Baby Rudin 3rd Ed

November 25, 2013

Chapter 1

1.1

(+): Take $r = \frac{a}{b}$ so $r + x = \frac{a}{b} + x = \frac{y}{z} \implies x = \frac{y}{z} - \frac{a}{b} = \frac{by-az}{bz}$ and x must be rational
(*): Take $r = \frac{a}{b}$ so $rx = \frac{xa}{b} = \frac{y}{z} \implies x = \frac{by}{az}$ and x must be rational

1.2

Notice this problem reduces to proving the irrationality of $\sqrt{3}$
Take a, b to be coprime integers, $b \neq 0 \implies \frac{a^2}{b^2} = 3 \implies a^2 = 3b^2$ a, b must be odd, since they would not be coprime otherwise. Now take $a = 2i + 1; b = 2n + 1; i \neq n \implies (2i + 1)^2 = 3(2n + 1)^2 \implies 4i^2 + 4i + 1 = 12n^2 + 12n + 3 \implies 2(i^2 + i) = 2(3n^2 + 3n) + 1$ Since the i terms describe an even integer, and the n terms describe an odd integer, it is impossible to choose i, n such that $a^2 = 3b^2$

1.3

(a) $xy = xz \implies y = 1 * y = (x^{-1}x)y = (x^{-1})xy = (x^{-1})xz = (x^{-1}x)z = 1 * z = z$
(b) As above, take $z = 1$
(c) As above, take $z = x^{-1}$
(d) From (c), $x^{-1}x = 1 \implies x = (x^{-1})^{-1}$

1.4

$\forall x \in E,\ \alpha \leq x \leq \beta \implies \alpha \leq \beta$

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