

Chapter 2 Statistical Learning

Statistical Learning

2.1.1 Why estimate F?

1. To make prediction
2. To understand the relationship:
sometimes, we care less about prediction and more about insight
-How does the salary depend on the experience?
-Which feature matter most?
Estimate the F help explain how X affect Y
3. Because the noise exist
Real world data is messy even with the same X, output vary. So, instead of learning noise, we aim for a function that capture the pattern, not randomness.

How can we estimate the F?

Step 1:

Choose a form of F (2 Approaches)

-Parametric method

Assume a specific shape for F

Ex: Linear regression

$$Y = B_0 + B_1X$$

Pros:

-Simple - fast - easy to interpret

Cons:

-can be too rigid (simple) (Underfitting)

-Non parametric method

do not assume a fixed form

Ex: kNN, neural network, Decision tree, Random forest

Pros:

-flexible - can model complex pattern

Cons:

-need more data - can overfitting

Step 2:

Define a loss function:

This measures how wrong our estimate is

Ex: -Mean Square Error (regression) - Cross-entropy loss (Classification)

Step 3: (Learn from data)

We find \hat{F} that minimize the loss

This can be done by gradient Descent and Closed-form Solution

2.2 Assessing Model Accuracy

2.2.1 Assessing the model accuracy

A model that works only on training data is useless

we care about: how well the model performs on unseen data.

→ Always evaluate models using *test error*, not *training error*

2.2.2 Bias-Variance trade-off

Why test error exists?

Even the best model makes mistakes because:

- Wrong assumption (Bias)
- Sensitivity to data (Variances)
- noise (irreducible error)

Bias = Error caused by oversimplifying the real relationship.

characteristic:

- Model too rigid
- Misses important pattern
- lead to underfitting

Ex: using a straight line to fit a curve pattern.

Variance = error caused by model sensitivity to training data.

Characteristic:

- Model changes a lot with small data change
- fits noise
- lead to overfitting

Irreducible error:

- Error caused by noise in data
- cannot be eliminated
- sets a lower bound on test error

Mathematical composition

Expected test MSE = $\text{Bias}^2 + \text{Variance} + \text{Irreducible error}$

2.2.3 Classification setting

Classification error rate

Error rate = incorrect prediction / Total prediction

Model sensitivity in classification

- +Simple classifier
- High bias
- Low variance
- +Complex classifier
- low bias
- High variance

Ex:

- K-NN with large K → Underfitting
- k-NN with small K → Overfitting