Reflection Models

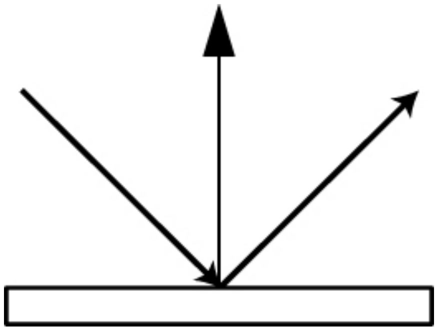
Last lecture

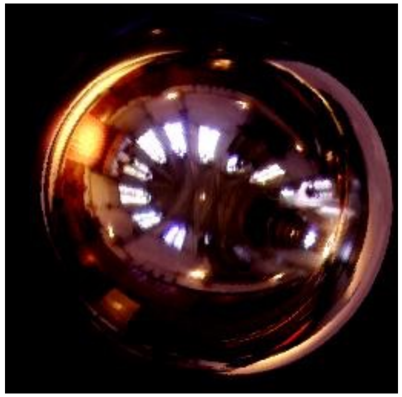
- The reflection equation and the BRDF
- Ideal reflection and refraction
- Diffuse surfaces and Lambert's Law

Today

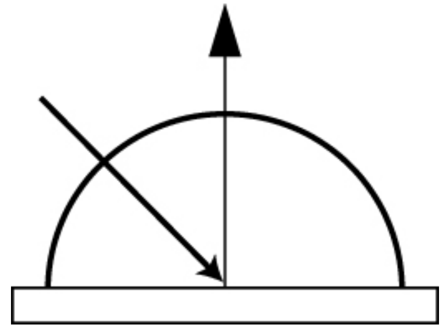
- Phong model
- Microfacet models
- Self-shadowing
- Torrance-Sparrow model

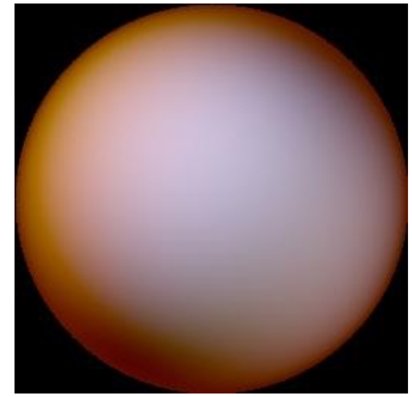
Glossy Surfaces



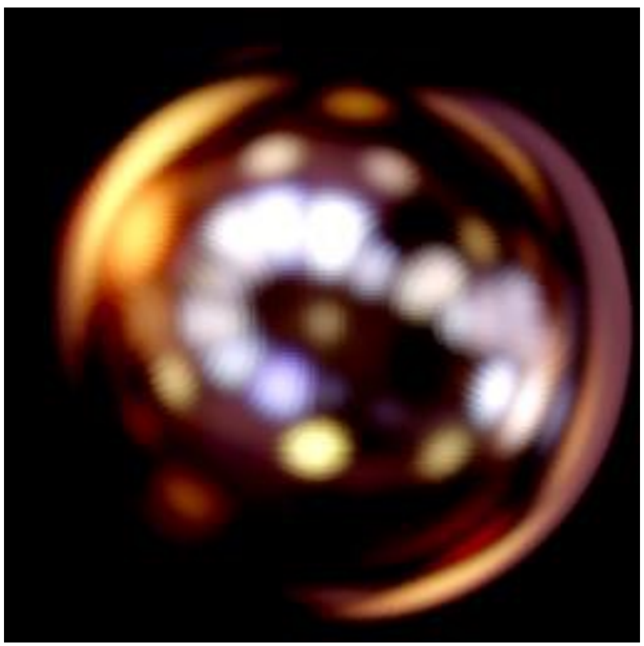


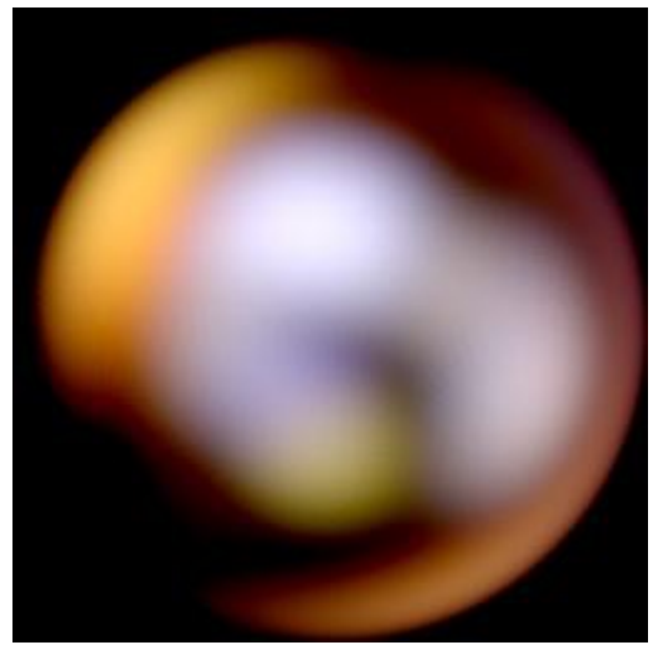
Mirror





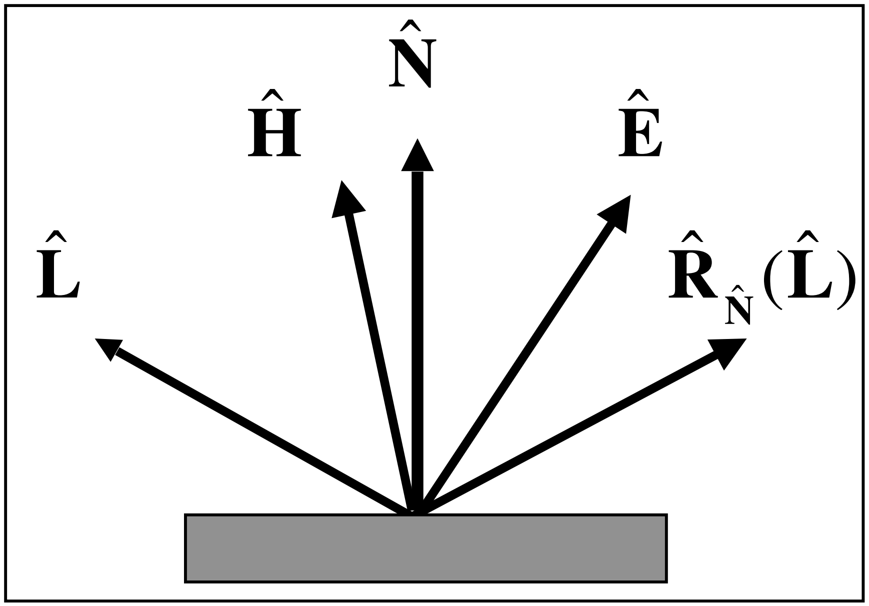
Diffuse

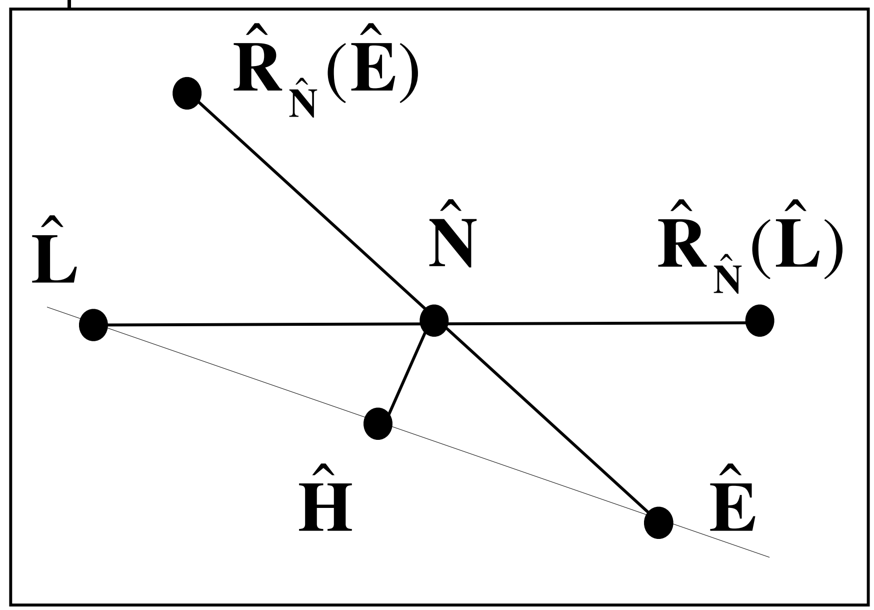




Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$$





$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

$$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$$

$$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$$

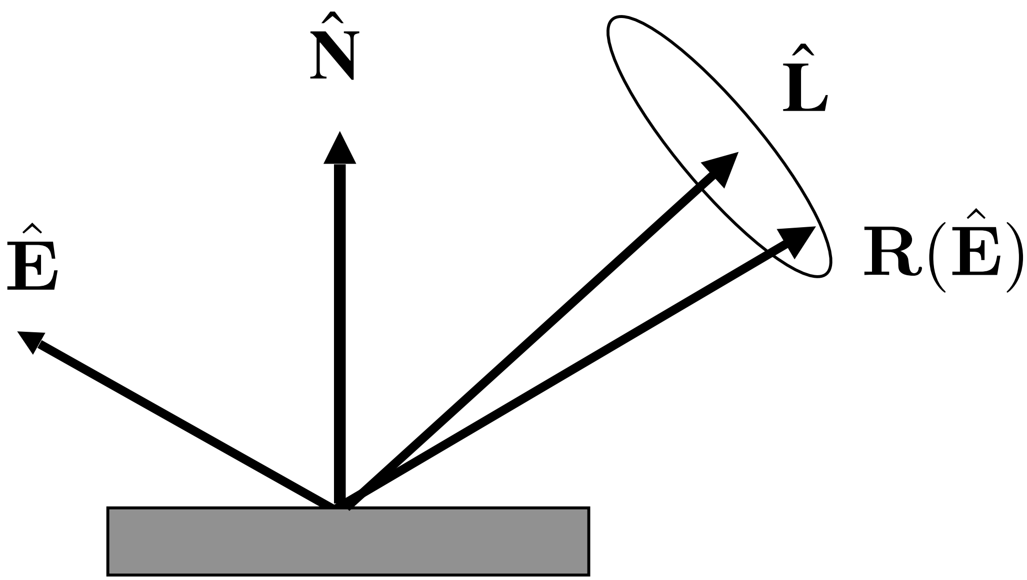
$$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$$

$$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$$

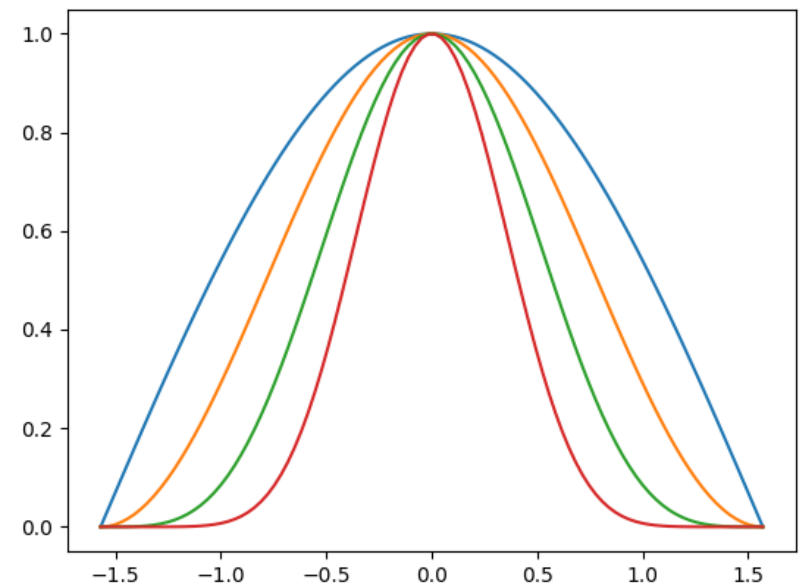
Phong Model

Phong Model

Phong Model is equivalent to a distributed light source



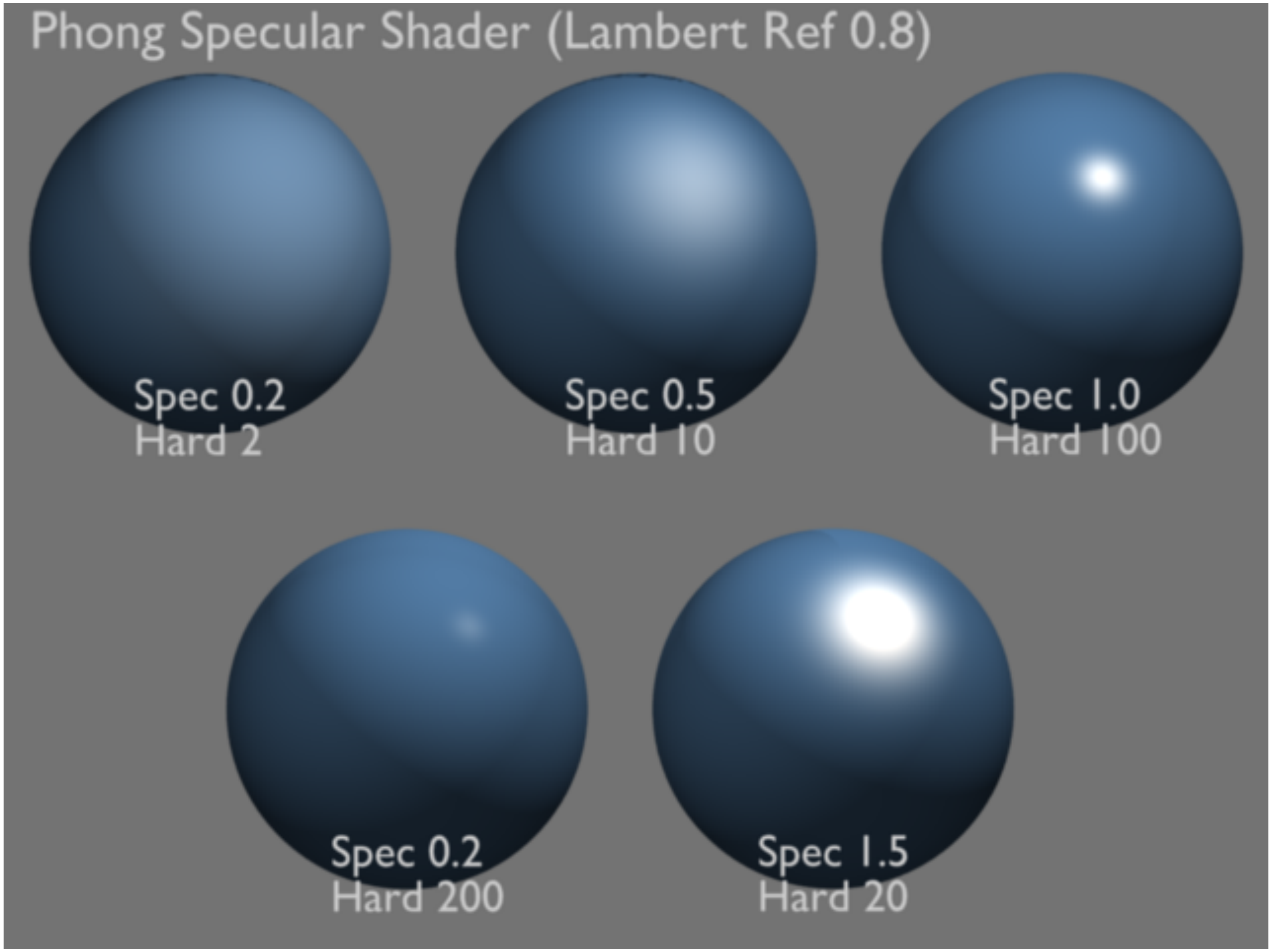
$$(\mathbf{R}(\hat{\mathbf{E}}) \cdot \hat{\mathbf{L}})^s$$



$\mathrm{R}(\hat{\mathrm{E}}) \cdot \hat{\mathrm{L}}$ **measures how far is** $\mathrm{R}(\hat{\mathbf{E}})$ **from the center of the light** $\hat{\mathrm{L}}$

Larger *s*, narrower source

Phong Model



http://wiki.blender.org/uploads/6/6e/Manual-Shaders-Phong.png

Energy Normalization

Energy normalize Phong Model

$$\rho(H^2 \to \omega_r) = \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s \cos\theta_i \, d\omega_i$$

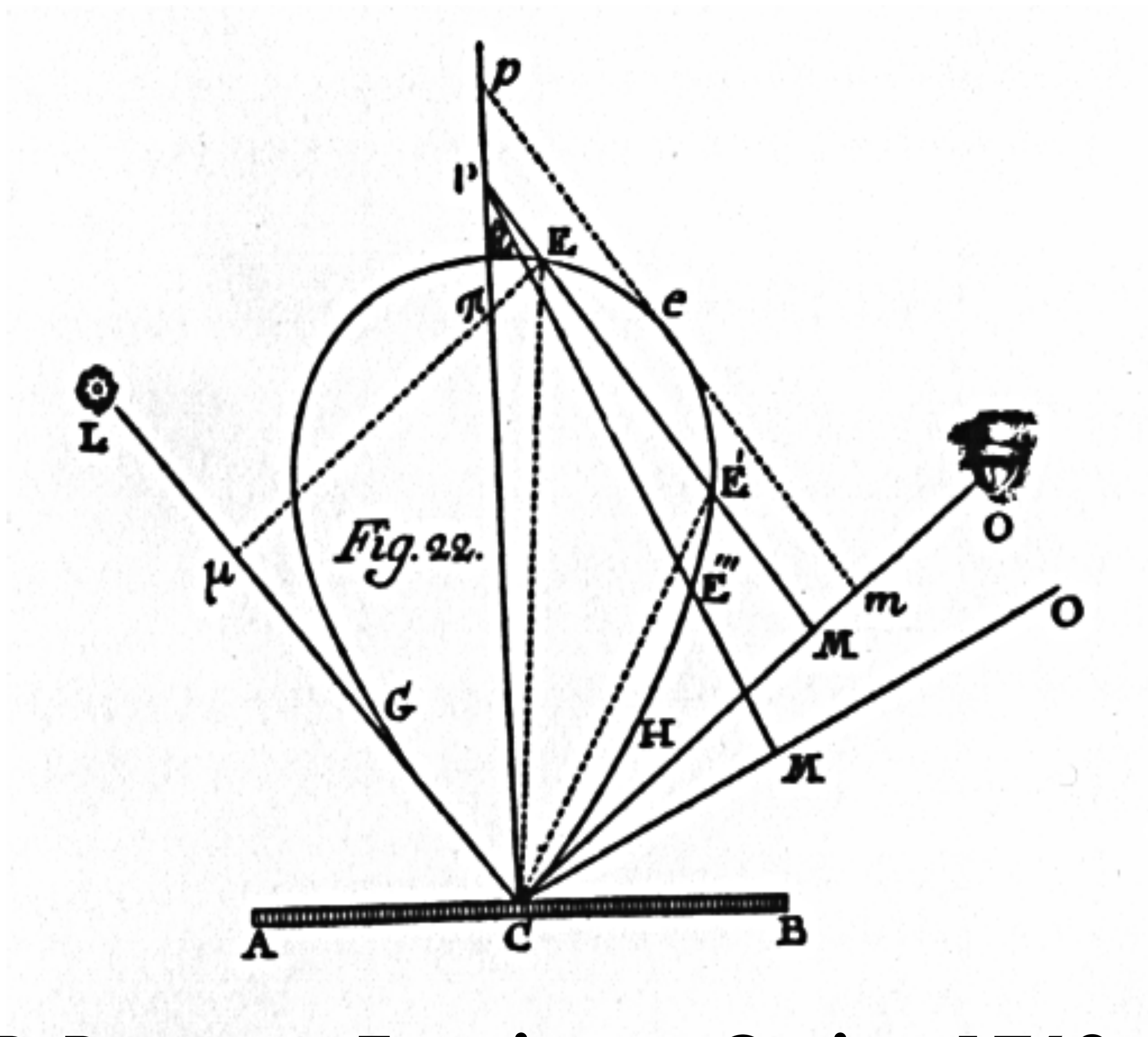
$$\leq \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_i$$

$$\leq \int_{H^2(\mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}))} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_R$$

$$= \int_{H^2} \cos^s\theta \, d\omega = \frac{2\pi}{s+1}$$

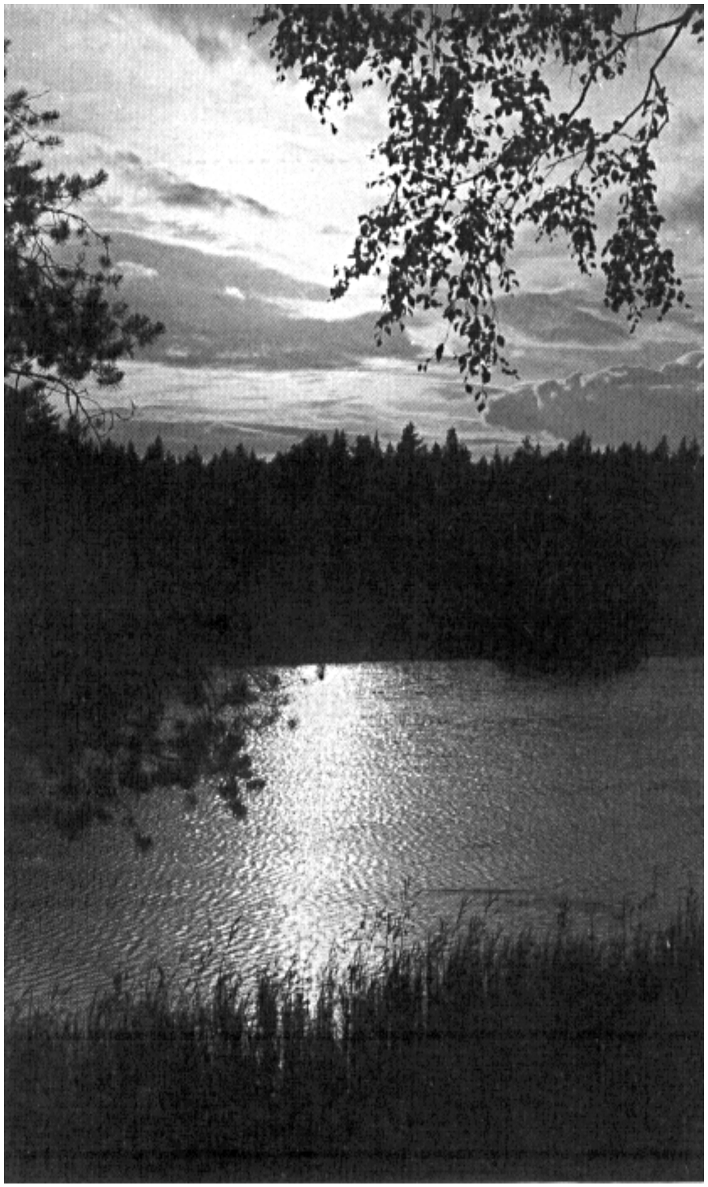
Microfacet Model

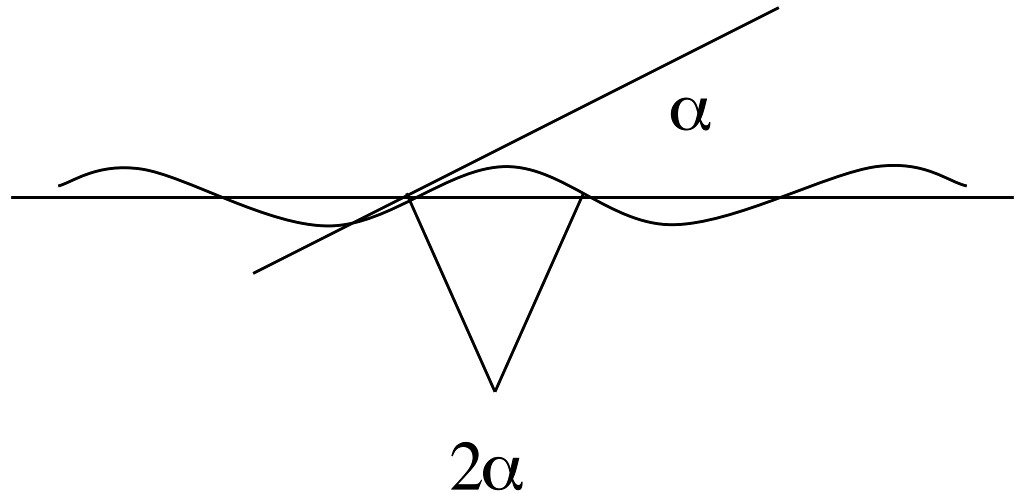
Bouguer's "little faces"



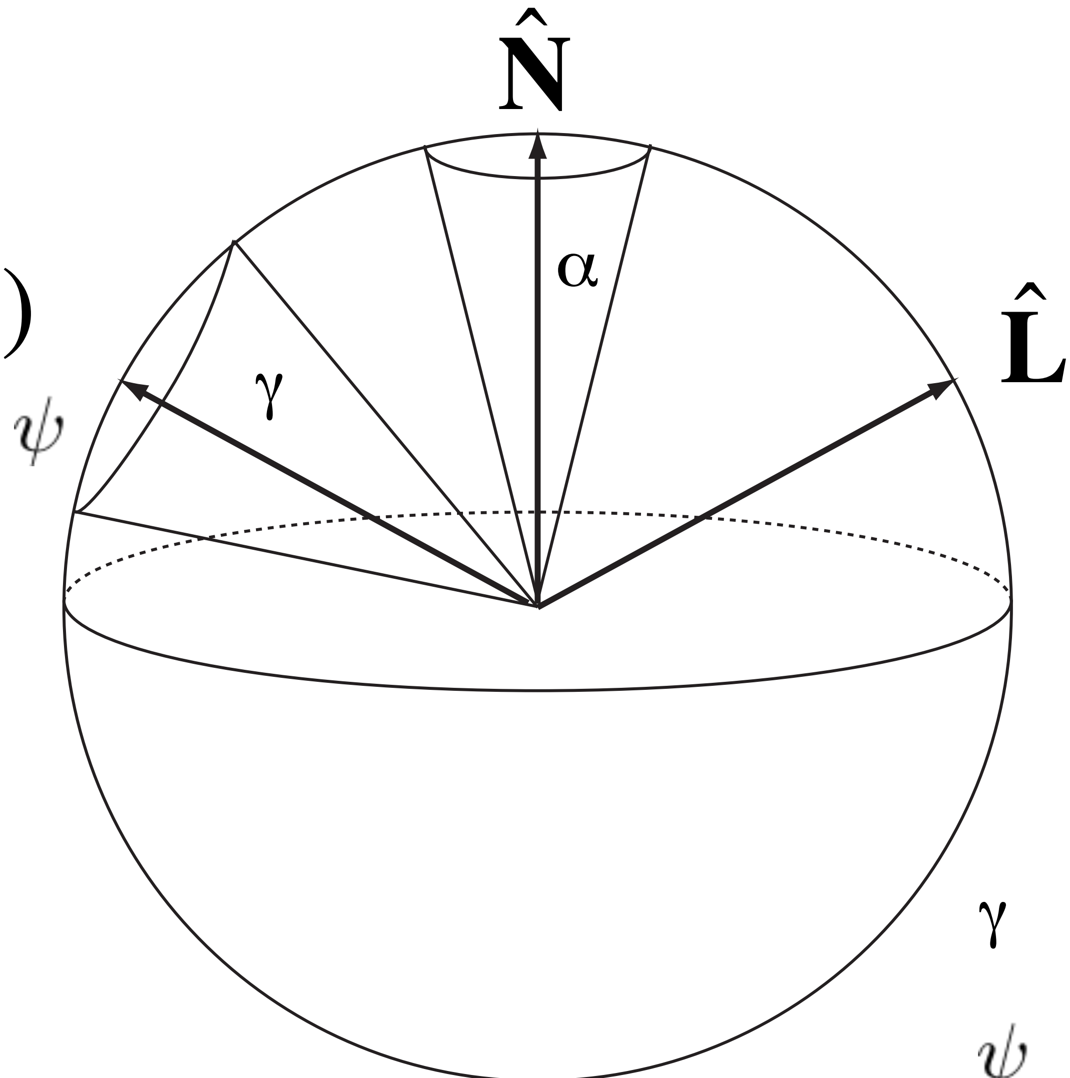
P. Bouguer, Treatise on Optics, 1760

Reflection of the Sun from Waves





Analysis on the Sphere



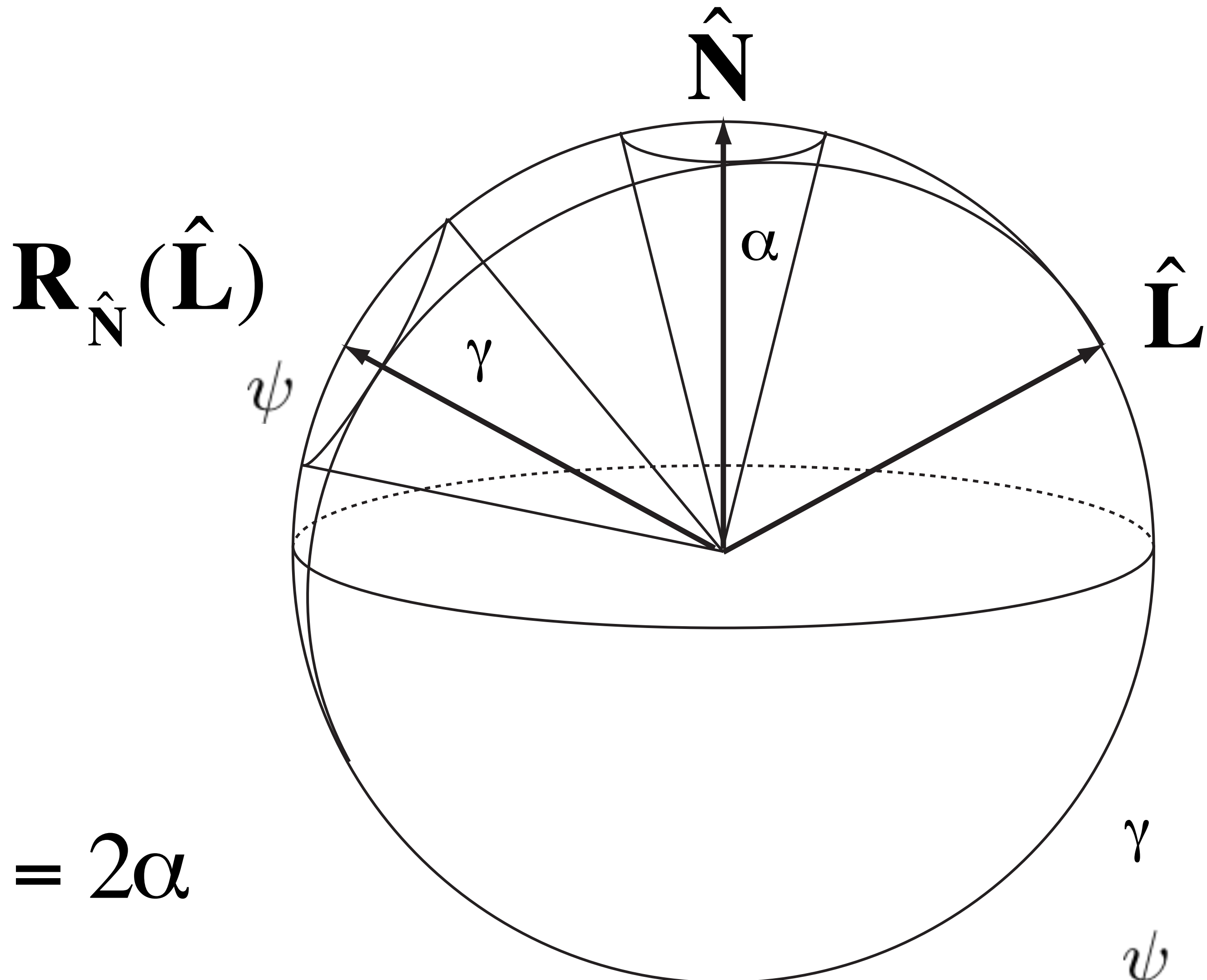
$$\gamma = 2\alpha$$

$$\tan\psi = \tan\alpha\cos\theta$$

γ **Vertical angle**

ψ **Horizontal angle**

Analysis on the Sphere



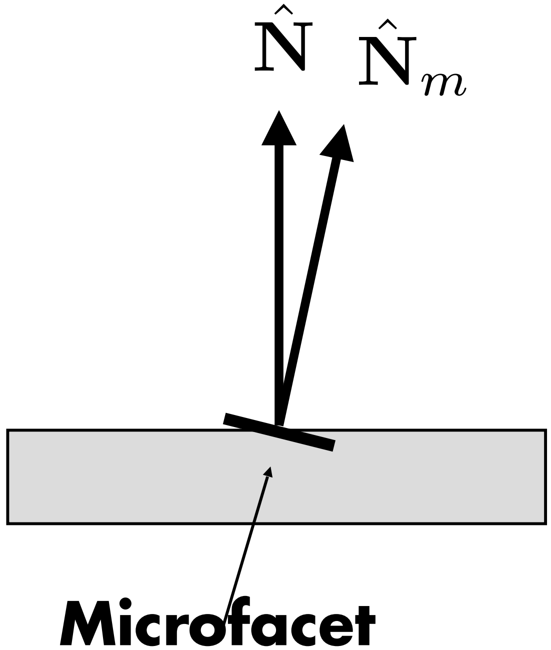
$$\gamma = 2\alpha$$

$$\tan\psi = \tan\alpha\cos\theta$$

γ **Vertical angle**

ψ **Horizontal angle**

Microfacet Distributions



Total projected area

$$\int_{H^2} dA(\omega_m)\, cos\theta_m\, d\omega_m = dA$$

Probability distribution

$$\int_{H^2} D(\omega_m)\, cos\theta_m\, d\omega_m = 1$$

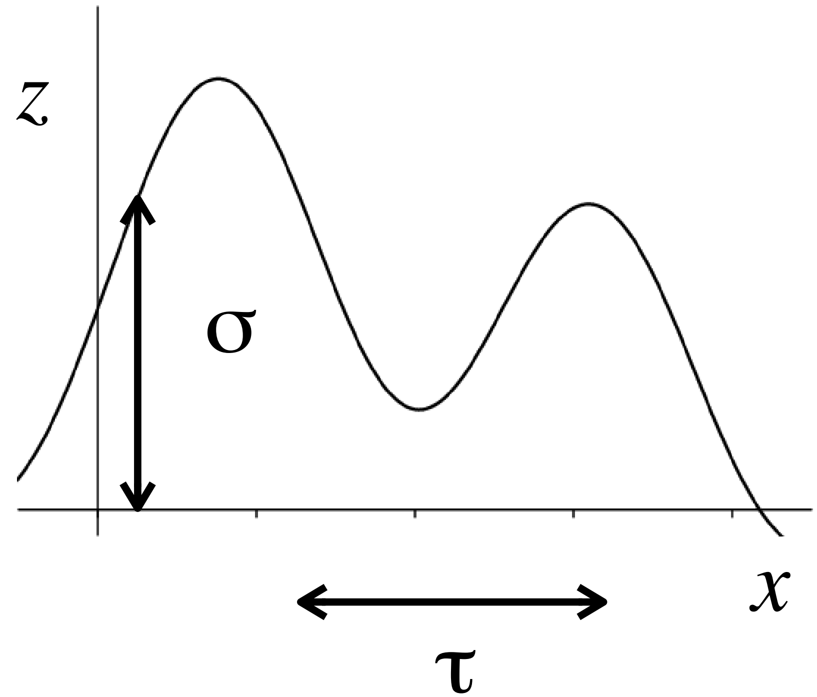
Area distribution $dA(\omega_m)$

Microfacet distribution $D(\omega_m) = dA(\omega_m)/dA$

Gaussian Rough Surface

Gaussian distribution of heights

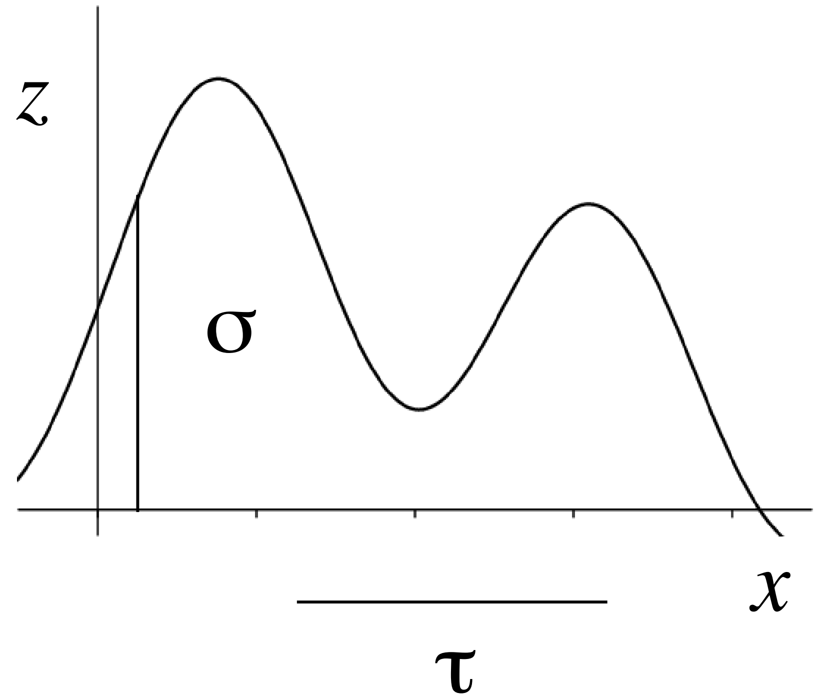
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



Beckmann Distribution

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



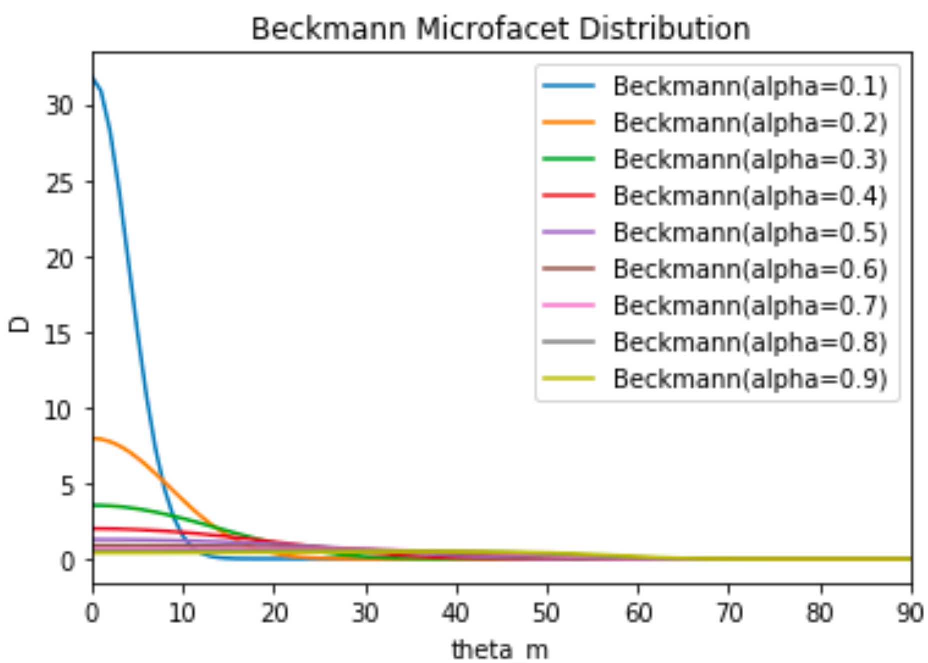
Beckmann distribution of normals (mirrors)

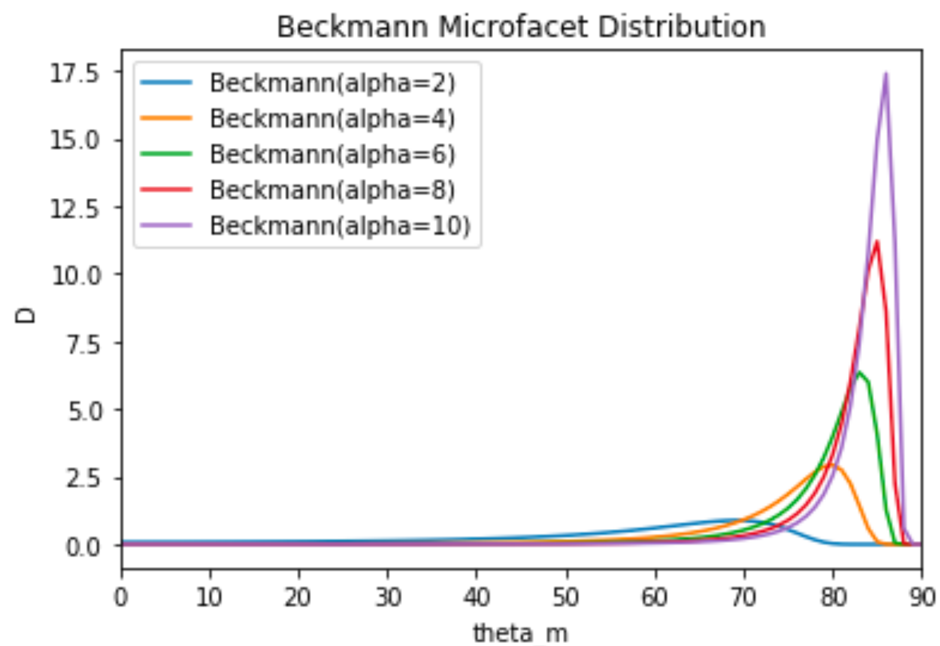
$$D(\omega_m) = \frac{e^{\frac{-\tan^2\theta_m}{\alpha^2}}}{\pi\alpha^2\cos^4\theta_m}$$

$$\alpha = \sqrt{2}\frac{\sigma}{\tau}$$

root mean square slope

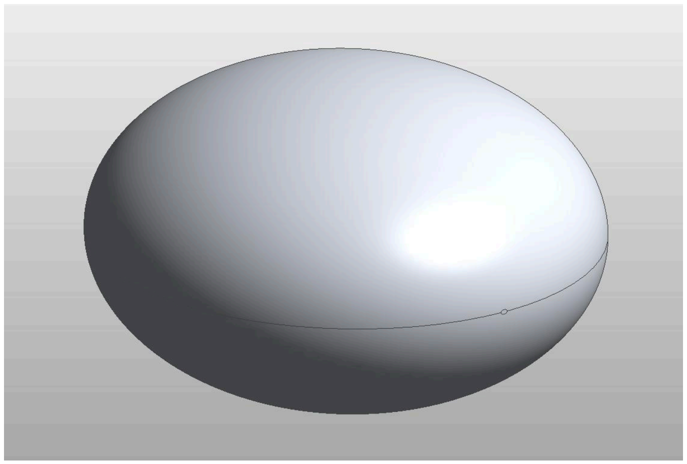
Beckmann Distribution





Trowbridge-Reitz (GGX) Distribution

Elipsoidal

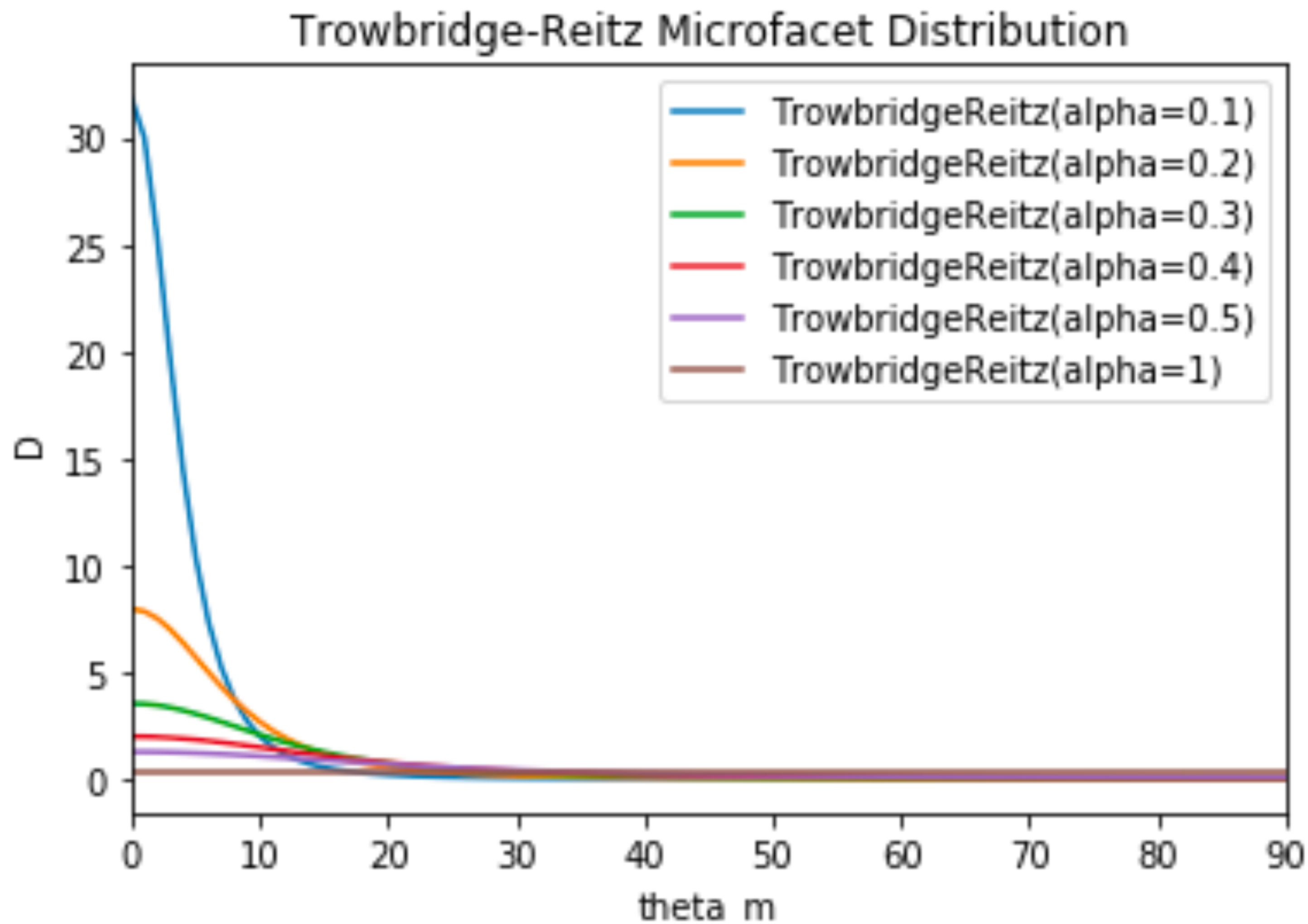


$$z = \alpha(1 - x^2 - y^2)^{(1/2)}$$

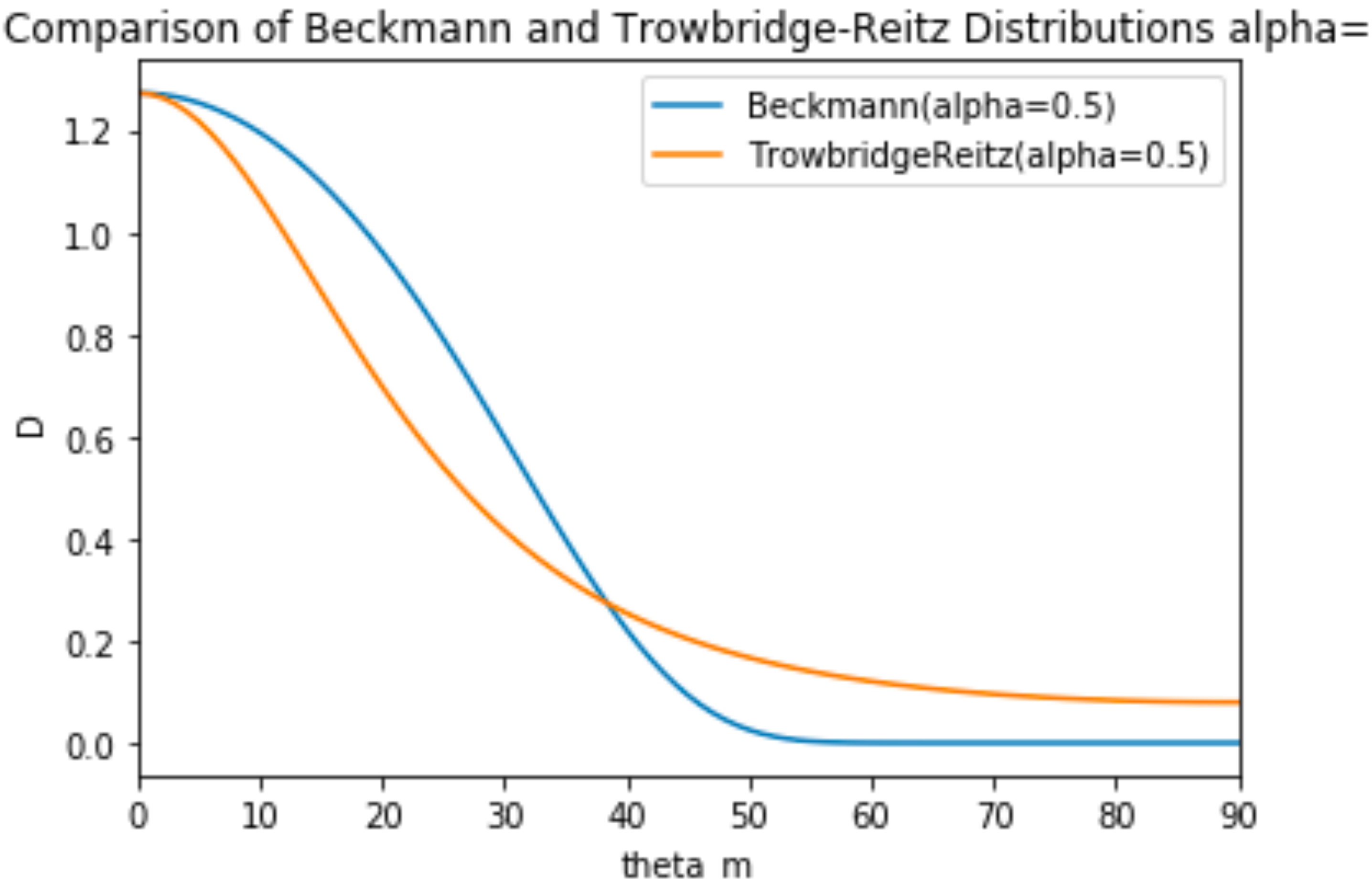
GGX distribution of normals

$$D(\omega_m) = \frac{1}{\pi\alpha^2 \cos^4\theta_m (1 + \frac{\tan^2\theta_m}{\alpha^2})^2}$$

Trowbridge-Reitz (GGX) Distribution



Comparison



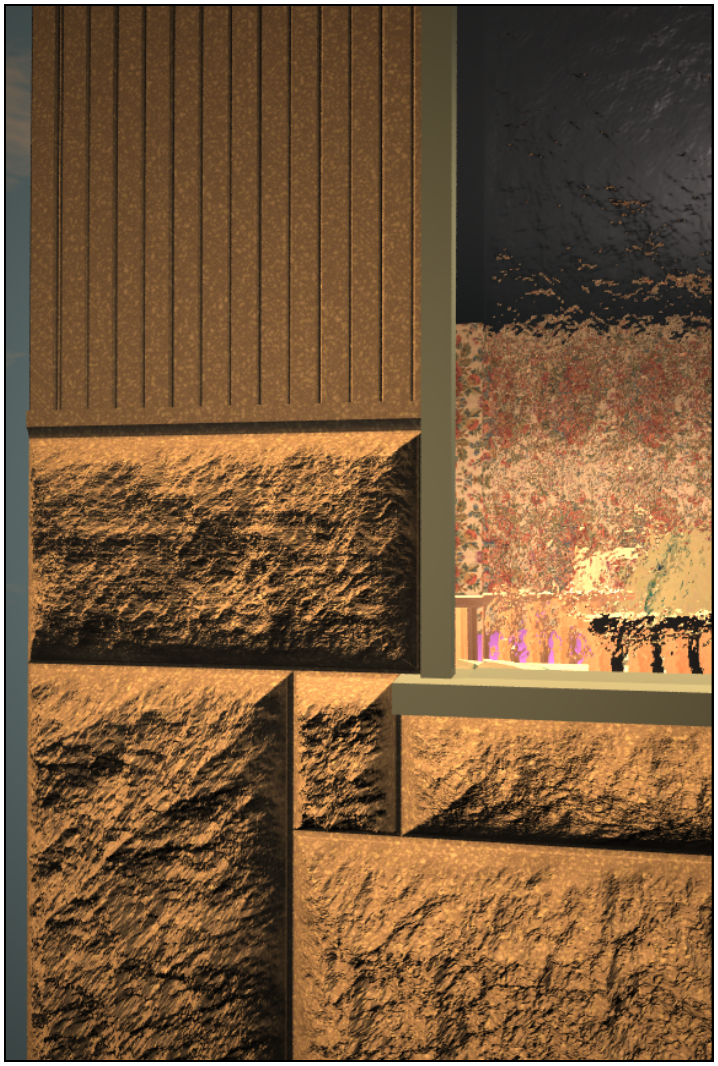
Trowbridge-Reitz has a longer tail

Trowbridge-Reitz matches experimental data better

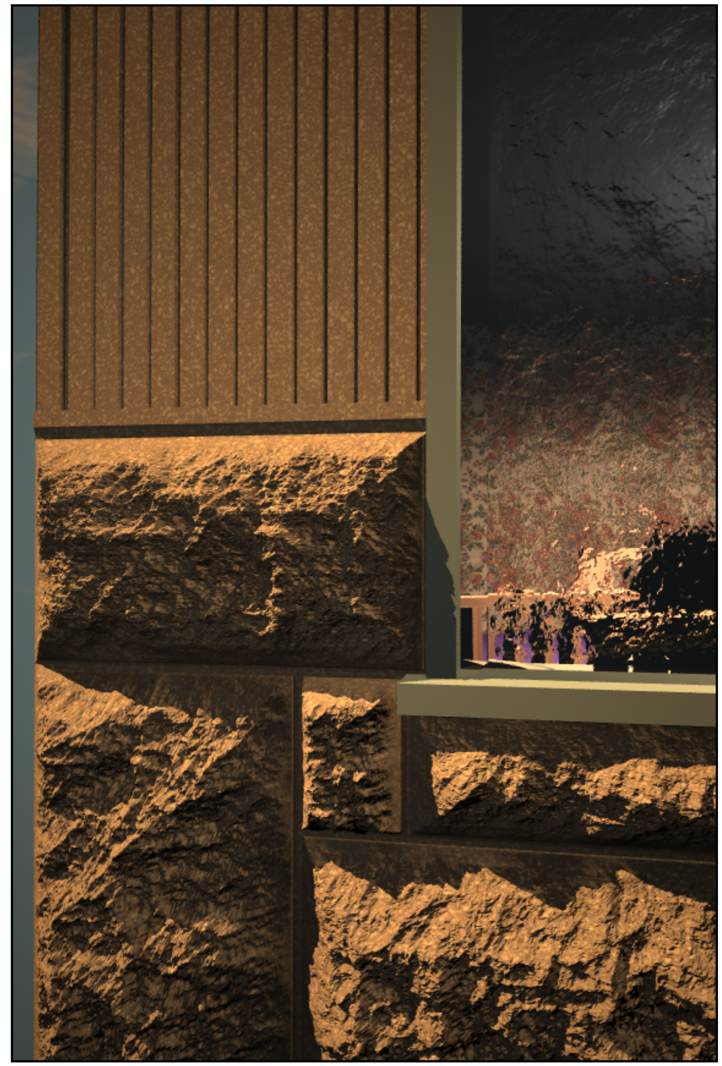
Self Shadowing

Shadowing Reduces Reflected Energy

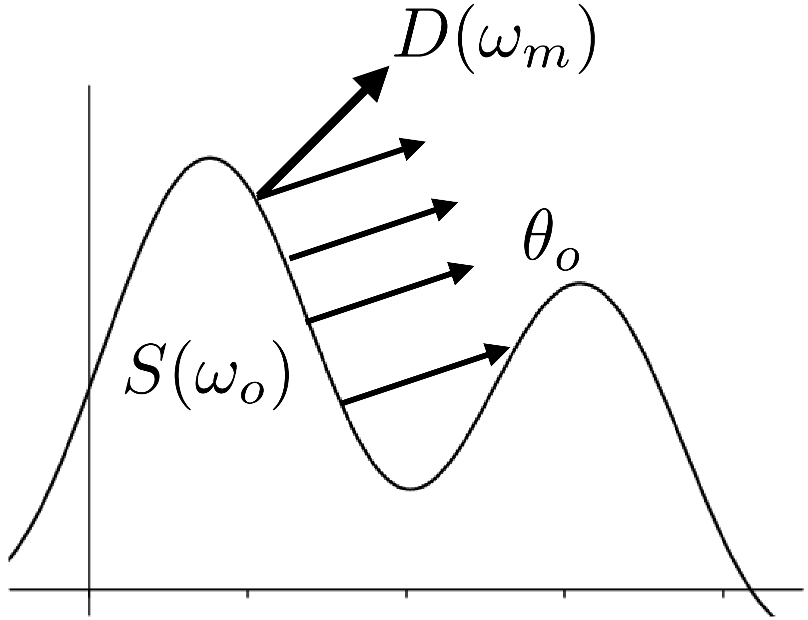
Without self-shadowing



With self-shadowing



Visible Projected Area

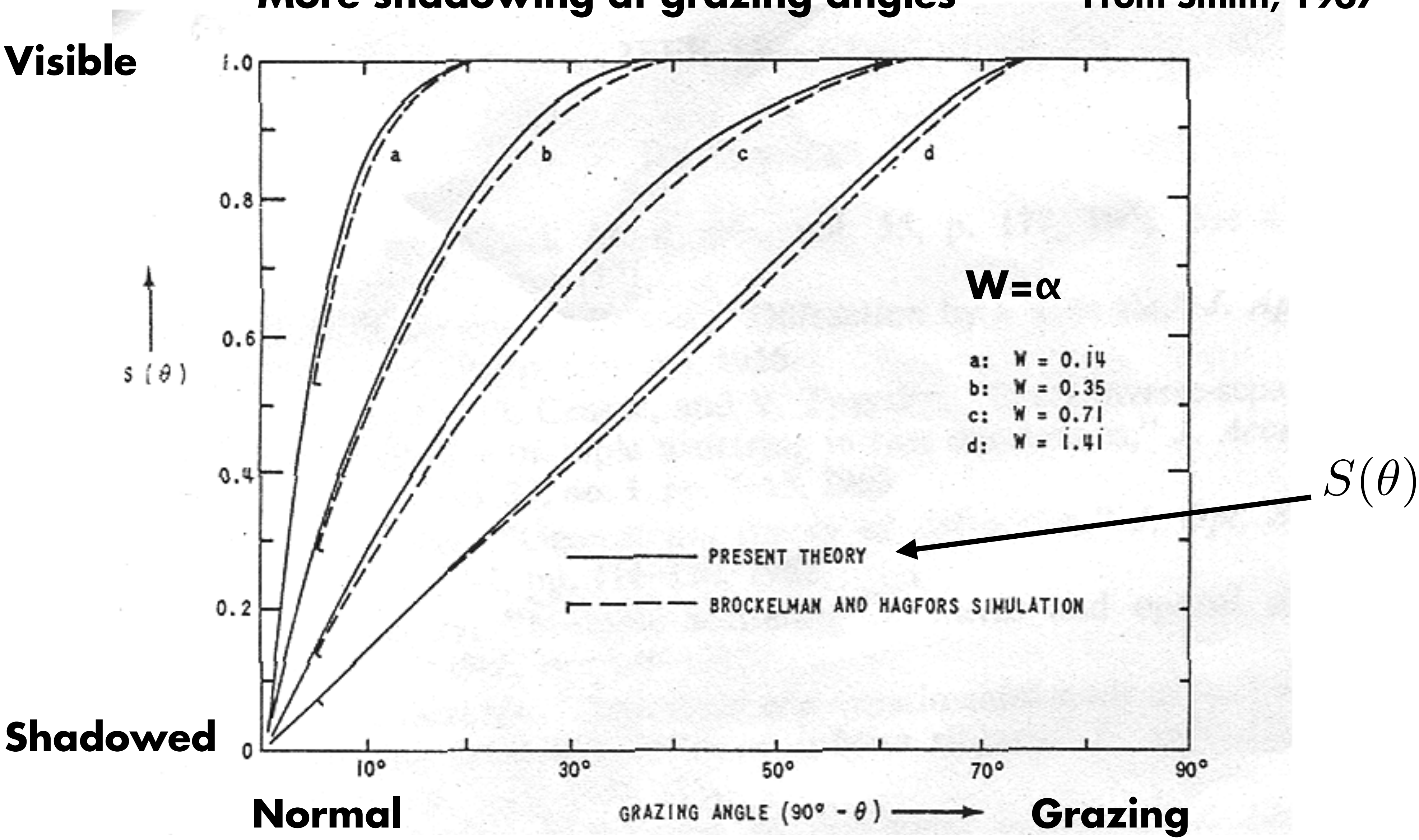


$$\int S(\omega_o)\,\max(0, \omega_{\mathrm{m}} \cdot \omega_{\mathrm{o}})\,\mathrm{D}(\omega_{\mathrm{m}})\,\mathrm{d}\omega_{\mathrm{m}} = \cos\theta_{\mathrm{o}}$$

The sum of the visible areas of the rough surface as viewed from the outgoing direction should equal the projected area of the underlying mean surface

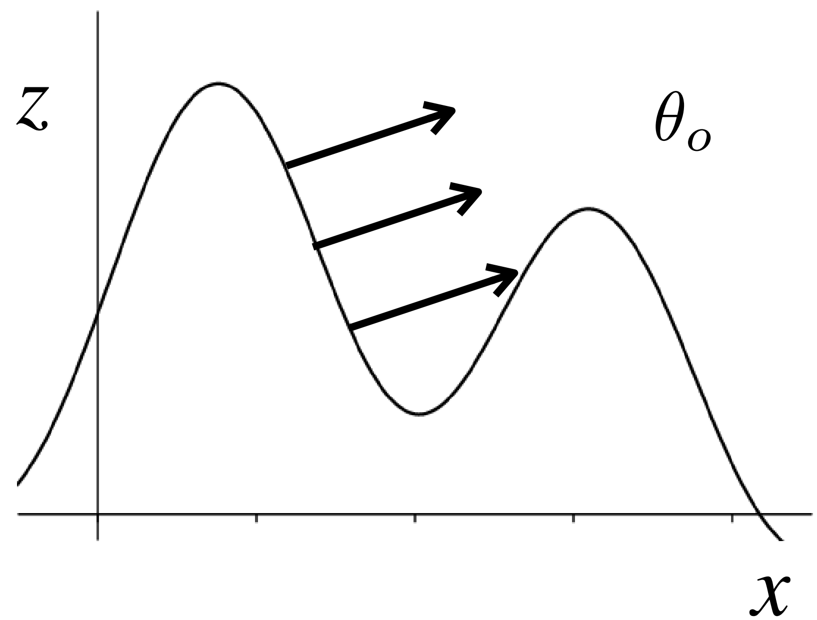
Smith Self-Shadowing Function

More shadowing at grazing angles From Smith, 1967



Smith Self-Shadowing Function

Assume probability of shadowing is independent of the normal



$$S(\theta_o) = \frac{1}{1 + \Lambda(\theta_o)}$$

$$\Lambda(\theta_o) = \frac{\mathrm{erf}(\mathrm{a}) - 1}{2} + \frac{1}{2a\sqrt{\pi}} \exp(-a^2)$$

$$a = \frac{1}{\alpha \tan \theta_o}$$

$$\mathrm{erf}(\mathrm{x}) = \frac{2}{\sqrt{\pi}} \int_0^{\mathrm{x}} \mathrm{e}^{-\mathrm{t}^2}\, \mathrm{dt}$$

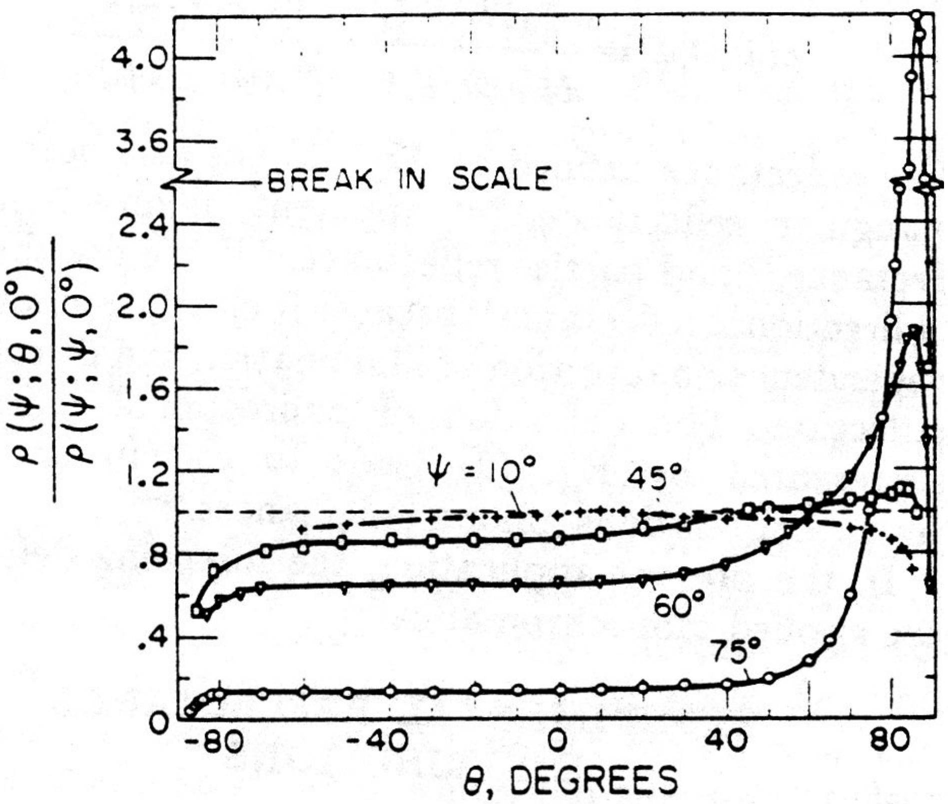
From Smith, 1967

Torrance-Sparrow Model

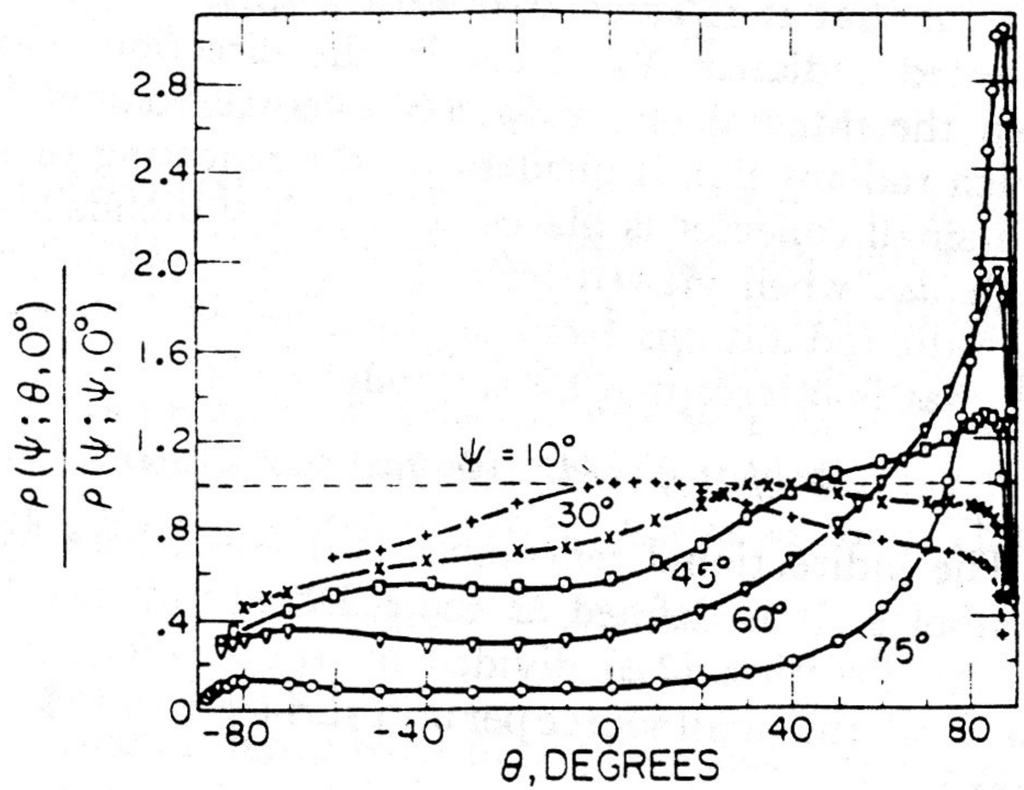
K. E. Torrance, E. M. Sparrow,
Theory of the off-specular reflection
from roughened surfaces,
JOSA 1967

Experiment: "Off-Specular" Peak

Peak of reflection is not at the angle of reflection



Magnesium Oxide
Dielectric



Aluminum
Metal

Torrance-Sparrow Theory

$$f_r(\omega_i \to \omega_r) = \frac{F(\theta_i')S(\theta_i)S(\theta_r)D(\alpha)}{4\cos\theta_i\cos\theta_r}$$

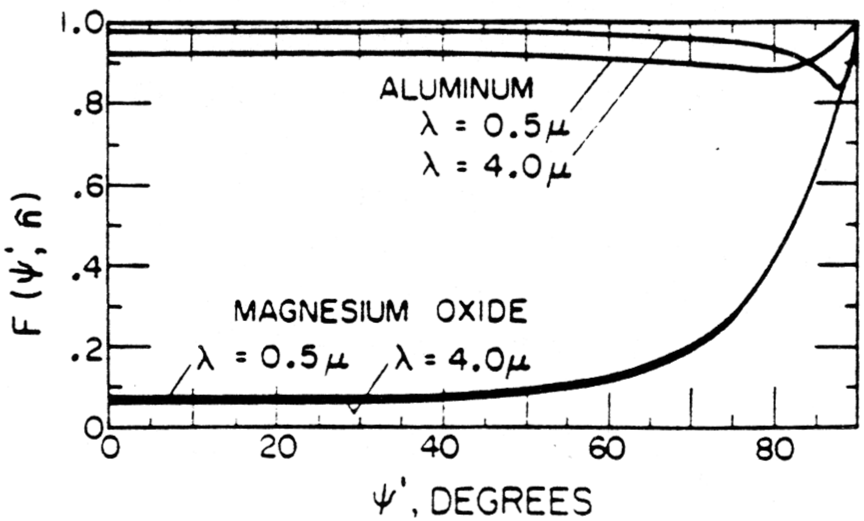


FIG. 6. Fresnel reflectance.

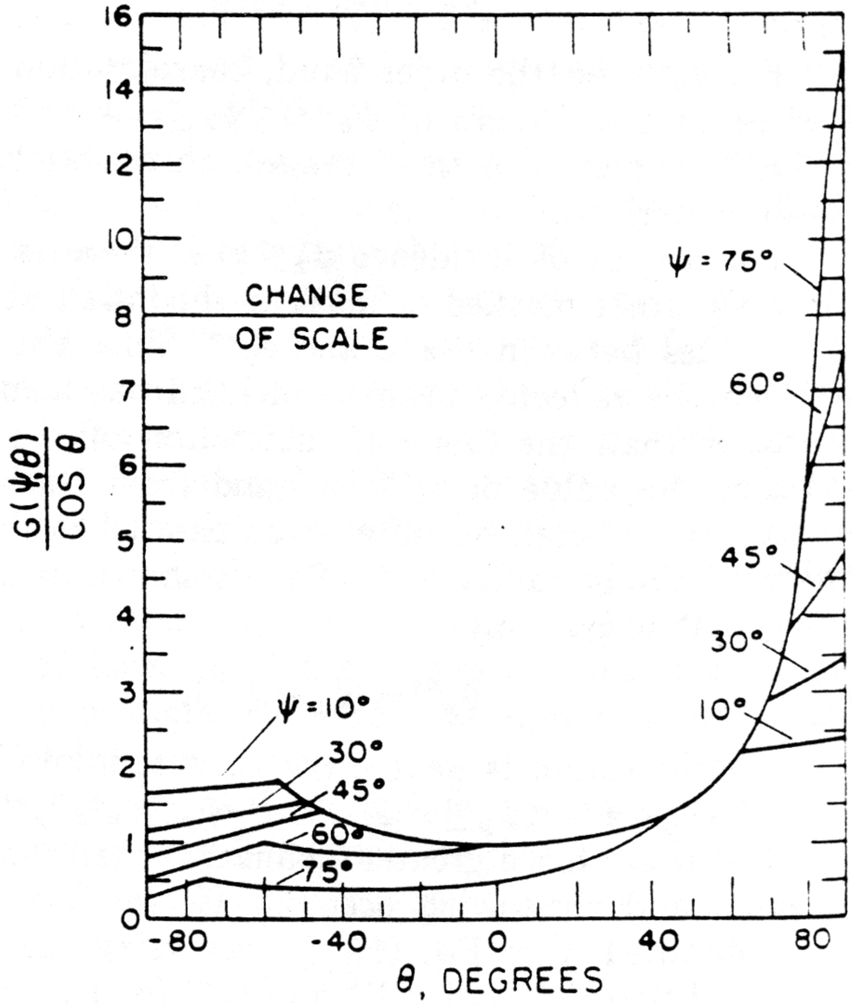
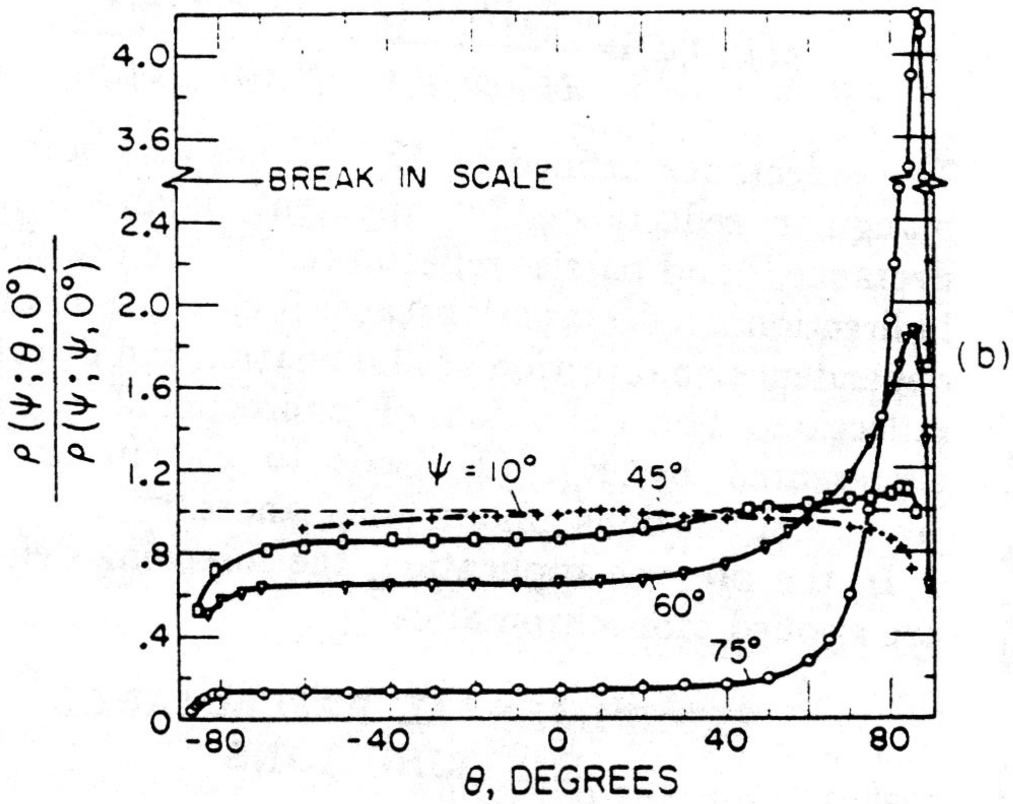
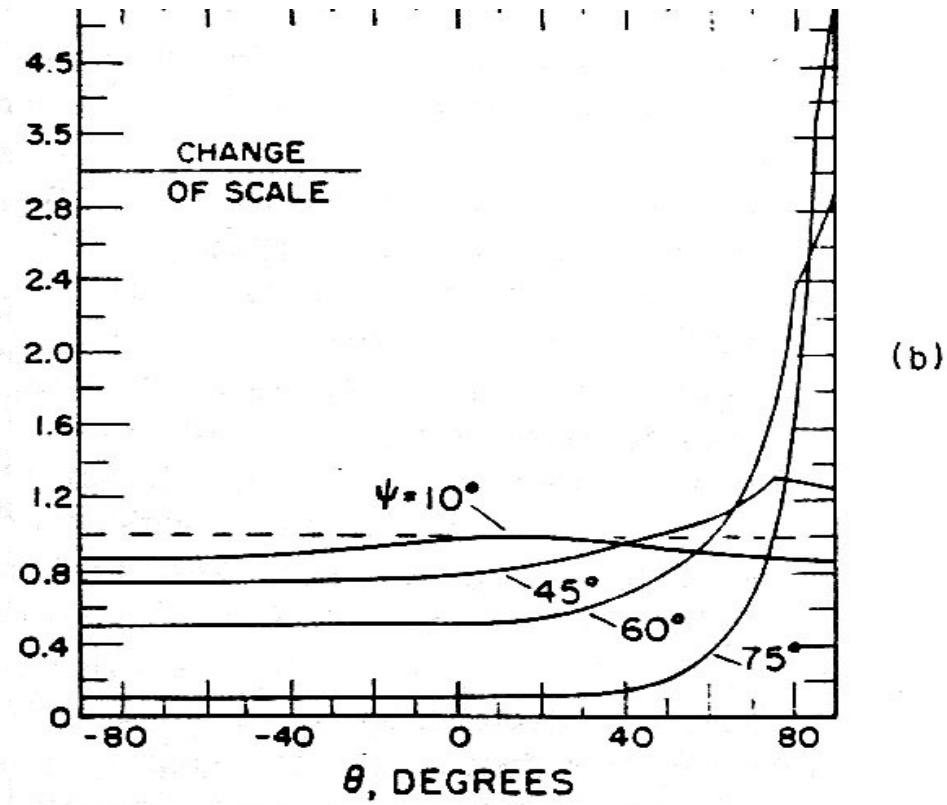


FIG. 7. The factor $G(\psi,\theta)/\cos\theta$ in the plane of incidence for various incidence angles ψ.

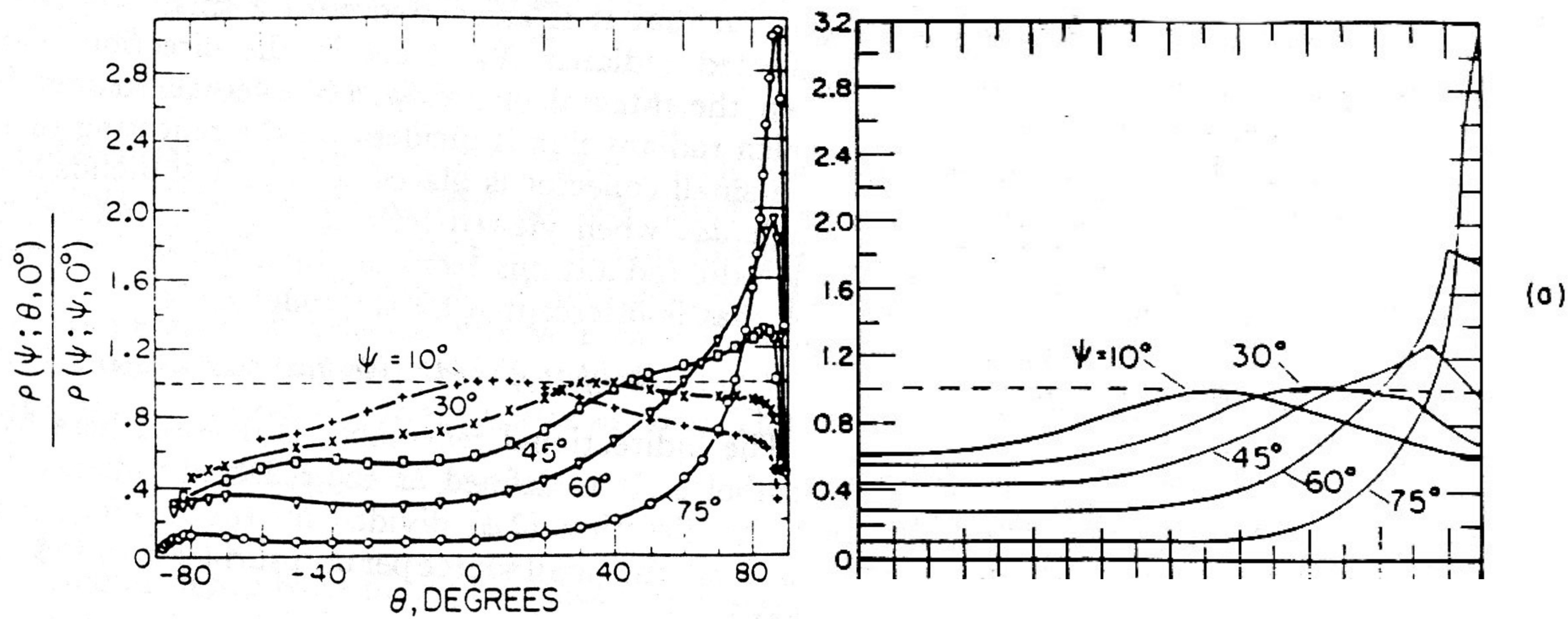
Torrance-Sparrow Model Prediction





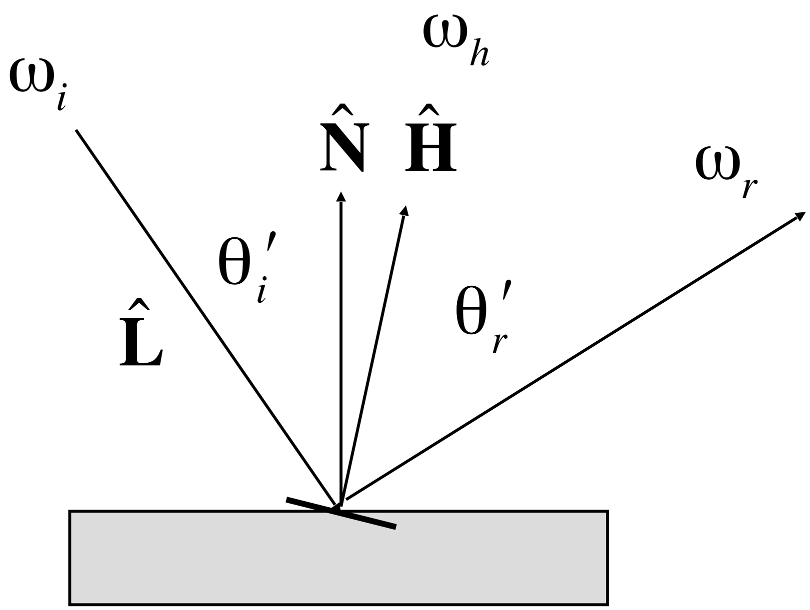
Magnesium Oxide

Torrance-Sparrow Model Prediction



Aluminum

Torrance-Sparrow Model



$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$
$$\cos\theta_i' = \hat{\mathbf{L}} \bullet \hat{\mathbf{H}}$$
$$d\omega_i' = d\omega_i$$

Prime indicates wrt H

$$d\Phi_h = L_i(\omega_i)\cos\theta_i' \, d\omega_i' \, dA(\omega_h)$$
$$dA(\omega_h) = D(\omega_h)\, d\omega_h \, dA$$
$$d\Phi_h = L_i(\omega_i)\cos\theta_i' \, d\omega_i' \, D(\omega_h)\, d\omega_h \, dA$$

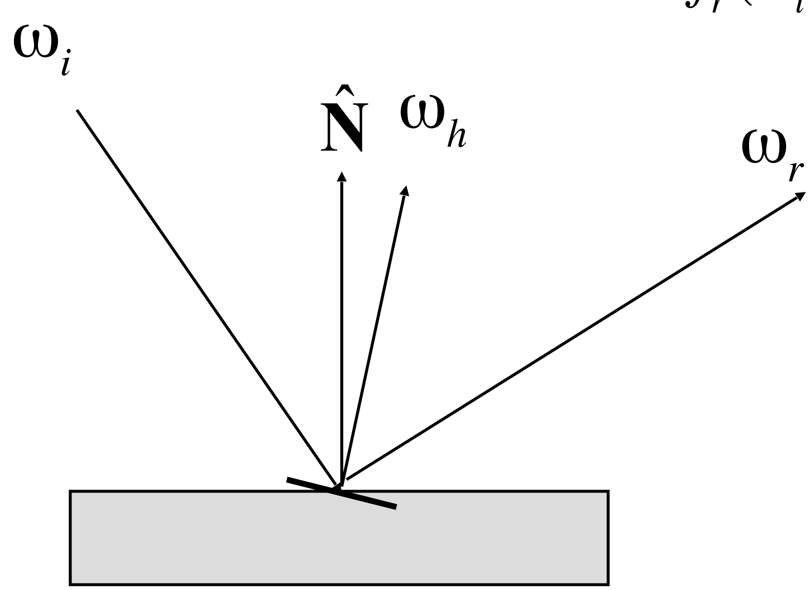
$$d\Phi_r = dL_r(\omega_i \to \omega_r)\cos\theta_r \, d\omega_r \, dA$$

$$d\Phi_r = d\Phi_h$$

$$\therefore dL_r(\omega_i \to \omega_r)\cos\theta_r \, d\omega_r \, dA$$
$$= L_i(\omega_i)\cos\theta_i' \, d\omega_i' \, D(\omega_h)\, d\omega_h \, dA$$

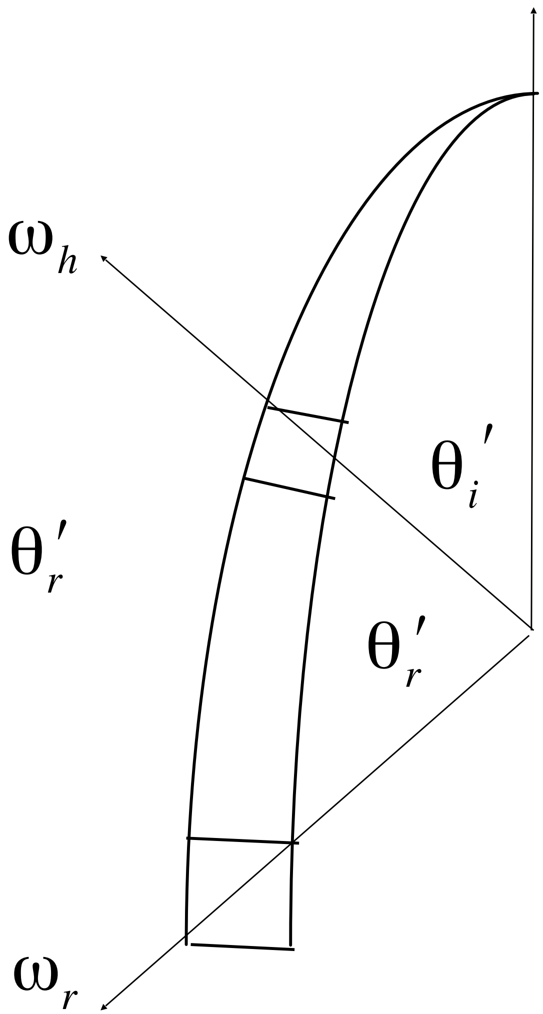
Torrance-Sparrow Model

$$dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$
$$= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA$$



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i(\omega_i)}$$
$$= \frac{L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA}{\left(\cos\theta_r d\omega_r dA\right)\left(L_i(\omega_i)\cos\theta_i\, d\omega_i\right)}$$
$$= \frac{D(\omega_h)}{\cos\theta_i \cos\theta_r}\cos\theta_i' \frac{d\omega_h}{d\omega_r}$$
$$= \frac{D(\omega_h)}{4\cos\theta_i \cos\theta_r}$$

Jacobian $\left|\frac{\partial \omega_h}{\partial \omega_r}\right|$



$$\theta_r' = \theta_i'$$

$$\theta_r = \theta_i' + \theta_r'$$
$$= 2\theta_i'$$

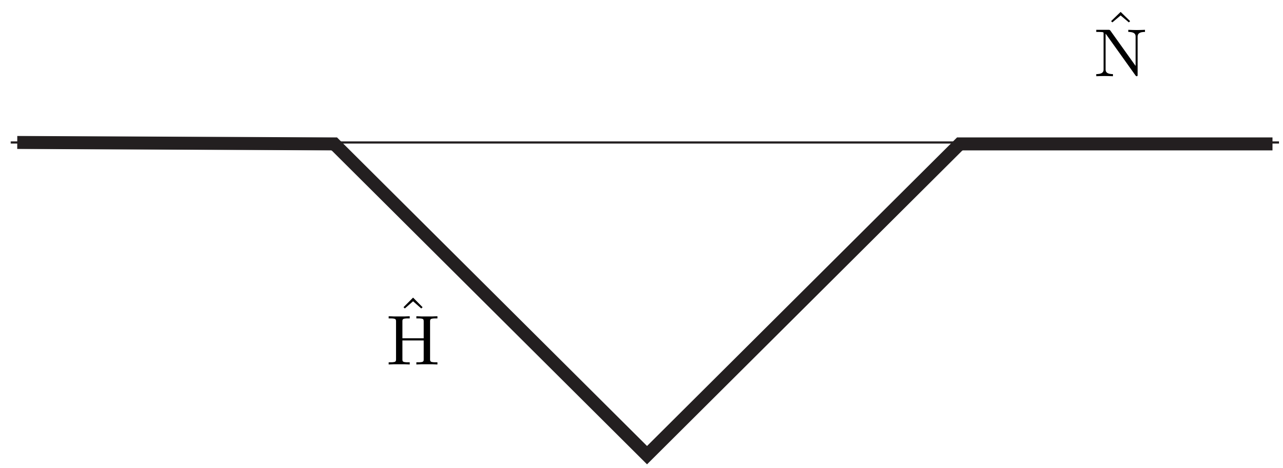
$$\varphi_r = \varphi_i$$

$$
\begin{aligned}
d\omega_r &= \sin\theta_r \, d\theta_r \, d\varphi_r \\
&= \left(\sin 2\theta_i'\right) 2 d\theta_i' \, d\varphi_i \\
&= \left(2 \sin\theta_i' \cos\theta_i'\right) 2 d\theta_i' \, d\varphi_i \\
&= 4 \cos\theta_i' \sin\theta_i' d\theta_i' \, d\varphi_i \\
&= 4 \cos\theta_i' \, d\omega_h
\end{aligned}
$$

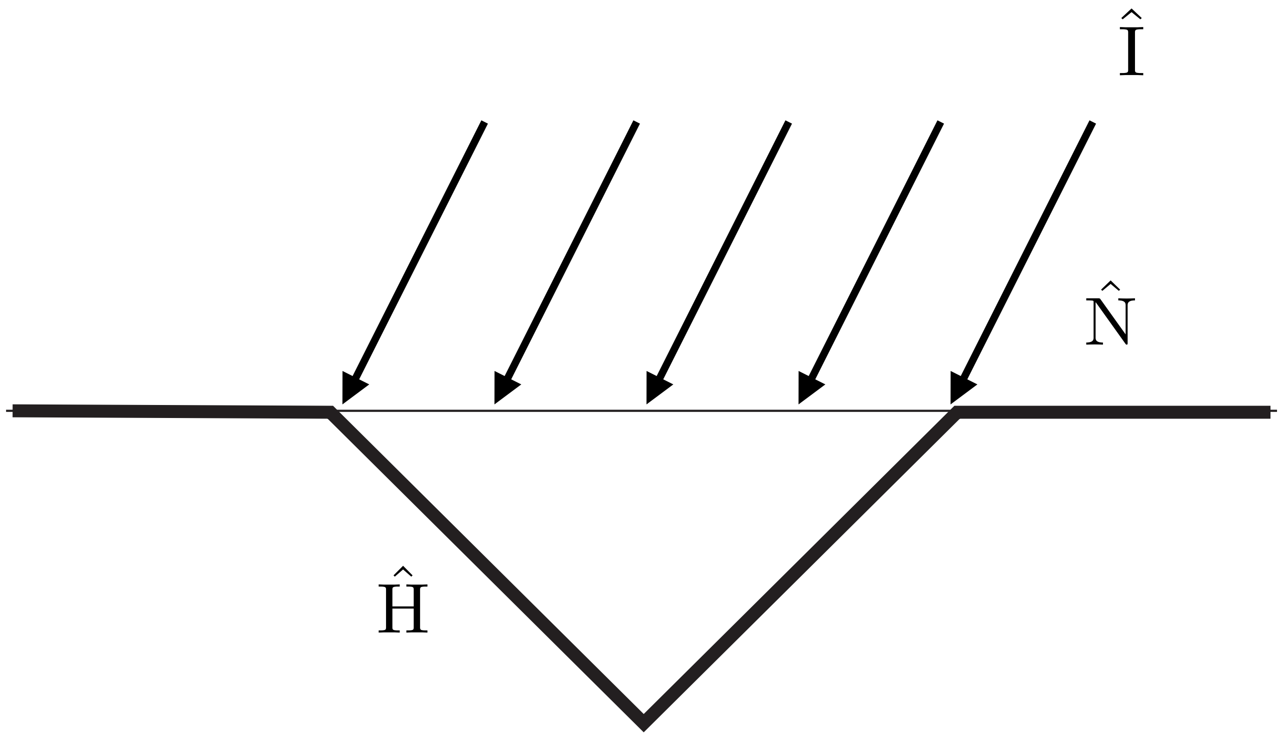
$$\frac{d\omega_h}{d\omega_r} = \frac{1}{4\cos\theta_i'}$$

V-Groove Self-Shadowing

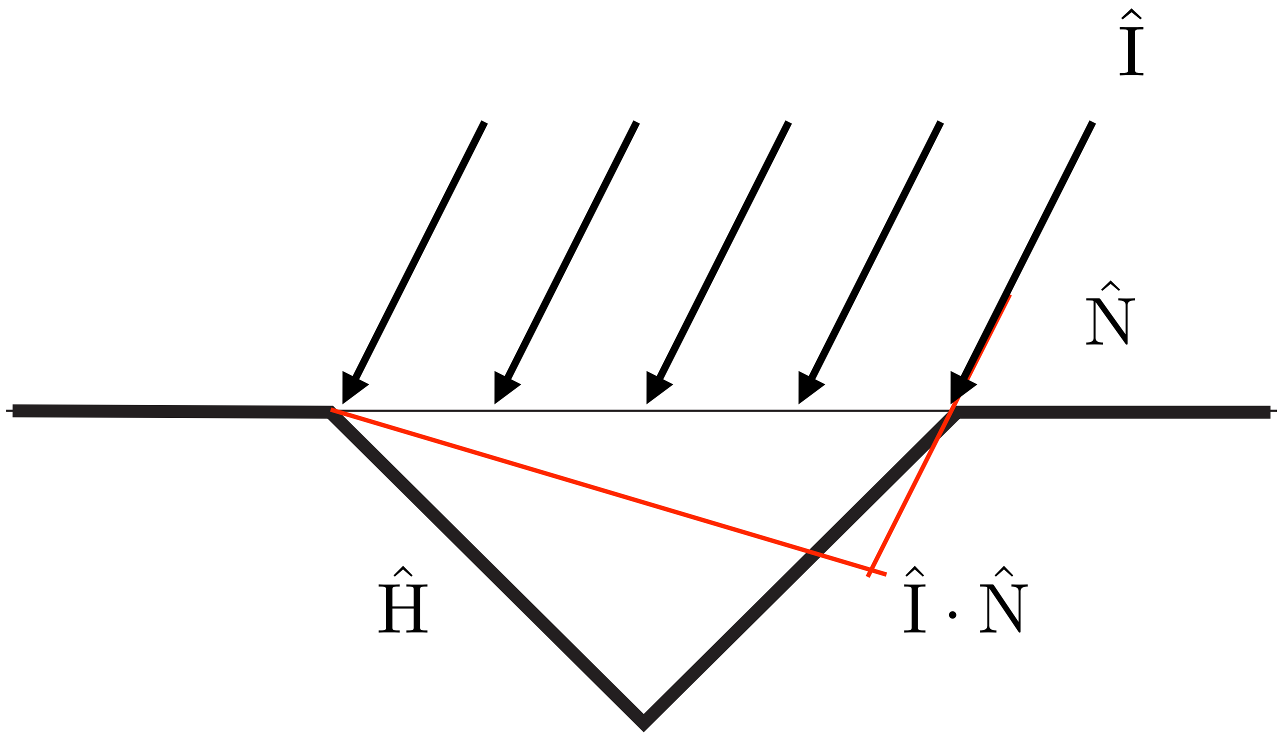
V-Groove



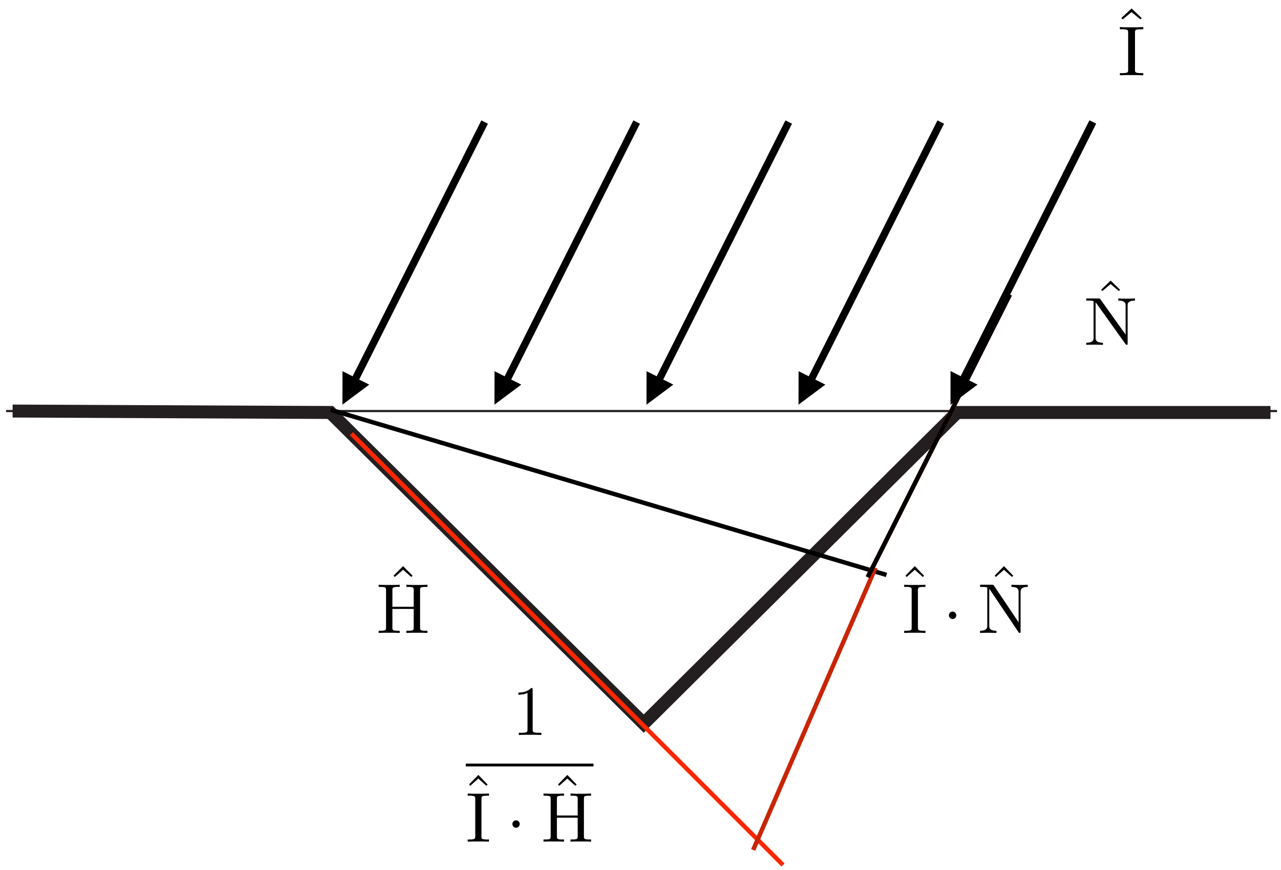
V-Groove



V-Groove

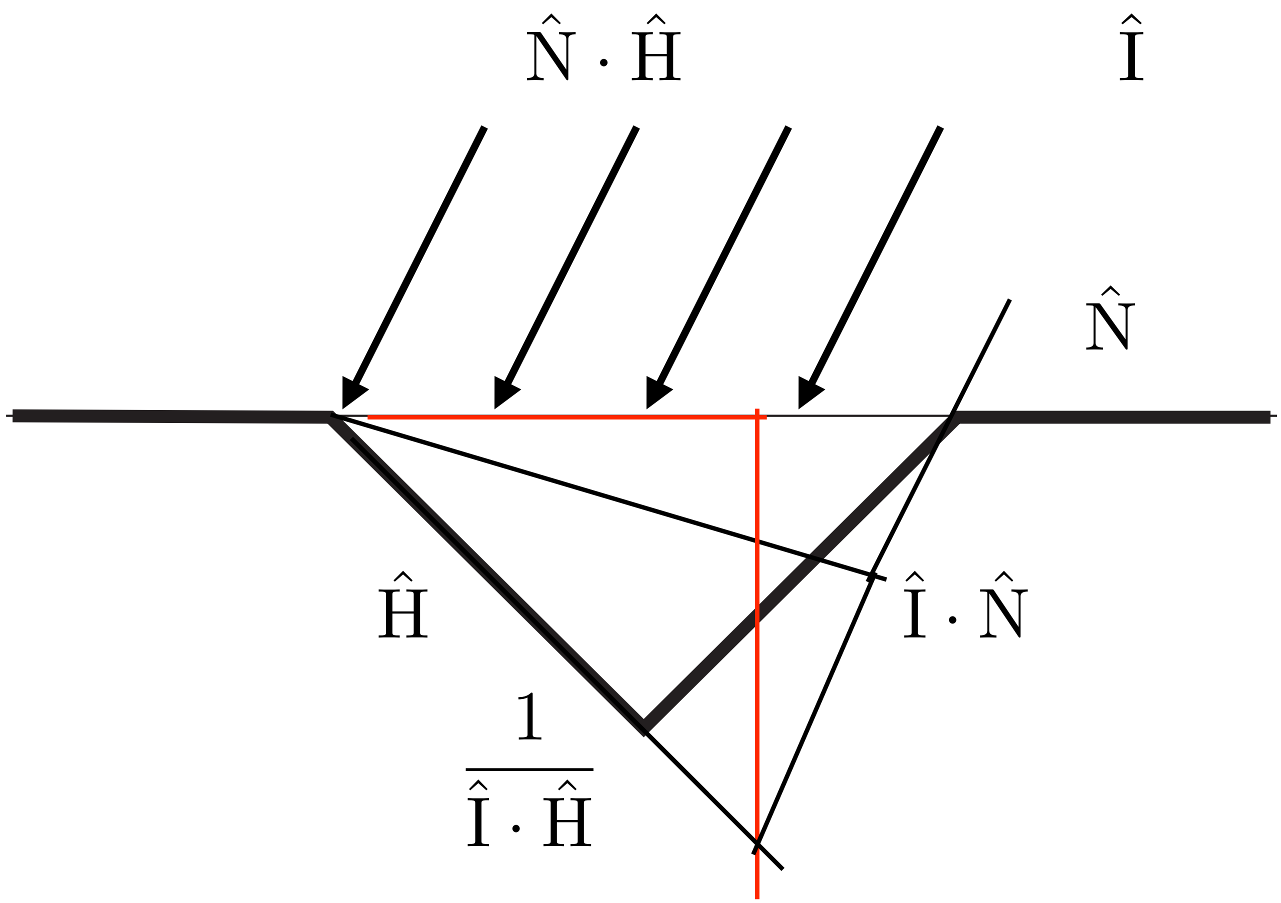


V-Groove

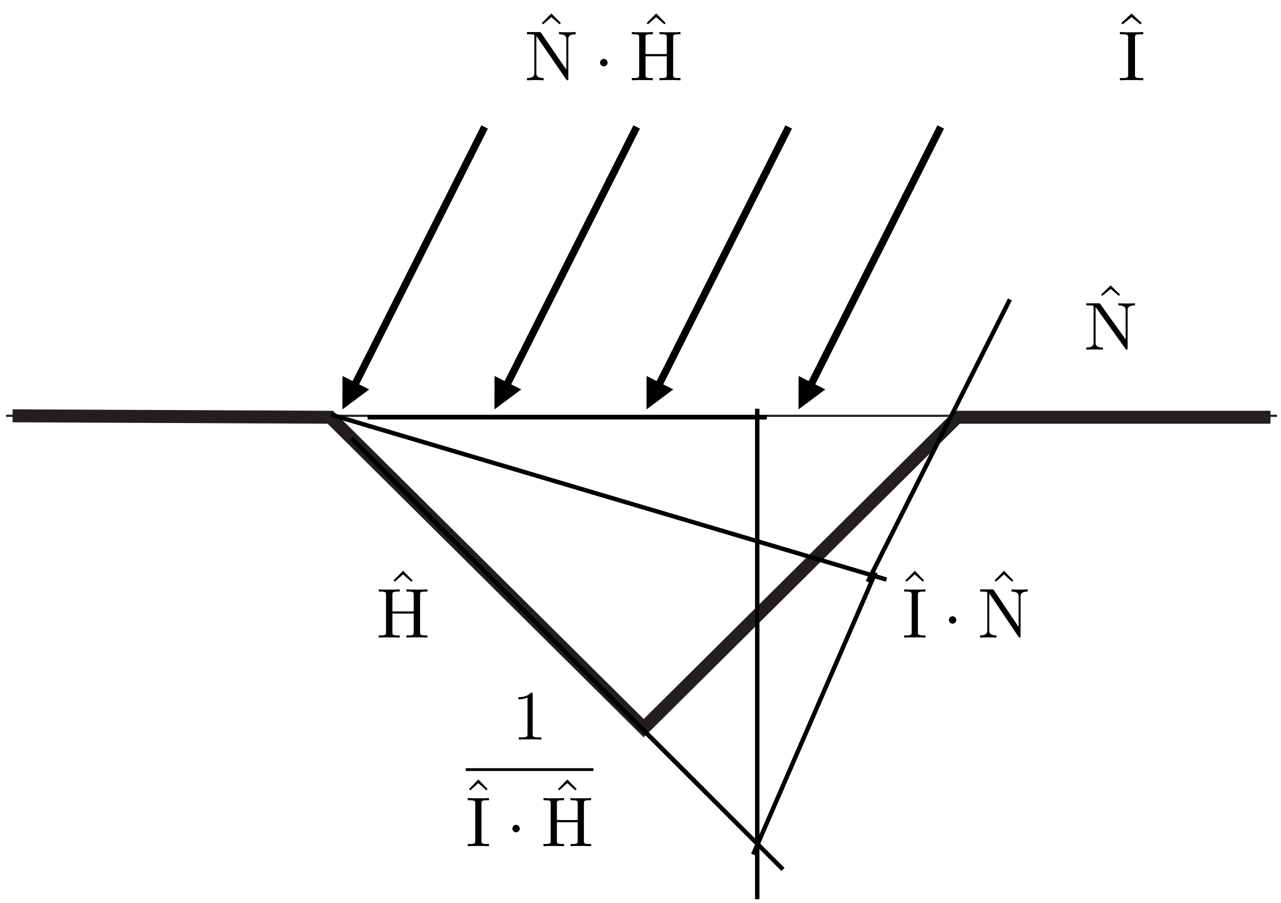


$\hat{\mathrm{I}}$
$\hat{\mathrm{N}}$
$\hat{\mathrm{H}}$
$\hat{\mathrm{I}} \cdot \hat{\mathrm{N}}$
$\frac{1}{\hat{\mathrm{I}} \cdot \hat{\mathrm{H}}}$

V-Groove

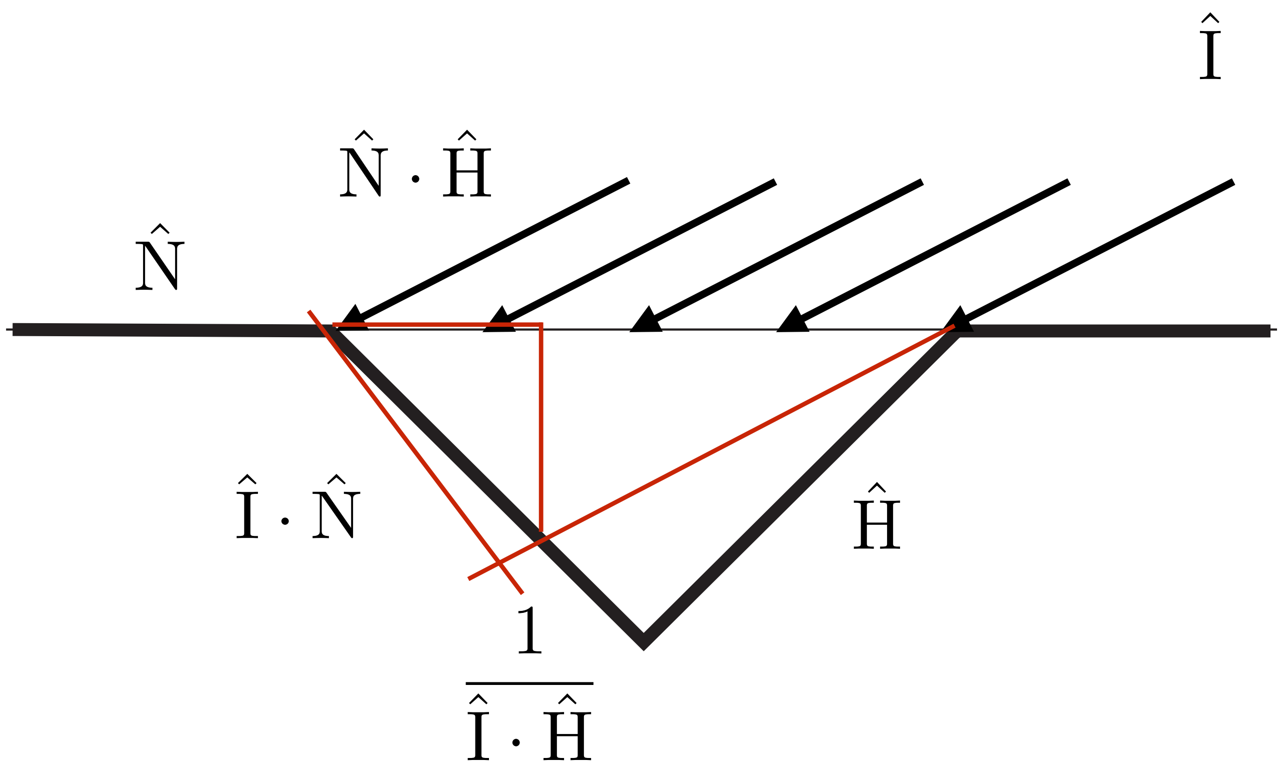

$\hat{N} \cdot \hat{H}$
$\hat{I}$
$\hat{N}$
$\hat{H}$
$\hat{I} \cdot \hat{N}$
$\frac{1}{\hat{I} \cdot \hat{H}}$

V-Groove



$$G(\hat{\mathrm{I}}, \hat{\mathrm{H}}) = \min(1, 2\frac{\hat{\mathrm{N}} \cdot \hat{\mathrm{H}}\ \hat{\mathrm{N}} \cdot \hat{\mathrm{I}}}{\hat{\mathrm{H}} \cdot \hat{\mathrm{I}}})$$

V-Groove



$$G(\hat{\mathrm{I}}, \hat{\mathrm{H}}) = \min(1, 2\frac{\hat{\mathrm{N}} \cdot \hat{\mathrm{H}} \ \hat{\mathrm{N}} \cdot \hat{\mathrm{I}}}{\hat{\mathrm{H}} \cdot \hat{\mathrm{I}}})$$