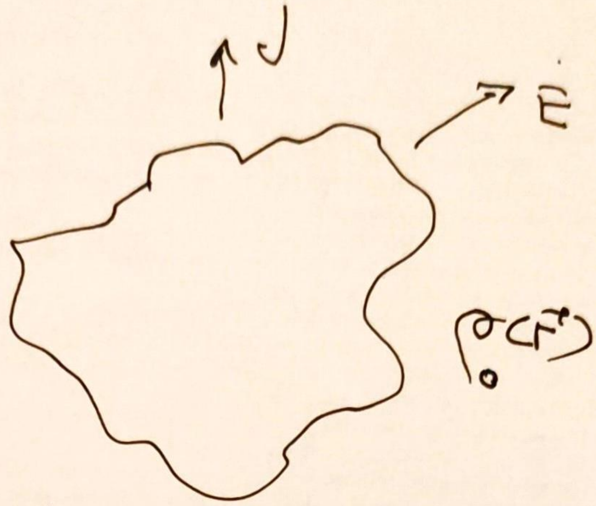
$\vec{j} = \sigma \vec{E}$

$\operatorname{div} E = 4\pi\rho$

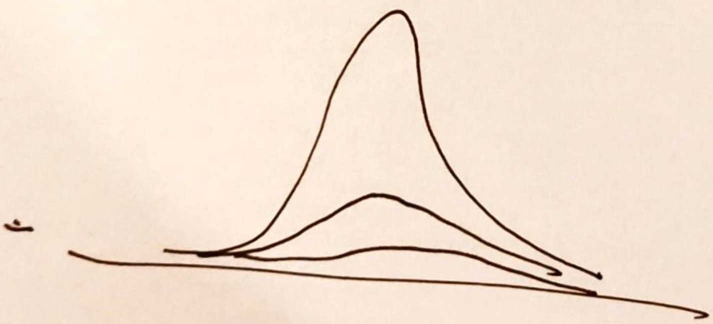


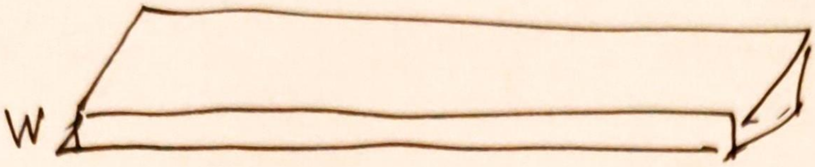
$$\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{j} = 0$$

$$\operatorname{div} \sigma \vec{E} = \sigma \operatorname{div} E = 4\pi\sigma\rho$$

$$\boxed{\frac{\partial \rho}{\partial t} = - 4\pi\sigma\rho} \qquad \rho(t) = \rho_0 e^{-4\pi\sigma t}$$

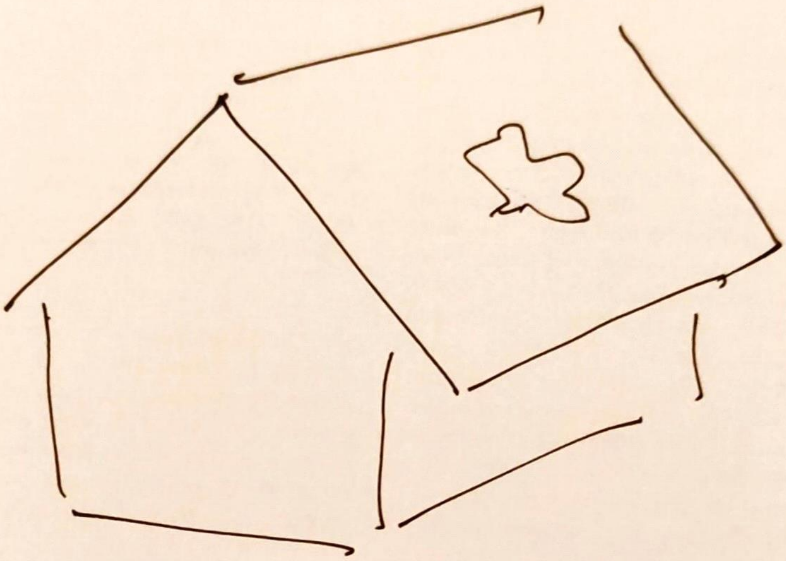
$[\sigma] = \frac{1}{sec.}$ $\qquad \frac{\sigma}{\epsilon_0}$

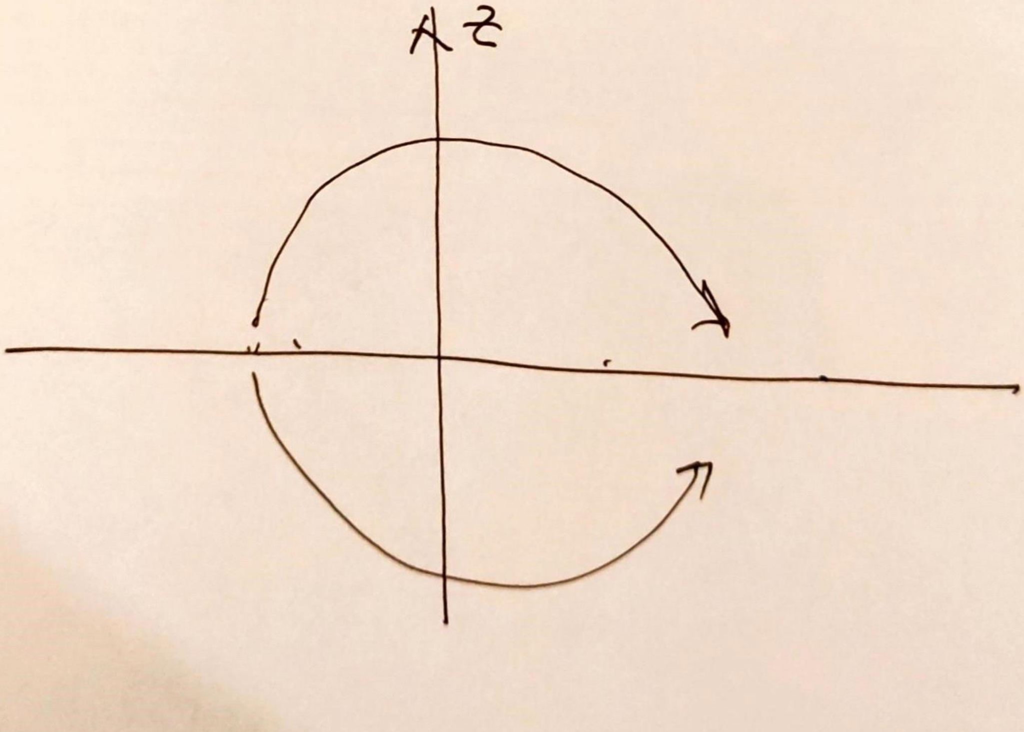




$$\vec{j}_2 = \sigma_2 \vec{E}$$

$$\sigma_2 = \sigma w \qquad [\sigma_2] = \frac{cm}{sec}$$





$$\nabla^2\phi(\vec{r},z) = -4\pi\rho(\vec{r},z) = -4\pi\rho_2(r)\delta(z)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \left(\frac{\partial}{\partial \vec{r}}\right)^2 + \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial}{\partial \vec{r}}\frac{\partial}{\partial \vec{r}}\phi + \frac{\partial^2\phi}{\partial z^2} = -4\pi\rho_2(r)\delta(z)$$

$$\phi(r,z) = \int \frac{d\vec{k}}{(2\pi)^2} e^{i\vec{k}\cdot\vec{r}}\phi(\vec{k},z)$$ ← Key idea

The Fourier Transform.

$\vec{r} \rightarrow \vec{k}$

$$\frac{\partial^2}{\partial r^2} = -k^2$$

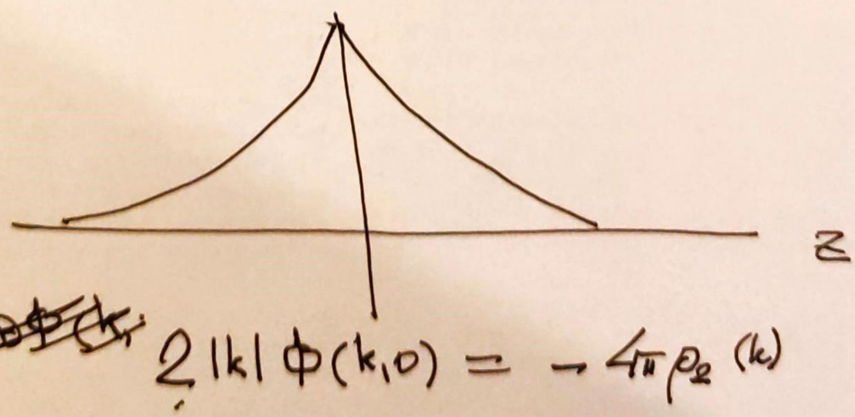
$$\underbrace{\left(-k^2 + \frac{\partial^2}{\partial z^2}\right)\phi(k,z) = -4\pi\rho_2(k)\delta(z).}$$

Green's Fn Problem.

$z \neq 0 \qquad \phi(k,z) = Ae^{|k|z} + Be^{-|k|z}$

Turns a PDE into an ODE

$$\boxed{\phi(k,z) = \phi(k,0)\, e^{-|k||z|}}$$



$$\int_{-\epsilon}^{+\epsilon} dz + \cancel{\partial\phi(k,}\ 2|k|\phi(k,0) = -4\pi\rho_2(k)$$

$$\boxed{\phi(k,0) = -\frac{2\pi}{|k|}\rho_2(k)}$$

$$\boxed{\phi(k,z) = -\frac{2\pi}{|k|} e^{-|k||z|}}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{j} = 0.$$

Solve it in k-space first.

$$\frac{\partial \rho(k)}{\partial t} + i\vec{k}\,\sigma_2 \vec{E} = 0$$

$$\vec{E} = -\nabla\phi = -i\vec{k}\,\phi(k,0) = i\vec{k}\,\frac{2\pi}{|k|}\rho_2(k)$$

$$\rho_2(k,t) = \rho_2(k,0)\, e^{-\frac{2\pi k^2}{|k|}\sigma_2 t}$$

$$\boxed{\rho_2(k,t) = \rho_2(k,0)\, e^{-2\pi k \sigma_2 t}}$$

$$\rho_2(r,t) = \int \rho_2(k,0)\, e^{-2\pi k\sigma_2 t + i\vec{k}\vec{r}} \frac{dk}{2\pi}$$

$$\int \frac{dk\, dr'}{2\pi} \rho_2(r',0)\, e^{-2\pi k\sigma_2 t + i\vec{k}(\vec{r}-\vec{r}')}$$

$$\rho_2(r,t) = \int K(r-r',t)\,\rho_2(r',0)\, dr'$$

$$K(\underline{r}-r', t) =$$

$$= \int \frac{dk}{2\pi} e^{-2\pi\sigma_2 k t + i k \vec{r}}$$

$$2\pi\sigma_2 t \cdot \vec{k} = \vec{\xi} \qquad k = \frac{\xi}{2\pi\sigma_2 t}$$

$$\int \frac{d\vec{\xi}}{2\pi\sigma_2 t} e^{-\xi + i \frac{\vec{\xi}\cdot\vec{r}}{2\pi\sigma_2 t}}$$

$$\tilde{K}\left(\frac{r - r'}{2\pi\sigma_2 t} \right)$$