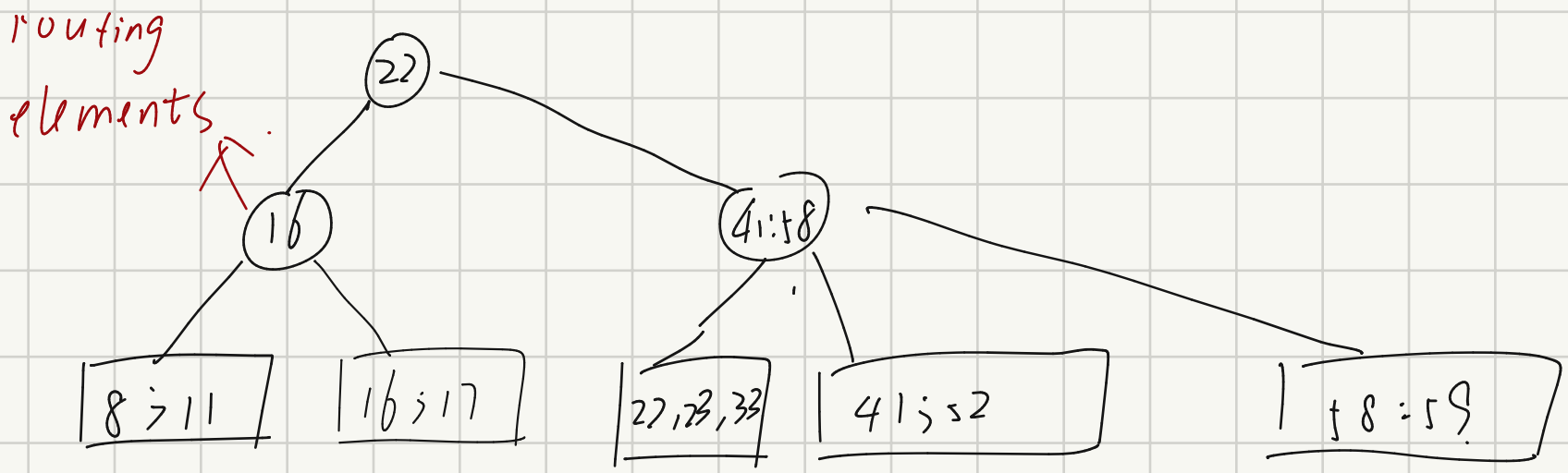
B+ Tree.

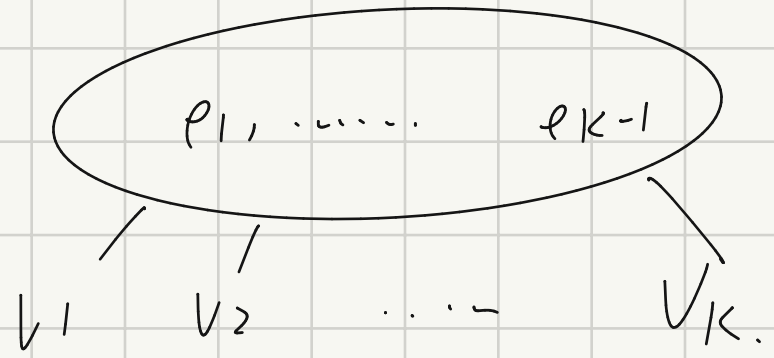
B+ tree of order 3.



B+ tree of order B.

Property:

1. Leaves at the same level
2. data elements stored in leaves
3.



e_1: min elements stored in leaves of $T_{V_{i+1}}$

e_i > every element in (left) Leaves of T_{V_i}

↳ relative.

4. $\lceil B/2 \rceil \leq$ fanout of an internal node $\leq B$

children or we can say pointers.

$2 \leq$ fanout of the root ≤ 3.

5 $\lceil B/2 \rceil \leq$ # elements in a leaf $\leq B$

If the leave is the root, then $1 \le$ # elements $\le B$

N: # data elements (Assume $N > B$)

leaves $\le \frac{N}{\lceil B/2 \rceil}$

space = $\frac{2N}{\lceil B/2 \rceil} \cdot B = 4N$.

⇓

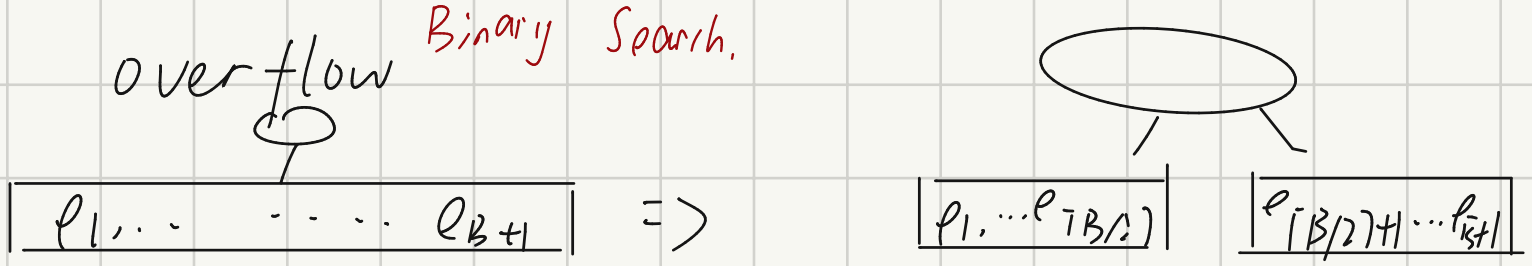
nodes less than twice # leaves

routing elements are different

$$\text{height} \le \underbrace{\log_{\lceil B/2 \rceil} \frac{N}{\lceil B/2 \rceil}}_{\text{except the first level, fanout of other levels are at least } \lceil B/2 \rceil \text{ that means the next level is at least } \lceil B/2 \rceil \text{ times of the prev, so we can find \# level by the leaves level}} + 1 = \log_{\lceil B/2 \rceil} N = \frac{\log_2 N}{\log_2 B - 1} = O(\log_B N)$$

find key. $O(\log_B N) \cdot \underbrace{O(\log B)}_{\text{Binary Search.}} = O(\log N)$

Insertion :



$O(\log_B N) \cdot O(\log B) + O(\log_B N) \cdot O(B) = O(\log N)$

- $O(\log_B N) \cdot O(\log B)$: find pos
- $O(\log_B N)$: divide times
- $O(B)$: time for one divide, $O(\log B)$

each leave can be divided into small 2-3 tree.

deletion

Underflow ($\lceil B/2 \rceil - 1$)

If some sibling has $\geq \lceil B/2 \rceil + 1$ children

take one from sibling

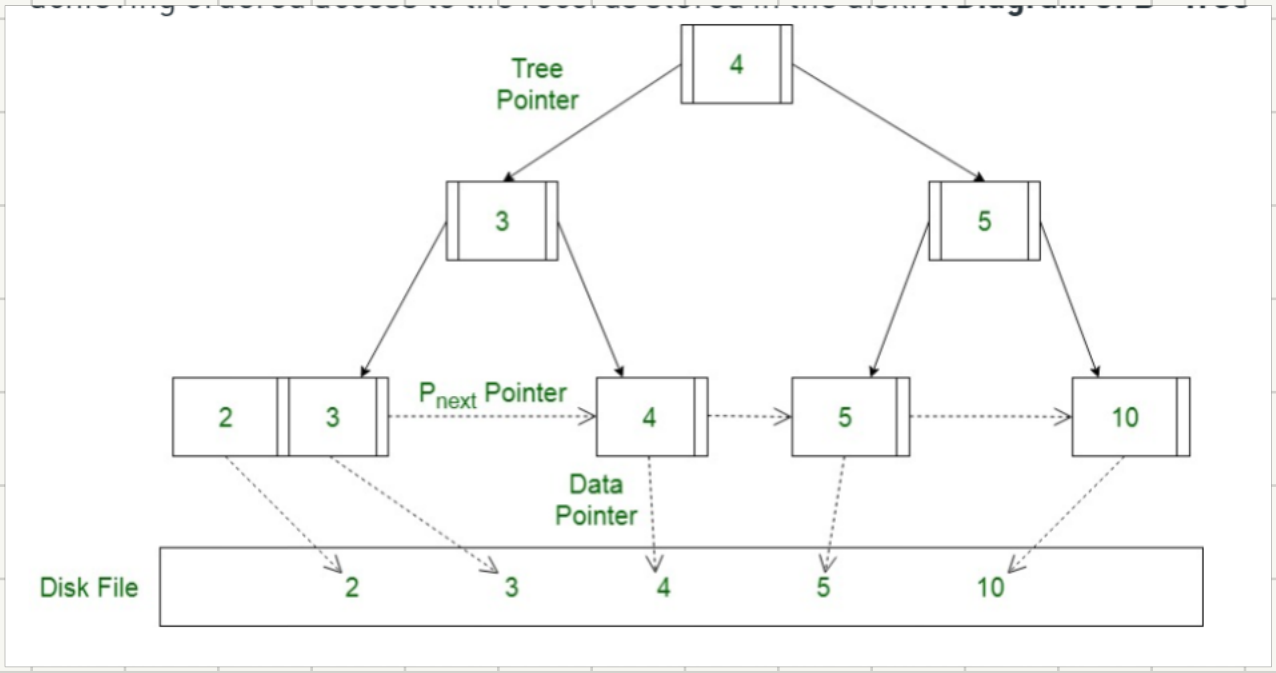
- else:

merge. // $\lceil B/2 \rceil + \lceil B/2 \rceil - 1 \leq B$.

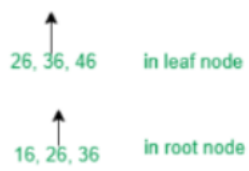
update counting element

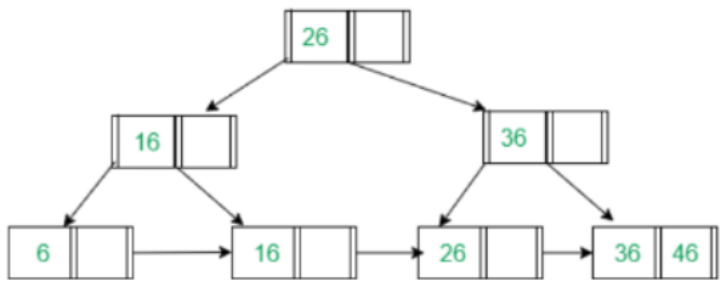
$O(\log N)$

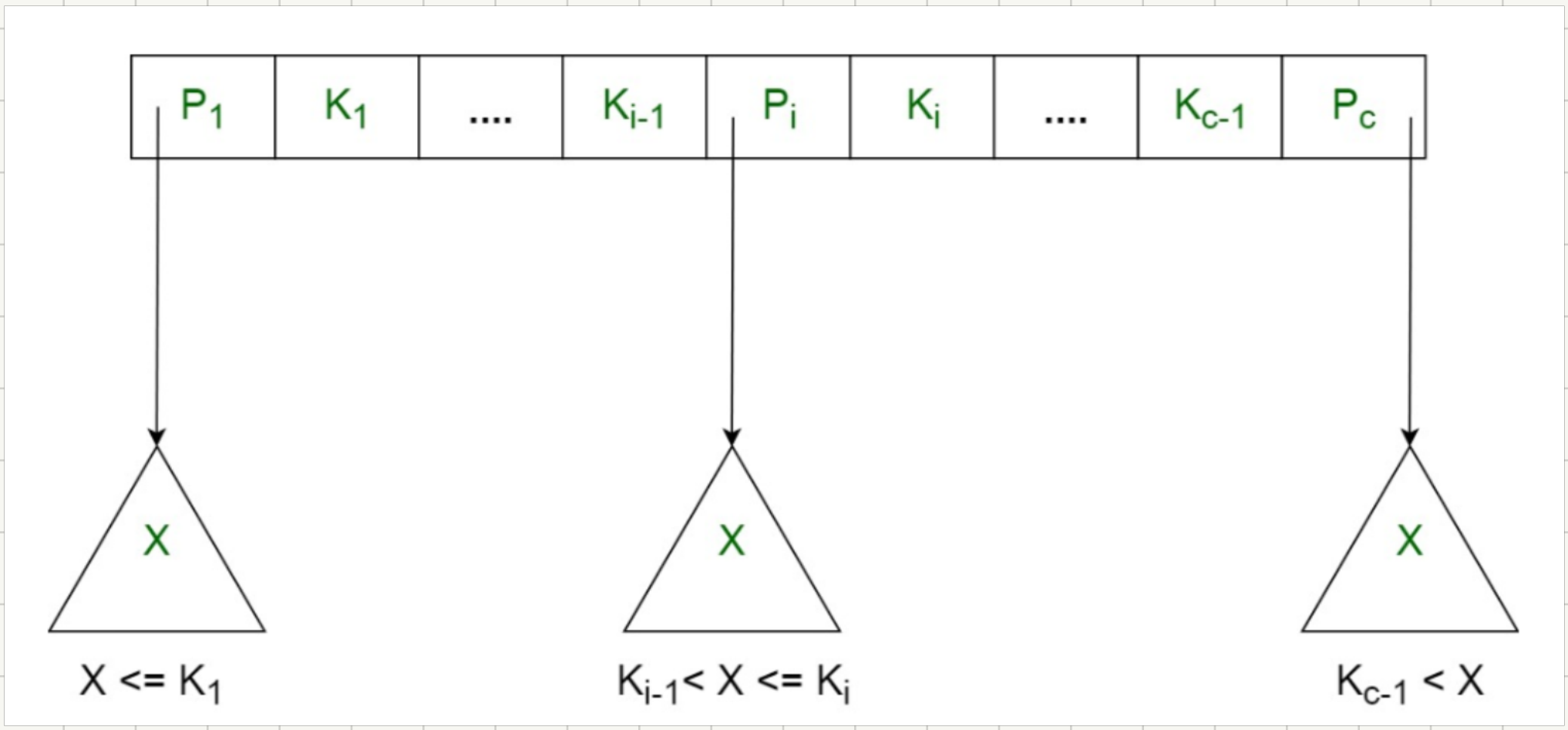
advantages: $B \uparrow$ IO $\downarrow$.

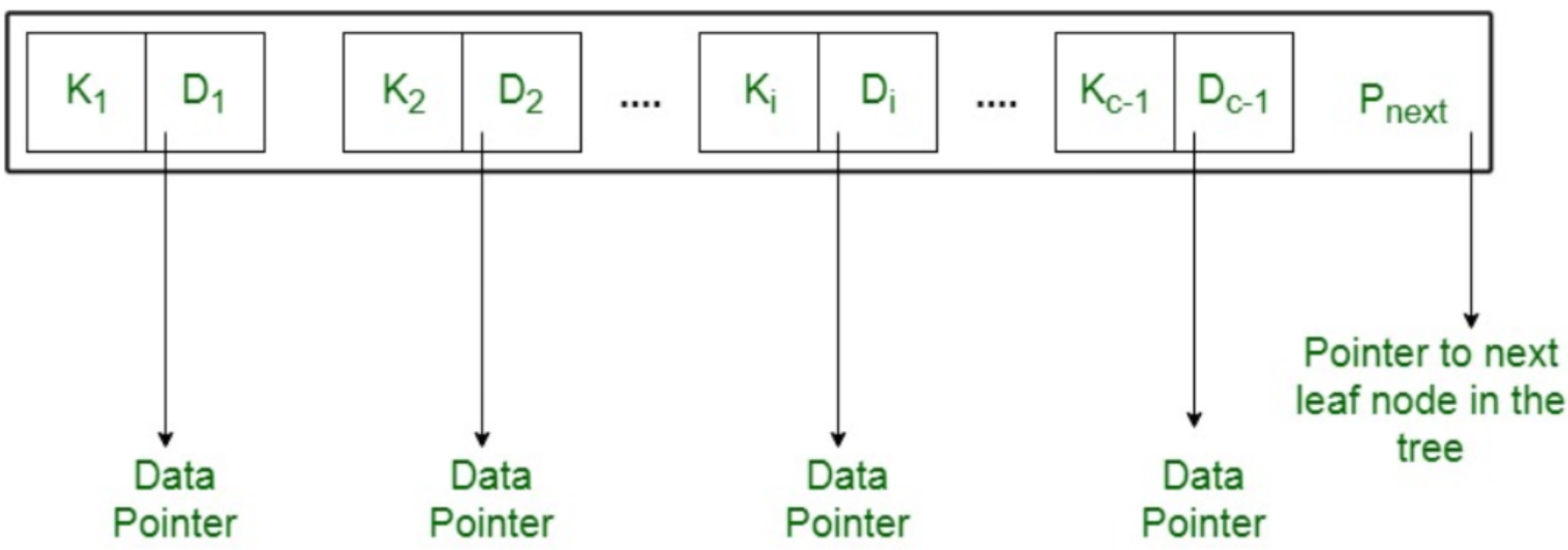

Tree
Pointer
4
3
5
2
3
P_{next} Pointer
4
5
10
Data
Pointer
Disk File
2
3
4
5
10

Insert 46


26, 36, 46
in leaf node
16, 26, 36
in root node


26
16
36
6
16
26
36
46


P1
K1
....
Ki-1
Pi
Ki
....
Kc-1
Pc
X
X
X
X <= K1
Ki-1< X <= Ki
Kc-1 < X


K1
D1
K2
D2
....
Ki
Di
....
Kc-1
Dc-1
Pnext
Data Pointer
Data Pointer
Data Pointer
Data Pointer
Pointer to next leaf node in the tree