CONTENT

1. Introduction

1.1 Background

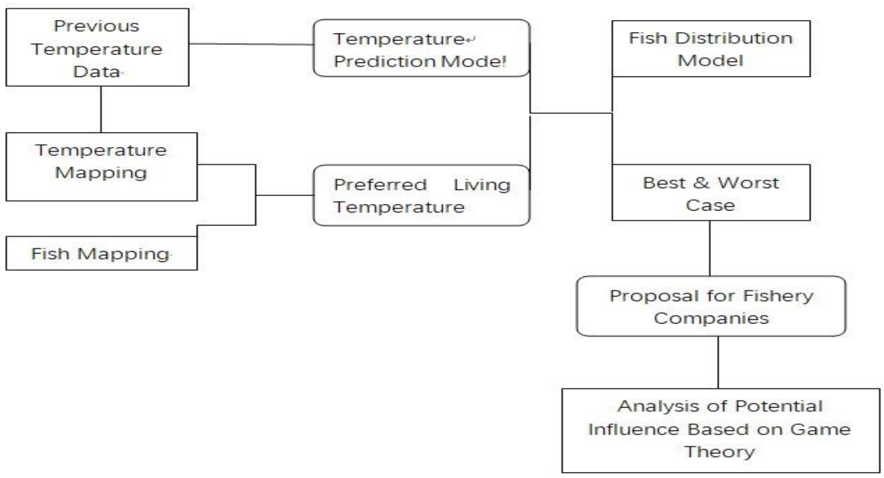
Fishery is an industry that benefits mainly by farming or seasoning fish and other sea animals (such as shrimp, or seals). People in this industry also produce related agricultural products. Among many of a coastal countries, fishing industry holds a quite important position. Especially for countries located on the coast of the world's four major fishing grounds, fishery is one of their pillar industries. As an area located along the North Atlantic fishing grounds, Scotland is listed.

Since last century, fishery has been playing an important role. However, over the decades, things have changed. With the release of carbon dioxide, the global temperature climbs up, the ocean temperature rises accordingly. The two main fish species caught in Scottish fishing grounds, herring and mackerel, have gradually moved North to seek suitable habitats. Therefore, it is important to help fishing company relocate.

1.2 Our Work

- We collect the past 50 years' ocean temperature data, build a prediction model, predict the possible temperature after 50 years or even more;
- Based on the annual production of herring and mackerel, we simulate their movements over the past 50 years and find the temperature in their relative habitable zones.
- Mapping the predicted ocean temperature, we locate the range of habitable zones, where the two species might migrate.
- From the perspective of small-scale fisheries, we simulate the process of fish leaving their fishing area, find the time when they completely lose their fry, and based on this, make economic planning and recommendations on whether they should move the fishery.
- We additionally conduct a game theory analysis on the cooperation and conflict between countries that jointly fish fry in the North Sea.

fig1: Analysis Flowchart



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2. Assumption and Justification

By adequate analysis of the problem, to simplify our model, we make the following well-justified assumptions.

- The data source is accurate and reliable, so our prediction is precise with little bias, which can be used to reasonably predict the ocean temperature in the future.
- We verify that these herring and mackerel basically exist at the same time, the temperature of the habitable zone is almost the same by consulting *The Annul Report of Fishery*, so we classify them as pelagic fishes and could conduct research together.
- We only consider the factors that have the greatest impact on the distribution of fishes——ocean temperature and currents. Since there are no recorded changes in currents, to simplify the model, only one factor, temperature, is considered.
- We divide the North Atlantic Ocean into a grid for each longitude and every 0.5 latitudes, thereby obtaining the following subregion map.

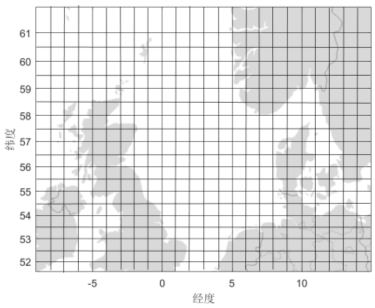


fig2: Subregion Map

- According to the coordinates of every temperature tested point and fish caught point, they are sorted into different subregions.
- Based on the classification, we calculate the average temperature of every subregion in each time period; also based on the sorting, we plot the amount of fish in every subregion.
- Assume that all fishing companies are located on the shore, we can draw the longest sailing distance based on the freshness time of the fish and the trawl speed, thereby delimiting a sector. In order to simplify the calculation, we normalize the fan-shaped area into the already defined longitude and latitude grids.
- Assume all the decision maker of fishery companies are rational; that is all there considerations are profit-based.
- Assume all the fishery companies and fishermen are profit-driven.
- Assume that fish in Scottish territorial waters flow into Norwegian territorial waters in one direction, so that profit-driven fishermen will have a tendency to enter other countries' territorial waters for fishing.

3. Analysis and Modeling

3.1 Fish coordinates prediction based on Gray Prediction Model

3.1.1 Basic analysis of the problem

According to biological analysis, due to the need of suitable temperature, pelagic fishes can not move directionless. The prior aim of their moving is to seek zones with proper temperature. This rule allows us to link fish migration lines and destinations with changing trends of temperature. We can make a bold assumption that the place where fish are gathered should have similar temperature characteristics.

Through data retrieval, we obtained the ocean temperature of North Atlantic from 1960 to 2010. First we choose five years as a time period, pre-process and get the average temperature of every subregion over each 5 years.

3.1.2 Ocean temperature prediction based on Gray Prediction Model

We have the original data of ocean temperature, mapping them in color, we have the following 10 ocean temperature map from 1964 to 2010.

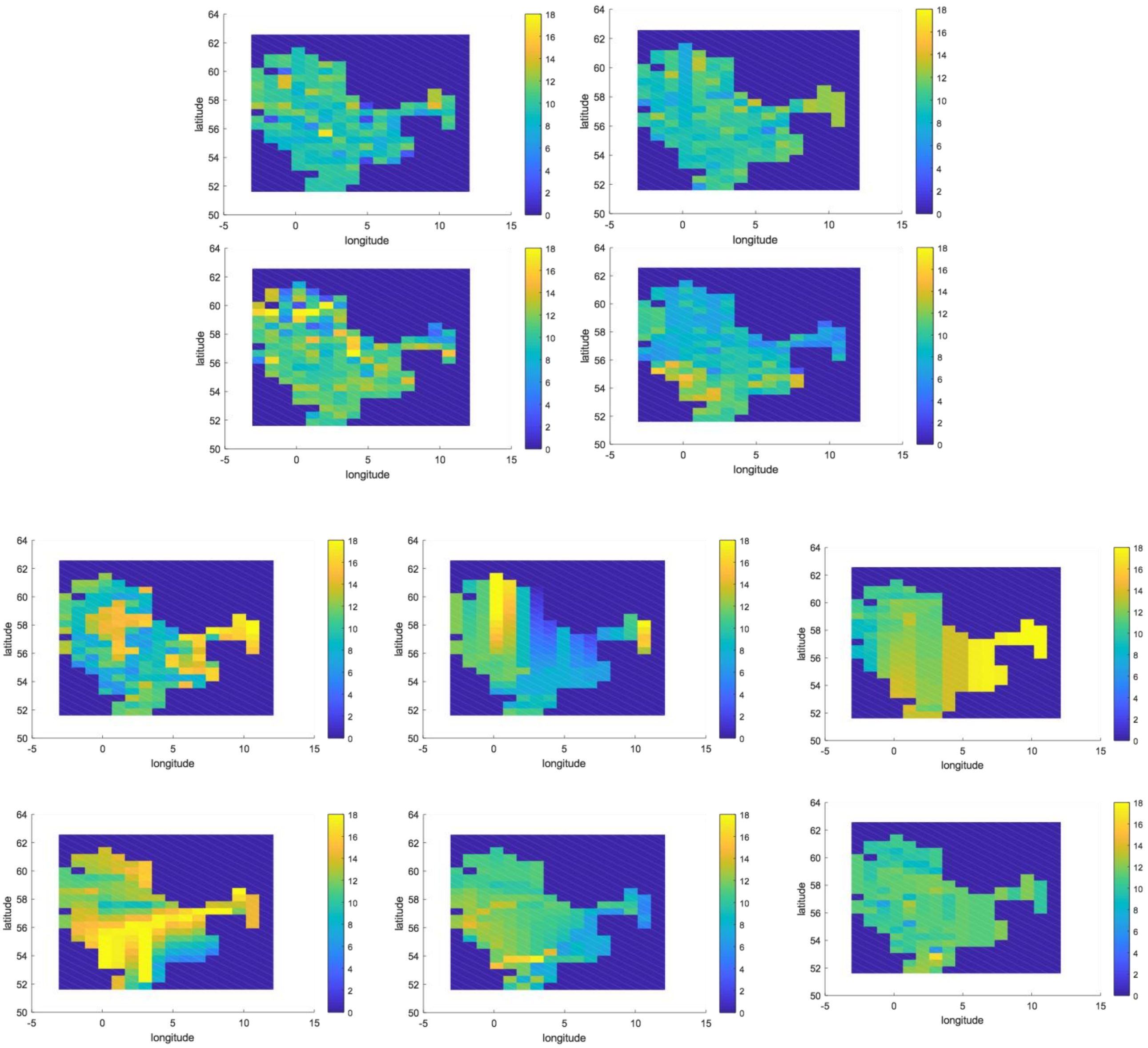


fig3: Ocean Temperature Map from 1964 to 2010

Now we know how the temperature changes along the ten time period, but these historical data are irregular and some fluctuate greatly, so we choose to realize the prediction through the gray prediction model.

In this study, the classic gray prediction model GM(1,1) model treats the ocean temperature as a gray quantity, and weakens the randomness of data by accumulating the sea surface temperature in each year, so that the data sequence presents a certain law. The differential equation is used to mine the nature.

Assume that subregion one is selected, a sequence of temperature changes from 1960 to 2010, $x^{(0)}$ is listed. This sequence is our original reference sequence. Then we give a one-time cumulative generation sequence $x^{(1)}$ and the mean generation sequence $z^{(1)}(k)$.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots x^{(0)}(n)) \quad (1\text{-}1)$$

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots x^{(1)}(n)) \quad (1\text{-}2)$$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1,2, \dots, n \quad (1\text{-}3)$$

$$z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \dots z^{(1)}(n)) \quad (1\text{-}4)$$

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2,3, \dots, n \quad (1\text{-}5)$$

Now give the gray differential equations, there is

$$x^{(0)}(k) + az^{(1)}(k) = b, k = 2,3, \dots, n \quad (1\text{-}6)$$

Then give the corresponding white differential equations

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b \quad (1\text{-}7)$$

At this time, due to the unknown development coefficient -a and gray interaction amount b, this equation is not yet solvable.

Next, specify $u = [a, b]^T$, construct a data vector $Y = [x^0(2), x^0(3), \dots, x^0(n)]^T$

and a data matrix $B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(2) & 1 \end{bmatrix}$, and then use the least squares method to find u, which makes $J(u) = (Y - Bu)^T(Y - Bu)$ reach the minimum.

The estimate value of $\hat{u} = [\hat{a}, \hat{b}]^T = (B^TB)^{-1}B^TY$

Now, we can get the above unknowns a and b, substitute them into the equations we can get :

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}\right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}}, k = 0,1, \dots, n-1, \dots \tag{1-8}$$

$\hat{x}^{(1)}(k+1)$ is the predicted temperature of this subregion one.

Among all the above calculation, we only choose one subregion to do the prediction. To predict the temperature trend of all subregions, we can combine the subregion distribution with temperature data and mathematically model it.

Define a matrix:

$$A_{ij} = \begin{bmatrix} a_{11}\ a_{12}\ \dots a_{1n} \\ a_{21}\ a_{22} \dots\ a_{2n} \\ \dots \\ a_{m1}\ a_{m2} \dots\ a_{mn} \end{bmatrix}$$

where i represents the number of the subregion, j represents the number of the year, that is, the same row of elements is the ocean temperature of the same subregion in the 10 time period. Modeling every element in A_{ij}, we can build the above gray prediction model.

After the modeling, we have the predicted average ocean temperature of every subregion in each 5 year period. Mapping that in color, we have the following ocean temperature picture in 2060.

3.1.3 Find the possible migration zones

First, by processing the coordinates of herring and mackerel, we can make fish distribution maps from 1980 to 2010, and we can clearly see the trends——moving north. By comparing the following six distribution maps of fishes with the color temperature maps in appendix, we can verify that the aggregation subregions have similar color blocks, which confirms our previous hypothesis.

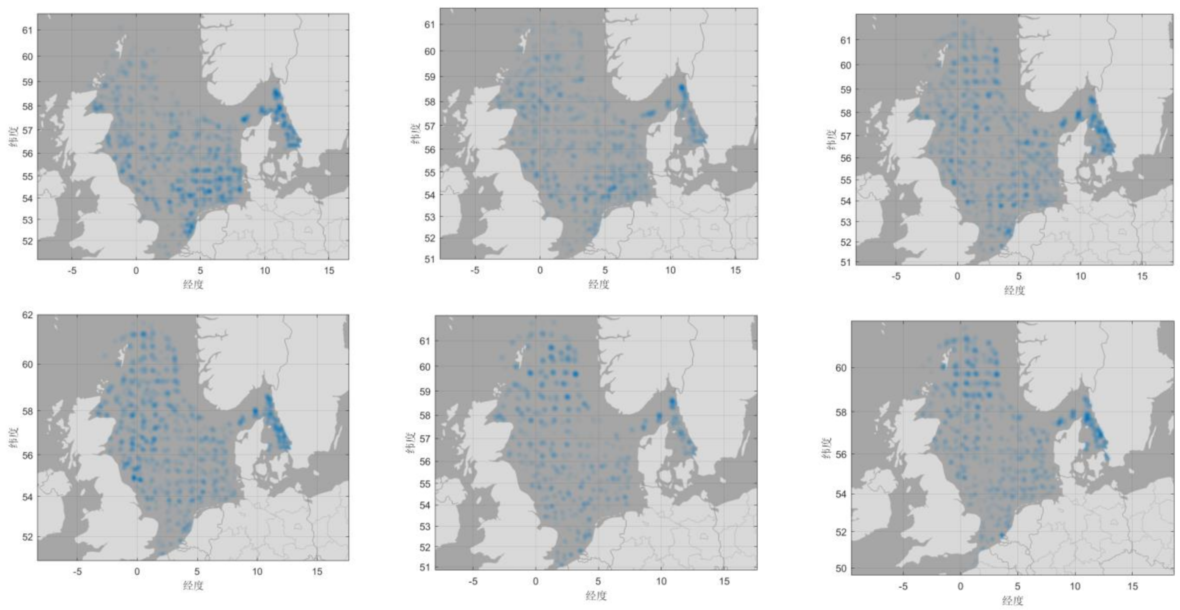


fig4: Fishes Distribution Maps from 1980 to 2010

Similarly, by dividing the sea area into multiple regions, we can find subregions where fish are gathered. By placing these sub-areas on the color temperature map made earlier, we can obtain the corresponding temperature interval, which is the preferred living temperature for fish.

Through prediction, we know the average temperature of each subregion in each period from 2010 to 2060. Dividing the fish's preferred survival temperature by the average temperature gives a coefficient. The closer the coefficient is to 1, the more likely it is that fish will migrate to the area. Dozens of zones with the largest coefficients are the migration destinations we choose.

Repeating this operation and accumulating them, we will find several subregions where the fish are most likely to gather. We can consider these areas as final destinations for fish migration.

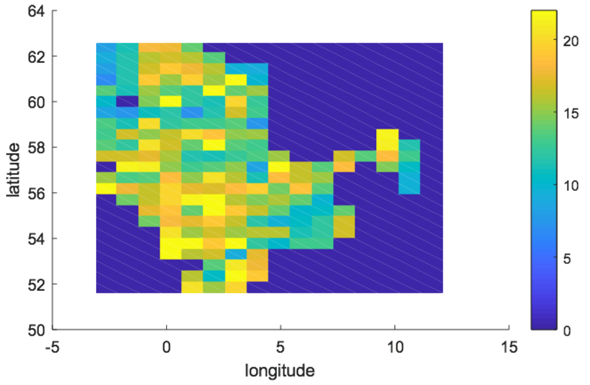


fig5: Predicted Temperature in 2060

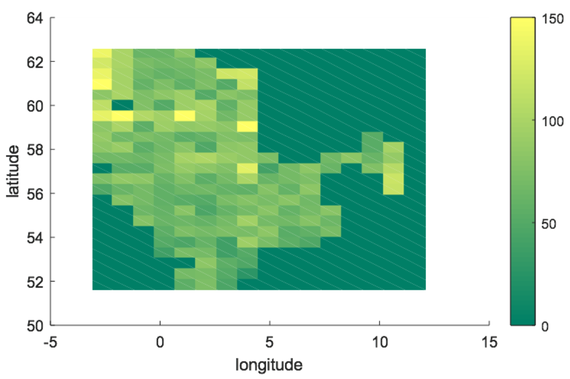


fig6: Predicted Distribution of Fish in 2060

3.2 Prediction on the best and worst case based on Gray Prediction Model

3.2.1 Basic analysis of the problem

By browsing the official website of the Scottish Government, we have confirmed that fishing companies generally use trawlers to catch pelagic fishes. Assume that there is no refrigeration equipment on trawlers, we are required to predict the best and worst case of those companies' fishing conditions in the next few years.

After the fish are caught, the fishermen must sort and refrigerate them in the shortest possible time. If they are not processed in time, fish will quickly lose their freshness. Since they do not have a refrigerator there, fish can only stay fresh for a few hours after being killed at room temperature. Therefore, they must be able to return to the shore and sell the fish at the specified time; because we know the speed of the trawler, this indicates that there is a limited distance that the fishing vessels can go.

According to that limitation, we can demarcate a new area in the sea we studied. Trawlers fishing outside this area will catch stale fishes. We call this demarcated area as Effective Fishing Area (EFA). Finally, we use the amount of fish available in EFA to measure the fishery condition. The more fish in EFA, the better the fishery condition is.

3.2.2 Model building

Fish is generally best served in 2-6 hours, and trawler generally sails at a speed of 3-6 nautical miles per hour. Based on this, we can mark the following area.

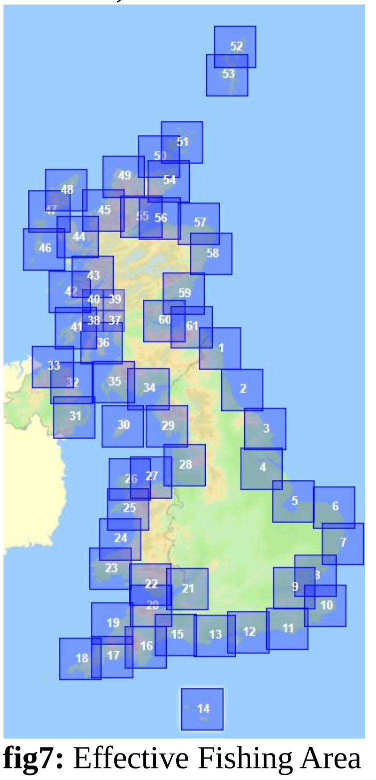


fig7: Effective Fishing Area

According to our temperature prediction model and the distribution of fish over pre-

vious years, we have obtained the preferred living temperature (PLT) of herring and mackerel. It is reasonable to believe that fishes have a tendency to gather in subregions whose temperature is near this range. In other words, we assume that there is more fish in the subregions with proper temperature. Therefore, we can indirectly predict the amount of fish in EFA based on temperature.
Next we introduce a ratio Q to measure the quantity of available fish in EFA, that is

$$Q = \frac{n_{pre}}{N} \tag{2-1}$$

where

- n_{pre} is the number of subregions with temperature near PLT
- N is the number of all subregions in EFA.

In this equation, a larger Q represents a larger proportion of the subregions whose temperature is near PLT in EFA.

Here gives our predicted value of Q in the following 50 years.

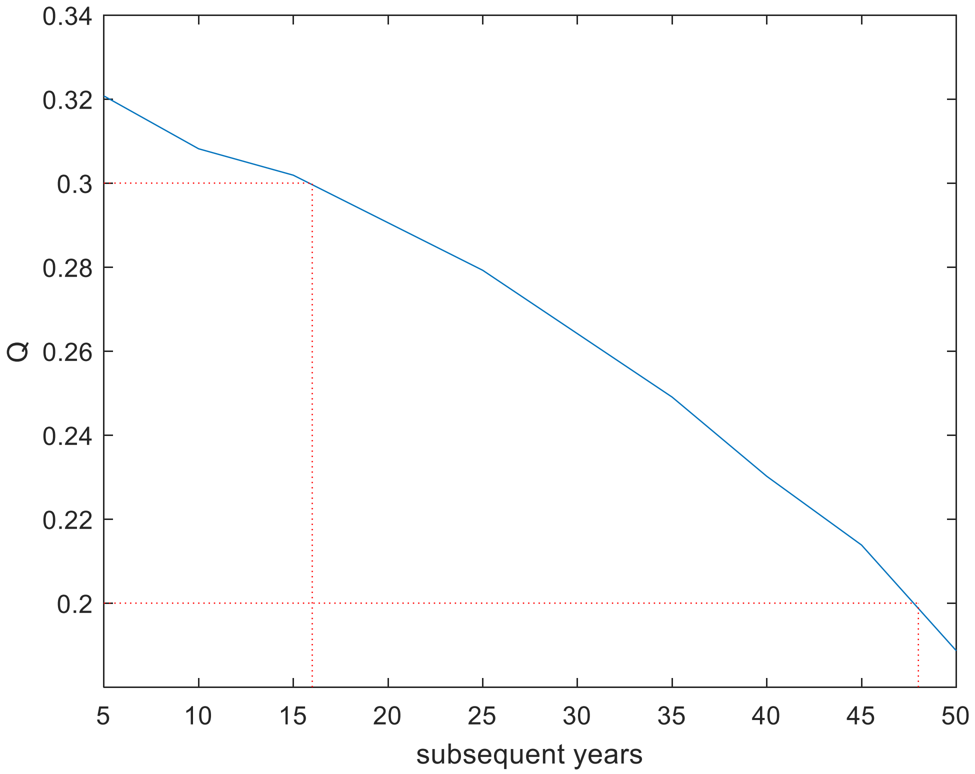


fig8: Value of Q

We assume that, when Q>0.3,that is in the next 16 years, companies can reach the best condition. By contrast, when Q<0.2, that is after 48 years, companies will be in the worst condition.

3.3 Strategies for small fishing companies based on Net Present Value (NPV) Rule[1]

3.3.1 Basic analysis of the problem

In 4.1, we have acknowledged that as the ocean temperature increases, herring and mackerel will gradually migrate northward and we develop the routine of this movement. Obviously, this migration does cause some small fisheries to lose fry. The problem here is, whether these small fisheries should relocate themselves to gain economic benefits or not? From reading the Scottish Sea Fisheries Statistics (2016), we find the Scottish fishing grounds are mainly distributed in the following location.

fig9:Scottish Fishing Grounds Distribution[2]

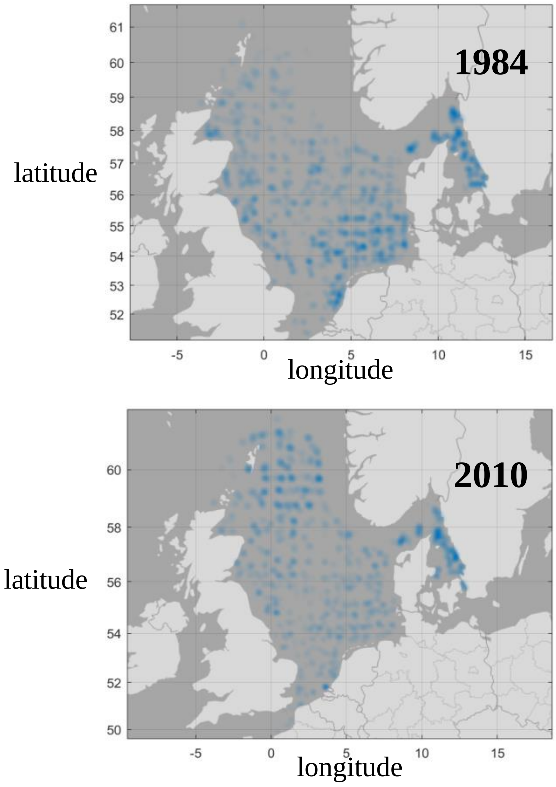


fig10: Pelagic Fish Migration Routine

Knowing the fishes will continuously move north in the coming decades, we can easily understand that there will be fewer and fewer fry at the southern part; if we can not take efficient measures to slow down the rising of ocean temperature, the southern fishing ground will eventually lose the grain. Therefore, these southern fishery companies is truly in the need of making changes to their operations.

3.3.2 Strategy 1 - Relocating some southern companies

First, regardless of relocating or not, fishery companies have some fixed costs each year unless they choose to shut down; such as salary payable, maintenance fee and cost of good sold. This kind of cost should not be taken into consideration.

To simply quantify the net present value of a relocating project, framed as NPV, which is measured in monetary terms, we consider expenditure (E) and income from the implementation of relocating (I). For fishery companies (F) here is

$$\mathrm{NPV(F) = I(F) - E(F)} \tag{3-1}$$

To define the income from the implementation of relocating (I), we think in terms of discounting future income, that is

$$\mathrm{I(F) = I(F)} = \sum_{i=1}^{n} \frac{C_i}{(1+r)^i} \tag{3-2}$$

where

- n is the total year after relocation when do the assessment.
- C_i is the cash flow earned by fishing herring and mackerel
- r is the discount rate regulated by natural market and government.

The NPV rule is that you take the action if your calculated NPV is positive, or in another word, larger than zero. We initially do the short run consideration. Within 5 years from 2010, we separately calculate the NPV of this relocating project.

If reject this project, then

$$\mathrm{NPV(F) = I(F)} = \sum_{i=1}^{5} \frac{C_i}{(1+r)^i} \tag{3-3}$$

although C_i will decrease as fish moving northward, those companies still have positive cash inflow and positive NPV.

If accept this project, then

$$\mathrm{NPV(F)} = \sum_{i=1}^{5} \frac{C_i}{(1+r)^i} - \mathrm{E(F)} \tag{3-4}$$

with cash inflow C_i decreases, there is extra expenditure E for moving the factory somewhere else. Therefore, the NPV is likely to decrease or even turn to a negative number. So in a relative short run, not relocating seems a better choice.

However, in the long run, things are completely different. From the modeling in 4.2, we assume that the best case occurred in the 16th year. We take i as our n here.

If accept this project, then

$$\mathrm{NPV(F)} = \sum_{i=1}^{20} \frac{C_i}{(1+r)^i} - \mathrm{E(F)} \tag{3-5}$$

even if in the first few years after relocation the NPV might be negative, as i increases, the expenditure will be evenly amortized to every year; then there will be a point when NPV turns positive, we win. Another thing we have to mention here, if we accept relocating, C_i will not decrease, for the lost resource of fishing is made up.

What if in the long run we do not accept this project? Then

$$\mathrm{NPV(F) = I(F)} = \sum_{i=1}^{20} \frac{C_i}{(1+r)^i} \tag{3-6}$$

it will keep positive in this time period, but when we enter the 21st year, we will have zero income; plus the fixed cost mentioned before, seems like these companies won't last long. Running a company must be a long-lasting business, therefore our suggestion is to relocate the southern companies somewhere north, after all there are still some vacant places along the shore. Meanwhile, the earlier this action is taken, the higher the profit.

3.3.2 Strategy 2 - Using some fishing vessels capable of operating and preserving fish themselves

In addition to relocating, the southern companies has a second option——sending some fishing vessels capable of preserving fish themselves for a certain period; but we suggest that this could be a supplementary option to relocation. If only choose to send capable vessels, buying new-type vessels, hiring extra workers and maintaining these ships will cost a lot; besides, there will still be one day that no fish in the area. So based on relocating and sending vessels, there will be

$$NPV(F) = \sum_{i=1}^{n} \frac{C_i}{(1+r)^i} - E(F_R) - E(F_V) \qquad (3\text{-}7)$$

where $E(F_v)$ is the cost of sending vessels.

As for whether the vessels should be sent on the basis of moving, this should depend on the operating conditions of the company itself. If the scale is small, it may be better to move only.

For fisheries in central and northern Scotland, relocation does not have to be considered, but sending capable vessels can be profit added. Since they do not have $E(F_R)$, after the action C_i will go up for more fish will be caught. Then

$$NPV(F) = \sum_{i=1}^{n} \frac{C_i}{(1+r)^i} - E(F_V) \qquad (3\text{-}8)$$

as income increases but expenditure is comparatively small, especially for the large scale companies, this option will be profitable.

3.4 Suggestions for companies on fishery entering other countries' territorial waters based on Game Theory[3]

3.4.1 Basic analysis of the problem

Through the proposal of sending vessels in 4.3, problems may arise. The range of sea area we study actually includes the territorial waters of many other countries. Fishes can freely move between the territorial seas of different countries without any restrictions; so marine fishery resource is a regional public resource. Based on our fish distribution model, a few years later, huge amount of fish will migrate from Scottish territorial sea to that of Norway.

This movement of fish means that Scottish companies will lose part of their grain. In order to maintain their income, theoretically they will enter the territorial seas of other countries for fishing; this will infringe the territorial right of other countries. Therefore, it is important to put forward a good policy, or the issue will inevitably lead to disputes.

3.4.2 Not adopting any politics

First, we quote an effectiveness function to quantify the change in available fish in Scotland and its impact. The quantity of fish is affected not only by the current catches in Scotland, but also by function T, which reflects the increase in fish in another country, that is

$$U^j = G^j(E^j, T(E^i)) \tag{4-1}$$

where

- U^j is the utility of the fish quantity level in the Scottish waters
- G^j is the function of the fish quantity in the Scottish waters
- E^j is the fish quantity in the Scottish waters
- $E^i$is the fish quantity in the Norwegian waters
- T is the diffusion function of the fish.

When E^iincreases, quantity of catches in Scottish waters decreases. This formula represents a one-way migration of fish.

In terms of these two mutually-affected countries, the increasing number of fishes in one country's waters and the declining in the other means an imbalance of resource allocation.

In our previous proposal, facing this problem, the focus is on the companies who send vessels. Therefore, if we do not take any measures, they will bear the losses if they do not go fishing; otherwise, if they go fishing, that will cause disputes. This is enough to illustrate that we need to take some actions.

3.4.3 Non-cooperative game

Under the non-cooperative game, each country with cross-border fisheries (ie, Norway and Scotland in this question) maximizes their utility and minimizes their cost individually.

Next, for the sake of simplifying the analysis, we only consider the cost minimization of different countries and define cost-minimized objective function of different countries respectively. Taking this question as an example, we assume that the migration of herring and mackerel is a one-way migration from Scotland to other countries. Cost of each country includes the cost of damage to fishery ecosystem and the cost of fish reductions. We define the cost function of damage to fisheries ecosystems as D=D(S). Also, we still apply the fish diffusion function T, where

- S represents the amount of catch
- $S_0^1$is the initial quantity of fish in Scotland
- S_r^1 is the number of fish cut for cost reduction in Scotland
- S_r^2 is the number of fish cut for cost reduction in Norway
- S_0^2 is the initial quantity of fish in Norway
- T is the diffusion function of the fish
- $C^1$is the cost of fish reductions.

Suppose that we have known the value of S_0^1 and S_0^2, and the number of fishes moving out of Scottish waters is equal to the number of fishes moving into Norwegian waters. For Scotland, we can write the cost-minimized objective function,

$$\min[D^1(S_0^1 - T(S_r^1) - S_r^1) + C^1(S_r^1)] \tag{4-2}$$

Take the derivative of this function to find its minimum cost condition, that is

$$-\frac{dD^1}{dS_r^1}\left(S_0^1 - T\left(\hat{S}_r^1\right) - \hat{S}_r^1\right) = \frac{dC^1}{dS_r^1}(\hat{S}_r^1) \tag{4-3}$$

Similarly, we can get the cost-minimized objective function of Norway

$$\min[D^2(S_0^2 + T(S_r^1) - S_r^2) + C^2(S_r^2)] \tag{4-4}$$

Take the derivative of this function to find its minimum cost condition, that is

$$-\frac{dD^2}{dS_r^2}\left(S_0^2 - T(S_r^1) - \hat{S}_r^2\right) = \frac{dC^2}{dS_r^2}(\hat{S}_r^2) \tag{4-5}$$

Now, we introduce a classic model called Pareto Optimality (PO). PO is an ideal state of resource allocation. Assume there is two people and assignable resources, during the changing from one state of allocation to another, make at least one person better off. In case of this question, the resource is fish. Because we have already got the minimum cost conditions of the two areas, the OP state can be achieved, which means the minimum cost of Scotland and Norway can be realized separately. The following image illustrates the optimal state of Pareto.

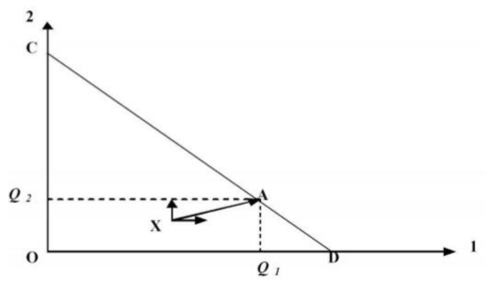


fig11: Pareto Optimality Condition

The horizontal coordinate represents the fishing volume in Scotland, and the vertical coordinate represents the that in Norway, so it can be fixed. Points on the line are optimized points, while others outside the line is not. However, by optimizing the condition, the original unoptimized point can be moved to the optimized position. But on this line, for points other than the midpoint, such as point A, the fishing volume of Scotland exceeds lots of Norway's, therefore we have the disputes again.

3.4.4 Cooperative game

Non-cooperative game is to minimize the cost of each country separately, while cooperative game is to consider the cost of two countries as a whole and minimize this overall cost.

In the same way, we can get the cost-minimized objective function and minimum cost condition of the whole of different countries, that is

$$\min[D^1(S_0^1 - T(S_r^1) - S_r^1) + D^2(S_0^2 - T(S_r^1) - S_r^2) + C^1(S_r^1) + C^2(S_r^2)] \quad (4\text{-}6)$$

$$-\frac{dD^1}{dS_r^1}\left(S_0^1 - T\left(\hat{S}_r^1\right) - \hat{S}_r^1\right) - \frac{dD^2}{dS_r^1}\left(S_0^2 + T\left(\hat{S}_r^1\right) - \hat{S}_r^2\right) = \frac{dC^1}{dS_r^1}(\hat{S}_r^1) \quad (4\text{-}7)$$

Comparing (4-6) and (4-7), with cooperative game strategy, Scotland's effort to reduce the number of catches (S_r^1) exceeds that using non-cooperative game strategy. That is, to achieve the goal of minimizing overall cost of the two countries requires Scotland to make more adjustments.

However, in the case of cooperation, no region will make large sacrifices at will. At this time, the policy under cooperation game——transfer payment, should be introduced.

During the negotiation process of the cooperative game between the two countries, an agreement can be reached to stipulate that the fish inflow country (Norway) should make corresponding compensation to Scotland. This rule guarantees that the overall cost of the two countries is minimized, the goal of balancing resources and benefits is achieved perfectly.

Through all the above analysis, the cooperative game theory between the two countries can fully ensure the companies' profits, and the fishermen will not do cross-border fishing. Meanwhile, there is a low possibility that territorial dispute is aroused.

4. Discussion and Conclusion

During the establishment of our model, we mainly adopted the method of dividing the latitude and longitude blocks in the North Sea. Based on these subregions, we locate the fish distribution and predict the corresponding temperature. This system makes our model as simple as possible, which is more conducive to our prediction and mapping. However, this is where we need to improve. Although the method is concise, it also causes a small amount of data to be missing during the data processing. After interpolation is used to complete the data sequence, it may have a certain impact on the accuracy of the model prediction.

From the perspective of business survival, our suggestions are of great significance. But environmental protection is not a profit-driven transaction, it requires our continuous and joint efforts. Each factory on the North Sea coast emits one ton less greenhouse gas each month, the change will be immeasurable. Over time, we will gradually optimize our environment, slow down the rise in ocean temperature, thus achieve a harmonious symbiosis between business and ecology.

Reference

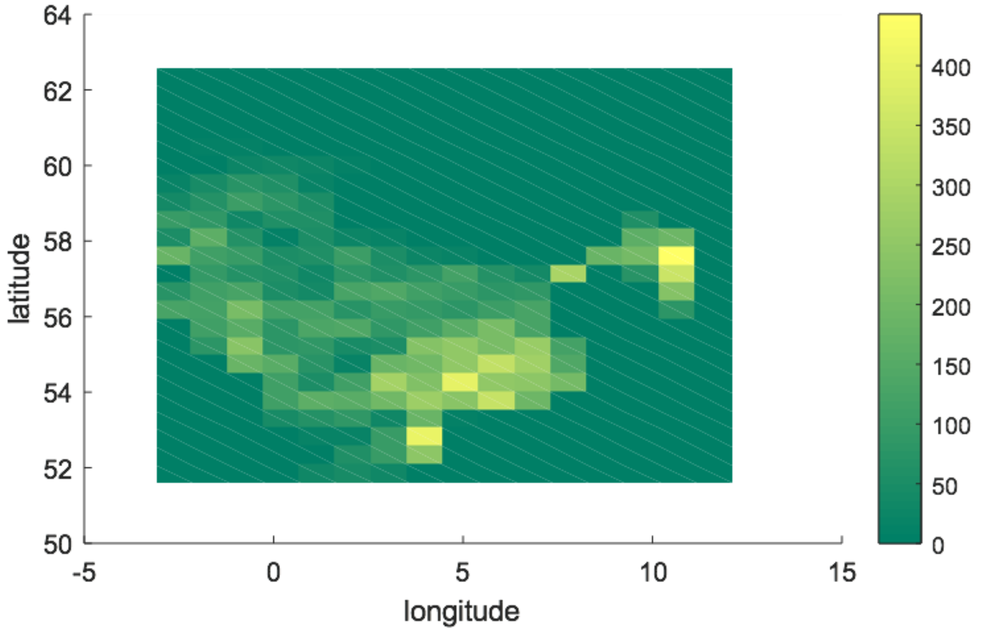
[1] Finance Markets & Corporate Strategy, Mark Grinblatt.

[2] Scottish Sea Fisheries Statistics 2018

[3] 徐敬俊.跨界海洋渔业管理研究——一个博弈论的分析框架[J].中国海洋大学学报(社会科学版),2011(03):33-35.

Appendix 1. Fish Allocation Graph from 1980

8084



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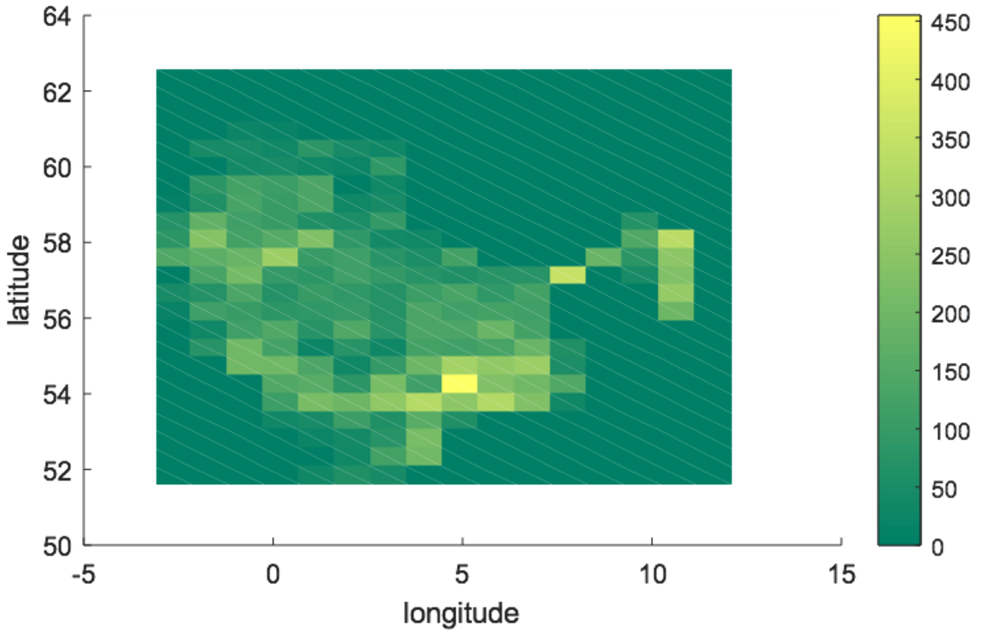

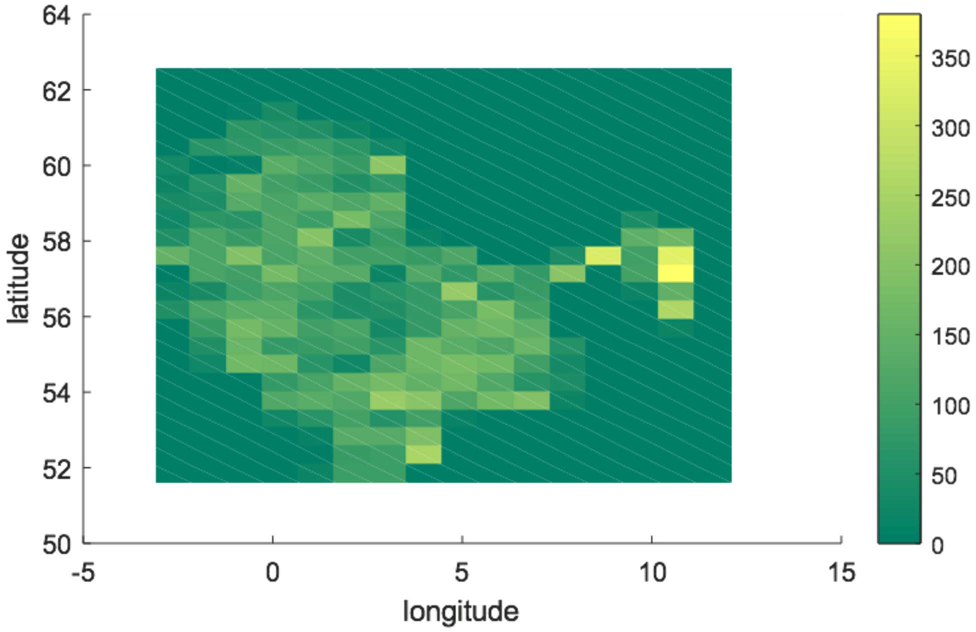

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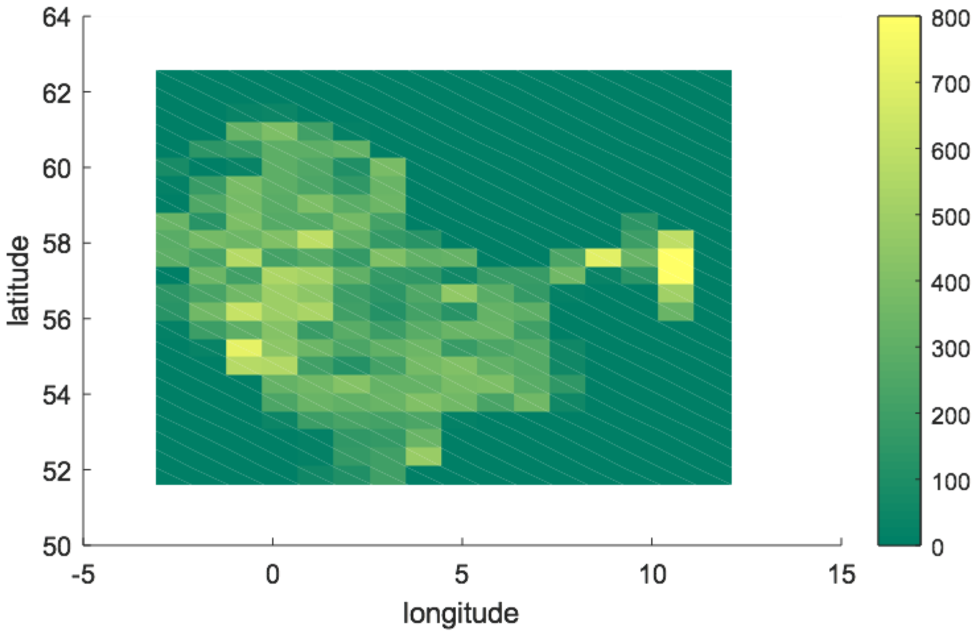

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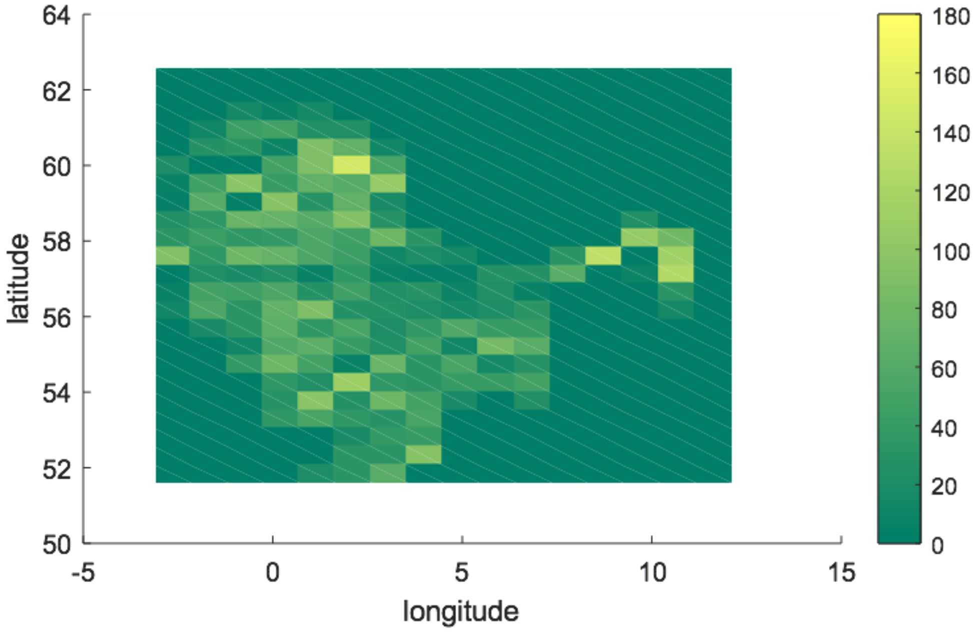
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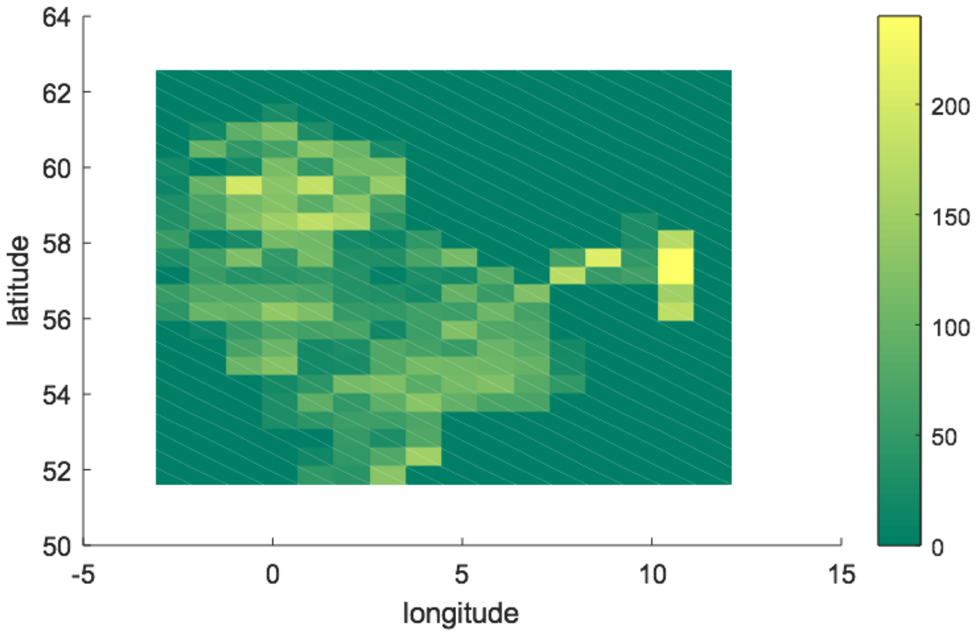
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