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110550062

Assignment 3

1. (20%) Use this ordinary difference table:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0.12	0.79168	-0.01834	-0.01129	0.00134	0.00038
0.24	0.77334	-0.02963	-0.00995	0.00172	0.00028
0.36	0.74371	-0.03958	-0.00823	0.00200	
0.48	0.70413	-0.04781	-0.00623		
0.60	0.65632	-0.05404			
0.72	0.60228				

Result:

```
>> Q1_110550062
The estimate of f(0.231) using Newton-Gregory polynomial of degree 2 with x0=0.12 is
   0.775107121875000

The estimate of f(0.231) using third-degree polynomial is
   0.775123777656250

The estimate of f(0.231) using fourth-degree polynomial is
   0.775121327454102

The error in the estimate of f(0.231) using Newton-Gregory polynomial of degree 2 with x0=0.12 is
     1.420557910158937e-05

The error in the estimate of f(0.231) using Newton-Gregory polynomial of degree 3 with x0=0.12 is
     2.450202148396308e-06
```

(a) Estimate f(0.231) from the Newton- Gregory polynomial of degree-2 with x0 = 0.12.

$$P_2(0.231) = f(0.12) + (0.231 - 0.12)\frac{\Delta f(0.12)}{0.12} + (0.231 - 0.12)(0.231 - 0.12 - 1)\frac{\Delta^2 f(0.12)}{2!\,0.12^2}$$

$$= \mathbf{0.77510712}$$

(b) Add one term to part (a) to get f(0.231) from the third-degree polynomial.

$$P_3(0.231) = P_2(0.231) + (0.231 - 0.12)(0.231 - 0.12 - 1)(0.231 - 0.12 - 2)\frac{\Delta^3 f(0.12)}{3!\,0.12^3}$$

$$= \mathbf{0.77512377}$$

(c) Estimate the errors of both parts (a) and (b).

$$\boldsymbol{Error = (x - x_0)(x - x_1)\ \dots\dots\ (x - x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}}$$

$\boldsymbol{E_2(0.231) = 1.4206e - 05}$

$\boldsymbol{E_3(0.231) = 2.4502e - 06}$

/ E2 = abs((s*(s-1)/2)*d2f(1)*(0.231 - 0.12)^2*(0.231 - 0.24)/(factorial(2))

/ E3 = abs((s*(s-1)*(s-2)/6)*d3f(1)*(0.231 - 0.12)^3*(0.231 - 0.24)^2*(0.231 - 0.36)/factorial(3))

(d) Is it better to start with x0 = 0.24 or with x0 = 0.36 when getting f(0.42) from quadratic? Justify your answer.

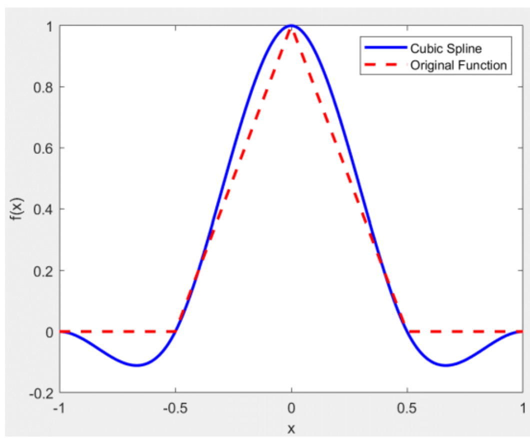
If interpolating **f(0.42)** from a quadratic using the given difference table, it is best to use the row with **x=0.36**.

The reason is that <u>the approximations tend to be better when the nodes of interpolation are centered around the interpolating point</u>.

If the row with x=0.24 is used, the interpolating nodes for a quadratic are 0.12, 0.24, and 0.36. If the row with x=0.36 is used, the interpolating nodes for a quadratic are 0.24, 0.36, and 0.48, and x=0.42 lies in the smallest interval containing 0.24, 0.36, and 0.48, meaning that using the row with x=0.36 is best centered.

2. Fit the function below with a natural cubic spline that matches to f(x) at five evenly spaced points in [-1, 1] and forces the slopes at the ends to be zero. Plot the spline curve together with f(x).

$$f(x) = \begin{cases} 0, & -1 < x < -0.5 \\ 1 - |2x|, & -0.5 < x < 0.5 \\ 0, & 0.5 < x < 1 \end{cases}$$



Plot of Natural Cubic Spline and Original Function

Q1_110550062.m   Q2_110550062.m

```
% Define the function f(x)
f = @(x) (x >= -1 & x < -0.5) .* 0 + (x >= -0.5 & x < 0.5) .* (1 - abs(2*x)) + (x >= 0.5 & x <= 1) .* 0;

% Define the evenly spaced points to match the spline to
x = linspace(-1, 1, 5);
y = f(x);

% Fit the natural cubic spline with zero end slopes
pp = spline(x, [0 y 0]);

% Evaluate the spline curve at a denser set of points for plotting
xx = linspace(-1, 1, 1000);
yy = ppval(pp, xx);

% Plot the spline curve together with f(x)
plot(xx, yy, 'b', 'LineWidth', 2);
hold on;
plot(xx, f(xx), 'r--', 'LineWidth', 2);
legend('Cubic Spline', 'Original Function');
xlabel('x');
ylabel('f(x)');

```

Matlab Code for Q2

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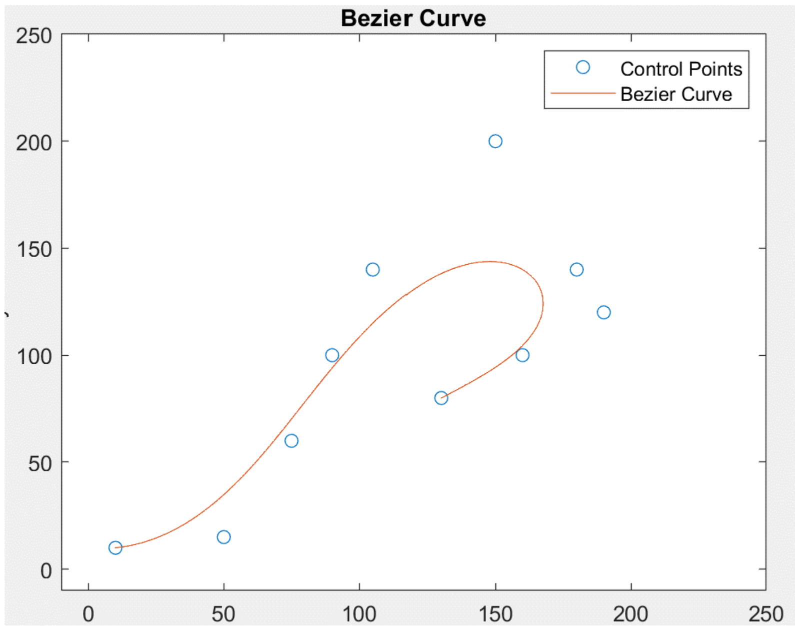
3. Compute the connected Bezier curve from this set of points:

	Point#									
	0	1	2	3	4	5	6	7	8	9
x	10	50	75	90	105	150	180	190	160	130
y	10	15	60	100	140	200	140	120	100	80

To calculate the nth-order Bezier curve that goes through $n+1$ data points with the first point indexed as 0, use $\sum_{i=0}^{n}\binom{n}{i}(1-u)^{n-i}u^i p_i$, where $p_i = (x_i, y_i)$.

(a) Draw a graph determined by the ten points.



(b) Why is the graph smoothly connected at points 3 and 6?

The reason why Points 3 and 6 are "**Smoothly connected**" in graph, is that a point lies on a curve such that a neighborhood of that point contains other points belonging to the curve, meaning the function is continuous and differentiable at both points.

Looking at the zig-zag lines, the points 3 and 6 lie on straight lines determined by two other points, so the slope (derivative) at these points are constant, which means that these points are smoothly connected since differentiability at a point implies continuity at a point.

Because the points 3 and 6 are smoothly connected on the zig-zag line, the Bezier curve near these control points locally appears to be a straight line.

(c) Rewrite the Bezier equations so that the parameter u is defined on [0, 1] for points 0 to 3, on [1, 2] for points 3 to 6, and on [2, 3] for points 6 to 9.

First, find the u-values that best fit the Bezier curve near the control point.

Looking at the plot, the Bezier curve near control point3 (90, 100) passes through the x-coordinate **90**, and the closest point on the Bezier curve near control point 6 passes through the x-coordinate **160**.

For Control Point3 (90, 100):

$$(1-u)^9*10+9*(1-u)^8*u*50+36*(1-u)^7*u^2*75+84*(1-u)^6*u^3*90+126*(1-u)^5*u^4*105+126*(1-u)^4*u^5*150+84*(1-u)^3*u^6*180+36*(1-u)^2*u^7*190+9*(1-u)*u^8*160+u^9*130=90$$

⇨ The root of equation above is $u = 0.319966$.

For Control Point6 (180, 140):

$$(1-u)^9*10+9*(1-u)^8*u*50+36*(1-u)^7*u^2*75+84*(1-u)^6*u^3*90+126*(1-u)^5*u^4*105+126*(1-u)^4*u^5*150+84*(1-u)^3*u^6*180+36*(1-u)^2*u^7*190+9*(1-u)*u^8*160+u^9*130=160$$

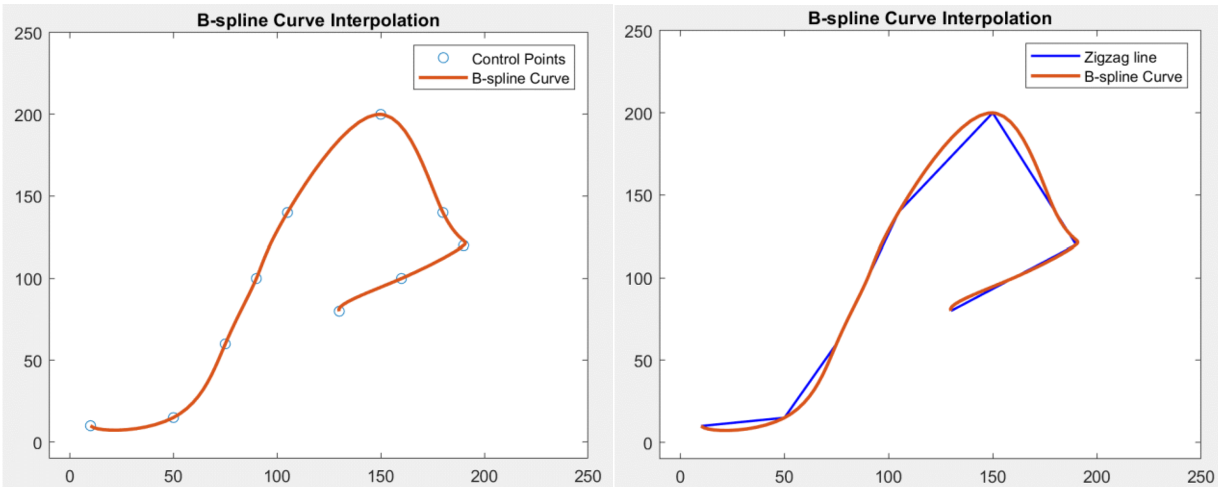
⇨ The root of equation above is $u = 0.659739$.

Therefore, <u>transform interval [0, 1], [1, 2] and [2, 3] to [0, 0.319966], [0.319966, 0.659739], [0.659739, 1], and do linear re-parameterization of *u*.</u>

Finally, we get $u = \begin{cases} 0.319966\hat{u}, & 0 \le \hat{u} \le 1 \\ 0.339773\hat{u} - 0.019807, & 1 \le \hat{u} \le 2 \\ 0.340261\hat{u} - 0.020783, & 2 \le \hat{u} \le 3 \end{cases}$

4. Repeat Problem 3 for a B-spline curve.

(a) Draw a graph determined by the ten points.



With Control Points   With Zigzag Line

(b) Why is the graph smoothly connected at points 3 and 6?

The points 3 and 6 lie on straight lines determined by two other points, so the slope (derivative) at these points are constant, nonchanging, meaning that these points are smoothly connected since differentiability at a point implies continuity at a point.

Because the points 3 and 6 are smoothly connected, the B-Spline curve near these control points locally appears to be a straight line.

(c) Rewrite the B-Spline equations so that the parameter u is defined on [0, 1] for points 0 to 3, on [1, 2] for points 3 to 6, and on [2, 3] for points 6 to 9.

B-spline equations can't be rewritten to satisfy the above request.
B-splines are piecewise continuous constructions where each individual piece is dependent on the parameter $u \in [0,1]$, so one cannot redefine the parameters for a B-spline around one control point individually without redefining all the parameters around all the control points,
However, the Bezier curve is one continuous piece that depends on $u \in [0,1]$, which allows for piecewise continuous mappings into $u \in [0,1]$ that will construct the Bezier curve.

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5. The equation of a plane is z = ax + by + c. We can fit experimental data to such a plane using the least-squares technique. Here are some data for z = f(x, y):

x	0.40	1.2	3.4	4.1	5.7	7.2	9.3
y	0.70	2.1	4.0	4.9	6.3	8.1	8.9
z	0.031	0.933	3.058	3.349	4.870	5.757	8.921

Result:

```
>> Q5_110550062

coeff =

   1.596092181530598
  -0.702381356586343
   0.220666028373471

Sum of squares of deviations = 0.3194
Normal equation: z = 1.5961x - 0.7024y + 0.2207
```

(a) Develop the normal equations to fit the (x, y) data to a plane.

$$\boldsymbol{A^T A r = A^T b}$$

```
[ AtA * r = Atb ]
AtA =

  200.7900  213.4900   31.3000
  213.4900  229.4200   35.0000
   31.3000   35.0000    7.0000


Atb =

  177.4348
  187.3327
   26.9190


r =

    1.5961
   -0.7024
    0.2207
```

(b) Use these equations to fit z = ax + by + c.

```
% Calculating the sum of the squares of the deviations
z_fit = a*x + b*y + c;
```

$$a = 1.5961$$
$$b = -0.7024$$
$$c = 0.2207$$
$$z = 1.5962x - 0.7024y + 0.2207$$

(c) What is the sum of the squares of the deviations of the points from the plane?

```
% Calculating the sum of the squares of the deviations
deviation = z - z_fit;
sum_squares = sum(deviation.^2);
```

Compute the sum of the squares of the deviations of the points.
Therefore, the sum of the squares of the deviations of the points from the plane is approximately 0.3194.

6. Find Pad´e approximations for these functions, with numerators and denominators each of the third degree:

- $cos^2(x)$

```
>> Q6_110550062
The numerator of Pad´e approximations for cos(x)^2 is
3 - 2*x^2

The denominator of Pad´e approximations for cos(x)^2 is
x^2 + 3
```

- $sin(x^4 - x)$

```
The numerator of Pad´e approximations for sin(x^4-x) is
-x*(21649*x^2 - 6120*x + 129180)

The denominator of Pad´e approximations for sin(x^4-x) is
128160*x^3 + 43179*x^2 - 6120*x + 129180
```

- xe^x

```
The numerator of Pad´e approximations for xexp(x) is
-3*x*(x^2 + 8*x + 20)

The denominator of Pad´e approximations for xexp(x) is
x^3 - 9*x^2 + 36*x - 60
```

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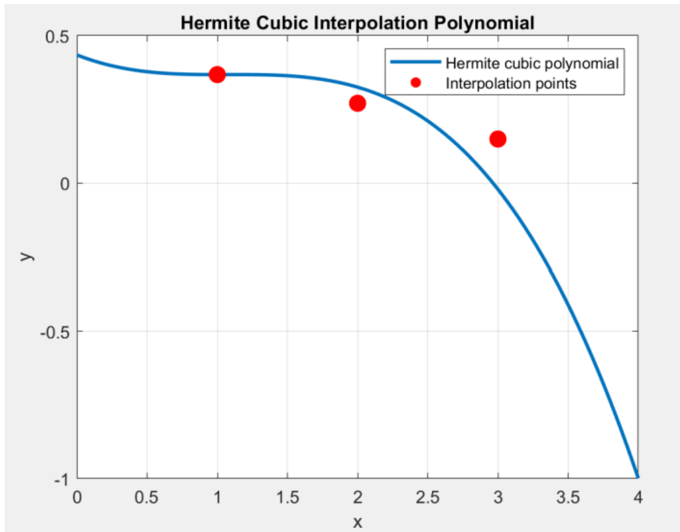
7. **Let f(x) = xe−x , x0 = 1, x1 = 2, x2 = 3**
(a) Construct the Hermite cubic interpolation polynomial for f at the specified interpolating points.
(b) Approximate f(1.5) using the polynomial from part (a)
Result:




```
>> bonus_110550062
p(1.5) = 0.36406331
```