

Electrical Network and Device Modelling Assignment 4

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1. (a) INSERT DIAGRAM HERE

Using KVL in loop 1:

$$\begin{aligned}\Sigma V_{drops} &= 0 \\ \implies V_i + I_i R_i &= 0 \\ \implies V_i &= -I_i R_i\end{aligned}$$

Using KVL in loop 2:

$$\begin{aligned}\Sigma V_{drops} &= 0 \\ \implies -V_o + I_o R_o + A_{voc} V_i &= 0 \\ \implies -V_o + I_o R_o + A_{voc} I_i R_i &= 0 \\ \implies I_i &= \frac{V_o - I_o R_o}{A_{voc} R_i} \\ \implies V_i &= \frac{V_o - I_o R_o}{A_{voc}}\end{aligned}$$

We know that for a two port network, the general matrix equation written in a parameters is:

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

From the equation obtained before, we can re-write them in matrix form:

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

Therefore, the A matrix of this voltage amplifier model is:

$$A = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix}$$

- (b) This circuit can be thought of as 3 cascaded two port networks forming a single two port network with a loaded output and a voltage input with source impedance. To find the A parameter matrix of the single two port network, first find the A parameter matrices of each amplifier stage and matrix multiply them together to.

$$\begin{aligned}
A_1 &= \begin{bmatrix} \frac{1}{10} & -\frac{1 \times 10^3}{10} \\ \frac{1}{(10)(1 \times 10^6)} & -\frac{1 \times 10^3}{(10)(1 \times 10^6)} \end{bmatrix} \\
A_2 &= \begin{bmatrix} \frac{1}{100} & -\frac{2 \times 10^3}{100} \\ \frac{1}{(100)(200 \times 10^3)} & -\frac{2 \times 10^3}{(100)(200 \times 10^3)} \end{bmatrix} \\
A_3 &= \begin{bmatrix} \frac{1}{2} & -\frac{50}{2} \\ \frac{1}{(2)(25 \times 10^3)} & -\frac{50}{(2)(25 \times 10^3)} \end{bmatrix} \\
A &= A_1 A_2 A_3 \\
&= \begin{bmatrix} 457 \times 10^{-6} & -22.885 \times 10^{-3} \Omega \\ 457 \times 10^{-12} \cup & -22.885 \times 10^{-9} \end{bmatrix}
\end{aligned}$$

Now the circuit can be modelled like this:

INSERT DIAGRAM HERE

Using Table 18.2 from the formula sheet, the formula

$$\frac{V_2}{V_g} = \frac{\mathbf{Z}_L}{(a_{11} + a_{21}\mathbf{Z}_g)\mathbf{Z}_L + a_{12} + a_{22}\mathbf{Z}_g}$$

can be used here.

$$\begin{aligned}
G &= \frac{v_L}{v_s} \\
&= \frac{R_L}{(a_{11} + a_{21}R_s)R_L + a_{12} + a_{22}R_s} \\
&= \frac{400}{(457 \times 10^{-6} + (457 \times 10^{-12})(50 \times 10^3))(400) + 22.885 \times 10^{-3} + (22.885 \times 10^{-9})(50 \times 10^3)} \\
&= 1.8496 \times 10^3
\end{aligned}$$

2. (a) i.
- ii.
- iii.

(b)

3. (a) Filter will have the structure:

INSERT DIAGRAM HERE!

For the filter stages, will use prototype filters and scaling to find values of components.

$$\begin{aligned}
R_p &= 1 \Omega \\
C_p &= 1 F
\end{aligned}$$

To find prototype cutoff frequency, must solve:

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

$$\begin{aligned}
& \left| \left(\frac{j\omega_c}{j\omega_c + 1} \right)^n \right| = \frac{1}{\sqrt{2}} \\
\Rightarrow & \frac{\omega_c^n}{\left(\sqrt{\omega_c^2 + 1} \right)^n} = \frac{1}{\sqrt{2}} \\
\Rightarrow & \frac{\omega_c^n}{(\omega_c^2 + 1)^{\frac{n}{2}}} = \frac{1}{2^{\frac{1}{2}}} \\
\Rightarrow & \frac{\omega_c^{2n}}{(\omega_c^2 + 1)^n} = \frac{1}{2} \\
\Rightarrow & \omega_c^2 - 2^{\frac{1}{n}} \omega_c^2 + 1 = 0 \\
\Rightarrow & \omega_c = \pm \frac{1}{\sqrt[n]{2} - 1} \\
& \omega_c > 0 \\
\Rightarrow & \omega_c = \frac{1}{\sqrt[n]{2} - 1}
\end{aligned}$$

Therefore, for our prototype cascaded filter:

$$\omega_{cp} = \frac{1}{\sqrt[n]{2} - 1} = 1.5538 \text{ rad/s}$$

Frequency scaling:

$$\begin{aligned}
K_f &= \frac{\omega_c}{\omega_{cp}} \\
&= \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{1.5538 \text{ rad/s}} \\
&= 4043.8220
\end{aligned}$$

Magnitude scaling:

$$\begin{aligned}
C &= \frac{1}{K_f K_m} C_p \\
\Rightarrow K_m &= \frac{C_p}{K_f C} \\
&= \frac{1 \text{ F}}{(4043.8220)(100 \text{ nF})} \\
&= 2472.9081
\end{aligned}$$

Resistor Value:

$$\begin{aligned}
R &= K_m R_p \\
&= (2472.9081)(1 \Omega) \\
&= 2472.9081 \Omega
\end{aligned}$$

Gain stage:

Resistors in the gain stage are independent of the frequency response of the circuit, and are just chosen to get the correct gain. For this configuration of op-amp:

$$G = 1 + \frac{R_f}{R_s}$$

For a gain of 10, choosing $R_f = 9 \text{ K}\Omega$ gives $R_s = 1 \text{ K}\Omega$.

INSERT DIAGRAM OF COMPLETED CIRCUIT WITH VALUES HERE:

- (b) Transfer function of the whole circuit is the product of the transfer function of each stage in the amplifier, noting that stage 1 and stage 2 are identical:

$$H(s) = (H_1(s))^2 H_3(s)$$

Where $H_1(s)$ is the transfer function for one amplification stage and $H_3(s)$ is the transfer function for the gain stage.

As $H_3(s)$ is frequency independent, we can say that $H_3(s) = 10$

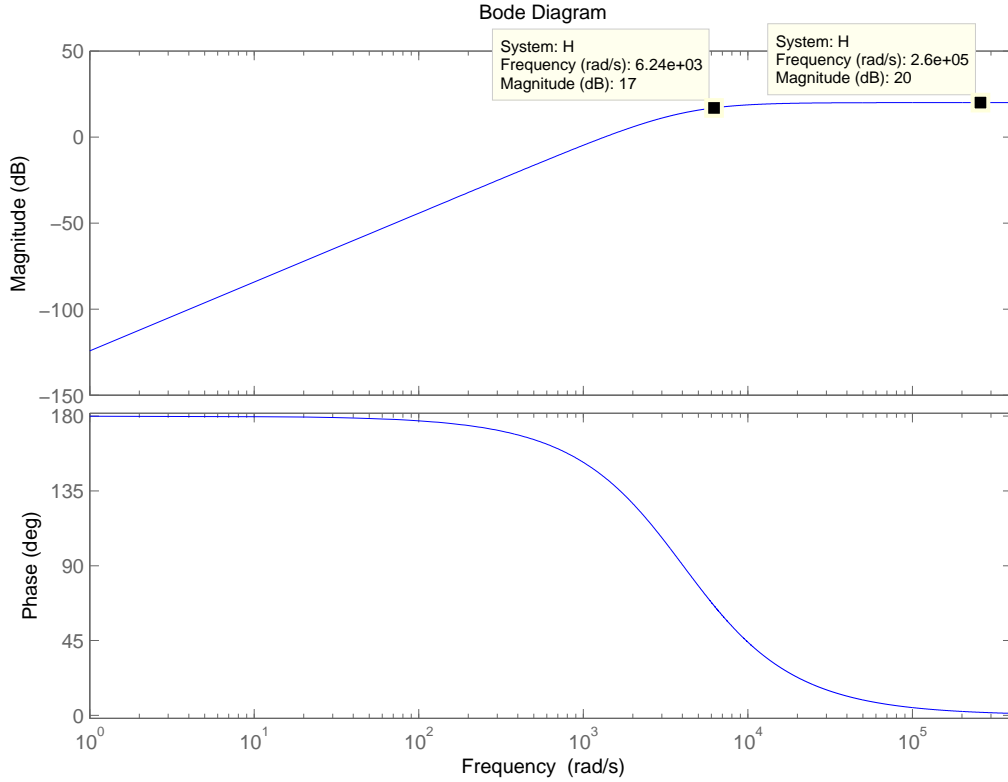
For the amplification stage: High pass filter has a transfer function in the form

$$H(s) = \frac{-R_f}{R_s} \frac{s}{s + \frac{1}{R_s C}}$$

For our amplification stages, $R_f = R_s = 2472.9081 \Omega$ and $C = 100 \text{ nF}$. Therefore:

$$\begin{aligned} H_1(s) &= \frac{-2472.9081 \Omega}{2472.9081 \Omega} \frac{s}{s + \frac{1}{(2472.9081 \Omega)(100 \text{ nF})}} \\ &= \frac{s}{s + 4.0438 \times 10^3} \\ H(s) &= (H_1(s))^2 H_3(s) \\ &= 10 \left(\frac{s}{s + 4.0438 \times 10^3} \right)^2 \\ &= \frac{10s^2}{s^2 + (8.0876 \times 10^3)s + 16.3525 \times 10^6} \end{aligned}$$

- (c)



$$\begin{aligned}
6.24 \times 10^3 \text{ rad/s} &= \frac{6.24 \times 10^3}{2\pi} \text{ Hz} \\
&= 993.127 \text{ Hz} \\
&\approx 1000 \text{ Hz}
\end{aligned}$$

Therefore, this transfer function satisfies the specifications.

4. i
- ii
- iii

5. Assuming an ideal op-amp, meaning virtual short circuit condition is true ($v_p = v_n = 0$) and no current flows into the non-inverting and inverting terminals of the op-amp.

Circuit in transformed s-domain is:

INSERT DIAGRAM HERE

Using KCL at node (1): $\Sigma I_{out} = 0$

$$\begin{aligned}
&\frac{V_P - V_I}{\frac{1}{sC_1} + R_1} + \frac{V_p - V_o}{\frac{1}{C_2 s} || R_2} = 0 \\
\Rightarrow &\frac{-V_I}{\frac{R_1 C_1 s + 1}{sC_1}} + \frac{-V_o}{\frac{R_2}{C_2 R_2 s + 1}} = 0 \\
\Rightarrow &\frac{V_o(C_2 R_2 s + 1)}{R_2} = -\frac{V_I(C_1 s)}{R_1 C_1 s + 1} \\
\Rightarrow &\frac{V_o}{V_I} = -\frac{R_2 C_1 s}{(R_1 C_1 s + 1)(C_2 R_2 s + 1)} \\
\Rightarrow &H(s) = -\frac{R_2 C_1 s}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2)s + 1} \\
\Rightarrow &H(s) = -\frac{\frac{1}{R_1 C_2} s}{s^2 + \frac{R_1 C_1 + R_2 C_2}{R_1 C_1 R_2 C_2} s + \frac{1}{R_1 C_1 R_2 C_2}} \\
\Rightarrow &H(s) = -\frac{\frac{1}{(500 \Omega)(10 \mu F)} s}{s^2 + \frac{(500 \Omega)(40 \mu F) + (4k\Omega)(10 \mu F)}{(500 \Omega)(40 \mu F)(4k\Omega)(10 \mu F)} s + \frac{1}{(500 \Omega)(40 \mu F)(4k\Omega)(10 \mu F)}} \\
\Rightarrow &H(s) = \frac{-200s}{s^2 + 75s + 1250}
\end{aligned}$$