$$\left(\frac{\partial A}{\partial B}\right)_C$$

$$A(B, C)$$

$$A(u, v) \qquad u(x, y) \quad v(x, y)$$

$$\left(\frac{\partial A}{\partial x}\right)_y$$

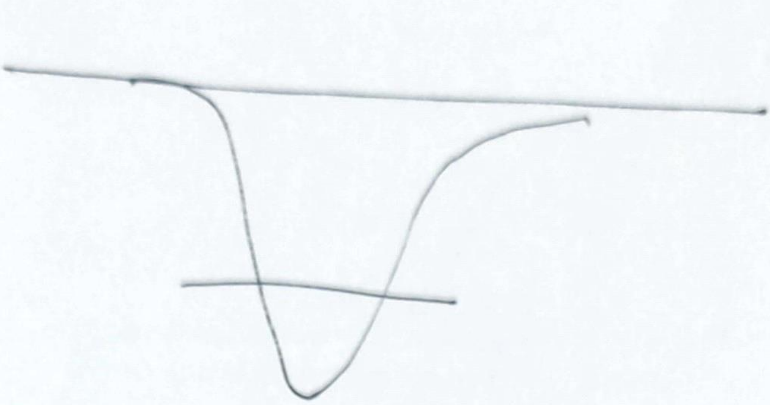
Jacobean $$\frac{D(A, B)}{D(x, y)} = \det \begin{Vmatrix} \frac{\partial A}{\partial x} & \frac{\partial A}{\partial y} \\ \frac{\partial B}{\partial x} & \frac{\partial B}{\partial y} \end{Vmatrix}$$

$$\left(\frac{\partial A}{\partial x}\right)_y = \frac{D(A, y)}{D(x, y)} = \begin{pmatrix} \frac{\partial A}{\partial x} & 0 \\ \frac{\partial A}{\partial y} & 1 \end{pmatrix} = \frac{\partial A}{\partial x}$$

$$\frac{D(A, B)}{D(x, y)} = -\frac{D(A, B)}{D(y, x)}$$

$$\frac{D(A, B)}{D(x, y)} = \frac{D(A, B)}{D(u, v)} \frac{D(u, v)}{D(x, y)}$$

$$\frac{D(A, B)}{D(x, y)} = \left[\frac{D(x, y)}{D(A, B)}\right]^{-1}$$



$\psi(x)$ $\qquad H = \dfrac{p^2}{2m} + U(x) = -\dfrac{\hbar^2}{2m}\dfrac{d^2}{dx^2} + U(x)$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi = -|E|\,\psi(x)$$

$$\psi(x) = \int \frac{dp}{2\pi\hbar}\, \phi(p)\, e^{ipx/\hbar}$$

$$\phi(p) = \int dx\, \psi(x)\, e^{-ipx/\hbar}$$

$$\frac{p^2}{2m}\phi(p) + (\hat{U}\psi)_p = -|E|\,\phi(p)$$

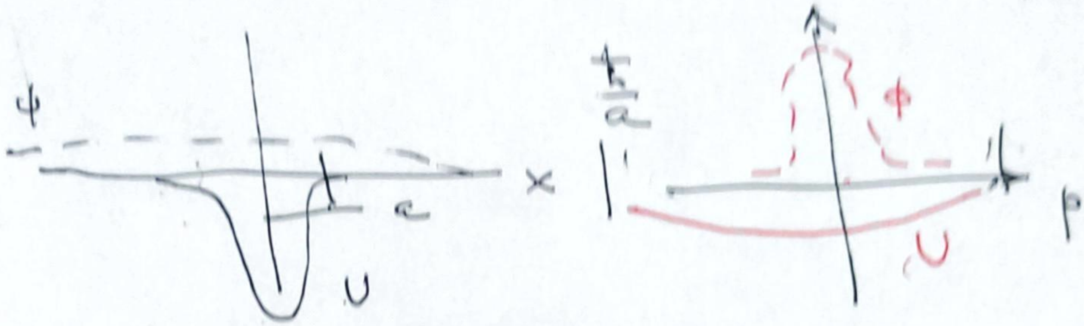
$$U(x) = \int U(p)\, e^{ipx/\hbar}\, \frac{dp}{2\pi\hbar}$$

$$\int dx\, e^{-ipx/\hbar}\, U(x)\psi(x) = \int \frac{dp_1}{2\pi\hbar}\int \frac{dp_2}{2\pi\hbar}\, e^{ip_1x/\hbar}\, e^{ip_2x/\hbar}\, U(p_1)\phi(p_2)$$

$$\exp\left[\frac{ix}{\hbar}(p_1 + p_2 - p)\right] \to \delta(p_1 - p + p_2)$$

$$(U\psi)_p = \int \frac{dp_2}{2\pi\hbar}\, U(p_2 - p)\, \phi(p_2)$$

$$\left(\frac{p^2}{2m}+|E|\right)\phi(p) = -\int\frac{dp_2}{2\pi\hbar}\,\phi(p_2)\,U(p-p_2)$$



$$\left(\frac{p^2}{2m}+|E|\right)\phi(p) \cong -U(0)\int\frac{dp_2}{2\pi\hbar}\,\phi(p_2)$$

$$\phi(p) = \frac{|U(0)|}{\frac{p^2}{2m}+|E|}\int\frac{dp_2}{2\pi\hbar}\,\phi(p_2)$$

$$\cancel{\int\frac{dp}{2\pi\hbar}\,\phi(p)} = |U(0)|\cancel{\int\frac{dp_2}{2\pi\hbar}\,\phi(p_2)}\int_{-\infty}^{+\infty}\frac{dp}{2\pi\hbar}\,\frac{2m}{p^2+2m|E|}$$

$$\boxed{1 = \frac{m|U(0)|}{\pi\hbar}\int_{-\infty}^{+\infty}\frac{dp}{p^2+2m|E|}} =$$

$$= \frac{m|U(0)|}{\cancel{\pi}\hbar}\,\frac{\cancel{\pi}}{\sqrt{2m|E|}} = \sqrt{\frac{m}{2\hbar^2}}\,\frac{|U(0)|}{\sqrt{|E|}}$$

$$|E| = -E = \frac{m}{2\hbar^2}\left(U(0)\right)^2 =$$

$$= \frac{m}{2\hbar^2}\left[\int_{-\infty}^{+\infty}U(x)\,dx\right]^2$$

$d=2$

$$1 = + \frac{|U(0)|}{(2\pi\hbar)^2} \int \frac{2m}{p^2+|E|2m} d^2p =$$

$$= + \frac{2m\,|U(0)|}{(2\pi\hbar)^2} \int_0^{\hbar/a} \frac{2\pi p\,dp}{p^2+|E|2m} =$$

$$+ \frac{\cancel{2\pi} m}{2\cancel{4\pi^2}\hbar^2} |U(0)| \int_0^{(\hbar/a)^2} \frac{d(p^2)}{p^2+|E|2m} =$$

$$= \frac{m}{2\pi\hbar^2} |U(0)| \ln \frac{\hbar^2}{a^2\,2m|E|}$$

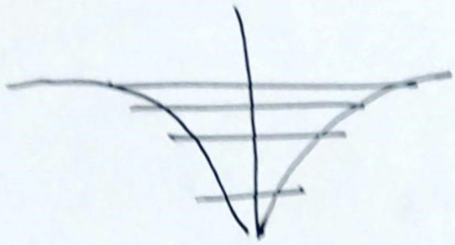
$$\ln \frac{\hbar^2}{2ma^2} \frac{1}{|E|} = \frac{2\pi\hbar^2}{m} \frac{1}{|U(0)|}$$

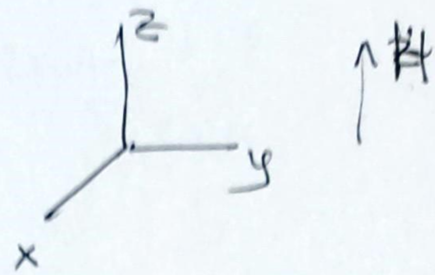
$$\frac{\hbar^2}{2ma^2} \frac{1}{|E|} = \exp\left[\frac{2\pi\hbar^2}{m|U(0)|}\right].$$

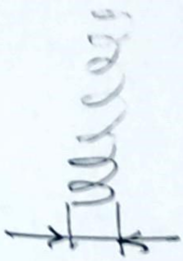
$$|E| = \frac{\hbar^2}{2ma^2} \exp\left[-\frac{2\pi\hbar^2}{m|U(0)|}\right]$$

$d=3$

$$\int_0^{\hbar/a} \frac{d^3p}{p^2+2m|E|}$$



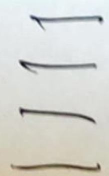




$x_H = \sqrt{\frac{c\hbar}{eH}}$

$E_n = \hbar\omega_c(n+1/2)$

$\omega_c = \frac{eH}{mc}$



$Ry = \frac{me^4}{\chi^2\hbar^2}$

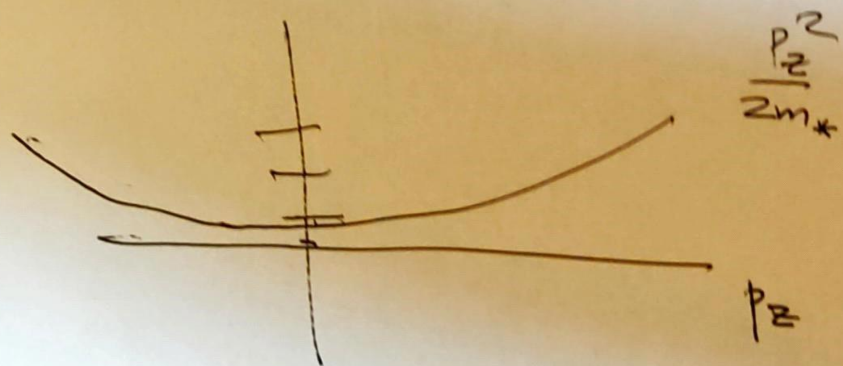
$m_* \ll m_e$

$\chi = 16$

GaAs

$\chi_{Ge} = 12.$

$\hbar\omega_c \gg Ry$



$\epsilon = \frac{\hbar\omega_c}{2} - \frac{me^4}{2\hbar^2}\ln^2.$

$\frac{p_z^2}{2m} \to -\frac{\hbar^2}{2m}\frac{d^2}{dz^2} - \frac{e^2}{|z|}$

$-\frac{e^2}{2m}\int dz|\psi(z)|^2\frac{e^2}{|z|}$