The multi-objective Minkowski Sum Problem - Theory and definitions

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Abstract

Some multi-objective optimization problems can be solved using decomposition methods where a set of smaller independent subproblems are solved. For this class of problems, the global objectives can be described as the sum of local objective values of the subproblems. We present some theoretical results for the nondominated sets of such problems and formulate so-called generator sets. The generator sets consists only of the necessarily nondominated points for each subproblem required for generating the global nondominated set. Using the generator sets we define improved upper bound sets that reduce the search area for nondominated (generator) points in the subproblems. Using these generator upper bound sets we present a method of solving the otherwise independent subproblems in a way which solve the global problem by sharing information between subproblems.

Report submitted to BSS, Aarhus University *30th January 2024*

1. Introduction

2. Literature review

Review of relevant literaure

1. Minkowski sum problems.
2. Kerbérénès phd.
3. filtering problems
4. Coupled problems.

3. Prerequisites/ methodology

3.1. General MO

- Relations (sets and vectors)
- Bound sets.
- Operator notation: $\oplus, \ominus, \bigoplus ... \times$
- Notation $\mathcal{X}, \mathcal{Y}, f, \mathcal{X}_E, \mathcal{Y}_\mathcal{N}$...

3.2. Minkowski Sum Problem

MSP definition for general p and S

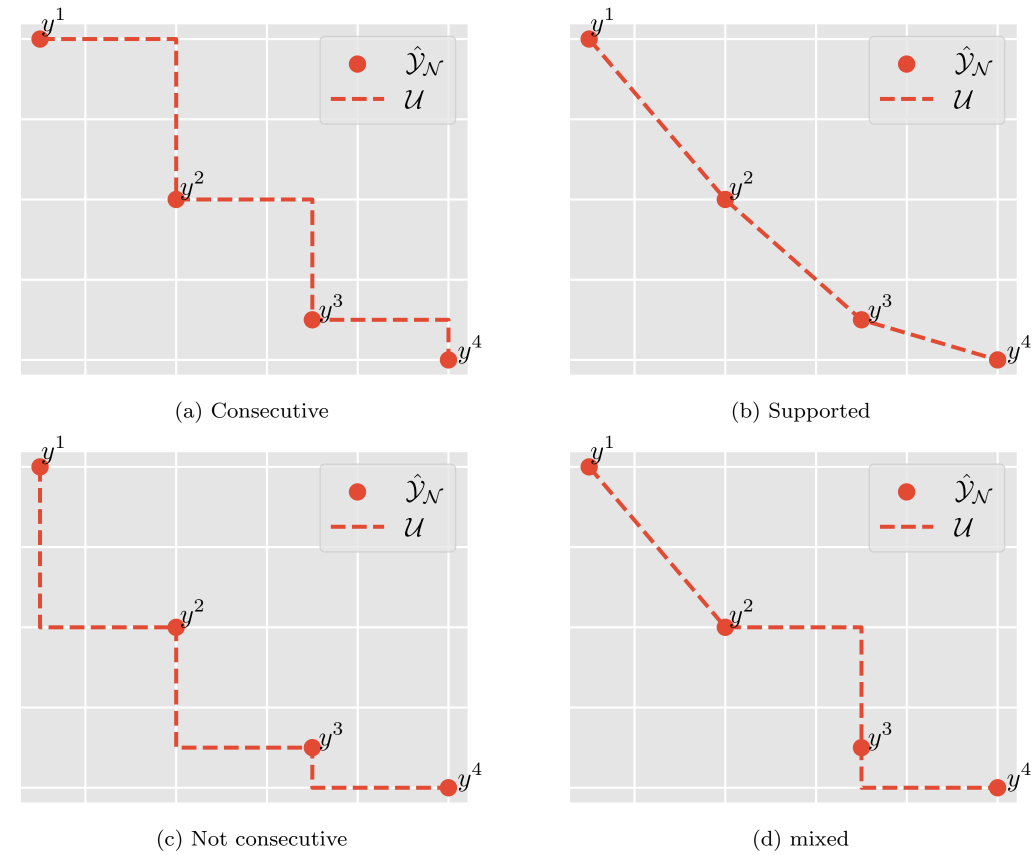
4. Theory

4.1. Theory for two subproblems S = 1,2

4.2. Generator sets

4.3. Generator upper bound sets

Consider different generator upper bound sets. $\bar{\mathcal{U}}^1$ is generally defined by some known solution $\hat{\mathcal{Y}}^1_\mathcal{N} \subseteq \mathcal{Y}^1_\mathcal{N}$ and $\hat{\mathcal{Y}}^1_\mathcal{N} \subseteq \mathcal{Y}^2_\mathcal{N}$. Consider how $\bar{\mathcal{U}}^1$ differs depending on the assumptions on the set of incumbent solutions, see Subsection 4.3.



(a) Consecutive

(b) Supported

(c) Not consecutive

(d) mixed

Figure 1: Induced upper bound sets based on varrying assumptions on the set $\hat{\mathcal{Y}}_{\mathcal{N}} \subseteq \mathcal{Y}_{\mathcal{N}}$.

5. Emperical study

***RQ? Formulate major research questions.
Consider sweep methods.

- Generate a new point from a subproblem and update all generator sets and upper bound sets.
- Periodically generate a point for each subproblem whereafter all generator sets and upper bound sets are updated.
- Sweep: generate all supported non-dominated points for each subproblem whereafter all generator sets and upper bound sets are updated.

5.1. Emperical study of Generator sets

When are generator sets small relative to the non-dominated sets.

5.2. Emperical study of Generator upper bound sets

When are upper bound sets from generators 'good'. (make precise).
Test cite (Adelgren et al., 2018)

References

N. Adelgren, P. Belotti, A. Gupte, Efficient storage of pareto points in biobjective mixed integer programming, INFORMS Journal on Computing 30 (2018) 324–338.