ELEN30009 - Electrical Network Analysis and Design
Assignment 4

David Lynch - 758863, Daniel Landgraf - 695683, Zixiang Ren - 765685

Tuesday - 2:15 PM

1. (a) INSERT DIAGRAM HERE
   Using KVL in loop 1:

$$\begin{aligned}
&\Sigma V_{drops} = 0 \\
\implies &V_i + I_i R_i = 0 \\
\implies &V_i = I_i R_i
\end{aligned}$$

   Using KVL in loop 2:

$$\begin{aligned}
&\Sigma V_{drops} = 0 \\
\implies &-V_o + I_o R_o + A_{voc} V_i = 0 \\
\implies &-V_o + I_o R_o + A_{voc} I_i R_i = 0 \\
\implies &I_i = \frac{V_o - I_o R_o}{A_{voc} R_i} \\
\implies &V_i = \frac{V_o - I_o R_o}{A_{voc}}
\end{aligned}$$

   We know that for a two port network, the general matrix equation written in a parameters is:

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

   From the equation obtained before, we can re-write them in matrix form:

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

   Therefore, the A matrix of this voltage amplifier model is:

$$A = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix}$$

   (b) This circuit can be thought of as 3 cascaded two port networks forming a single two port network with a loaded output and a voltage input with source impedance.
   To find the A parameter matrix of the single two port network, first find the A parameter matrices of each amplifier stage and matrix multiply them together to.

$$A_1 = \begin{bmatrix} \frac{1}{10} & -\frac{1\times 10^3}{10} \\ \frac{1}{101\times 10^6} & -\frac{1\times 10^3}{101\times 10^6} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \frac{1}{100} & -\frac{2\times 10^3}{100} \\ \frac{1}{100200\times 10^3} & -\frac{2\times 10^3}{100200\times 10^3} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \frac{1}{2} & -\frac{50\times 10^3}{2} \\ \frac{1}{225\times 10^3} & -\frac{50\times 10^3}{225\times 10^3} \end{bmatrix}$$

$$\begin{aligned} A &= A_1 A_2 A_3 \\ &= \begin{bmatrix} 457\times 10^{-6} & 22.885\ \Omega \\ 457\times 10^{-12}\ \mho & 22.885\times 10^{-6} \end{bmatrix} \end{aligned}$$

2. (a) i. We know that in a Thevenin equivalent circuit, maximum power transfer to the load occurs when $Z_L = Z_{Th}^*$, or for entirely resistive circuits, $R_L = R_{Th}$.

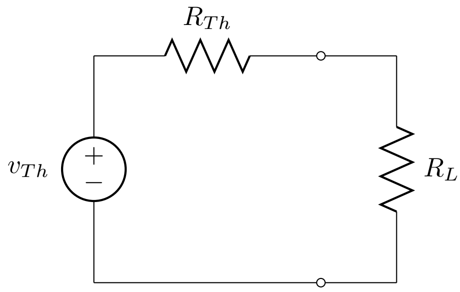


Figure 1: Thevenin equivalent circuit

Now by the formula sheet:

$$\begin{aligned} R_{Th} &= \frac{a_{12} + a_{22}\cdot R_g}{a_{11} + a_{21}\cdot R_g} \\ &= \frac{10 + 1.5\cdot 2}{4 + 0.5\cdot 2} \\ &= 2.6\ \Omega \end{aligned}$$

Therefore, when $R_L = 2.6\ \Omega$, maximum power is transferred to the load resistor.

ii. Maximum power transferred to load can be found from the formula:

$$P_{L\ max} = \frac{v_L^2}{R_L}$$

For the Thevenin equivalent circuit, v_L, the voltage drop across the load, can be found by voltage division. However, since $R_L = R_{Th}$, we know half of v_{Th} will drop across R_L, the other half dropped across R_{Th}.

Now by the formula sheet:

$$\begin{aligned} v_{Th} &= \frac{v_g}{a_{11} + a_{21}\cdot R_g} \\ &= \frac{10}{4 + 0.5\cdot 2} \\ &= 2\ \text{V} \end{aligned}$$

$$\therefore v_L = \frac{v_{Th}}{2} = 1 \text{ V}$$

$$\therefore P_{L\ max} = \frac{1^2}{2.6} = 384.62 \text{ mW}$$

The maximum power delivered to the load is $P_{L\ max} = 384.62$ mW.

iii. From (2a ii), $R_L = 2.6\ \Omega \quad \therefore v_L = 1$ V.
From this, i_L can be found by Ohm's law:

$$i_L = \frac{v_L}{R_L} = \frac{1}{2.6} = 384.62 \text{ mA}$$

Now by the formula sheet:

$$\frac{i_2}{i_1} = \frac{-1}{a_{21} \cdot R_L + a_{22}}$$

And from this, i_1 can be found from i_L, where we note $i_2 = -i_L$:

$$\begin{aligned} i_1 &= -i_2 \left(a_{21} \cdot R_L + a_{22}\right) \\ &= i_L \left(a_{21} \cdot R_L + a_{22}\right) \\ &= 384.62 * 10^{-3} \left(0.5 \cdot 2.6 + 1.5\right) \\ &= 1.077 \text{ A} \end{aligned}$$

And so, the current flowing into port 1 is $i_1 = 1.077$ A.

(b) For measurement 1, we have the constraint that $V_2 = 0$ V.
For measurement 2, we have the constraing that $V_1 = 0$ V.

Therefore, we want to find a set of two-port network parameters that have V_1 and V_2 as the independent variables.

We find the y parameters meet this condition:

$$i_1 = y_{11} \cdot v_1 + y_{12} \cdot v_2 \qquad i_2 = y_{21} \cdot v_1 + y_{22} \cdot v_2$$

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} \text{ S} \qquad y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} \text{ S}$$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0} \text{ S} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} \text{ S}$$

By measurement 1:

$$y_{11} = \frac{1}{10} = 100 \text{ mS} \qquad y_{21} = \frac{-0.5}{10} = -50 \text{ mS}$$

By measurement 2:

$$y_{12} = \frac{-1}{20} = -50 \text{ mS} \qquad y_{22} = \frac{3}{20} = 150 \text{ mS}$$

Now for cascade of two-port networks, $[A_T] = [A_1] \times [A_2]$, where these are the a parameter matrices for the overall network, network 1 and network 2 respectively.

Therefore convert above y parameters to a parameters to make finding overall network parameters easier (note $\Delta[Y]$ is the discriminant of the y parameter matrix):

$$a_{11} = -\frac{y_{22}}{y_{21}} = 3 \qquad\qquad a_{12} = -\frac{1}{y_{21}} = 20\ \Omega$$
$$a_{21} = -\frac{\Delta[Y]}{y_{21}} = 0.25\ \text{S} \qquad\qquad a_{22} = -\frac{y_{11}}{y_{21}} = 2$$

As stated above, we can find the a parameters of the overall two-port network by matrix multiplying the a parameters of the constituent cascaded two-port networks:

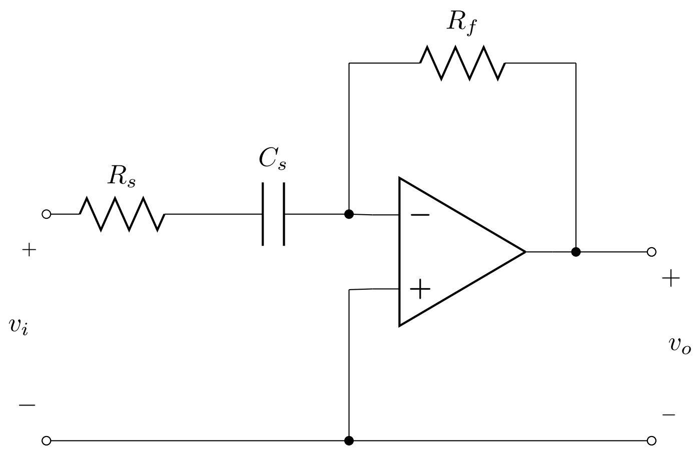
$$\begin{aligned}
[A_T] &= [A_1] \times [A_2] \\
&= \begin{bmatrix} 4 & 10\ \Omega \\ 0.5\ \text{S} & 1.5 \end{bmatrix} \begin{bmatrix} 3 & 20\ \Omega \\ 0.25\ \text{S2} & \end{bmatrix} \\
&= \begin{bmatrix} 14.5 & 100\ \Omega \\ 1.875\ \text{S} & 13 \end{bmatrix}
\end{aligned}$$

And so we find that the a parameters of the cascaded network are $a_{11} = 14.5$, $a_{12} = 100\ \Omega$, $a_{21} = 1.875$ S and $a_{22} = 13$.

3. (a) Design 2^{nd} order filter by cascading two first order filters together, followed by a gain stage.

Design the filter by starting with prototype stages then frequency and magnitude scaling to get desired properties.

Now a prototype first order active high pass filter has unity gain in the passband and a cutoff frequency of $\omega_c = 1$ rad/s, and is of form:



Noting that a circuit of this form has transfer function:

$$H_L(s) = \frac{-s}{s+1}$$

Now when two of these are connected in cascade, the overall transfer function is:

$$\begin{aligned}
H(s) &= H_L(s) \cdot H_L(s) \\
&= \frac{s^2}{(s+1)^2}
\end{aligned}$$

(b)

(c)

4. i

ii

iii

5.