# Electrical Network and Device Modelling Assignment 4

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1. (a) INSERT DIAGRAM HERE

   Using KVL in loop 1:

   $$\Sigma V_{drops} = 0$$
   $$\implies V_i + I_i R_i = 0$$
   $$\implies V_i = I_i R_i$$

   Using KVL in loop 2:

   $$\Sigma V_{drops} = 0$$
   $$\implies -V_o + I_o R_o + A_{voc} V_i = 0$$
   $$\implies -V_o + I_o R_o + A_{voc} I_i R_i = 0$$
   $$\implies I_i = \frac{V_o - I_o R_o}{A_{voc} R_i}$$
   $$\implies V_i = \frac{V_o - I_o R_o}{A_{voc}}$$

   We know that for a two port network, the general matrix equation written in $a$ parameters is:

   $$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

   From the equation obtained before, we can re-write them in matrix form:

   $$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

   Therefore, the $A$ matrix of this voltage amplifier model is:

   $$A = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix}$$

   (b) This circuit can be thought of as 3 cascaded two port networks forming a single two port network with a loaded output and a voltage input with source impedance.
   To find the $A$ parameter matrix of the single two port network, first find the $A$ parameter matrices of each amplifier stage and matrix multiply them together to.

$$A_1 = \begin{bmatrix} \frac{1}{10} & -\frac{1 \times 10^3}{10} \\ \frac{1}{(10)(1 \times 10^6)} & -\frac{1 \times 10^3}{(10)(1 \times 10^6)} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \frac{1}{100} & -\frac{2 \times 10^3}{100} \\ \frac{1}{(100)(200 \times 10^3)} & -\frac{2 \times 10^3}{(100)(200 \times 10^3)} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \frac{1}{2} & -\frac{50}{2} \\ \frac{1}{(2)(25 \times 10^3)} & -\frac{50}{(2)(25 \times 10^3)} \end{bmatrix}$$

$$A = \mathbf{A_1 A_2 A_3} \text{ (Bold indicating no minus signs on } A_{i_{j2}})$$

$$= \begin{bmatrix} 542.7 \times 10^{-6} & -27.135 \times 10^{-3} \ \Omega \\ 542.7 \times 10^{-12} \ \mho & -27.1350 \times 10^{-6} \end{bmatrix}$$

Now the circuit can be modelled like this:

INSERT DIAGRAM HERE

Using Table 18.2 from the formula sheet, the formula

$$\frac{V_2}{V_g} = \frac{\mathbf{Z}_L}{(a_{11} + a_{21}\mathbf{Z}_g)\mathbf{Z}_L + a_{12} + a_{22}\mathbf{Z}_g}$$

can be used here.

$$G = \frac{v_L}{v_s}$$

$$= \frac{R_L}{(a_{11} + a_{21}R_s)R_L + a_{12} + a_{22}R_s}$$

$$= \frac{400}{(542.7 \times 10^{-6} + (542.7 \times 10^{-12})(50 \times 10^3))(400) + 27.135 \times 10^{-3} + (27.1350 \times 10^{-6})(50 \times 10^3)}$$

$$= 2.0056 \times 10^3$$

2. (a)   i.
      ii.
      iii.

   (b)

3. (a) Filter will have the structure:
      INSERT DIAGRAM HERE!
      For the filter stages, will use prototype filters and scaling to find values of components.

$$R_p = 1 \ \Omega$$
$$C_p = 1 \ F$$

To find prototype cutoff frequency, must solve:

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

$$\left| \left( \frac{j\omega_c}{j\omega_c + 1} \right)^n \right| = \frac{1}{\sqrt{2}}$$

$$\implies \frac{\omega_c^n}{\left( \sqrt{\omega_c^2 + 1} \right)^n} = \frac{1}{\sqrt{2}}$$

$$\implies \frac{\omega_c^n}{(\omega_c^2 + 1)^{\frac{n}{2}}} = \frac{1}{2^{\frac{1}{2}}}$$

$$\implies \frac{\omega_c^{2n}}{(\omega_c^2 + 1)^n} = \frac{1}{2}$$

$$\implies \omega_c^2 - 2^{\frac{1}{n}}\omega_c^2 + 1 = 0$$

$$\implies \omega_c = \pm \frac{1}{\sqrt{\sqrt[n]{2} - 1}}$$

$$\omega_c > 0$$

$$\implies \omega_c = \frac{1}{\sqrt{\sqrt[n]{2} - 1}}$$

Therefore, for our prototype cascaded filter:

$$\omega_{cp} = \frac{1}{\sqrt{\sqrt[2]{2} - 1}} = 1.5538 \; rad/s$$

Frequency scaling:

$$K_f = \frac{\omega_c}{\omega_{cp}}$$

$$= \frac{(2\pi \; rad)(1000 \; Hz)}{1.5538 \; rad/s}$$

$$= 4043.8220$$

Magnitude scaling:

$$C = \frac{1}{K_f K_m} C_p$$

$$\implies K_m = \frac{C_p}{K_f C}$$

$$= \frac{1 \; F}{(4043.8220)(100 \; nF)}$$

$$= 2472.9081$$

Resistor Value:

$$R = K_m R_p$$

$$= (2472.9081)(1 \; \Omega)$$

$$= 2472.9081 \; \Omega$$

Gain stage:
Resistors in the gain stage are independent of the frequency response of the circuit, and are just chosen to get the correct gain. For this configuration of op-amp:

$$G = 1 + \frac{R_f}{R_s}$$

For a gain of 10, choosing $R_f = 9 \; K\Omega$ gives $R_s = 1 \; K\Omega$.
INSERT DIAGRAM OF COMPLETED CIRCUIT WITH VALUES HERE:

(b) Transfer function of the whole circuit is the product of the transfer function of each stage in the amplifier, noting that stage 1 and stage 2 are identical:

$$H(s) = (H_1(s))^2 H_3(s)$$

Where $H_1(s)$ is the transfer function for one amplification stage and $H_3(s)$ is the transfer function for the gain stage.

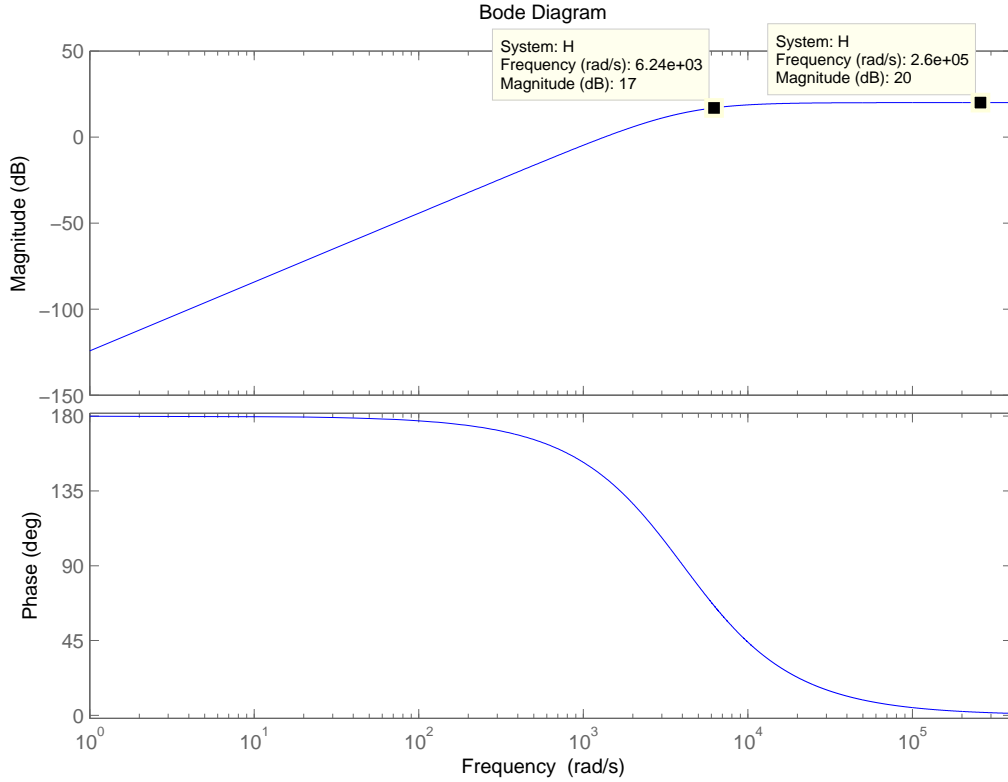As $H_3(s)$ is frequency independent, we can say that $H_3(s) = 10$

For the amplification stage: High pass filter has a transfer function in the form

$$H(s) = \frac{-R_f}{R_s} \frac{s}{s + \frac{1}{R_s C}}$$

For our amplification stages, $R_f = R_s = 2472.9081 \ \Omega$ and $C = 100 \ nF$. Therefore:

$$H_1(s) = \frac{-2472.9081 \ \Omega}{2472.9081 \ \Omega} \frac{s}{s + \frac{1}{(2472.9081 \ \Omega)(100 \ nF)}}$$

$$= \frac{s}{s + 4.0438 \times 10^3}$$

$$H(s) = (H_1(s))^2 H_3(s)$$

$$= 10 \left( \frac{s}{s + 4.0438 \times 10^3} \right)^2$$

$$= \frac{10s^2}{s^2 + (8.0876 \times 10^3)s + 16.3525 \times 10^6}$$

(c)



Bode Diagram

System: H
Frequency (rad/s): 6.24e+03
Magnitude (dB): 17

System: H
Frequency (rad/s): 2.6e+05
Magnitude (dB): 20

$$6.24 \times 10^3 \ rad/s = \frac{6.24 \times 10^3}{2\pi} \ Hz$$
$$= 993.127 \ Hz$$
$$\approx 1000 \ Hz$$

Therefore, this transfer function satisfies the specifications.

4. i

   ii

   iii

5. Assuming an ideal op-amp, meaning virtual short circuit condition is true ($v_p = v_n = 0$) and no current flows into the non-inverting and inverting terminals of the op-amp.

Circuit in transformed s-domain is:

INSERT DIAGRAM HERE

Using KCL at node ①: $\Sigma I_{out} = 0$

$$\frac{V_P - V_I}{\frac{1}{sC_1} + R_1} + \frac{V_p - V_o}{\frac{1}{C_2 s} || R_2} = 0$$

$$\implies \frac{-V_I}{\frac{R_1 C_1 s + 1}{sC_1}} + \frac{-V_o}{\frac{R_2}{C_2 R_2 s + 1}} = 0$$

$$\implies \frac{V_o(C_2 R_2 s + 1)}{R_2} = -\frac{V_I(C_1 s)}{R_1 C_1 s + 1}$$

$$\implies \frac{V_o}{V_I} = -\frac{R_2 C_1 s}{(R_1 C_1 s + 1)(C_2 R_2 s + 1)}$$

$$\implies H(s) = -\frac{R_2 C_1 s}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2)s + 1}$$

$$\implies H(s) = -\frac{\frac{1}{R_1 C_2} s}{s^2 + \frac{R_1 C_1 + R_2 C_2}{R_1 C_1 R_2 C_2} s + \frac{1}{R_1 C_1 R_2 C_2}}$$

$$\implies H(s) = -\frac{\frac{1}{(500 \ \Omega)(10\mu F)} s}{s^2 + \frac{(500 \ \Omega)(40\mu F) + (4k\Omega)(10\mu F)}{(500 \ \Omega)(40\mu F)(4k\Omega)(10\mu F)} s + \frac{1}{(500 \ \Omega)(40\mu F)(4k\Omega)(10\mu F)}}$$

$$\implies H(s) = \frac{-200s}{s^2 + 75s + 1250}$$

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