

Electrical Network and Device Modelling Assignment 2

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1. (a)
(b)

$$\begin{aligned} Z_{eq1} &= \left(\frac{1}{Z_{R_1}} + \frac{1}{Z_{C_1}} \right)^{-1} \\ &= \left(\frac{1}{R_1} + \frac{1}{sC_1} \right)^{-1} \\ &= \left(\frac{1}{1.25 \times 10^6} + s \cdot 4 \times 10^{-12} \right)^{-1} \Omega \\ &= \left(\frac{1 + 1.25 \times 10^6 \cdot (s \cdot 4 \times 10^{-12})}{1.25 \times 10^6} \right)^{-1} \Omega \\ &= \frac{1.25 \times 10^6}{1 + 1.25 \times 10^6 \cdot (s \cdot 4 \times 10^{-12})} \Omega \\ &= \frac{2.5 \times 10^{11}}{s + 2 \times 10^5} \Omega \end{aligned}$$

- (c)

$$\begin{aligned} Z_{eq2} &= \left(\frac{1}{Z_{R_2}} + \frac{1}{Z_{C_2}} \right)^{-1} \\ &= \left(\frac{1}{R_2} + \frac{1}{sC_2} \right)^{-1} \\ &= \left(\frac{1}{5 \times 10^6} + s \cdot 16 \times 10^{-12} \right)^{-1} \Omega \\ &= \left(\frac{1 + 5 \times 10^6 \cdot (s \cdot 16 \times 10^{-12})}{5 \times 10^6} \right)^{-1} \Omega \\ &= \frac{5 \times 10^6}{1 + 5 \times 10^6 \cdot (s \cdot 16 \times 10^{-12})} \Omega \\ &= \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} A \end{aligned}$$

(d) Using voltage division:

$$\begin{aligned}
V_o(s) &= V_{in}(s) \cdot \frac{Z_{eq2}}{Z_{eq1} + Z_{eq2}} \\
&= \frac{10}{s} \cdot \frac{\frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}}{\frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} + \frac{2.5 \times 10^{11}}{s + 2 \times 10^5}} V \\
&= \frac{10}{s} \cdot \frac{\frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}}{\frac{(6.25 \times 10^{10})(s + 2 \times 10^5) + (2.5 \times 10^{11})(s + 1.25 \times 10^4)}{(s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}} V \\
&= \frac{10}{s} \cdot \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} \cdot \frac{(s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}{(6.25 \times 10^{10})(s + 2 \times 10^5) + (2.5 \times 10^{11})(s + 1.25 \times 10^4)} V \\
&= \frac{6.25 \times 10^{11} \cdot (s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}{3.125 \times 10^{11} \cdot s \cdot (s + 1.25 \times 10^4) \cdot (s + 5 \times 10^4)} V \\
&= \frac{2 \cdot (s + 2 \times 10^5)}{s \cdot (s + 5 \times 10^4)} V \\
&= \frac{2s + 4 \times 10^5}{s^2 + 5 \times 10^4 \cdot s} V
\end{aligned}$$

Using Partial fractions to continue.

$$\begin{aligned}
\frac{2s + 4 \times 10^5}{s^2 + 5 \times 10^4 \cdot s} &= \frac{A}{s} + \frac{B}{s + 50000} \\
A(s + 50000) + Bs &= 2(s + 200000) \\
(s = 0) : 50000A &= 400000 \\
&\implies A = 8 \\
(s = -50000) : -50000B &= 300000 \\
&B = -6 \\
\implies V_o(s) &= \frac{8}{s} - \frac{6}{s + 50000} V
\end{aligned}$$

Using inverse Laplace transform to find $v_o(t)$.

$$\begin{aligned}
v_o(t) &= \mathcal{L}^{-1}[V_o(s)] \\
&= \mathcal{L}^{-1} \left[\frac{8}{s} - \frac{6}{s + 50000} \right] V \\
&= (8 - 6e^{-50000t})u(t) V
\end{aligned}$$

(e) Using Ohm's Law in frequency domain:

$$\begin{aligned}
 V_{in}(s) &= Z_{eq} \cdot I_o(s) \\
 &= (Z_{eq1} + Z_{eq2}) \cdot I_o(s) \\
 \implies I_o(s) &= \frac{V_{in}(s)}{Z_{eq1} + Z_{eq2}} \\
 &= \frac{\frac{10}{s}}{\frac{2.5 \times 10^{11}}{s + 2 \times 10^5} + \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}} A \\
 &= \frac{\frac{10}{s}}{\frac{3.125 \times 10^{11} \cdot (s + 50000)}{(s + 12500) \cdot (s + 200000)}} A \\
 &= \frac{(s + 12500) \cdot (s + 200000)}{3.125 \times 10^{10} \cdot s \cdot (s + 50000)} A
 \end{aligned}$$

After a partial fractions expansion (add working in here)

$$I_o(s) = \frac{9}{2500000 \cdot (s + 50000)} + \frac{1}{625000 \cdot s} + \frac{1}{3.125 \times 10^{10}} A$$

Using inverse Laplace transform to find $i_o(t)$.

$$\begin{aligned}
 i_o(t) &= \mathcal{L}^{-1}[I_o(s)] \\
 &= \mathcal{L}^{-1} \left[\frac{9}{2500000 \cdot (s + 50000)} + \frac{1}{625000 \cdot s} + \frac{1}{3.125 \times 10^{10}} \right] \\
 &= 3.6 \times 10^{-6} e^{-50000t} \cdot u(t) + 1.6 \times 10^{-6} \cdot u(t) + 3.2 \times 10^{-11} \delta(t) A
 \end{aligned}$$

If we assume $t > 0$ only then:

$$i_o(t) = (3.6 \cdot e^{-50000t} + 1.6) \mu A$$

2. (a) Using KCL at the node between capacitor, resistor and inductor: $\Sigma i_{out} = 0$

$$\begin{aligned}
 &\frac{V_o(s) - \frac{5}{s}}{2.5s} + \frac{V_o(s)}{2} + V_o(s) \cdot s \cdot 0.1 = 0 \\
 \implies &\frac{2 \cdot V_o(s)}{5s} - \frac{2}{s^2} + \frac{V_o(s)}{2} + \frac{s \cdot V_o(s)}{10} = 0 \\
 \implies &V_o(s) \left(\frac{2}{5s} + \frac{s}{10} + \frac{1}{2} \right) = \frac{2}{s^2} \\
 \implies &V_o(s) \left(\frac{s^2}{10} + \frac{s}{2} + \frac{2}{5} \right) = \frac{2}{s} \\
 \implies &V_o(s) \cdot (s^2 + 5s + 4) = \frac{20}{s} \\
 \implies &V_o(s) = \frac{20}{s \cdot (s^2 + 5s + 4)} \\
 \implies &V_o(s) = \frac{20}{s \cdot (s + 4) \cdot (s + 1)}
 \end{aligned}$$

Now using partial fraction expansion:

$$\begin{aligned}
 \frac{20}{s \cdot (s+4) \cdot (s+1)} &= \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1} \\
 \Rightarrow A(s+4)(s+1) + B(s)(s+1) + C(s)(s+4) &= 20 \\
 (s=0) \Rightarrow A &= 5 \\
 (s=-1) \Rightarrow 12B &= 20 \\
 \Rightarrow B &= \frac{5}{3} \\
 (s=-1) \Rightarrow -3C &= 20 \\
 \Rightarrow C &= \frac{-20}{3} \\
 \Rightarrow V_o(s) &= \frac{5}{s} + \frac{5}{3(s+4)} - \frac{20}{3(s+1)}
 \end{aligned}$$

(b)

$$\begin{aligned}
 v_o(t) &= \mathcal{L}^{-1}[V_o(s)] \\
 &= \mathcal{L}^{-1} \left[\frac{5}{s} + \frac{5}{3(s+4)} - \frac{20}{3(s+1)} \right] \\
 &= \left(5 + \frac{5}{3}e^{-4t} - \frac{20}{3}e^{-t} \right) u(t)
 \end{aligned}$$

(c)

3. (a) NEEDS DIAGRAM!!!!

$$\begin{aligned}
 Z_{R||L} &= \left(\frac{1}{5} + \frac{1}{2s} \right)^{-1} \\
 &= \left(\frac{2s+5}{10s} \right)^{-1} \\
 &= \frac{10s}{2s+5} \\
 \Rightarrow H(s) &= \frac{V_o(s)}{V_i(s)} \\
 &= \frac{z_{R||L}}{Z_{R||L} + Z_C} \\
 &= \frac{\frac{10s}{2s+5}}{\frac{10s}{2s+5} + \frac{10}{s}} \\
 &= \frac{\frac{10s}{2s+5}}{\frac{10s^2 + 20s + 50}{2s^2 + 5s}} \\
 &= \frac{s^2}{s^2 + 2s + 5}
 \end{aligned}$$

(b) NEEDS DIAGRAM!!!!

Using phasor analysis:

$$\begin{aligned} V_o &= V_i \cdot \frac{\left(\frac{1}{R} + \frac{1}{j\omega L}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{j\omega L}\right)^{-1} + \frac{1}{j\omega C}} \\ &= 10\angle 0^\circ \cdot \frac{\left(\frac{1}{5} + \frac{1}{40j}\right)^{-1}}{\left(\frac{1}{5} + \frac{1}{40j}\right)^{-1} + \frac{1}{2j}} \\ &= 10.075\angle 5.78^\circ \text{ V} \end{aligned}$$

This means that the steady state response for the system when $v_i(t) = 10 \cos(20t)$ is $v_o(t) = 10.075 \cos(20t + 5.78^\circ) \text{ V}$

4. Q4

5. Q5

(a)

(b)

(c)

6. Q6