ETM 540 Week 2 Homework

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Problem 3.1

Ok, we're making an explicit model with 16 choice variables, one for each combination of source and destination. Note that this is a supply constrained model since we have more capacity than supply, therefore we'll want to ship our entire supply, and we'll be optimizing for the minimum cost to ship that entire supply. Here is our table for easy reference:

Node	PDX	SEA	MSP	ATL	**Supply**
Chicago	20	21	8	12	**500**
Beaverton	6	7	18	24	**500**
Eugene	8	10	22	28	**500**
Dallas	16	26	15	5	**600**
Capacity	**700**	**500**	**500**	**600**	

a (Formulation)

Our 16 choice variables will be named in the format recommended in the book, so that XCP refers to the path from Chicago to Portland. How fortunate that all of these locations start with different letters!

- XCP = Amount to ship from Chicago to PDX
- XCS = Amount to ship from Chicago to SEA
- XCM = Amount to ship from Chicago to MSP
- XCA = Amount to ship from Chicago to ATL
- XBP = Amount to ship from Beaverton to PDX
- XBS = Amount to ship from Beaverton to SEA
- XBM = Amount to ship from Beaverton to MSP
- XBA = Amount to ship from Beaverton to ATL
- XEP = Amount to ship from Eugene to PDX
- XES = Amount to ship from Eugene to SEA
- XEM = Amount to ship from Eugene to MSP
- XEA = Amount to ship from Eugene to ATL
- XDP = Amount to ship from Dallas to PDX
- XDS = Amount to ship from Dallas to SEA
- XDM = Amount to ship from Dallas to MSP
- XDA = Amount to ship from Dallas to ATL

Now let's formulate the LP:

$$
\begin{aligned}
\text{Minimize } & 20*XCP+21*XCS+8*XCM+12*XCA+6*XBP+7*XBS+18*XBM+24*XBA+ \\
& 8*XEP+10*XES+22*XEM+28*XEA+16*XDP+26*XDS+15*XDM+5*XDA \\
\text{subject to } & XCP+XCS+XCM+XCA=500 \\
& XBP+XBS+XBM+XBA=500 \\
& XEP+XES+XEM+XEA=500 \\
& XDP+XDS+XDM+XDA=600 \\
& XCP+XBP+XEP+XDP\leq 700 \\
& XCS+XBS+XES+XDS\leq 500 \\
& XCM+XBM+XEM+XDM\leq 500 \\
& XCA+XBA+XEA+XDA\leq 600 \\
& XCP,XCS,XCM,XCA,XBS,XBM,XBA,XEP,XES,XEM,XEA, \\
& XDP,XDS,XDM,XDA\geq 0
\end{aligned}
$$

b (Implementation)

Let's implement the model we've described above as in HW1:

```r
transmodel <- MIPModel() %>%
  add_variable(XCP, type = "continuous", lb = 0) %>% #decision variables are amount to ship from each s
  add_variable(XCS, type = "continuous",lb = 0) %>%  #to each destination
  add_variable(XCM, type = "continuous", lb = 0) %>%
  add_variable(XCA, type = "continuous", lb = 0) %>%
  add_variable(XBP, type = "continuous", lb = 0) %>%
  add_variable(XBS, type = "continuous",lb = 0) %>%
  add_variable(XBM, type = "continuous", lb = 0) %>%
  add_variable(XBA, type = "continuous", lb = 0) %>%
  add_variable(XEP, type = "continuous", lb = 0) %>%
  add_variable(XES, type = "continuous",lb = 0) %>%
  add_variable(XEM, type = "continuous", lb = 0) %>%
  add_variable(XEA, type = "continuous", lb = 0) %>%
  add_variable(XDP, type = "continuous", lb = 0) %>%
  add_variable(XDS, type = "continuous",lb = 0) %>%
  add_variable(XDM, type = "continuous", lb = 0) %>%
  add_variable(XDA, type = "continuous", lb = 0) %>%

  set_objective(20*XCP+21*XCS+8*XCM+12*XCA+6*XBP+7*XBS+18*XBM+24*XBA+
                8*XEP+10*XES+22*XEM+28*XEA+16*XDP+26*XDS+15*XDM+5*XDA, "min") %>% #minimize cost

  add_constraint(XCP+XCS+XCM+XCA == 500) %>% #ship Chicago supply
  add_constraint(XBP+XBS+XBM+XBA == 500) %>% #ship Beaverton supply
  add_constraint(XEP+XES+XEM+XEA == 500) %>% #ship Eugene supply
  add_constraint(XDP+XDS+XDM+XDA == 600) %>% #ship Dallas supply
  add_constraint(XCP+XBP+XEP+XDP <= 700) %>% #Portland capacity
  add_constraint(XCS+XBS+XES+XDS <= 500) %>% #Seattle capacity
  add_constraint(XCM+XBM+XEM+XDM <= 500) %>% #Beaverton capacity
  add_constraint(XCA+XBA+XEA+XDA <= 600) %>% #Dallas capacity


  solve_model(with_ROI(solver = "glpk"))
```

c (Solution and discussion)

Let's examine the outputs from solving our transportation model, starting with the solver status:

```
print(solver_status(transmodel))
```

```
## [1] "optimal"
```

Looks like the solver has reached an optimum solution. How much stuff should we ship?

```
pander(transmodel$solution)
```

Table 2: Table continues below

XBA	XBM	XBP	XBS	XCA	XCM	XCP	XCS	XDA	XDM	XDP	XDS	XEA
0	0	200	300	0	500	0	0	600	0	0	0	0

XEM	XEP	XES
0	500	0

This looks like a valid solution. Each warehouse has shipped its full supply (for example, Beaverton has a supply of 500 and has shipped 200 to Portland and 300 to Seattle), and no distributor has received more than its capacity (for example Portland is full having received 500 goods from Eugene and 200 from Beaverton). How much did it cost to ship all these?

```
print(objective_value(transmodel))
```

```
## [1] 14300
```

Just as a sanity check we were shipping 2100 total goods at a cost of around 7 each, so this looks like a reasonable number.

Problem 3.2

Let's do this again, the easy way.

a (Formulation)

We have NW warehouses and ND distributors. The i-th warehouse has S_i supply and the j-th distributor has D_j capacity. For each path between a warehouse and a distributor, we have $C_{i,j}$, the cost of shipping one unit of product along that path. We want to ship our entire supply while minimizing total cost. We'll write the model for the supply-constrained case. We are deciding $x_{i,j}$, how many units of product to ship along each path. The formal models is as follows:

$$\begin{aligned}
\text{Minimize the total shipment cost } & \sum_{i=1}^{NW} \sum_{j=1}^{ND} C_{i,j} x_i \\
\text{subject to shipping all goods } & \sum_{j=1}^{ND} x_{i,j} = S_i \ \forall \ i \\
\text{and not overflowing distributors } & \sum_{i=1}^{NW} x_{i,j} \leq D_j \ \forall \ j \\
\text{and not shipping negative stuff } & x_{i,j} \geq 0 \ \forall \ i, j
\end{aligned}$$

Isn't that nicer?

b (Implementation)

Let's write that in code:

```
NW<- 4
ND<- 4
PathNames<-list(c("XCP", "XCS", "XCM", "XCA", "XBS", "XBM", "XBA", "XEP", "XES", "XEM", "XEA", "XDP", "X

WarehouseNames <- list(c("Chicago", "Beaverton", "Eugene", "Dallas"))


DistributorNames<- list(c("PDX", "SEA", "MSP", "ATL"))


PathCosts <- matrix(c(20, 21, 8, 12, 6, 7, 18, 24, 8, 10, 22, 28, 16, 26, 15, 5),
              ncol=ND,byrow=TRUE,dimnames=c(WarehouseNames,DistributorNames))

Supply<- matrix(c(500, 500, 500, 600),
              ncol=NW,dimnames=c("Supply",WarehouseNames))

Demand <- matrix(c(700, 500, 500, 600),
              ncol=ND,dimnames=c("Demand",DistributorNames))

transmodel2 <- MIPModel() %>%
  add_variable (x[i, j], i=1:NW,j=1:ND, type="continuous", lb=0) %>%
  set_objective (sum_expr(sum_expr(PathCosts[i, j] * x[i, j] , i=1:NW), j=1:ND ), "min") %>% #minimize
  add_constraint (sum_expr(x[i, j], j=1:ND) == Supply[i],  i=1:NW) %>%  # NW constraints on supply
  add_constraint (sum_expr(x[i, j], i=1:NW) <= Demand[j],  j=1:ND) %>%  # ND constraints on demand
  solve_model(with_ROI(solver = "glpk"))

transmodel2
```

```
## Status: optimal
## Objective value: 14300
```

c (Solution and discussion)

We have obtained an optimal solution again, and it looks like total shipping costs remain the same.

Let's check to see if we get the same results for our decision variables x:

```
solution2display<-matrix(transmodel2$solution, ncol=ND, dimnames=(c(WarehouseNames,DistributorNames)))#
solution2display<-rbind(solution2display, Demand) ##add demand for checking
solution2display<-cbind(solution2display, c(Supply, "-")) ##add supply for checking
dimnames(solution2display)[[2]][5]<-"Supply" #fixing a name
pander(solution2display)
```

	PDX	SEA	MSP	ATL	Supply
Chicago	0	0	500	0	500
Beaverton	200	300	0	0	500
Eugene	500	0	0	0	500
Dallas	0	0	0	600	600
Demand	700	500	500	600	-

This does look like the same solution as above. Note that we have shipped the full supply from each warehouse adn that each distributor is not exceeding its capacity. Looks like Portland filled all the way up and Seattle didn't because it's cheaper to ship from Beaverton to Portland, but Beaverton didn't ship everything to Portland since it's even cheaper to ship from Eugene to Portland.