Electrical Network and Device Modelling Assignment 2

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1. (a)

   (b)

$$
\begin{aligned}
Z_{eq1} &= \left(\frac{1}{Z_{R_1}} + \frac{1}{Z_{C_1}}\right)^{-1} \\
&= \left(\frac{1}{R_1} + \frac{1}{\frac{1}{sC_1}}\right)^{-1} \\
&= \left(\frac{1}{1.25 \times 10^6} + s \cdot 4 \times 10^{-12}\right)^{-1} \Omega \\
&= \left(\frac{1 + 1.25 \times 10^6 \cdot (s \cdot 4 \times 10^{-12})}{1.25 \times 10^6}\right)^{-1} \Omega \\
&= \frac{1.25 \times 10^6}{1 + 1.25 \times 10^6 \cdot (s \cdot 4 \times 10^{-12})} \Omega \\
&= \frac{2.5 \times 10^{11}}{s + 2 \times 10^5} \Omega
\end{aligned}
$$

   (c)

$$
\begin{aligned}
Z_{eq2} &= \left(\frac{1}{Z_{R_2}} + \frac{1}{Z_{C_2}}\right)^{-1} \\
&= \left(\frac{1}{R_2} + \frac{1}{\frac{1}{sC_2}}\right)^{-1} \\
&= \left(\frac{1}{5 \times 10^6} + s \cdot 16 \times 10^{-12}\right)^{-1} \Omega \\
&= \left(\frac{1 + 5 \times 10^6 \cdot (s \cdot 16 \times 10^{-12})}{5 \times 10^6}\right)^{-1} \Omega \\
&= \frac{5 \times 10^6}{1 + 5 \times 10^6 \cdot (s \cdot 16 \times 10^{-12})} \Omega \\
&= \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} A
\end{aligned}
$$

(d) Using voltage division:

$$
\begin{aligned}
V_o(s) &= V_{in}(s) \cdot \frac{Z_{eq2}}{Z_{eq1} + Z_{eq2}} \\
&= \frac{10}{s} \cdot \frac{\dfrac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}}{\dfrac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} + \dfrac{2.5 \times 10^{11}}{s + 2 \times 10^5}}\ V \\
&= \frac{10}{s} \cdot \frac{\dfrac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}}{\dfrac{(6.25 \times 10^{10})(s + 2 \times 10^5) + (2.5 \times 10^{11})(s + 1.25 \times 10^4)}{(s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}}\ V \\
&= \frac{10}{s} \cdot \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4} \cdot \frac{(s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}{(6.25 \times 10^{10})(s + 2 \times 10^5) + (2.5 \times 10^{11})(s + 1.25 \times 10^4)}\ V \\
&= \frac{6.25 \times 10^{11} \cdot (s + 1.25 \times 10^4) \cdot (s + 2 \times 10^5)}{3.125 \times 10^{11} \cdot s \cdot (s + 1.25 \times 10^4) \cdot (s + 5 \times 10^4)}\ V \\
&= \frac{2 \cdot (s + 2 \times 10^5)}{s \cdot (s + 5 \times 10^4)}\ V \\
&= \frac{2s + 4 \times 10^5}{s^2 + 5 \times 10^4 \cdot s}\ V
\end{aligned}
$$

Using Partial fractions to continue.

$$
\begin{aligned}
\frac{2s + 4 \times 10^5}{s^2 + 5 \times 10^4 \cdot s} &= \frac{A}{s} + \frac{B}{50000} \\
A(s + 50000) + Bs &= 2(s + 200000) \\
(s = 0):\ 50000A &= 400000 \\
\implies A &= 8 \\
(s = -50000):\ -50000B &= 300000 \\
B &= -6 \\
\implies V_o(s) = \frac{8}{s} - \frac{6}{s + 50000}\ V
\end{aligned}
$$

Using inverse Laplace transform to find $v_o(t)$.

$$
\begin{aligned}
v_o(t) &= \mathcal{L}^{-1}[V_o(s)] \\
&= \mathcal{L}^{-1}\left[\frac{8}{s} - \frac{6}{s + 50000}\right]\ V \\
&= (8 - 6e^{-50000t})u(t)\ V
\end{aligned}
$$

(e) Using Ohm's Law in frequency domain:

$$
\begin{aligned}
V_{in}(s) &= Z_{eq} \cdot I_o(s) \\
&= (Z_{eq1} + Z_{eq2}) \cdot I_o(s) \\
\implies I_o(s) &= \frac{V_{in}(s)}{Z_{eq1} + Z_{eq2}} \\
&= \frac{\frac{10}{s}}{\frac{2.5 \times 10^{11}}{s + 2 \times 10^5} + \frac{6.25 \times 10^{10}}{s + 1.25 \times 10^4}} \ A \\
&= \frac{\frac{10}{s}}{\frac{3.125 \times 10^{11} \cdot (s + 50000)}{(s + 12500) \cdot (s + 200000)}} \ A \\
&= \frac{(s + 12500) \cdot (s + 200000)}{3.125 \times 10^{10} \cdot s \cdot (s + 50000)} \ A
\end{aligned}
$$

After a partiaal fractions expansion (add working in here)

$$
I_o(s) = \frac{9}{2500000 \cdot (s + 50000)} + \frac{1}{625000 \cdot s} + \frac{1}{3.125 \times 10^{10}} \ A
$$

Using inverse Laplace transform to find $i_o(t)$.

$$
\begin{aligned}
i_o(t) &= \mathcal{L}^{-1}\left[I_o(s)\right] \\
&= \mathcal{L}^{-1}\left[\frac{9}{2500000 \cdot (s + 50000)} + \frac{1}{625000 \cdot s} + \frac{1}{3.125 \times 10^{10}}\right] \\
&= 3.6 \times 10^{-6} e^{-50000t} \cdot u(t) + 1.6 \times 10^{-6} \cdot u(t) + 3.2 \times 10^{-11} \delta(t) \ A
\end{aligned}
$$

If we assume $t > 0$ only then:

$$
i_o(t) = (3.6 \cdot e^{-50000t} + 1.6) \ \mu A
$$

2. Q2

(a) Using KCL at the node between capacitor, resistor and inductor: $\Sigma i_{out} = 0$

$$
\begin{aligned}
& \frac{V_o(s) - \frac{5}{s}}{2.5s} + \frac{V_o(s)}{2} + V_o(s) \cdot s \cdot 0.1 = 0 \\
\implies & \frac{2 \cdot V_o(s)}{5s} - \frac{2}{s^2} + \frac{V_o(s)}{2} + \frac{s \cdot V_o(s)}{10} = 0 \\
\implies & V_o(s) \left(\frac{2}{5s} + \frac{s}{10} + \frac{1}{2} \right) = \frac{2}{s^2} \\
\implies & V_o(s) \left(\frac{s^2}{10} + \frac{s}{2} + \frac{2}{5} \right) = \frac{2}{s} \\
\implies & V_o(s) \cdot (s^2 + 5s + 4) = \frac{20}{s} \\
\implies & V_o(s) = \frac{20}{s \cdot (s^2 + 5s + 4)} \\
\implies & V_o(s) = \frac{20}{s \cdot (s + 4) \cdot (s + 1)}
\end{aligned}
$$

Now using partial fraction expansion:

$$\begin{aligned}
\frac{20}{s\cdot(s+4)\cdot(s+1)} &= \frac{A}{s} + \frac{B}{s+4} + \frac{c}{s+1} \\
&\implies A(s+4)(s+1) + B(s)(s+1) + C(s)(s+4) = 20 \\
(s=0) &\implies A = 5 \\
(s=-1) &\implies 12B = 20 \\
&\implies B = \frac{5}{3} \\
(s=-1) &\implies -3C = 20 \\
&\implies C = \frac{-20}{3} \\
&\implies V_o(s) = \frac{5}{s} + \frac{5}{3(s+4)} - \frac{20}{3(s+1)}
\end{aligned}$$

(b)

$$\begin{aligned}
v_o(t) &= \mathcal{L}^{-1}[V_o(s)] \\
&= \mathcal{L}^{-1}\left[\frac{5}{s} + \frac{5}{3(s+4)} - \frac{20}{3(s+1)}\right] \\
&= (5 + \frac{5}{3}e^{-4t} - \frac{20}{3}e^{-t})u(t)
\end{aligned}$$

(c)

3. Q3

   (a)

   (b)

4. Q4

5. Q5

   (a)

   (b)

   (c)

6. Q6