New topic: recursion.

Background: proofs by induction.

Induction is not quite a "normal" proof.

Normal proof: Goal: $A \Rightarrow E$.

$A \Rightarrow B$ (Axiom/defn)

$B \Rightarrow C$ (SAS Theorem)

$C \Rightarrow D$ (Lemma ..)

$D \Rightarrow E$ (other theorem) ✓

With inductive proofs, usually we are trying to prove a parameterized statement, call it $T(n)$, for $n \in \mathbb{N} = \{1,2,3,\ldots\}$.

E.g. $T(n)$ could be the statement that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

In broad strokes, induction proceeds in 2 phases:

1. Prove $T(n)$ explicitly for small n. (e.g. $n=1$).

   For $T(n)$ as above:

   $$\sum_{i=1}^{1} i = \frac{1(1+1)}{2} = \frac{2}{2} = 1 \checkmark$$

2. Show that $T(n) \Rightarrow T(n+1)$.

For $T(n)$ as above:

Assume $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Then
$$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1)$$
$$= \frac{n(n+1)}{2} + n+1$$
$$= \frac{n(n+1)}{2} + \frac{2(n+1)}{2}$$
$$= \frac{(n+2)(n+1)}{2} \quad \checkmark$$

③ with ① + ②, we can conclude that $T(n)$ is true <u>for all</u> n. Indeed, for any particular value, we've shown that there is a "normal" proof of the following form, with say $n=5$:

$$\underbrace{\underset{1}{T(1)} \xRightarrow[2]{} T(2) \xRightarrow[2]{} T(3) \xRightarrow[2]{} T(4) \xRightarrow[2]{} T(5)}_{3}$$

Recursion: basically it is induction, where

$T(n) \equiv$ "my program/function works on inputs of size n".

Example: a function to compute $n! = \prod_{i=1}^{n} i = 1 \cdot 2 \cdots n$

```
int fac(int n)
{
    if(n==0) return 1;  // (1)
    // now assume fac(n-1) gives
    // the right answer ((n-1)!).
    return fac(n-1) * n;
}
```

What happens when we call fac(3)?