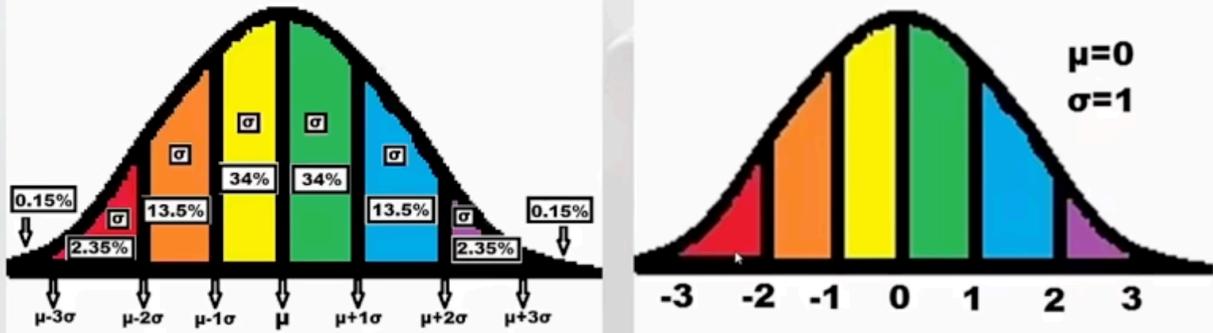


## 7.2 & 7.3 Z-score and Probabilities

Z-score is the Standard Normal Distribution

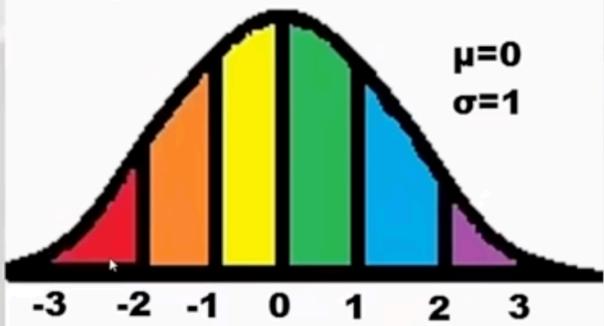
### Z is the Standard Normal Distribution



### Z-scores

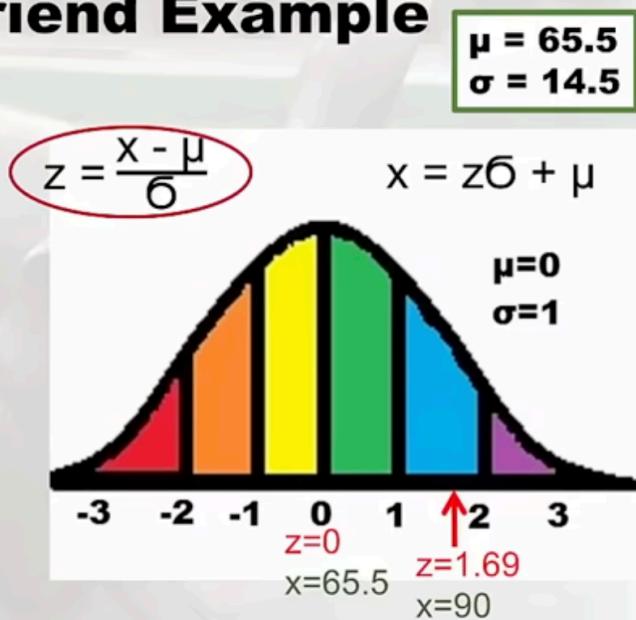
- Every value on a normal distribution (every “x”) can be converted to a z-score.
- You must know the following to use formula:
  - The “x” – what you want to convert to z
  - The  $\mu$  of the distribution
  - The  $\sigma$  of the distribution

$$z = \frac{x - \mu}{\sigma} \quad x = z\sigma + \mu$$



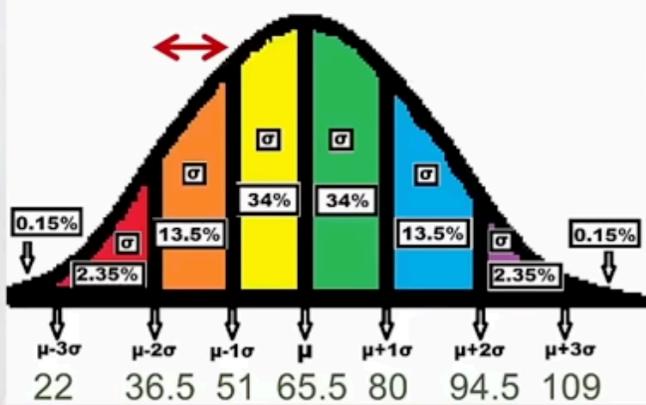
## Z-scores: Smart Friend Example

- Remember our  $n=100$  students?
- Let's say your friend got a 90. What is the z-score for 90?
  - $x=90$
  - $\mu = 65.5$
  - $\sigma = 14.5$
- $(90-65.5)/14.5 = 1.69$



Z-score probability

## Remember “Probability” from the Empirical Rule?



Question: What is the *probability* I will select a student with a score between 36.5 and 51?

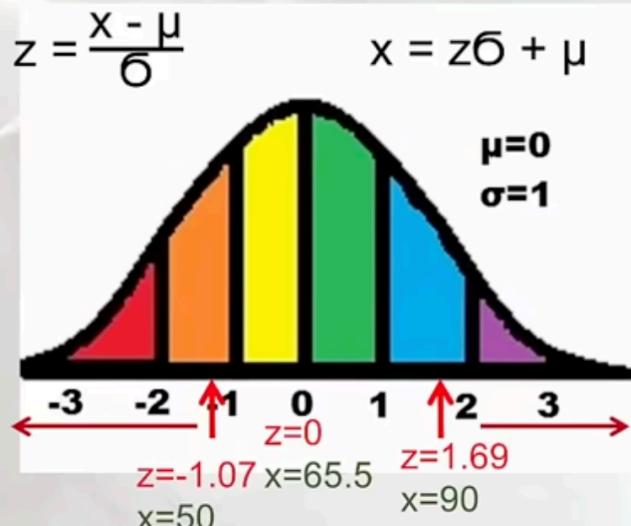
Answer: 13.5%

But what if you have z-scores of 1.69 (smart friend) and -1.07 (not-so-smart friend)?

## Questions about Z-Score Probabilities

$$\mu = 65.5$$
$$\sigma = 14.5$$

- What is the *probability* that students scored *above* the smart friend?
  - In other words – what is the area under the curve from  $z=1.69$  all the way up?
- What is the *probability* that students scored *below* the not-so-smart friend?
  - In other words – what is the area under the curve from  $z=-1.07$  all the way down?



We will look these up using the Z table.

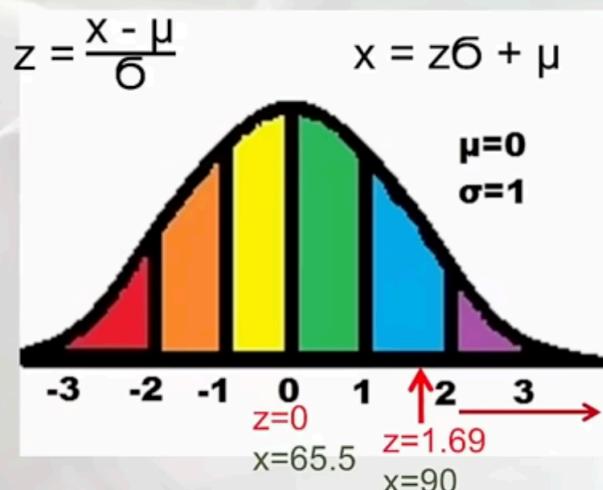
If you are looking for areas to the right, it is more efficient to use the opposite z value

## How to Use the Z Table

$$\mu = 65.5$$
$$\sigma = 14.5$$

- Let's do the smart friend's probability.
  - What is the *probability* that students scored *above* the smart friend ( $x=90$ ,  $z=1.69$ )?
- For areas to the *right* of a specified z value, either:
  - Look up in table, then subtract result from 1, or
  - Use the opposite z (-1.69)

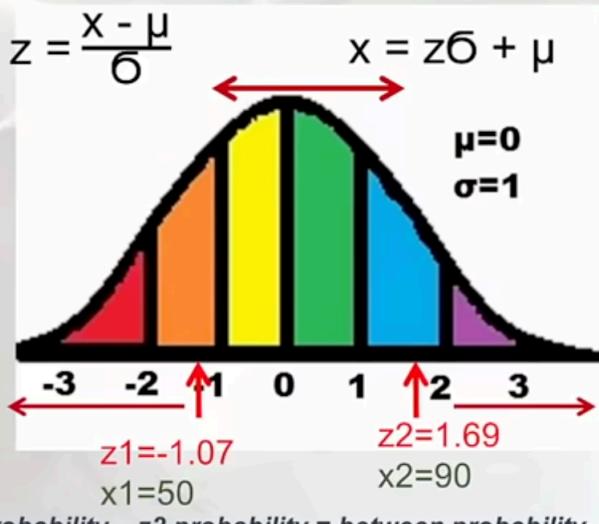
$$p = 0.0455, \text{ or } 4.55\%$$



## Calculating Probability Between Scores

$$\mu = 65.5$$
$$\sigma = 14.5$$

1. Note that you have  $x_1$  and  $x_2$  (two  $x$ 's)
2. Calculate  $z_1$  and  $z_2$
3. For  $z_1$ , find the probability to the *left (below) z*
  - Remember, direct probability from z table
4. For  $z_2$ , find the probability to the *right (above) z*
  - Use one of the 2 methods
5. Subtract both  $z_1$  and  $z_2$  probabilities from 1.



$$1 - z_1 \text{ probability} - z_2 \text{ probability} = \text{between probability}$$

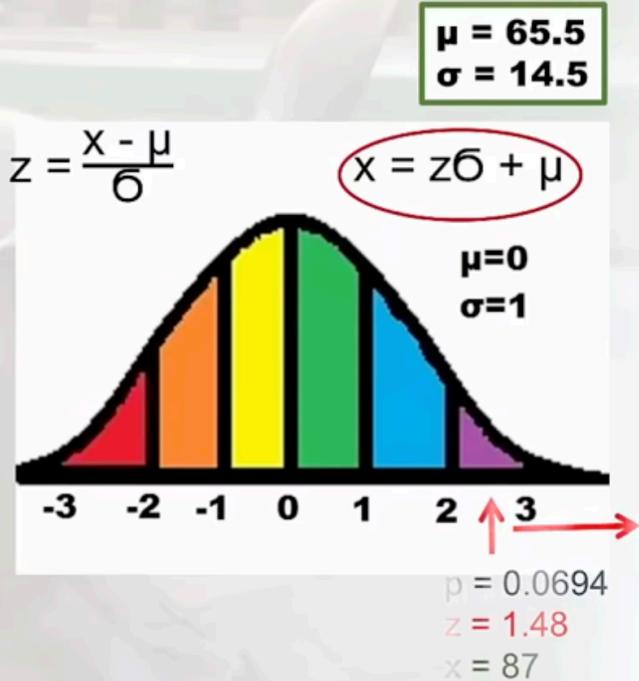
## Note About Z Table

1. Treat all areas (probabilities) to the left of  $z = -3.49$  as  $p = 0.0000$
2. Treat all areas (probabilities) to the right of  $z = 3.49$  as  $p = 1.000$



## Calculating x Questions

- What is the score that marks the top 7% of the scores?
- We are looking for the z at  $p=0.0700$ 
  - Closest p in table is 0.0694
  - That maps to  $z = -1.48$
- Since we want "top 7%" we want positive z:  $z = 1.48$
- $x = (1.48 * 14.5) + 65.5 = 87$
- 87 is the score that marks the top 7% of scores



## Calculating x Questions

- What scores mark the middle 20% of the data?
- Strategy is to find the z-score for  $(1-0.2000)/2 = 0.4000$
- For  $p = 0.4013$ ,  $z = -0.25$
- Also,  $z = 0.25$  is on the positive side.
- $x$  for the left side:
  - $x = (-0.25 * 14.5) + 65.5 = 61.9$
- $x$  for the right side:
  - $x = (0.25 * 14.5) + 65.5 = 69.1$
- 61.9 and 69.1 mark the middle 20% of the data.

