Rank

rank = number of dimensions in the column space.
if the rank is as high as it can be, meaning it equals the number of columns === Full rank
1 dim --> rank 1
2dim --> rank 2

"Rank" --> Dimensions

The rank of a matrix is the maximum number of linearly independent rows or columns in a matrix. it essentially determines the dimentionality of the vector space formed by the rows or columns of the matrix.

It helps determine:

- If a system of linear equations has solutions.
- The "usefulness" of rows or columns contributes to the matrix's information.

Example: Find the rank of a matrix $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$ using the normal form method.

Solution:

Given $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$

Apply $R_2 = R_2 - R_1$, $R_3 = R_3 - 2R_1$ *and* $R_4 = R_4 - 3R_1$

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Apply $R_1 = R_1 - 2R_2$ *and* $R_4 = R_4 - R_2$

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Apply $R_1 = R_1 + R_3$ *and* $R_2 = R_2 - R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Apply $C_4 \rightarrow C_4 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus A can be written as $\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$.

Thus, ρ(A) = 3