Electrostatics

$\text{curl}\, \vec{E} = 0 \qquad \text{div}\, \vec{E} = 4\pi\rho$

$\hookrightarrow \qquad \vec{E} = -\vec{\nabla}\phi \qquad \phi = \phi' + \alpha$

$\alpha = \text{const}$

$\text{div}(\nabla\phi) = \nabla^2\phi = -4\pi\rho$

$$\phi(\vec{r}) = \int d\vec{r}_1 \frac{\rho(\vec{r}_1)}{|\vec{r} - \vec{r}_1|}$$

$$\vec{E} = \int d\vec{r}_1 \frac{\rho(\vec{r}_1)(\vec{r} - \vec{r}_1)}{|r - r_1|^3}$$

Magnetostatics

$\text{div}\, \vec{H} = 0, \quad \text{curl}\, \vec{H} = \frac{4\pi}{c}\vec{j}$

$\vec{H} = \text{curl}\, \vec{A} \longrightarrow A' = A + \nabla f$

$\text{curl}(\text{curl}\, \vec{A}) = \frac{4\pi}{c}\vec{j}$

$[\vec{\nabla} \times [\vec{\nabla} \times \vec{A}]] = \qquad [a \times [b \times c]] = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(a \cdot \vec{b})$

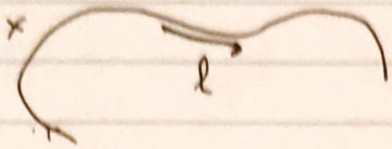
$= \vec{\nabla}(\text{div}\, \vec{A}) - \nabla^2\vec{A} \qquad$ quage $\text{div}\, \vec{A} = 0$

$\nabla^2\vec{A} = -\frac{4\pi}{c}\vec{j}$

$$A(\vec{r}) = \frac{1}{c}\int d\vec{r}_1 \frac{\vec{j}(\vec{r}_1)}{|\vec{r} - \vec{r}_1|}$$

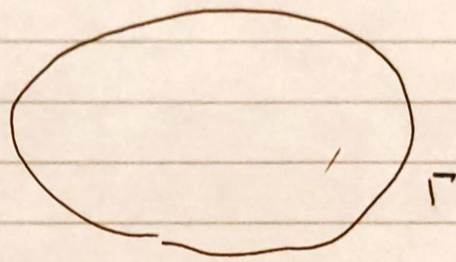
$$\vec{H}(r) = \frac{1}{c}\int d\vec{r}_1 \frac{[\vec{j}(\vec{r}_1) \times (\vec{r} - \vec{r}_1)]}{|\vec{r} - \vec{r}_1|^3}$$

$|\vec{l}|=1 \qquad \vec{r}_1(x)$



$\vec{j} = I\vec{l}\,j(x)\,\delta(\vec{r}_\perp)$

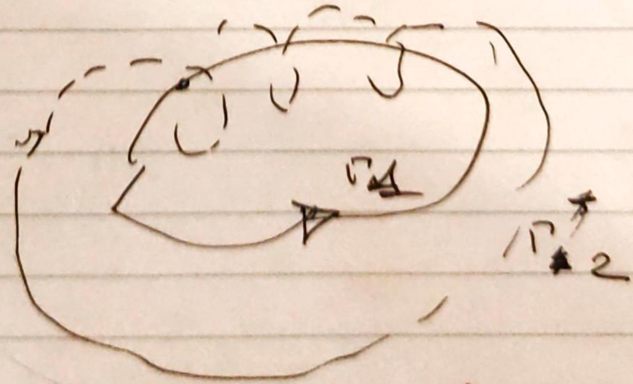
$$H(\vec{r}) = \frac{I}{c}\int_C dx_1 \frac{[\vec{l}(x_1)\times(\vec{r}(x_1)-\vec{r})]}{|\vec{r}-\vec{r}(x_1)|^3}$$



$$\vec{H}(r) = \frac{I}{c}\oint_\Gamma dx_1 \frac{[\vec{l}(x_1)\times(\vec{r}(x_1)-\vec{r})]}{|\vec{r}-\vec{r}(x_1)|^3} =$$

$dr(x_1) = l(x_1)dx_1$

$$= \frac{I}{c}\oint \frac{[d\vec{r}(x_1)\times(\vec{r}(x_1)-\vec{r})]}{|r-r(x_1)|^3}$$

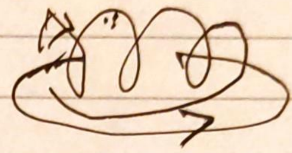


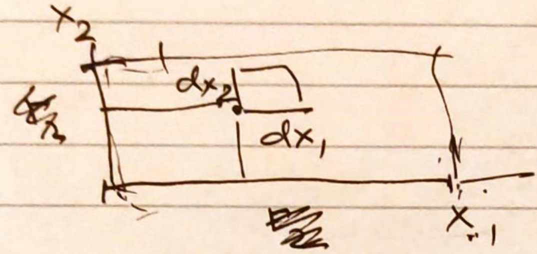
$$\oint_{\Gamma_2} \vec{H}\,d\vec{r}_2 = \int_S \mathrm{curl}\,H \cdot \vec{n}\,dS$$

$$\operatorname{curl} H = \cancel{\frac{4\pi}{c} j} \quad 4\pi I N_{12}$$

$$\boxed{N_{12} = \frac{1}{4\pi} \int_{r_1}\int_{r_2} \frac{[d\vec{r}(x_1) \times d\vec{r}(x_2)]\cdot(r_1 - r_2)}{|r_1 - r_2|^3}}$$

C. F. Gauss

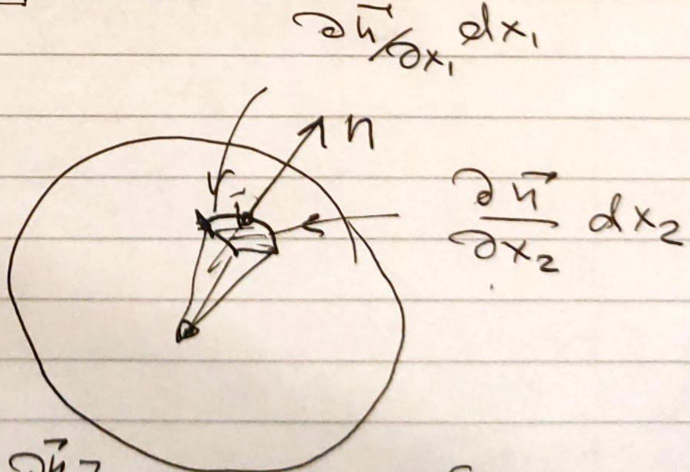




$\vec{r}(x_1)$

$0 < x_1 < 1$

$$\vec{n}_{12} = \frac{\vec{r}(x_1) - \vec{r}(x_2)}{|r(x_1) - r(x_2)|} \qquad \vec{n}_{12}^{\,2} = 1$$



$$d\Omega = \vec{n}\cdot\left[\frac{\partial\vec{n}}{\partial x_1} \times \frac{\partial\vec{n}}{\partial x_2}\right] dx_1 dx_2$$

$$\int_0^1\int_0^1 \vec{n}(x_1,x_2)\left[\frac{\partial\vec{n}}{\partial x_1} \times \frac{\partial\vec{n}}{\partial x_2}\right] dx_1 dx_2 = \int d\Omega =$$

$$= 4\pi N_{12}$$