Electrical Network Analysis and Design Assignment 2

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Tuesday 2:15pm - EDS 8

1. (a) INSERT DIAGRAM HERE

   (b)

$$
\begin{aligned}
Z_{eq1} &= \left(\frac{1}{Z_{R_1}} + \frac{1}{Z_{C_1}}\right)^{-1} \\
&= \left(\frac{1}{R_1} + \frac{1}{\frac{1}{sC_1}}\right)^{-1} \\
&= \left(\frac{1}{1.25 \times 10^6} + s \cdot 4 \times 10^{-12}\right)^{-1} \Omega \\
&= \left(\frac{1 + 1.25 \times 10^6 \cdot (s \cdot 4 \times 10^{-12})}{1.25 \times 10^6}\right)^{-1} \Omega \\
&= \frac{1.25 \times 10^6}{1 + 1.25 \times 10^6 \cdot (s \cdot 4 \times 10^{-12})} \Omega \\
&= \frac{2.5 \times 10^{11}}{s + 2 \times 10^5} \Omega
\end{aligned}
$$

   (c)

$$
\begin{aligned}
Z_{eq2} &= \left(\frac{1}{Z_{R_2}} + \frac{1}{Z_{C_2}}\right)^{-1} \\
&= \left(\frac{1}{R_2} + \frac{1}{\frac{1}{sC_2}}\right)^{-1} \\
&= \left(\frac{1}{5 \times 10^6} + s \cdot 16 \times 10^{-12}\right)^{-1} \Omega \\
&= \left(\frac{1 + 5 \times 10^6 \cdot (s \cdot 16 \times 10^{-12})}{5 \times 10^6}\right)^{-1} \Omega \\
&= \frac{5 \times 10^6}{1 + 5 \times 10^6 \cdot (s \cdot 16 \times 10^{-12})} \Omega \\
&= \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} A
\end{aligned}
$$

(d) Using voltage division:

$$\begin{aligned}
V_o(s) &= V_{in}(s) \cdot \frac{Z_{eq2}}{Z_{eq1} + Z_{eq2}} \\
&= \frac{10}{s} \cdot \frac{\dfrac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}}{\dfrac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} + \dfrac{2.5 \times 10^{11}}{s + 2 \times 10^5}} \; V \\
&= \frac{10}{s} \cdot \frac{\dfrac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}}{\dfrac{(6.25 \times 10^{10})(s + 2 \times 10^5) + (2.5 \times 10^{11})(s + 1.25 \times 10^4)}{(s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}} \; V \\
&= \frac{10}{s} \cdot \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} \cdot \frac{(s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}{(6.25 \times 10^{10})(s + 2 \times 10^5) + (2.5 \times 10^{11})(s + 1.25 \times 10^4)} \; V \\
&= \frac{6.25 \times 10^{11} \cdot (s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}{3.125 \times 10^{11} \cdot s \cdot (s + 1.25 \times 10^4) \cdot (s + 5 \times 10^4)} \; V \\
&= \frac{2 \cdot (s + 2 \times 10^5)}{s \cdot (s + 5 \times 10^4)} \; V \\
&= \frac{2s + 4 \times 10^5}{s^2 + 5 \times 10^4 \cdot s} \; V
\end{aligned}$$

Using Partial fractions to continue.

$$\begin{aligned}
\frac{2s + 4 \times 10^5}{s^2 + 5 \times 10^4 \cdot s} &= \frac{A}{s} + \frac{B}{50000} \\
A(s + 50000) + Bs &= 2(s + 200000) \\
(s = 0) : \; 50000A &= 400000 \\
\implies A &= 8 \\
(s = -50000) : \; -50000B &= 300000 \\
B &= -6 \\
\implies V_o(s) = \frac{8}{s} - \frac{6}{s + 50000} \; V
\end{aligned}$$

Using inverse Laplace transform to find $v_o(t)$.

$$\begin{aligned}
v_o(t) &= \mathcal{L}^{-1}[V_o(s)] \\
&= \mathcal{L}^{-1}\left[\frac{8}{s} - \frac{6}{s + 50000}\right] \; V \\
&= (8 - 6e^{-50000t})u(t) \; V
\end{aligned}$$

(e) Using Ohm's Law in frequency domain:

$$\begin{aligned}
V_{in}(s) &= Z_{eq} \cdot I_o(s) \\
&= (Z_{eq1} + Z_{eq2}) \cdot I_o(s) \\
\implies I_o(s) &= \frac{V_{in}(s)}{Z_{eq1} + Z_{eq2}} \\
&= \frac{\frac{10}{s}}{\frac{2.5 \times 10^{11}}{s + 2 \times 10^5} + \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}} \; A \\
&= \frac{\frac{10}{s}}{\frac{3.125 \times 10^{11} \cdot (s + 50000)}{(s + 12500) \cdot (s + 200000)}} \; A \\
&= \frac{(s + 12500) \cdot (s + 200000)}{3.125 \times 10^{10} \cdot s \cdot (s + 50000)} \; A
\end{aligned}$$

After a partiaal fractions expansion (add working in here)

$$I_o(s) = \frac{9}{2500000 \cdot (s + 50000)} + \frac{1}{625000 \cdot s} + \frac{1}{3.125 \times 10^{10}} \; A$$

Using inverse Laplace transform to find $i_o(t)$.

$$\begin{aligned}
i_o(t) &= \mathcal{L}^{-1}\left[I_o(s)\right] \\
&= \mathcal{L}^{-1}\left[\frac{9}{2500000 \cdot (s + 50000)} + \frac{1}{625000 \cdot s} + \frac{1}{3.125 \times 10^{10}}\right] \\
&= 3.6 \times 10^{-6} e^{-50000t} \cdot u(t) + 1.6 \times 10^{-6} \cdot u(t) + 3.2 \times 10^{-11} \delta(t) \; A
\end{aligned}$$

If we assume $t > 0$ only then:

$$i_o(t) = (3.6 \cdot e^{-50000t} + 1.6) \; \mu A$$

2. (a) Convert circtuit to its s-domain equivalent:

ADD CIRCUIT DIAGRAM

The circuit is a voltage divider:

$$\begin{aligned}
V_o(s) &= V_i(s) * \frac{Z_R || Z_C}{Z_R || Z_C + Z_L} \\
&= \frac{5}{s} * \frac{\frac{20s}{s(2s+10)}}{\frac{20s}{s(2s+10)} + 2.5s} \\
&= \frac{5}{s} * \frac{1}{1 + 0.125(2s^2 + 10s)} \\
&= \frac{5}{s(0.25s^2 + 1.25s + 1)} \\
&= \frac{20}{s(s+4)(s+1)}
\end{aligned}$$

Perform partial fraction expansion:

$$\frac{20}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1}$$

$$\therefore 20 = A(s+4)(s+1) + Bs(s+1) + Cs(s+4)$$

Now solve for A, B and C:

$$s = 0 \implies 20 = A(4)(1) \implies A = 5$$

$$s = -4 \implies 20 = B(-4)(-3) \implies B = \frac{5}{3}$$

$$s = -1 \implies 20 = C(-1)(3) \implies C = -\frac{20}{3}$$

And we arrive at $V_o(s)$ in partial fraction expanded form:

$$V_o(s) = \frac{5}{s} + \frac{5}{3(s+4)} - \frac{20}{3(s+1)} \text{ V}$$

(b) Perform the inverse Laplace transform:

$$\begin{aligned} v_o(t) &= \mathcal{L}^{-1}\left[V_o(s)\right] \\ &= \left(5 + \frac{5}{3}e^{-4t} - \frac{20}{3}e^{-t}\right) u(t) \text{ V} \end{aligned}$$

(c) UPDATE TO IMPROVE FLOW OF THIS PASSAGE FROM NOTES, also improve explanation for second paragraph...

Circuit elements the same, therefore s_1, s_2 and form of equation will remain the same. Input voltage source (forcing function) the same therefore steady state component of response will remain the same.

One or both of the exponential terms will change, as these are the only parts of the response equation that are determined by the initial conditions.

3. (a) Convert circuit to its s-domain equivalent, and assume no energy stored at $t = 0^-$:

INSERT CIRCUIT DIAGRAM HERE

Find $V_o(s)$ by recognising circuit is a voltage divider:

$$V_o(s) = V_i(s) * \frac{\frac{5*2s}{5+2s}}{\frac{5*2s}{5+2s} + \frac{10}{s}}$$

$$\begin{aligned} \therefore H(s) &= \frac{10s}{10s + \frac{50}{s} + 20} \\ &= \frac{10s^2}{10s^2 + 20s + 50} \end{aligned}$$

$$\therefore H(s) = \frac{s^2}{s^2 + 2s + 5}$$

(b) We note that the steady state response to the sinusoidal input will be given by the following equation:

$$v_{oSS}(t) = 10 * |H(j20)| \cos(20t + \theta(20))$$

Where $H(j\omega) = |H(j\omega)|e^{j\theta(\omega)}$

Therefore, find $|H(j20)|$ and $\theta(20)$:

$$H(j20) = \frac{-400}{-395 + j40}$$

$$\begin{aligned}\therefore |H(j20)| &= \frac{400}{\sqrt{(-395)^2 + 40^2}} \\ &= 1.008\end{aligned}$$

$$\begin{aligned}\text{And } \theta(20) &= \arctan\left(\frac{0}{-400}\right) - \arctan\left(\frac{40}{-395}\right) \\ &= 180^\circ - 174.22^\circ \\ &= 5.78^\circ\end{aligned}$$

Finally sub these values into the equation for v_{oSS}:

$$v_{oSS}(t) = 10.075 \cos(20t + 5.78^\circ)$$

4. Start by converting the output response equation to its s-domain equivalent:

$$\begin{aligned}V_o(s) &= \mathcal{L}[v_o(t)] \\ &= \frac{10}{s+3} - \frac{20}{s+4} + \frac{10}{s+5} \\ &= \frac{10(s+4)(s+5) - 20(s+3)(s+5) + 10(s+3)(s+4)}{(s+3)(s+4)(s+5)} \\ &= \frac{20}{(s+3)(s+4)(s+5)}\end{aligned}$$

Now by definition, $V_i(s) = \frac{V_o(s)}{H(s)}$, therefore:

$$\begin{aligned}V_i(s) &= \frac{20}{(s+3)(s+4)(s+5)} * \frac{(s+3)(s+4)}{2(s+5)} \\ &= \frac{10}{(s+5)^2}\end{aligned}$$

Now convert the input function back to the time domain:

$$\begin{aligned}v_i(t) &= \mathcal{L}^{-1}[V_i(s)] \\ &= 10te^{-5t}u(t) \text{ V}\end{aligned}$$

5. (a)

$$\begin{aligned}F(s) &= \frac{20s^2 + 141s + 315}{s\left(s^2 + 10s + 21\right)} \\ &= \frac{20s^2 + 141s + 315}{s(s+7)(s+3)}\end{aligned}$$

Perform partial fraction expansion:

$$\frac{20s^2+141s+315}{s(s+7)(s+3)} = \frac{A}{s} + \frac{B}{s+7} + \frac{C}{s+3}$$

$$\therefore 20s^2+141s+315 = A(s+7)(s+3) + Bs(s+3) + Cs(s+7)$$

Now solve for A, B and C:

$$s=0 \implies 315 = A(7)(3) \implies A = 15$$

$$s=-7 \implies 308 = B(-7)(-4) \implies B = 11$$

$$s=-3 \implies 72 = A(-3)(4) \implies C = -6$$

And we arrive at $F(s)$ in partial fraction expanded form:

$$F(s) = \frac{15}{s} + \frac{11}{s+7} - \frac{6}{s+3}$$

Now perform the inverse Laplace transform:

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}[F(s)] \\ &= \left(15 + 11e^{-7t} - 6e^{-3t}\right) u(t) \end{aligned}$$

(b)

$$\begin{aligned} F(s) &= \frac{14s^2+56s+152}{(s+6)\left(s^2+4s+20\right)} \\ &= \frac{14s^2+56s+152}{(s+6)(s+2-j4)(s+2+j4)} \end{aligned}$$

Perform partial fraction expansion:

$$\frac{14s^2+56s+152}{(s+6)(s+2-j4)(s+2+j4)} = \frac{A}{s+6} + \frac{B}{s+2-j4} + \frac{B^*}{s+2+j4}$$

$$\therefore 14s^2+56s+152 = A(s+2-j4)(s+2+j4) + B(s+6)(s+2+j4) + B^*(s+6)(s+2-j4)$$

Now solve for A and B:

$$s=-6 \implies 320 = A(-4-j4)(-4+j4) \implies A = 10$$

$$s=-2+j4 \implies -128 = B(4+j4)(j8) \implies B = 2+j2$$

And we arrive at $F(s)$ in partial fraction expanded form:

$$\begin{aligned} F(s) &= \frac{10}{s+6} + \frac{2+j2}{s+2-j4} + \frac{2-j2}{s+2+j4} \\ &= \frac{10}{s+6} + \frac{2\sqrt{2}\angle 45^\circ}{s+2-j4} + \frac{2\sqrt{2}\angle -45^\circ}{s+2+j4} \end{aligned}$$

Now perform the inverse Laplace transform:

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}[F(s)] \\ &= \left(10e^{-6t} + 4\sqrt{2}e^{-2t}\cos(4t+45^\circ)\right) u(t) \end{aligned}$$

(c)

$$F(s) = \frac{25(s+4)^2}{s^2(s+5)^2}$$

Perform partial fraction expansion:

$$\frac{25(s+4)^2}{s^2(s+5)^2} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+5)^2} + \frac{D}{s+5}$$

$$\therefore 25(s+4)^2 = A(s+5)^2 + Bs(s+5)^2 + Cs^2 + Ds^2(s+5)$$

Now solve for A, B, C and D:

$$s = -5 \implies 25 = C(-5)^2 \implies C = 1$$

$$s = 0 \implies 400 = A(5)^2 \implies A = 16$$

$$s = 1 \implies 625 = 36(16) + B(36) + 1 + D(6)$$

$$\begin{aligned} \therefore 36B + 6D &= 48 \\ 6B + D &= 8 \end{aligned} \tag{1}$$

$$s = -1 \implies 225 = 16(16) - B(16) + 1 + D(4)$$

$$\begin{aligned} \therefore -16B + 4D &= -32 \\ -4B + D &= -8 \end{aligned} \tag{2}$$

Solving equations (1) and (2) gives $B = \frac{8}{5}$ and $D = -\frac{8}{5}$. We then arrive at $F(s)$ in partial fraction expanded form:

$$F(s) = \frac{16}{s^2} + \frac{8}{5s} + \frac{1}{(s+5)^2} - \frac{8}{5(s+5)}$$

Now perform the inverse Laplace transform:

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}[F(s)] \\ &= \left(\frac{8}{5} + 16t + e^{-5t}\left(t - \frac{8}{5}\right)\right) u(t) \end{aligned}$$

6. Q6