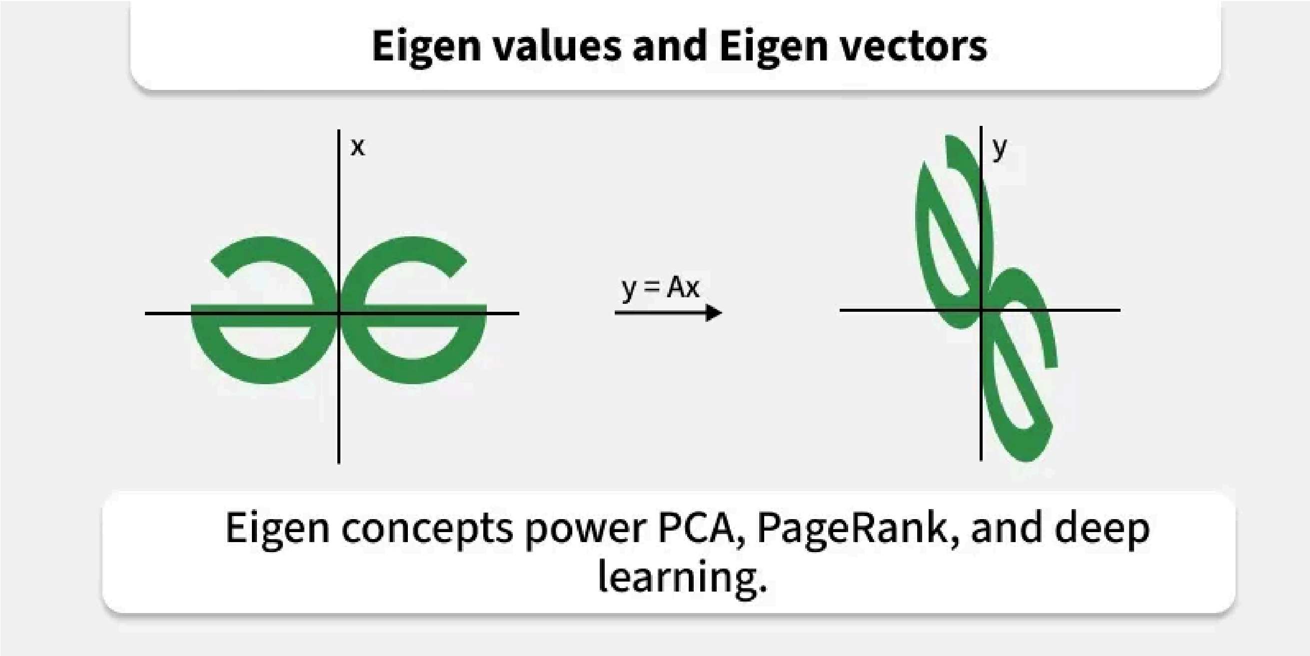
Eigenvalue and Eigenvector



1. **Eigenvalue** = Scalar shows how much eigenvector get compressed or stretched during the transformation.

   Note: The eigenvector's direction remains unchanged unless the eigenvalue is negative, in which case the direction is simply reversed.

   The equation for eigenvalue is given by,

   $$Av = \lambda v$$

   Where,

   - A is the matrix,
   - v is associated eigenvector and
   - λ is scalar eigenvalue.

2. Eigenvector = non-zero vectors that, when multiplied by a matrix, only stretch or shrink without changing direction.

Types of Eigenvector

The eigenvectors calculated for the square matrix are of two types which are,

- Right Eigenvector
- Left Eigenvector

Right Eigenvector: The eigenvector which is multiplied by the given square matrix from the right-hand side is called the right eigenvector. It is calculated by using the following equation,

$$AV_R = \lambda V_R$$

Where,

- **A** is given square matrix of order n×n,
- **λ** is one of the eigenvalues and
- $\mathbf{V_R}$ is the column vector matrix

The value of V_R is, $\mathbf{V_R} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ . \\ . \\ v_n \end{bmatrix}$

Left Eigenvector: The eigenvector which is multiplied by the given square matrix from the left-hand side is called the left eigenvector. It is calculated by using the following equation,

$$V_L A = V_L \lambda$$

Where,

- **A** is given square matrix of order n×n,
- **λ** is one of the eigenvalues and
- V_L is the row vector matrix.

The value of V_L is,

$$V_L = [v_1, v_2, v_3, \ldots, v_n]$$

Eigenvector of a 3 × 3 Matrix

An example of eigenvector of 3x3 matrix is,

Example: Find the eigenvalues and the eigenvector for the matrix A = $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

Solution:

If eigenvalues are represented using λ and the eigenvector is represented as v = $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Then the eigenvector is calculated by using the equation,

$|A - \lambda I| = 0$

$$\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{bmatrix} = 0$$

Simplifying the above determinant we get

$\Rightarrow (2-\lambda)(\lambda^2) + 2\lambda^2 + 2\lambda^2 = 0 \quad \Rightarrow (-\lambda^3) + 6\lambda^2 = 0$

$\Rightarrow \lambda^2(6 - \lambda) = 0$

$\Rightarrow \lambda = 0, \lambda = 6, \lambda = 0$

For λ = 0

(A - λI) v = 0

$$\Rightarrow \begin{bmatrix} 2-0 & 2 & 2 \\ 2 & 2-0 & 2 \\ 2 & 2 & 2-0 \end{bmatrix} . \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} . \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

Simplifying the above equation we get

$2a + 2b + 2c = 0$

$\Rightarrow a + b + c = 0$

Let $b = k_1$ *and* $c = k_2$

$\Rightarrow a + k_1 + k_2 = 0$

$\Rightarrow a = -(k_1 + k_2)$

Thus, the eigenvector is,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -(k_1 + k_2) \\ k_1 \\ k_2 \end{bmatrix}$$

taking $k_1 = 1$ *and* $k_2 = 0$*, the eigenvector is,*

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

taking $k_1 = 0$ *and* $k_2 = 1$*, the eigenvector is,*

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

For λ = 6

(A - λI) v = 0

$$\Rightarrow \begin{bmatrix} 2-6 & 2 & 2 \\ 2 & 2-6 & 2 \\ 2 & 2 & 2-6 \end{bmatrix} . \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix} . \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

Simplifying the above equation we get,

$-4a + 2b + 2c = 0 \quad \Rightarrow 2(-2a + b + c) = 0$

$\Rightarrow 2a = b + c$

Let $b = k_1$ *and* $c = k_2$ *and taking* $k_1 = k_2 = 1$ *we get,*

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Thus, the eigenvector is,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$