Electrical Network and Device Modelling Assignment 4

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1. (a) INSERT DIAGRAM HERE
   Using KVL in loop 1:

$$\begin{aligned}
&\Sigma V_{drops} = 0\\
\implies &V_i + I_i R_i = 0\\
\implies &V_i = I_i R_i
\end{aligned}$$

   Using KVL in loop 2:

$$\begin{aligned}
&\Sigma V_{drops} = 0\\
\implies &-V_o + I_o R_o + A_{voc} V_i = 0\\
\implies &-V_o + I_o R_o + A_{voc} I_i R_i = 0\\
\implies &I_i = \frac{V_o - I_o R_o}{A_{voc} R_i}\\
\implies &V_i = \frac{V_o - I_o R_o}{A_{voc}}
\end{aligned}$$

   We know that for a two port network, the general matrix equation written in a parameters is:

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

   From the equation obtained before, we can re-write them in matrix form:

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

   Therefore, the A matrix of this voltage amplifier model is:

$$A = \begin{bmatrix} \frac{1}{A_{voc}} & -\frac{R_o}{A_{voc}} \\ \frac{1}{A_{voc} R_i} & -\frac{R_o}{A_{voc} R_i} \end{bmatrix}$$

   (b) This circuit can be thought of as 3 cascaded two port networks forming a single two port network with a loaded output and a voltage input with source impedance.
   To find the A parameter matrix of the single two port network, first find the A parameter matrices of each amplifier stage and matrix multiply them together to.

$$A_1 = \begin{bmatrix} \frac{1}{10} & -\frac{1\times10^3}{10} \\ \frac{1}{101\times10^6} & -\frac{1\times10^3}{101\times10^6} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} \frac{1}{100} & -\frac{2\times10^3}{100} \\ \frac{1}{100200\times10^3} & -\frac{2\times10^3}{100200\times10^3} \end{bmatrix}$$
$$A_3 = \begin{bmatrix} \frac{1}{2} & -\frac{50\times10^3}{2} \\ \frac{1}{225\times10^3} & -\frac{50\times10^3}{225\times10^3} \end{bmatrix}$$
$$\begin{aligned} A &= A_1A_2A_3 \\ &= \begin{bmatrix} 457\times10^{-6} & 22.885\ \Omega \\ 457\times10^{-12}\ \mho & 22.885\times10^{-6} \end{bmatrix} \end{aligned}$$

Now the circuit can be modelled like this:
INSERT DIAGRAM HERE
Using Table 18.2 from the formula sheet, the formula

$$\frac{V_2}{V_g} = \frac{\mathbf{Z}_L}{(a_{11}+a_{21}\mathbf{Z}_g)\mathbf{Z}_L + a_{12} + a_{22}\mathbf{Z}_g}$$

can be used here.

$$\begin{aligned} \frac{v_L}{v_s} &= \frac{R_L}{(a_{11}+a_{21}R_s)R_L + a_{12} + a_{22}R_s} \\ &= \frac{400}{(457\times10^{-6} + (457\times10^{-12})(50\times10^3))(400) + 22.885 + (22.885\times10^{-6})(50\times10^3)} \\ &= 16.5143 \end{aligned}$$

2. (a) i.
   ii.
   iii.
   (b)

3. (a) Filter will have the structure:
   INSERT DIAGRAM HERE!
   For the filter stages, will use prototype filters and scaling to find values of components.

$$\begin{aligned} R_p &= 1\ \Omega \\ C_p &= 1\ F \end{aligned}$$

   To find prototype cutoff frequency, must solve:

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}H_{max}$$

$$
\begin{aligned}
&\left|\left(\frac{j\omega_c}{j\omega_c+1}\right)^n\right| = \frac{1}{\sqrt{2}} \\
\implies &\frac{\omega_c^n}{\left(\sqrt{\omega_c^2+1}\right)^n} = \frac{1}{\sqrt{2}} \\
\implies &\frac{\omega_c^n}{(\omega_c^2+1)^{\frac{n}{2}}} = \frac{1}{2^{\frac{1}{2}}} \\
\implies &\frac{\omega_c^{2n}}{(\omega_c^2+1)^n} = \frac{1}{2} \\
\implies &\omega_c^2 - 2^{\frac{1}{n}}\omega_c^2 + 1 = 0 \\
\implies &\omega_c = \pm\frac{1}{\sqrt{\sqrt[n]{2}-1}} \\
&\omega_c > 0 \\
\implies &\omega_c = \frac{1}{\sqrt{\sqrt[n]{2}-1}}
\end{aligned}
$$

Therefore, for our prototype cascaded filter:

$$\omega_{cp} = \frac{1}{\sqrt{\sqrt[2]{2}-1}} = 1.5538\ rad/s$$

Frequency scaling:

$$
\begin{aligned}
K_f &= \frac{\omega_c}{\omega_{cp}} \\
&= \frac{(2\pi\ rad)(1000\ Hz)}{1.5538\ rad/s} \\
&= 4043.8220
\end{aligned}
$$

Magnitude scaling:

$$
\begin{aligned}
C &= \frac{1}{K_f K_m} C_p \\
\implies K_m &= \frac{C_p}{K_f C} \\
&= \frac{1\ F}{(4043.8220)(100\ nF)} \\
&= 2472.9081
\end{aligned}
$$

Resistor Value:

$$
\begin{aligned}
R &= K_m R_p \\
&= (2472.9081)(1\ \Omega) \\
&= 2472.9081\ \Omega
\end{aligned}
$$

Gain stage:
Resistors in the gain stage are independent of the frequency response of the circuit, and are just chosen to get the correct gain. For this configuration of op-amp:

$$G = 1 + \frac{R_f}{R_s}$$

For a gain of 10, choosing $R_f = 9\ K\Omega$ gives $R_s = 1\ K\Omega$.
INSERT DIAGRAM OF COMPLETED CIRCUIT WITH VALUES HERE:

(b) Transfer function of the whole circuit is the product of the transfer function of each stage in the amplifier, noting that stage 1 and stage 2 are identical:

$$H(s) = (H_1(s))^2 H_3(s)$$

Where $H_1(s)$ is the transfer function for one amplification stage and $H_3(s)$ is the transfer function for the gain stage.
As $H_3(s)$ is frequency independent, we can say that $H_3(s) = 10$

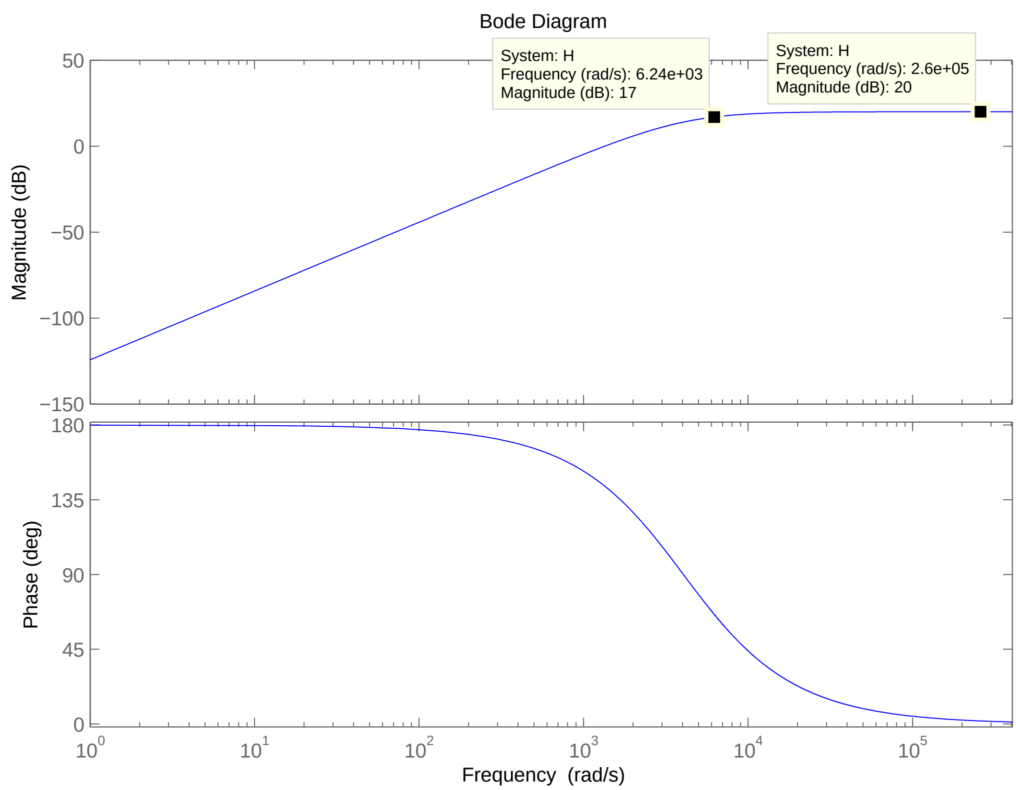
For the amplification stage: High pass filter has a transfer function in the form

$$H(s) = \frac{-R_f}{R_s} \frac{s}{s + \frac{1}{R_s C}}$$

For our amplification stages, $R_f = R_s = 2472.9081\ \Omega$ and $C = 100\ nF$. Therefore:

$$\begin{aligned} H_1(s) &= \frac{-2472.9081\ \Omega}{2472.9081\ \Omega} \frac{s}{s + \frac{1}{(2472.9081\ \Omega)(100\ nF)}} \\ &= \frac{s}{s + 4.0438 \times 10^3} \\ H(s) &= (H_1(s))^2 H_3(s) \\ &= 10 \left(\frac{s}{s + 4.0438 \times 10^3} \right)^2 \\ &= \frac{10s^2}{s^2 + (8.0876 \times 10^3)s + 16.3525 \times 10^6} \end{aligned}$$

(c)



$$
\begin{aligned}
6.24 \times 10^3 \; rad/s &= \frac{6.24 \times 10^3}{2\pi} \; Hz \\
&= 993.127 \; Hz \\
&\approx 1000 \; Hz
\end{aligned}
$$

4. i

   ii

   iii

5.