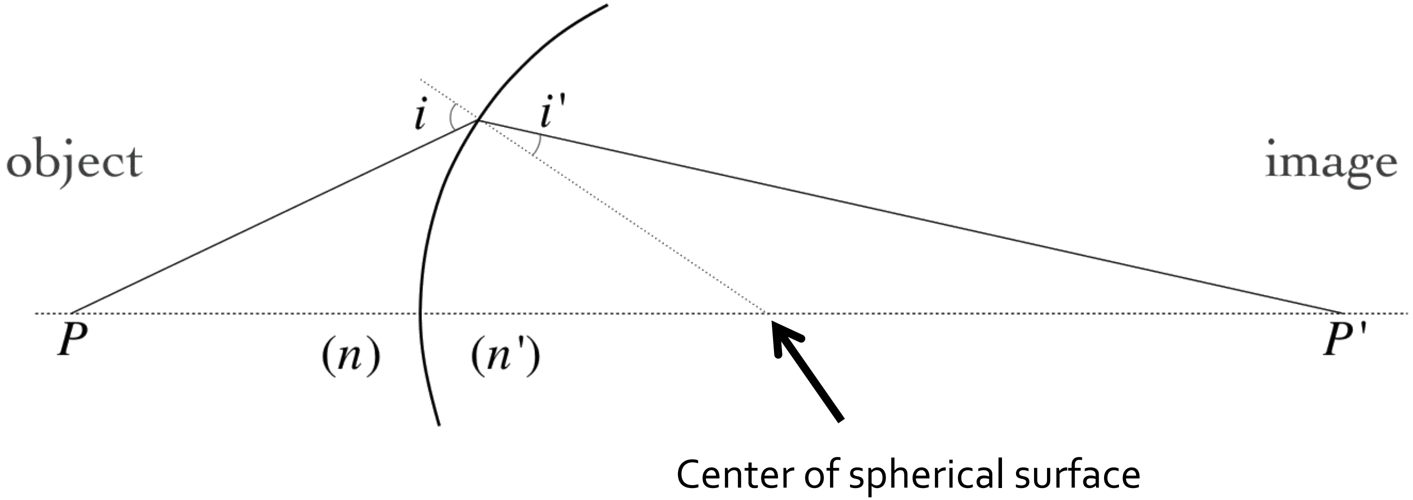
Where do these two equations come from?

Paraxial Focusing



How can we relate angles i and i' ?

Paraxial Focusing



Snell's law:

$$n \sin i = n' \sin i'$$

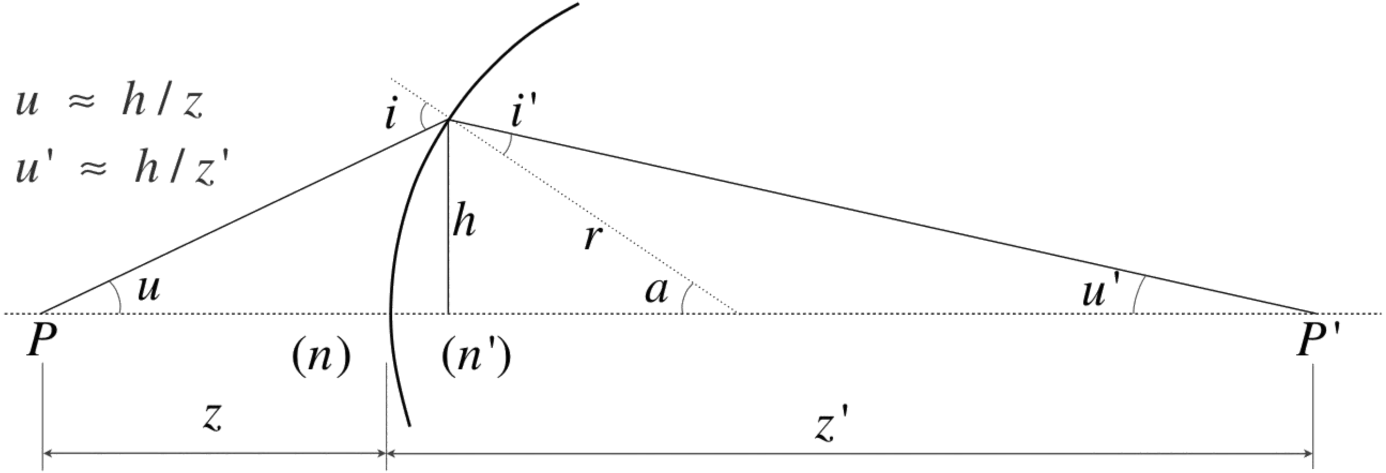
paraxial approximation:

$$n\, i \approx n'\, i'$$

Paraxial Focusing

$$u \approx h / z$$

$$u' \approx h / z'$$

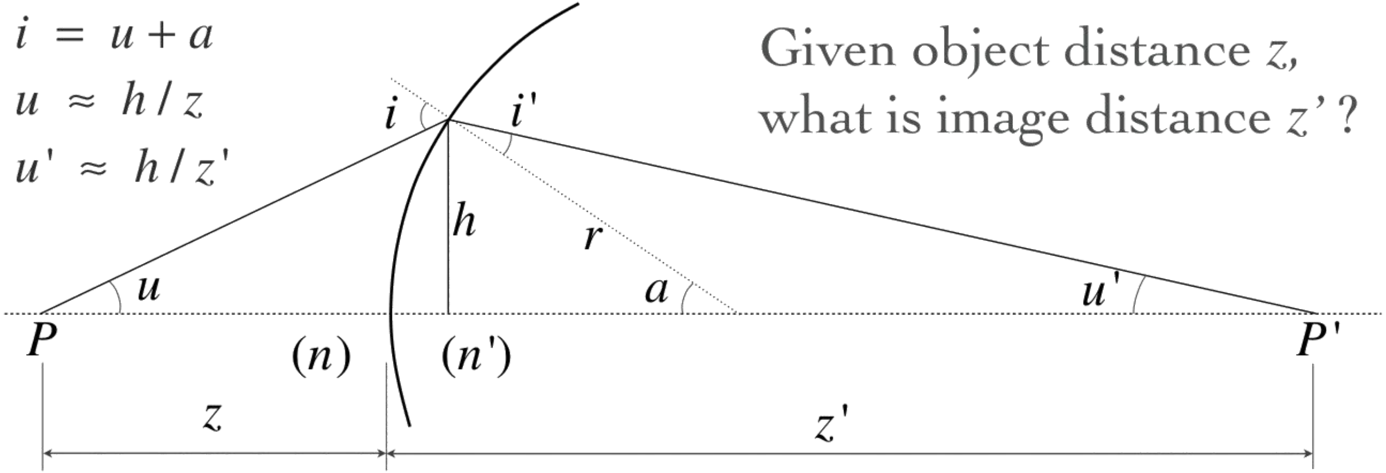


$$n\, i \approx n'\, i'$$

What is angle i equal to?

Paraxial Focusing

$$i = u + a$$
$$u \approx h / z$$
$$u' \approx h / z'$$



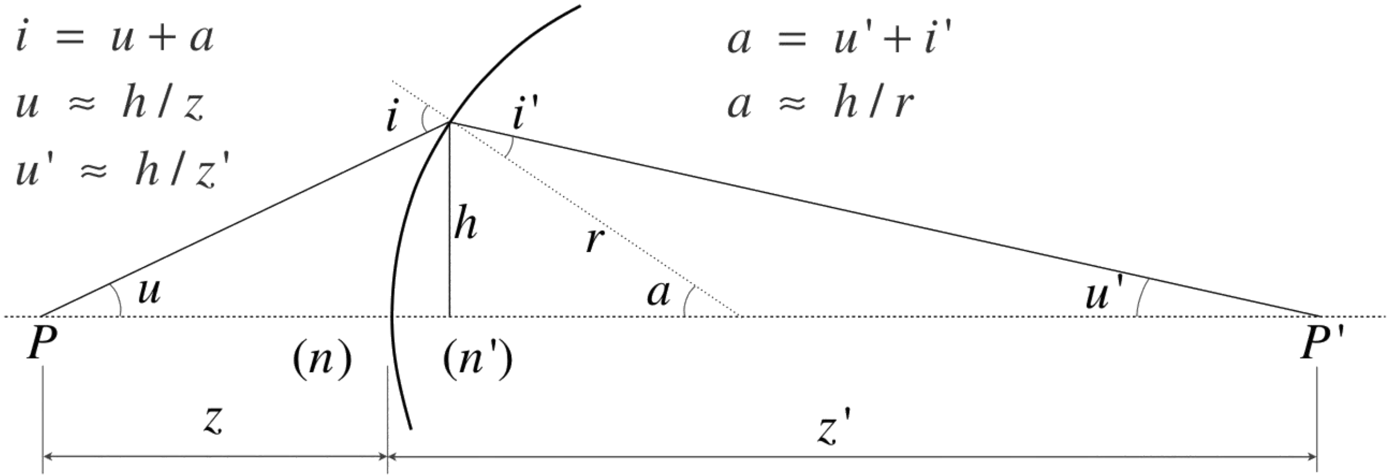
Given object distance z, what is image distance z'?

$$n\,i \approx n'\,i'$$

Paraxial Focusing

$$i = u + a$$
$$u \approx h / z$$
$$u' \approx h / z'$$

$$a = u' + i'$$
$$a \approx h / r$$

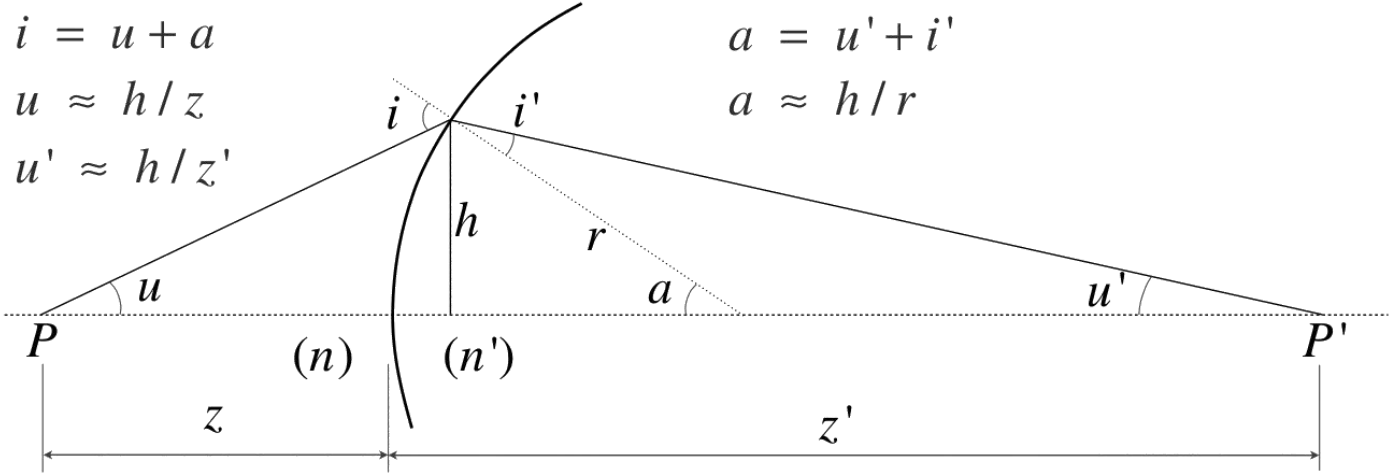


$$n\, i \approx n'\, i'$$

Paraxial Focusing

$$i = u + a$$
$$u \approx h / z$$
$$u' \approx h / z'$$

$$a = u' + i'$$
$$a \approx h / r$$



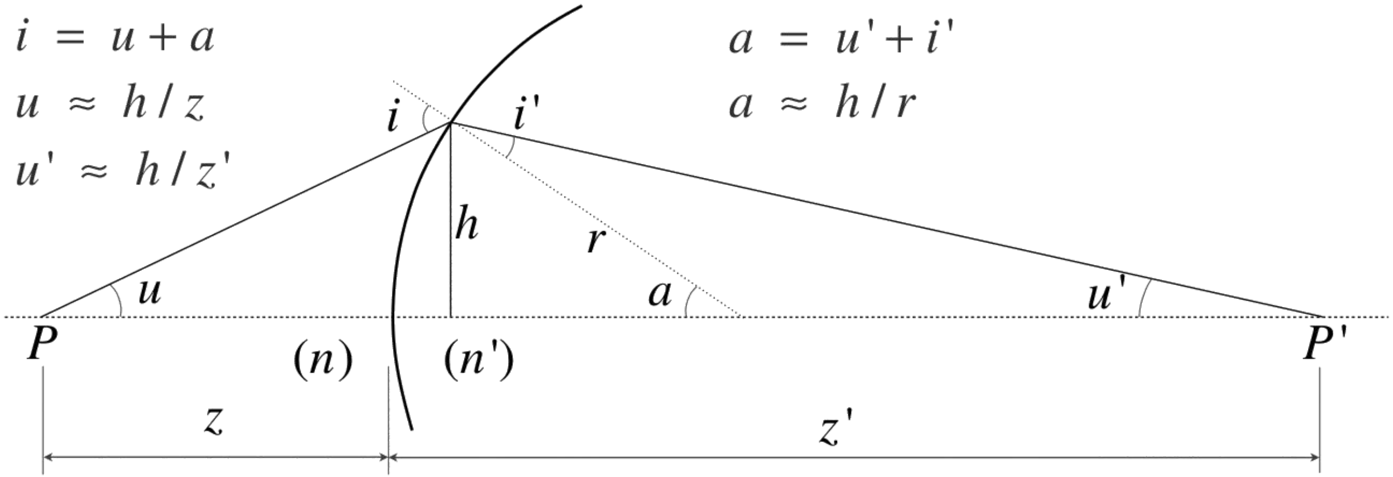
$$n(u + a) \approx n'(a - u')$$
$$n(h/z + h/r) \approx n'(h/r - h/z')$$
$$n/z + n/r \approx n'/r - n'/z'$$

What does this last equation imply?

$$n\,i \approx n'\,i'$$

Paraxial Focusing

$$i = u + a$$
$$u \approx h / z$$
$$u' \approx h / z'$$



$$a = u' + i'$$
$$a \approx h / r$$

h has been cancelled out, so any ray from P will focus at P'.

$$n\,i \approx n'\,i'$$

$$n(u + a) \approx n'(a - u')$$
$$n(h/z + h/r) \approx n'(h/r - h/z')$$
$$n/z + n/r \approx n'/r - n'/z'$$

Paraxial Focusing

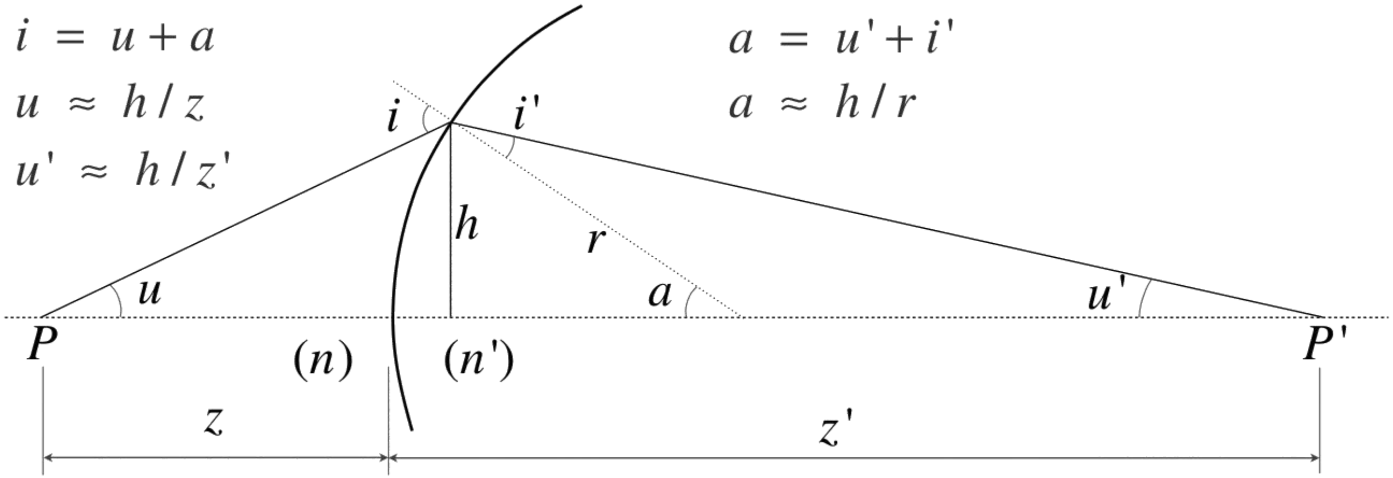
$$i = u + a$$

$$u \approx h / z$$

$$u' \approx h / z'$$

$$a = u' + i'$$

$$a \approx h / r$$



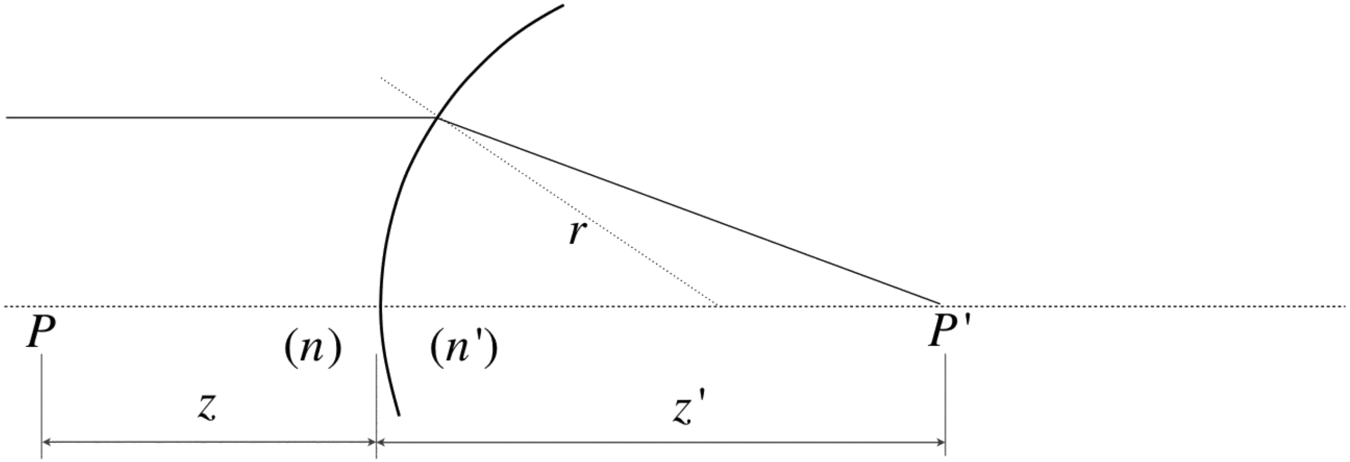
$$n\,(u + a) \approx n'\,(a - u')$$

$$n\,(h/z + h/r) \approx n'\,(h/r - h/z')$$

$$n/z + n/r \approx n'/r - n'/z'$$

$$n\,i \approx n'\,i'$$

Focal Length



What happens as z tends to infinity?

$$n / z + n / r \approx n' / r - n' / z'$$

$$n / r \approx n' / r - n' / z'$$

$$z' \approx (r\, n') / (n' - n)$$

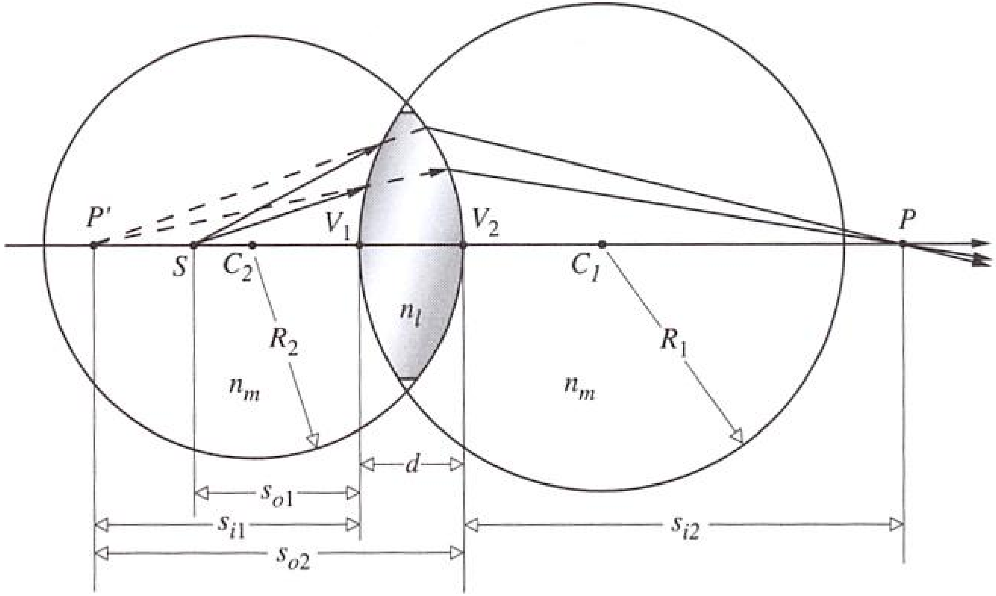
Gaussian Lens Formula

Using similar derivations, we can extend these results to two spherical interfaces.

- We obtain a spherical lens in air.

- Thin lens approximation: d close to zero.
- Under this approximation, we obtain the *lensmaker's equation*.

$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$



Gaussian Lens Formula

- Starting from the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

- and recalling that as object distance s_o is moved to infinity, image distance s_i becomes focal length f_i, we get

$$\frac{1}{f_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

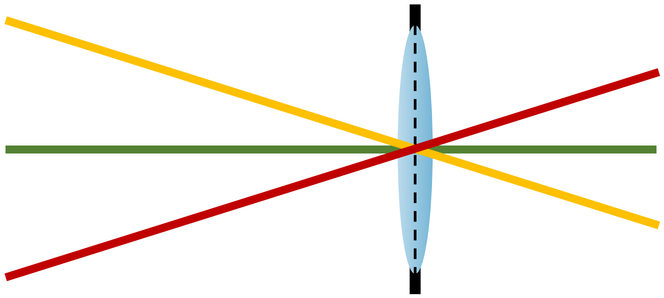
Looks familiar?

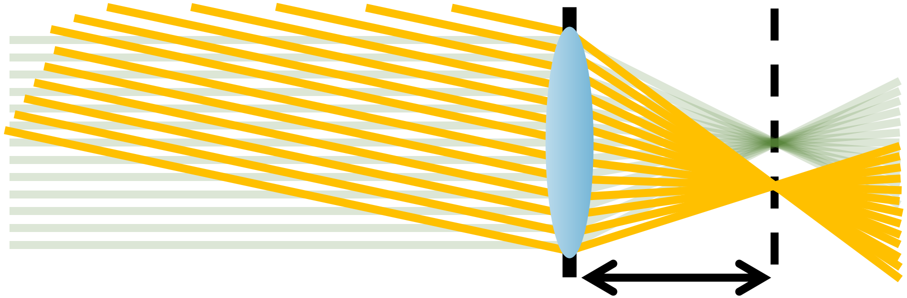
- Equating these two, we get the Gaussian lens formula

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}. \qquad \text{(Hecht, eqn 5.17)}$$

Thin Lens Model

Simplification of geometric optics for well-designed lenses.





Two assumptions:

1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.

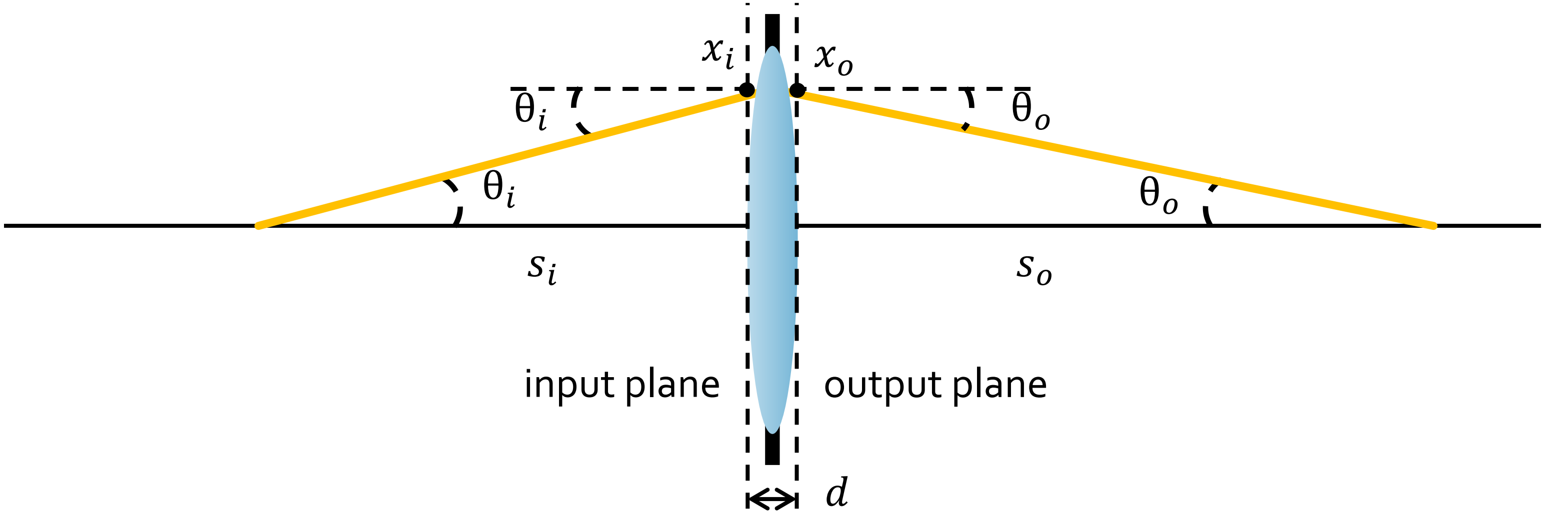
Same result! as what we obtained last time using ray tracing assumptions.

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

$$m = \frac{S' - f}{f}$$

Ray Rransfer Matrix Analysis

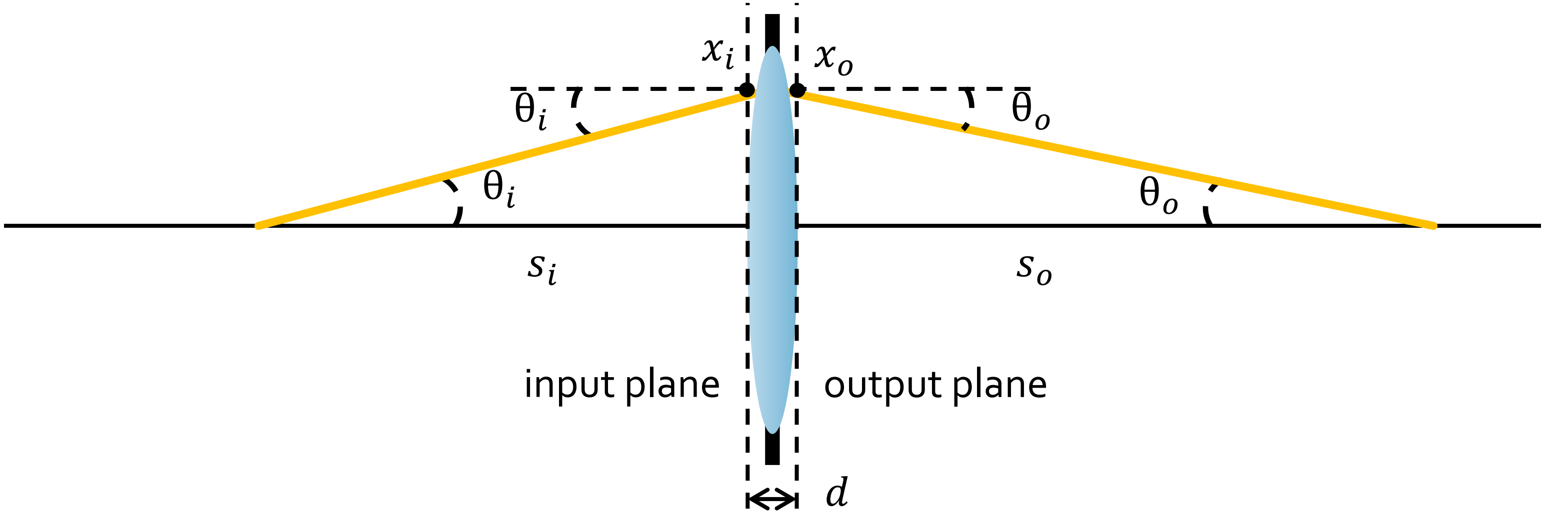
Revisit Thin Lenses Again



Assumptions:

- ➢ Paraxial ⟶
- ➢ Thin Lens ⟶

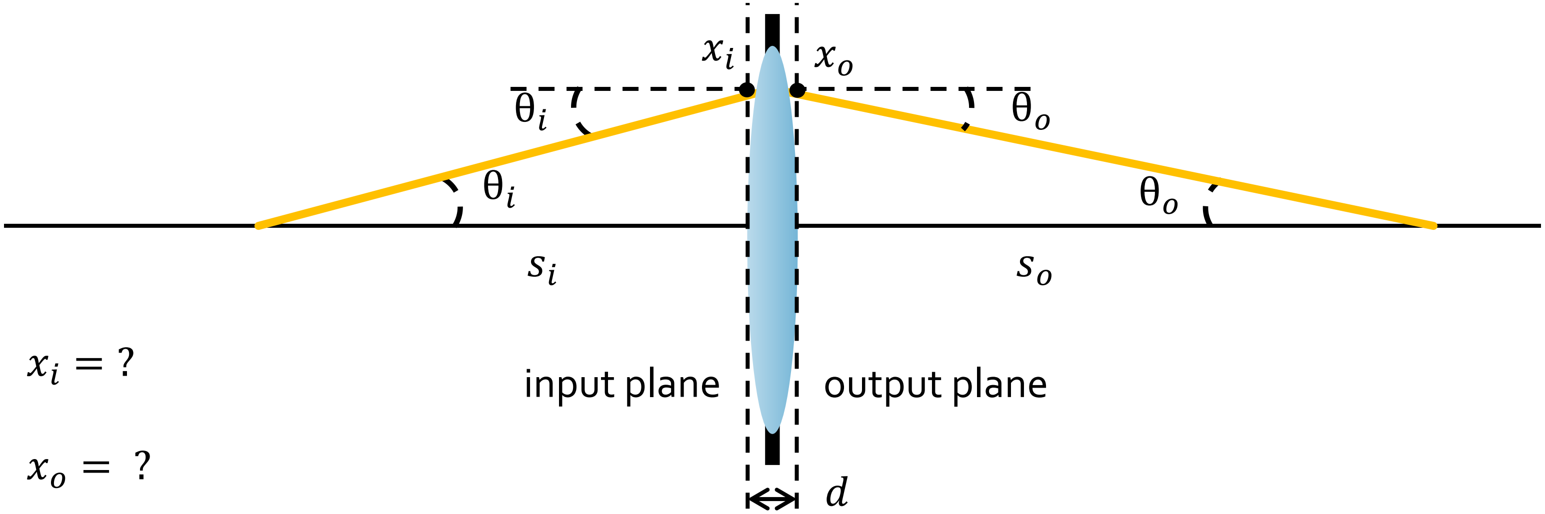
Revisit Thin Lenses Again



Assumptions:

- ➢ Paraxial ⟶ angles θ are small, thus first-order approximations for sinθ, cosθ, and tanθ apply.
- ➢ Thin Lens ⟶ width of lens is negligible (d ≃ o) relative to distances s.

Revisit Thin Lenses Again

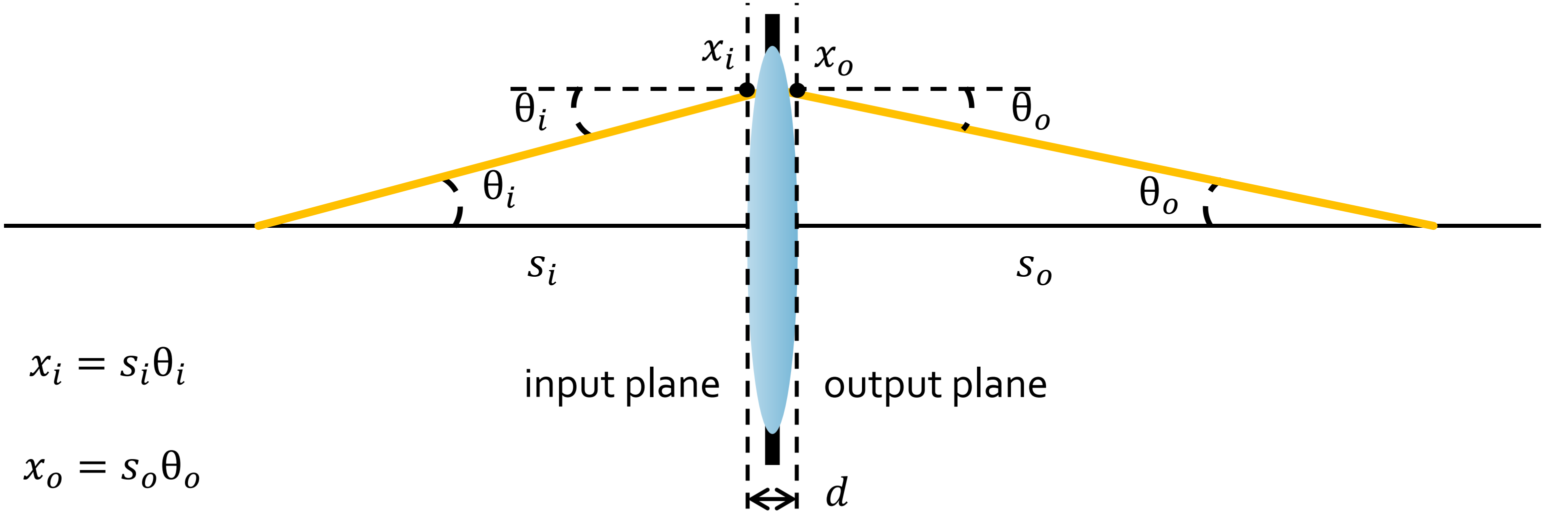


$x_i = ?$

$x_o = \ ?$

$$\frac{1}{s_o} + \frac{1}{s_i} = ?$$

Revisit Thin Lenses Again



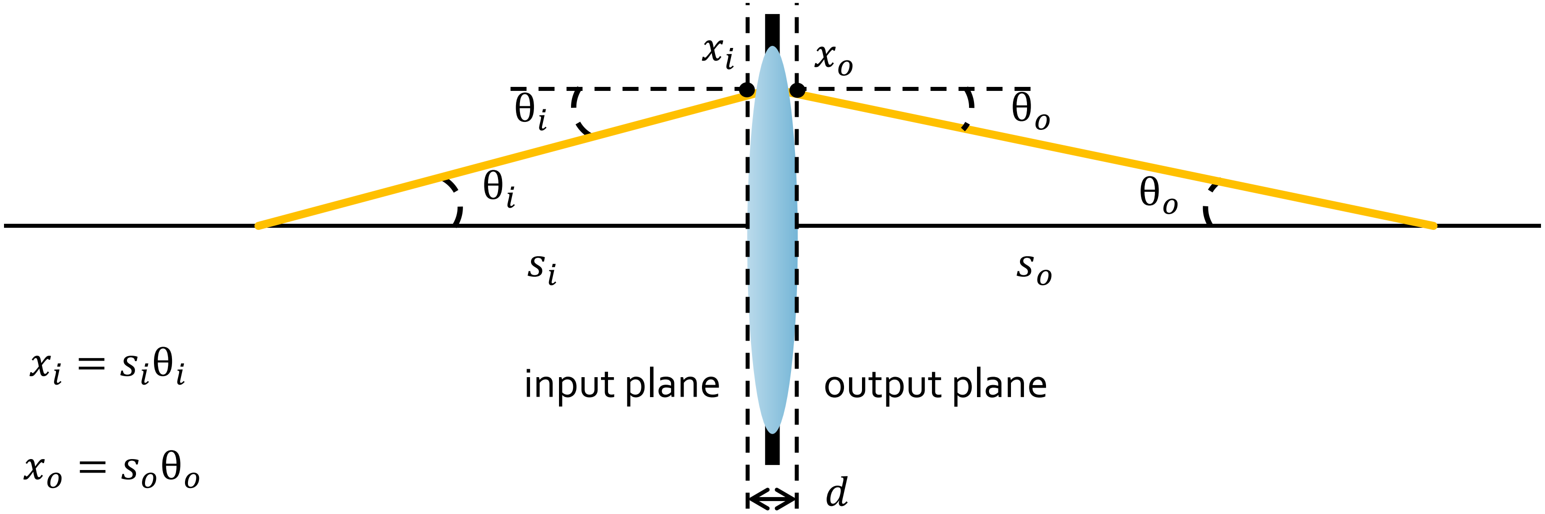
$$x_i = s_i \theta_i$$

$$x_o = s_o \theta_o$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$d \simeq 0 \Rightarrow x_i = x_o$$

Revisit Thin Lenses Again



$$x_i = s_i \theta_i$$

$$x_o = s_o \theta_o$$

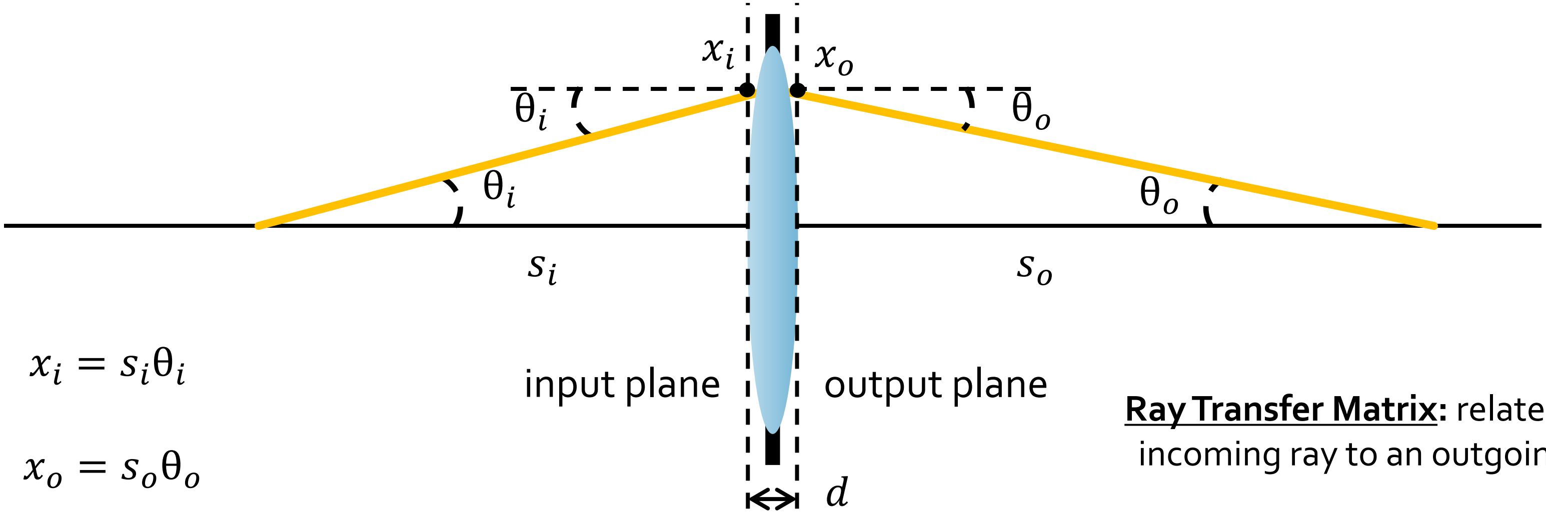
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$d \simeq 0 \Rightarrow x_i = x_o$$

Putting it all together, we can write:

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

Revisit Thin Lenses Again



$$x_i = s_i\theta_i$$

$$x_o = s_o\theta_o$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

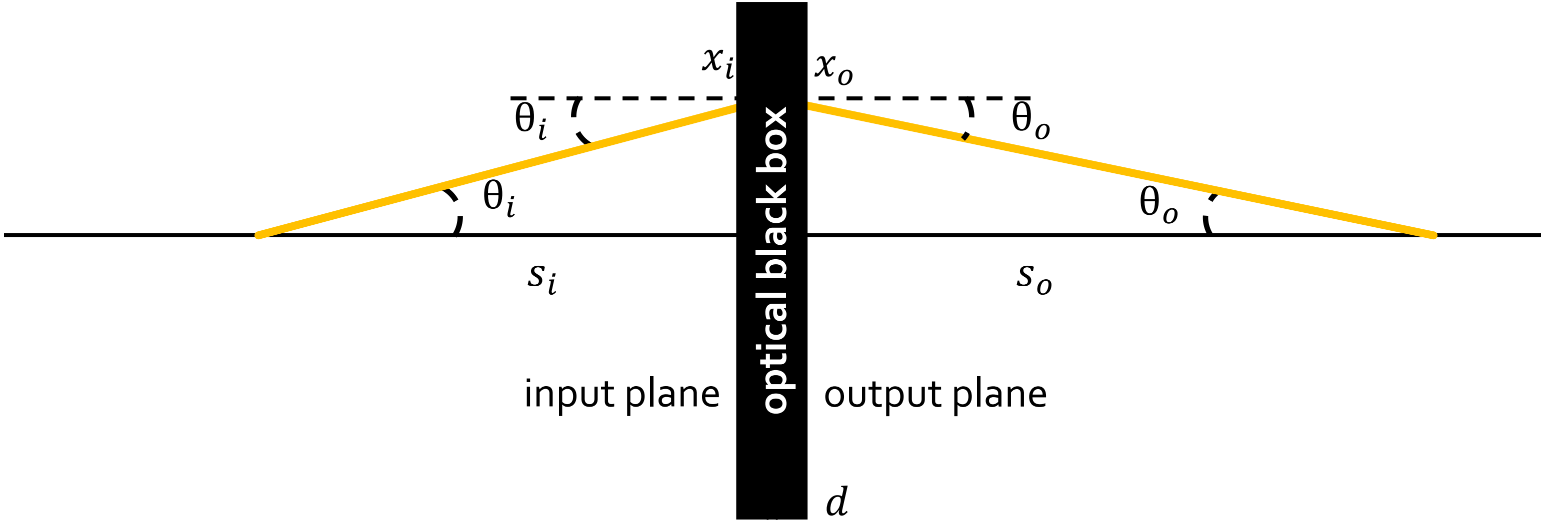
$$d \simeq 0 \Rightarrow x_i = x_o$$

Putting it all together, we can write:

Ray Transfer Matrix: relates each incoming ray to an outgoing ray

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

Ray Transfer Matrix Analysis



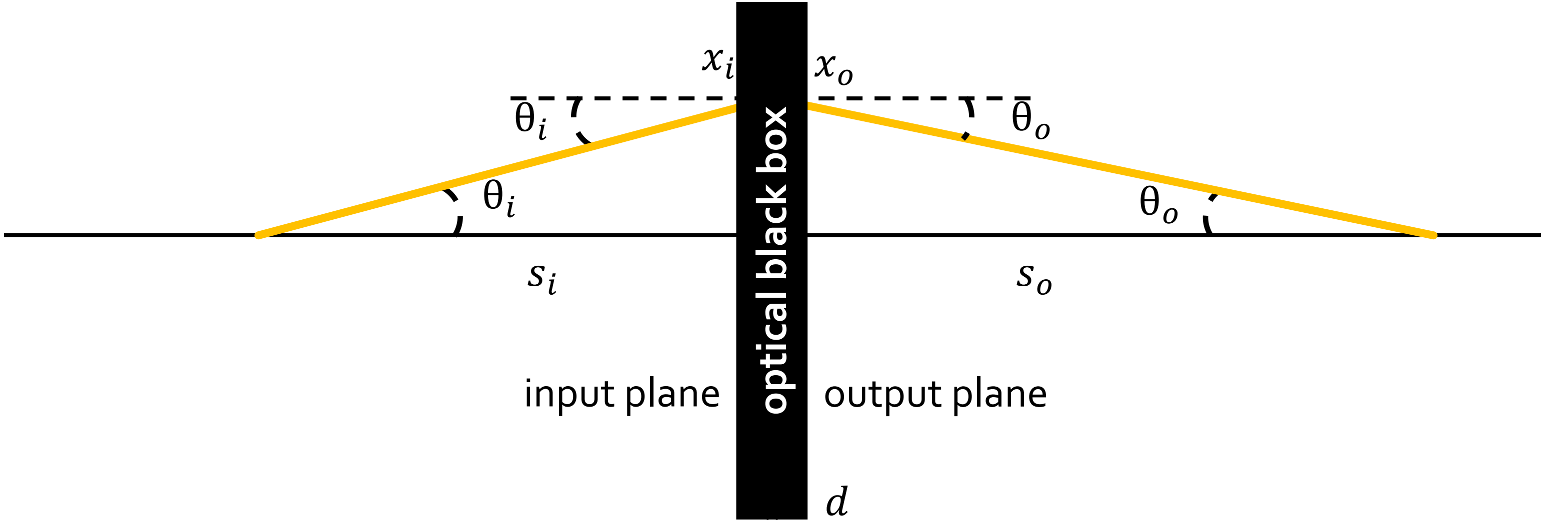
Every optical system implements a (generally non-linear) ray mapping of the form:

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} f(x_i, \theta_i) \\ g(x_i, \theta_i) \end{bmatrix}$$

How do we go from here to a ray transfer matrix?

- Paraxial approximation: Use first-order approximation around axial ray.

Ray Transfer Matrix Analysis



Under paraxial approximation:

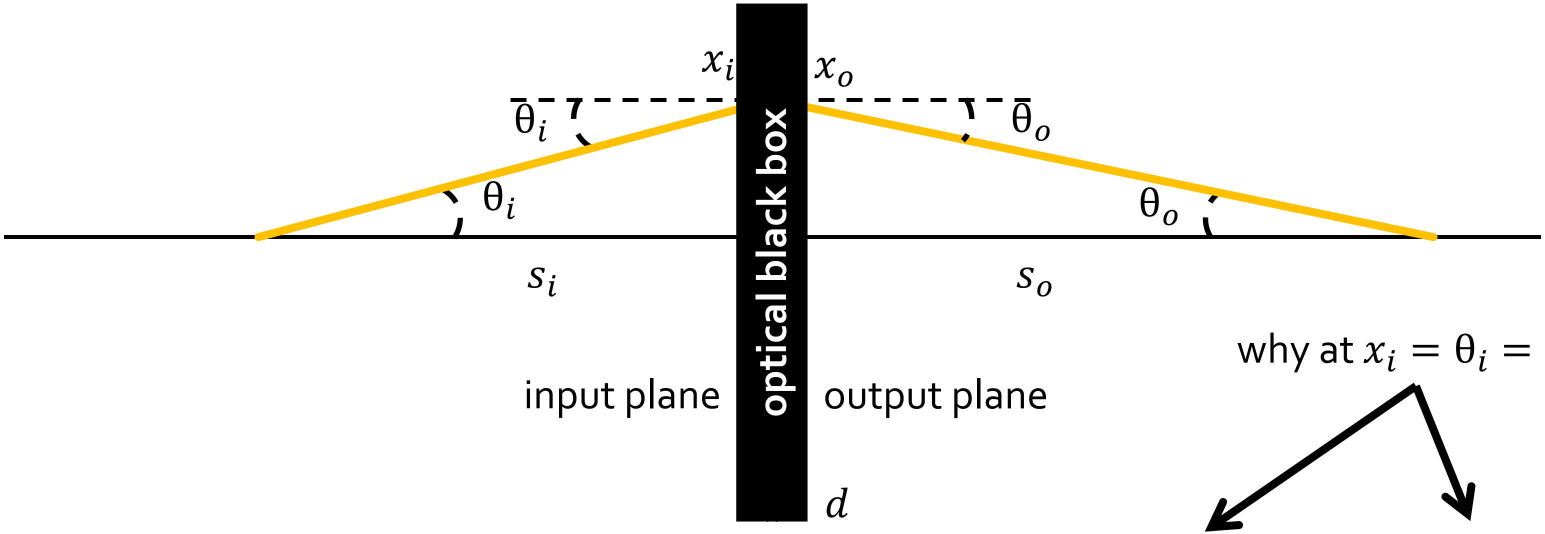
$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} f(x_i, \theta_i) \\ g(x_i, \theta_i) \end{bmatrix} \simeq \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

where

$A = ?$ $\quad B = ?$

$C = ?$ $\quad D = ?$

Ray Transfer Matrix Analysis



why at $x_i = \theta_i = 0$?

Under paraxial approximation:

$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \begin{bmatrix} f(x_i, \theta_i) \\ g(x_i, \theta_i) \end{bmatrix} \simeq \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

where

$$A = \left.\frac{\partial f}{\partial x_i}\right|_{x_i = \theta_i = 0} \qquad B = \left.\frac{\partial f}{\partial \theta_i}\right|_{x_i = \theta_i = 0}$$

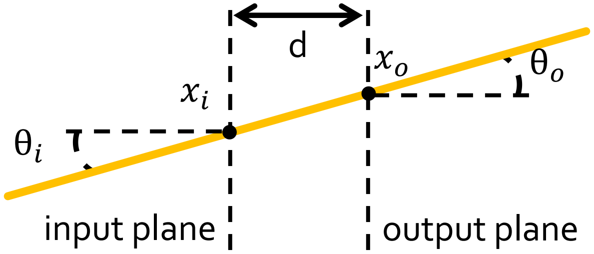
$$C = \left.\frac{\partial g}{\partial x_i}\right|_{x_i = \theta_i = 0} \qquad D = \left.\frac{\partial g}{\partial \theta_i}\right|_{x_i = \theta_i = 0}$$

definition of ray transfer matrix, a.k.a. ABCD matrix

What is the ABCD matrix of...

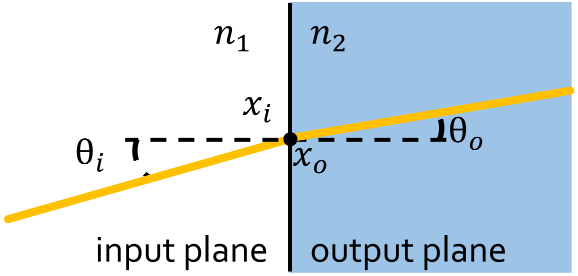
- Free space propagation?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$



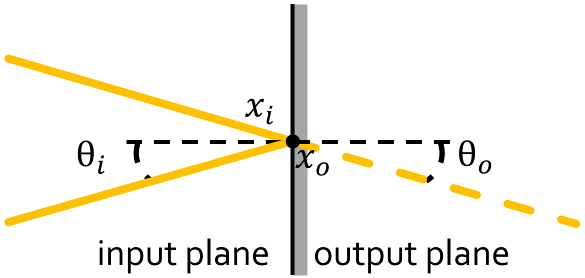
- Planar refractive interface?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

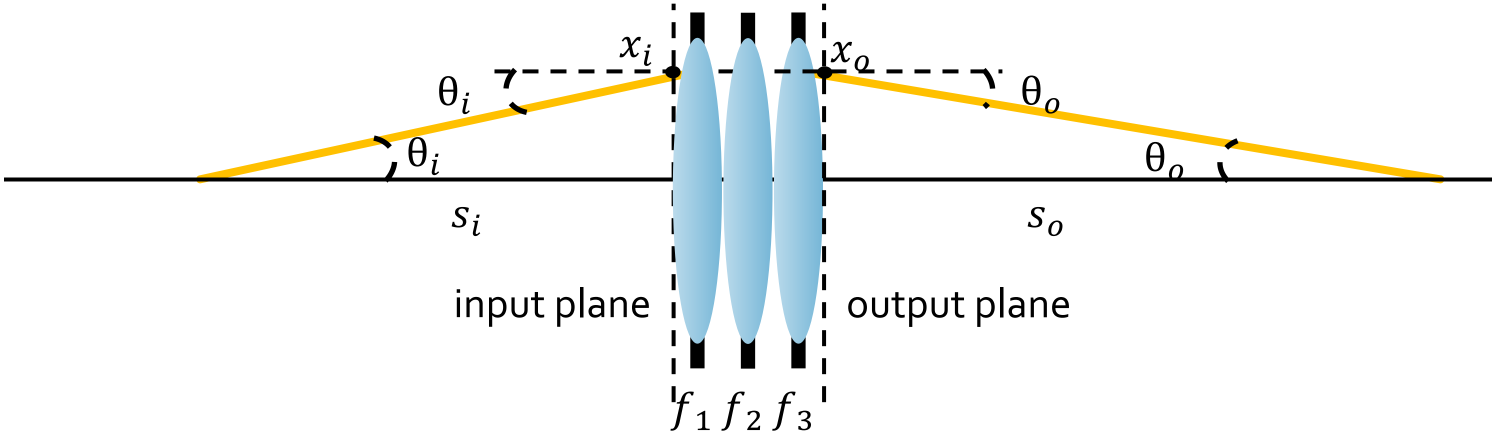


- Planar mirror?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Cascaded Optical Systems



Let's say we stack together three lenses.

- What is the total ray transfer matrix?
- Notice the matrix ordering.

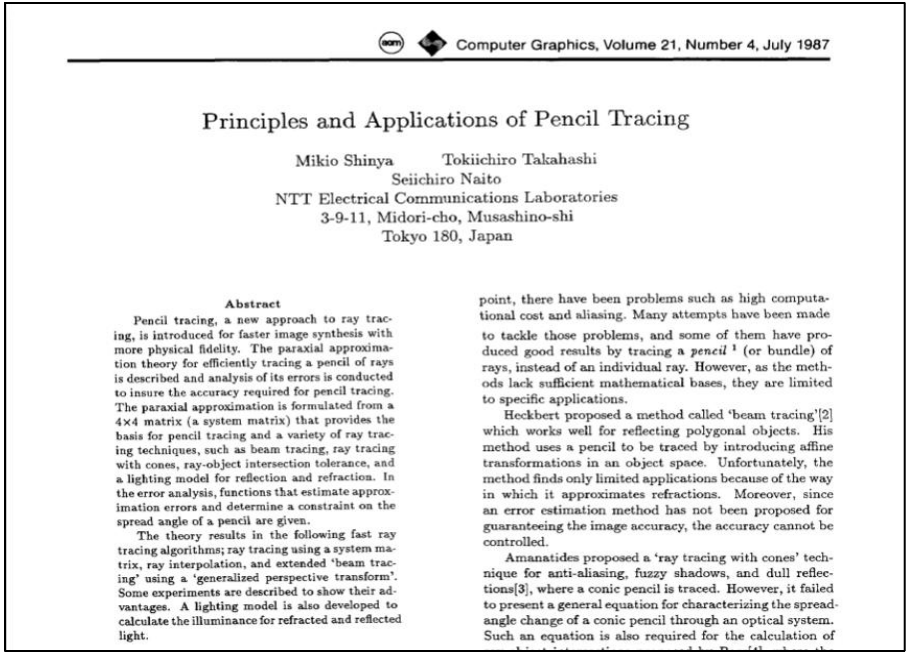
$$\begin{bmatrix} x_o \\ \theta_o \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_3} & 1 \end{bmatrix}}_{\text{lens 3}} \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix}}_{\text{lens 2}} \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}}_{\text{lens 1}} \begin{bmatrix} x_i \\ \theta_i \end{bmatrix}$$

Ray Transfer Matrix Analysis

- Also known as ABCD matrix analysis (from the form of the ray transfer matrix).
- Any optical system, no matter how complicated, can be described by its ray transfer matrix.
- A cascaded optical system has a ray transfer matrix that is the product of the ray transfer matrices of its components.
- All of the above hold assuming paraxial rays, no aberrations, and no diffraction (**geometric optics**).

Graphics Perspective on Ray Transfer Matrix

- How can I use ray transfer matrix analysis to make ray tracing faster?
- How can I use ray transfer matrix analysis to make Monte Carlo rendering faster?



Computer Graphics, Volume 21, Number 4, July 1987

Principles and Applications of Pencil Tracing

Mikio Shinya Tokiichiro Takahashi
Seiichiro Naito
NTT Electrical Communications Laboratories
3-9-11, Midori-cho, Musashino-shi
Tokyo 180, Japan

Abstract

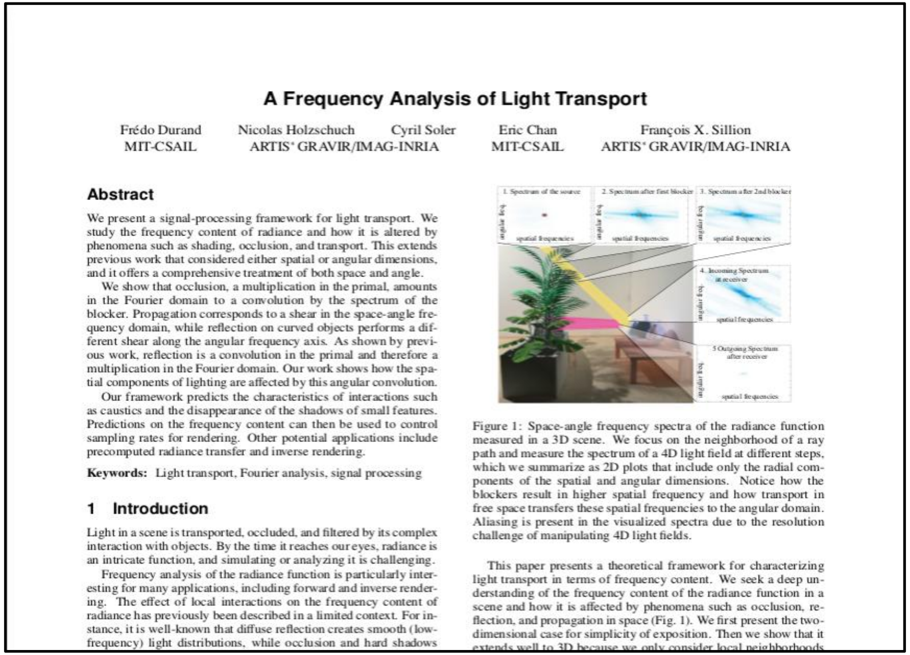
Pencil tracing, a new approach to ray tracing, is introduced for faster image synthesis with more physical fidelity. The paraxial approximation theory for efficiently tracing a pencil of rays is described and analysis of its errors is conducted to insure the accuracy required for pencil tracing. The paraxial approximation is formulated from a 4×4 matrix (a system matrix) that provides the basis for pencil tracing and a variety of ray tracing techniques, such as beam tracing, ray tracing with cones, ray-object intersection tolerance, and a lighting model for reflection and refraction. In the error analysis, functions that estimate approximation errors and determine a constraint on the spread angle of a pencil are given.

The theory results in the following fast ray tracing algorithms; ray tracing using a system matrix, ray interpolation, and extended 'beam tracing' using a 'generalized perspective transform'. Some experiments are described to show their advantages. A lighting model is also developed to calculate the illuminance for refracted and reflected light.

point, there have been problems such as high computational cost and aliasing. Many attempts have been made to tackle those problems, and some of them have produced good results by tracing a *pencil* [1] (or bundle) of rays, instead of an individual ray. However, as the methods lack sufficient mathematical bases, they are limited to specific applications.

Heckbert proposed a method called 'beam tracing'[2] which works well for reflecting polygonal objects. His method uses a pencil to be traced by introducing affine transformations in an object space. Unfortunately, the method finds only limited applications because of the way in which it approximates refractions. Moreover, since an error estimation method has not been proposed for guaranteeing the image accuracy, the accuracy cannot be controlled.

Amanatides proposed a 'ray tracing with cones' technique for anti-aliasing, fuzzy shadows, and dull reflections[3], where a conic pencil is traced. However, it failed to present a general equation for characterizing the spread-angle change of a conic pencil through an optical system. Such an equation is also required for the calculation of



A Frequency Analysis of Light Transport

Frédo Durand, MIT-CSAIL
Nicolas Holzschuch, Cyril Soler, ARTIS* GRAVIR/IMAG-INRIA
Eric Chan, MIT-CSAIL
François X. Sillion, ARTIS* GRAVIR/IMAG-INRIA

Abstract

We present a signal-processing framework for light transport. We study the frequency content of radiance and how it is altered by phenomena such as shading, occlusion, and transport. This extends previous work that considered either spatial or angular dimensions, and it offers a comprehensive treatment of both space and angle.

We show that occlusion, a multiplication in the primal, amounts in the Fourier domain to a convolution by the spectrum of the blocker. Propagation corresponds to a shear in the space-angle frequency domain, while reflection on curved objects performs a different shear along the angular frequency axis. As shown by previous work, reflection is a convolution in the primal and therefore a multiplication in the Fourier domain. Our work shows how the spatial components of lighting are affected by this angular convolution.

Our framework predicts the characteristics of interactions such as caustics and the disappearance of the shadows of small features. Predictions on the frequency content can then be used to control sampling rates for rendering. Other potential applications include precomputed radiance transfer and inverse rendering.

Keywords: Light transport, Fourier analysis, signal processing

1 Introduction

Light in a scene is transported, occluded, and filtered by its complex interaction with objects. By the time it reaches our eyes, radiance is an intricate function, and simulating or analyzing it is challenging.

Frequency analysis of the radiance function is particularly interesting for many applications, including forward and inverse rendering. The effect of local interactions on the frequency content of radiance has previously been described in a limited context. For instance, it is well-known that diffuse reflection creates smooth (low-frequency) light distributions, while occlusion and hard shadows

Figure 1: Space-angle frequency spectra of the radiance function measured in a 3D scene. We focus on the neighborhood of a ray path and measure the spectrum of a 4D light field at different steps, which we summarize as 2D plots that include only the radial components of the spatial and angular dimensions. Notice how the blockers result in higher spatial frequency and how transport in free space transfers these spatial frequencies to the angular domain. Aliasing is present in the visualized spectra due to the resolution challenge of manipulating 4D light fields.

This paper presents a theoretical framework for characterizing light transport in terms of frequency content. We seek a deep understanding of the frequency content of the radiance function in a scene and how it is affected by phenomena such as occlusion, reflection, and propagation in space (Fig. 1). We first present the two-dimensional case for simplicity of exposition. Then we show that it

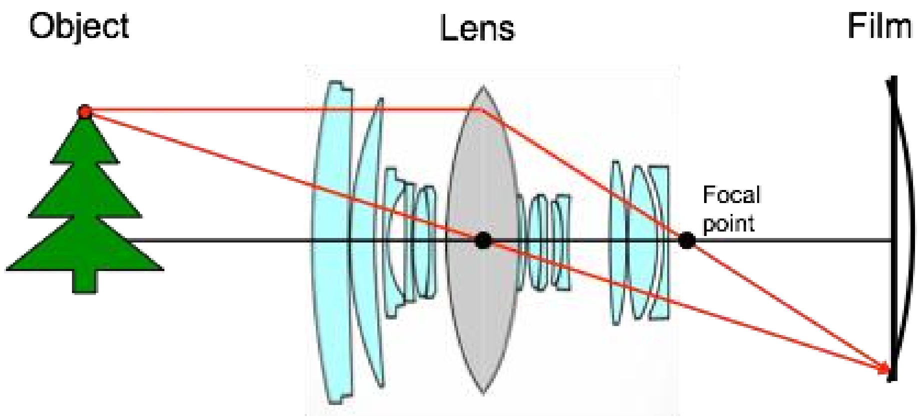
Compound Lenses and Aberrations

Thin Lenses are a Fiction

The thin lens model assumes that the lens has no thickness, but this is rarely true…

➢ Even though we have multiple lenses, the entire optical system can be (paraxially) described using a single thin lens of some equivalent focal length and aperture number.

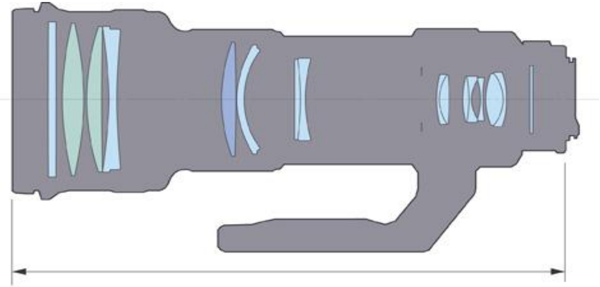
➢ Where and what exactly this lens is is difficult to determine?



To make real lenses behave like ideal thin lenses, we have to use combinations of multiple lens elements (compound lenses).

Thin Lenses are a Fiction

The thin lens model assumes that the lens has no thickness, but this is rarely true...

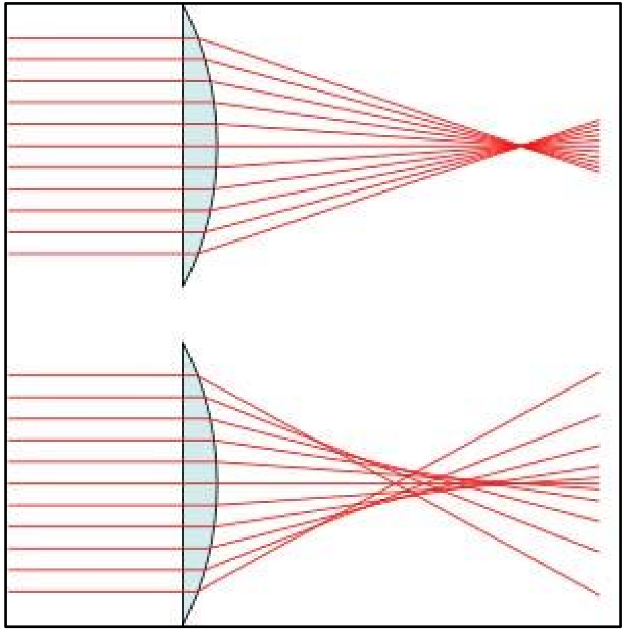


To make real lenses behave like ideal thin lenses, we have to use combinations of multiple lens elements (compound lenses).

Spherical Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

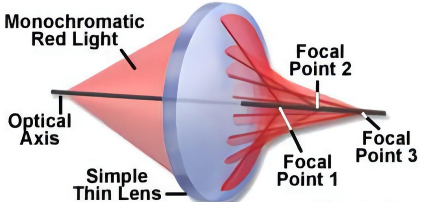
➢ Example: spherical aberration.

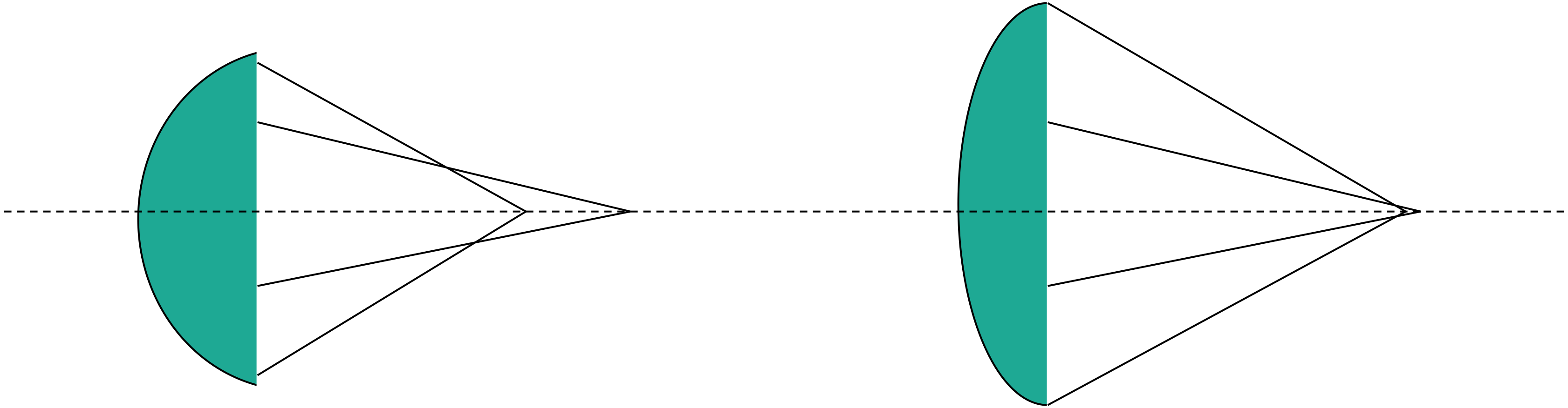


Spherical Aberrations

For lenses made with spherical surfaces, rays which are parallel to the optic axis but at different distances from the optic axis fail to converge to the same point.

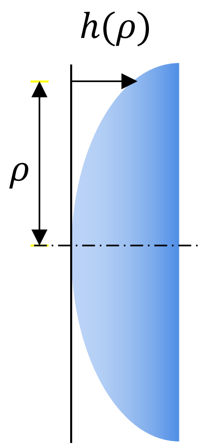
Spherical aberration in the human eye is reduced by the **aspheric** shape of the lens and the cornea





Spherical Aberration- Correction

- Use Compound Lens： use multiple convex (+) and concave lens (-) to reduce spherical abberation.
- Smaller aperture
- Graded-index optical lens
- Asperical Lens:



$$h(\rho) = \frac{c\rho}{1 + \sqrt{1 - \alpha\rho}} + \sum_{i=2}^{n} a_{2i}\rho^{i}$$

$$\alpha = (1 + \kappa)c^2$$

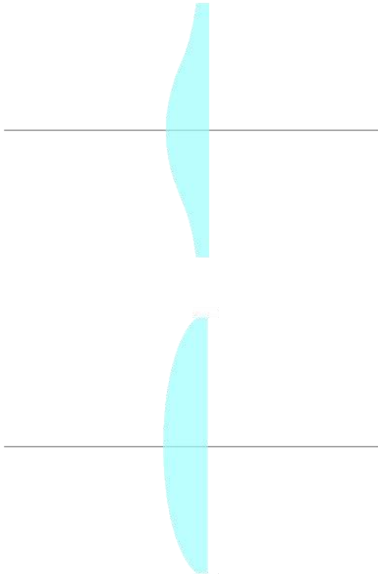


Figure 4 – Example of fixing spherical aberration using an aspherical lens

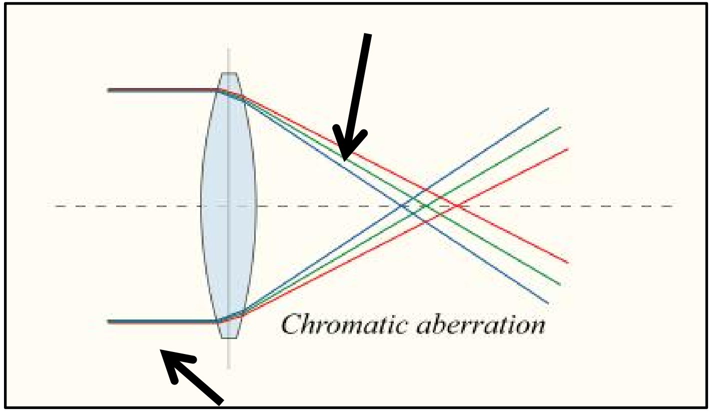
slide credit： Qilin Sun; https://www.lumenera.com/

Chromatic Aberration

Deviations from ideal thin lens behavior (e.g., imperfect focus).

- Example: chromatic aberration.

focal length shifts with wavelength



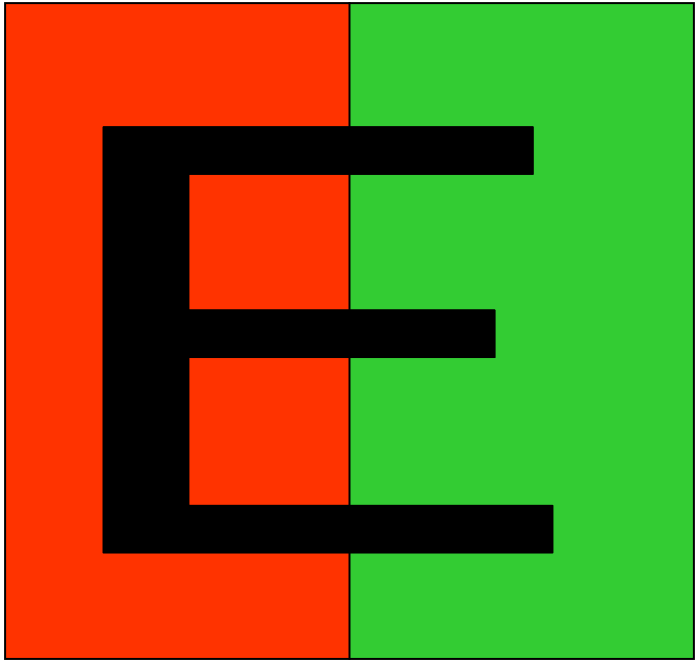
glass has dispersion

(refractive index changes with wavelength)

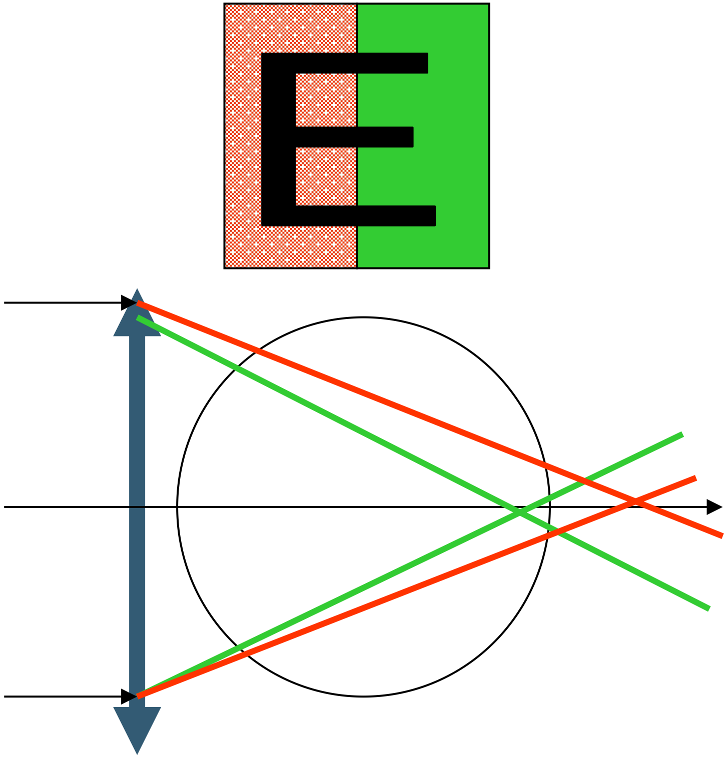
- Dispersive power (**abbe value**) is based on change in index for different wavelengths
- If the index is the same for all wavelengths, there is *NO DISPERSION*
- Short wavelength has higher n and is refracted more than long wavelength.The **n ↑ = wavelength ↓**
- Dispersion usually increases in high index

Chromatic Aberration — Duochrome Test

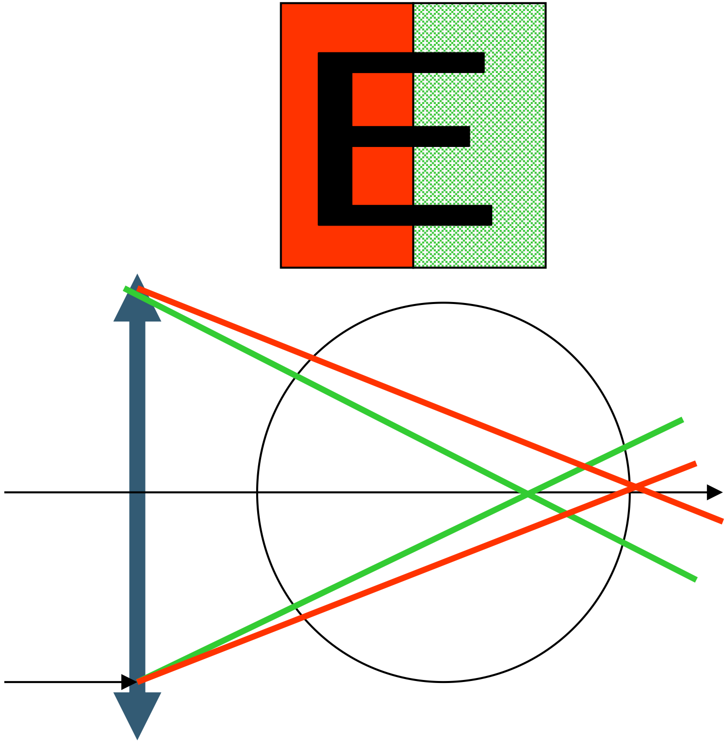
- Duochrome test helps you determine position of focal point with respect to the fovea
- Useful to avoid overminusing pt.



Chromatic Aberration — Duochrome Test



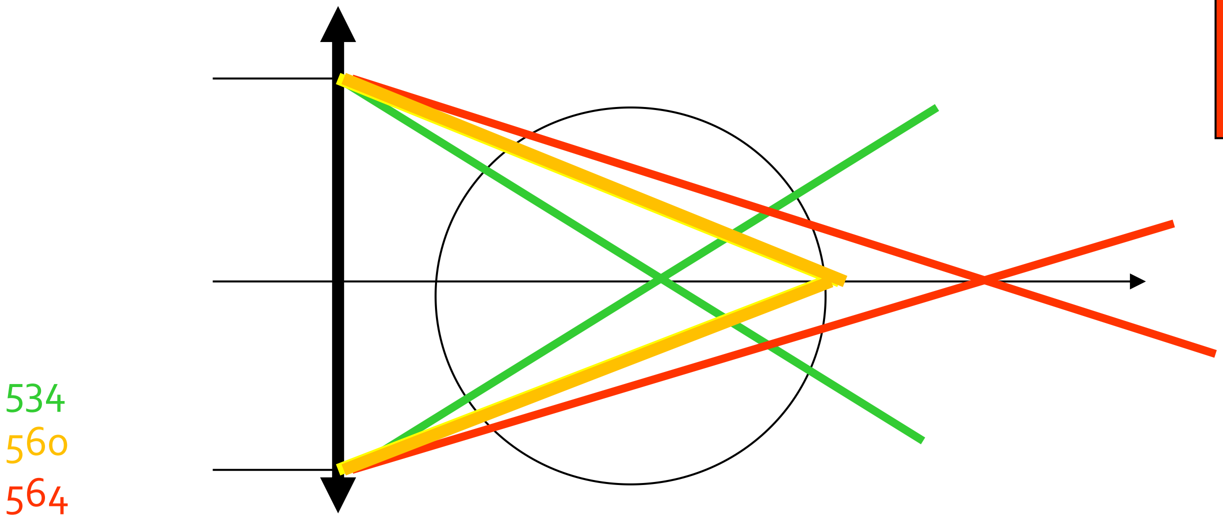
Too much minus green is clearer

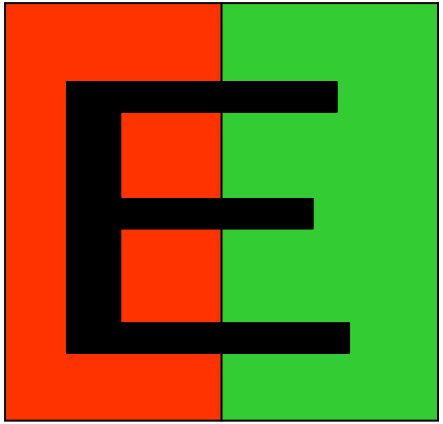


Too much plus red is clearer

Chromatic Aberration — Duochrome Test

Red clarity= green clarity then image is positioned correctly.

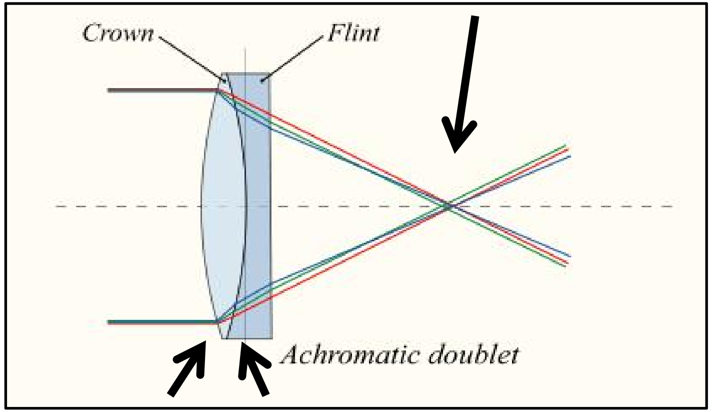




Correction of Chromatic Aberration

- An achromat doublet does not completely eliminate chromatic aberration, but can eliminate it for two colors, say red and blue.
- The idea is to use a lens pair – a strong lens of low dispersion coupled with a weaker one of high dispersion calculated to match the focal lengths for two chosen wavelengths.
- Cemented doublets of this type are a mainstay of lens design.

Cancels out dispersion of each other



glasses of different refractive index

Using a doublet (two-element compound lens), we can reduce chromatic aberration.

Correction of Chromatic Aberration

APOCHROMATIC LENS

The addition of a third lens corrects for three colors (red, blue and green), greatly reducing the fuzziness caused by the colors uncorrected in the achromatic doublet.

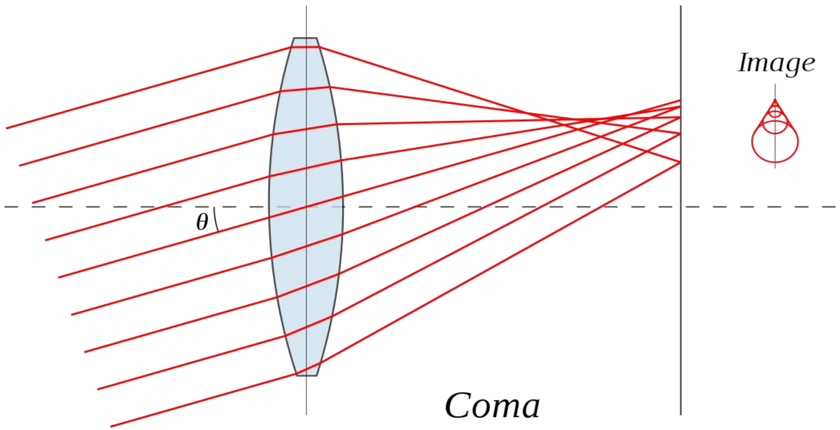
In the human eye, chromatic aberration is reduced by the lens, which changes index from the nucleus outward.

Chromatic Aberration

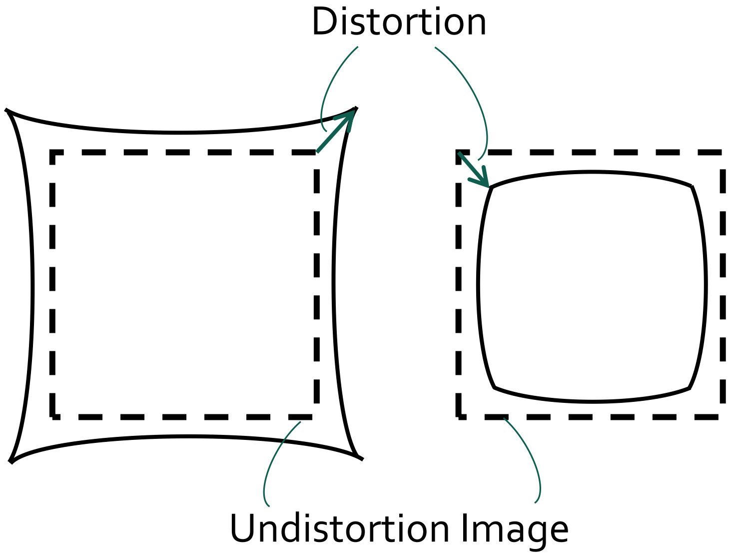
Oblique Aberrations

These appear only as we move further from the center of the field of view.

- Contrast with spherical and chromatic, which appear everywhere.
- Many other examples (astigmatism, field curvature, etc.).



Coma

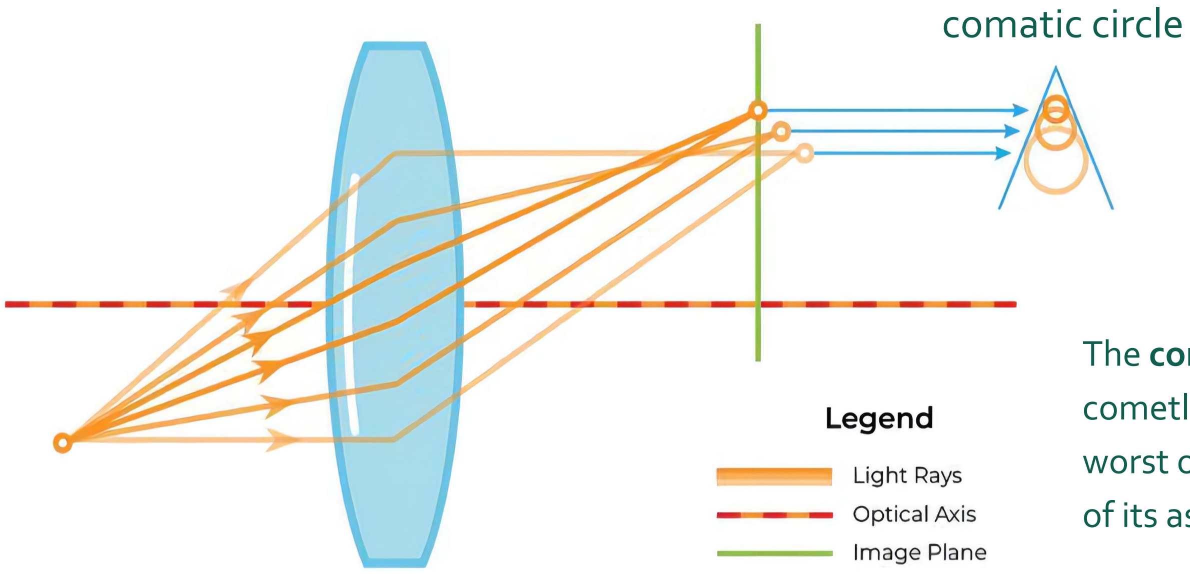


Pincushion

Barrel

Coma

Coma is an aberration which causes rays from an off-axis point of light in the object plane to create a trailing "comet-like" blur directed away from the optic axis.



The **coma flare**, which owes its name to its cometlike tail, is often considered the worst of all aberrations, primarily because of its asymmetric configuration.

Coma- Correction

- Use Compound Lens： use lenses with different curvatures.
- Smaller aperture.
- Set entrance pupil at the center of the sphere.

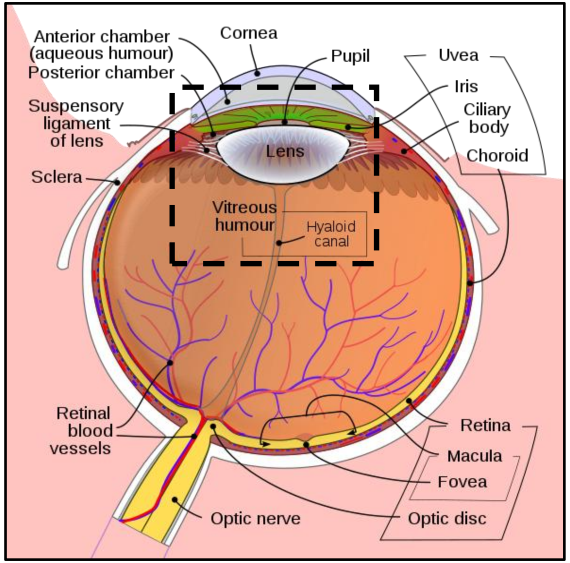
- For a single lens, coma can be partially corrected by bending the lens. More complete correction can be achieved by using a combination of lenses symmetric about a central stop.
- Coma is not well compensated for in the human eye.

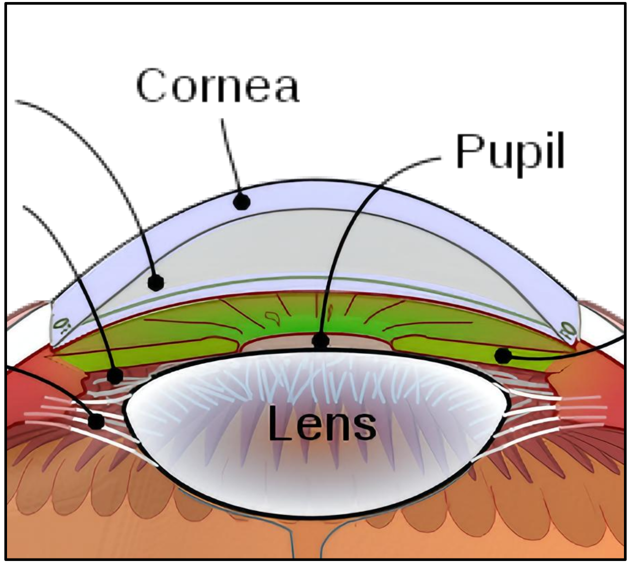
Distortion Example

The human eye is already a compound lens

As the human eye is a liquid lens, and water has dispersion, it has chromatic aberration.

- The combined cornea, anterior chamber, and crystalline lens form an achromatic doublet.
- Our brain further reduces *perceived* aberration by “cleverly” processing LMS cone responses.





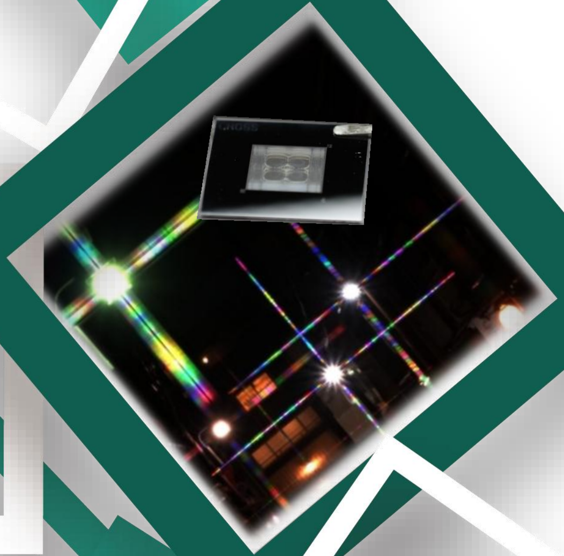
A Costly Aberration

Hubble telescope originally suffered from severe spherical aberration.

- COSTAR mission inserted optics to correct the aberration.

Today's Topic

- Paraxial optics.
- Ray Transfer Matrix
- Aberrations and compound lenses.



GAMES 204

Thank You!

Qilin Sun（孙启霖）

香港中文大学（深圳）

点昀技术（Point Spread Technology）