Lecture 11

Inference for p

agenda

HW 4	Any questions? Also, a small typo!
Exam revisions	Discuss common misunderstandings with Midterm 1 and give instructions for revisions
Review	Quick review on previous material
Lecture Pt 1	Introduction to hypothesis testing (for p).
Lecture Pt 2	If time, lecture on CLT for a mean.

Sample Population

Statistics vs. Parameters

Statistic	Parameter
$\bar{x}$	μ
s	σ
$\hat{p}$	p
$\hat{\beta}_0$	β_0
$\hat{\beta}_1$	β_1

independent

For a large enough sample size (n), the distribution of the sample proportion (which is essentially a mean) will be approximately normal.

$$\hat{p} \dot{\sim} N\left(\mu_{\hat{p}} = p, \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\right)$$

The Central Limit Theorem (CLT) for a proportion

Conditions

Independent Observations

Two ways to satisfy:

1) Random sample
2) Sample size is less than 10% of population size

Enough successes and failures

$np \geq 10$

$n(1-p) \geq 10$

Suppose the Acme Drug Company develops a new drug, designed to prevent colds. They claim a 70% success rate. They choose a simple random sample of 100 women from a population of 50,000 volunteers. At the end of the study, 38% of the women caught a cold. Does it seems likely that 70% or more *did not* catch a cold?

$p = 0.7 \quad n = 100 \quad \hat{p}$

(1) Independence:

- SRS ✓

(2) $np = 100 \times 0.7 = 70 \geq 10$ ✓

$n(1-p) = 30 \geq 10$ ✓

CLT satisfied!

$$\hat{p} \dot{\sim} N\left(\mu_{\hat{p}} = 0.7, \; \sigma_{\hat{p}} = \sqrt{\frac{0.7(0.3)}{100}}\right)$$

$\rightarrow P(\hat{p} \geq 0.7) = \text{pnorm}\left(0.7, \underline{0.7}, \sqrt{\dfrac{0.7(0.3)}{100}}, \text{lower.tail=F}\right)$

$= 0.5$

$\rightarrow$ Based on our sample data

$\text{pnorm}\left(0.7, 0.62, \sqrt{\dfrac{0.62(0.38)}{100}}, \text{lower.tail=F}\right)$

$= \boxed{< 0.5}$

What if we do not know p?

Most of the time, we will not know the true population proportion (it's a parameter!).

When we do not know p, our best guess is $\hat{p}$. Let's plug it in!

Standard error: $s_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ → guess of $\sigma_{\hat{p}}$

New success/failure condition: $n\hat{p} \geq 10$

$$n(1-\hat{p}) \geq 10$$

Review

- Confidence intervals: where do they come from and how do we use them?

sample

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Plausible range of values for population parameters

$\hookrightarrow$ perform inference

Inference

Hypothesis testing

Purpose: to test a claim and determine if there is sufficient evidence in favor of that claim

Main components

Hypotheses (null & alternative)

H_0: skeptical perspective - knot

H_A: claim we are interested in

Test Statistic

comparing our data to the null

p-value

Probability of seeing our data if the null was true

Conclusion

decision: reject or fail to reject the null

Note: we can use a confidence interval instead of a p-value (see hw 4)

Step 1: hypotheses

(innocent)

null hypothesis

$$H_0 : p = \underline{p_0}$$

$\uparrow$

=

$H_0 : \underline{p} = p_0$

(guilty)

alternative hypothesis

$$H_A : \begin{array}{l} p < \underline{p_0} \\ p > \underline{p_0} \\ p \neq \underline{p_0} \end{array}$$

$H_A : p < p_0$

Step 2: checking conditions

Independence

are the observational units randomly selected?

Enough data

$\hat{p}$

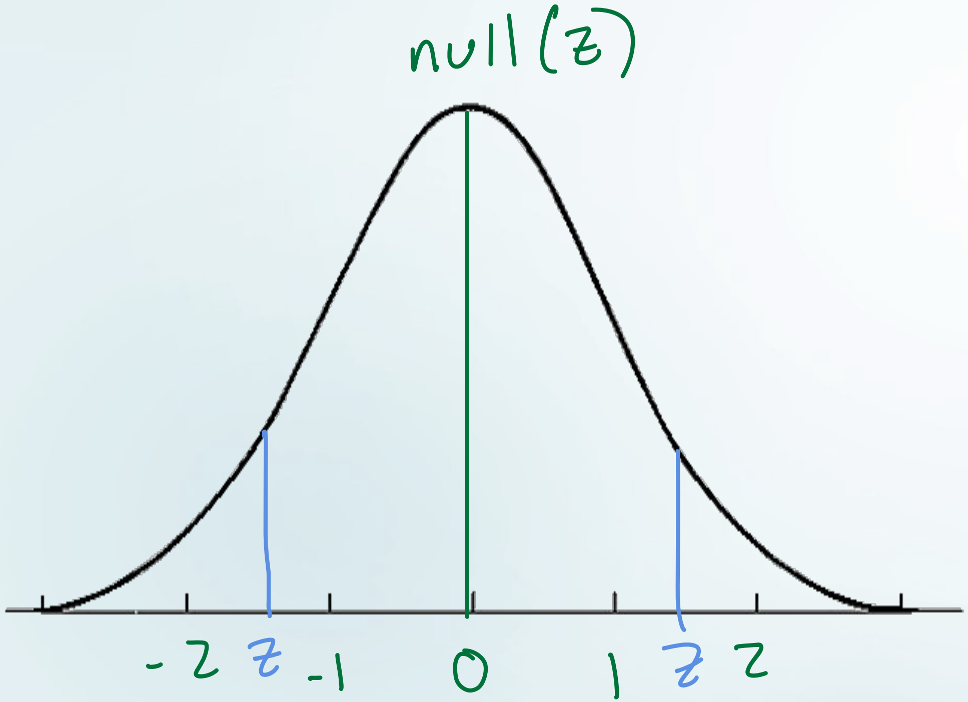
$$np_0 \geq 10$$

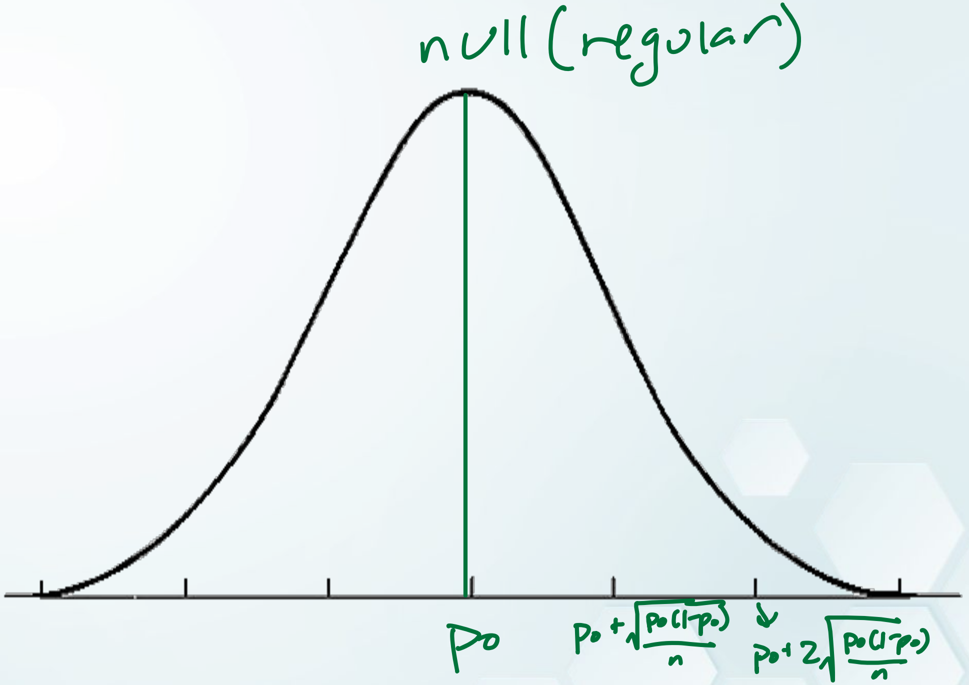
$$n(1 - p_0) \geq 10$$

Step 3: test statistic

$$z = \frac{\text{statistic - null value}}{\text{se of statistic}}$$

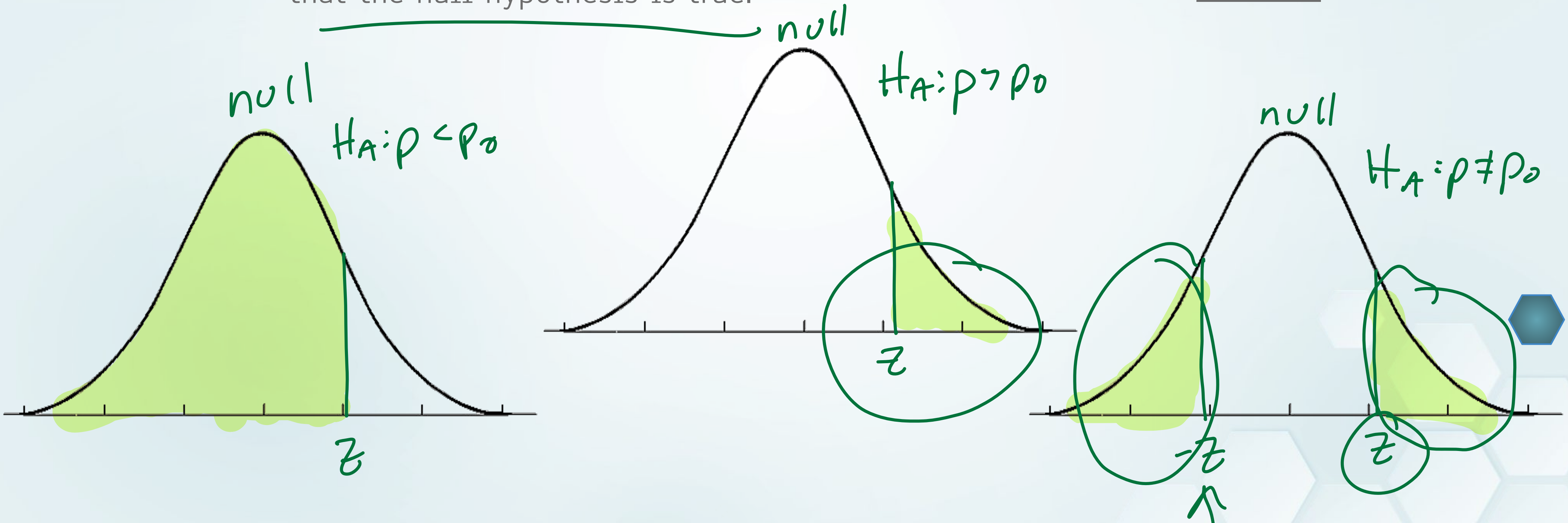
$$= \frac{\hat{p} - p_0}{\underbrace{\sqrt{\frac{p_0(1-p_0)}{n}}}_{\text{se assuming null is true}}}$$





Step 4: p-value

"The probability of seeing the data we observed or something more extreme (in the direction of the alternative hypothesis) given that the null hypothesis is true."



Step 4: p-values in R

$>$ $P(Z \geq z | H_0 \text{ true })$

$<$ $P(Z \leq z | H_0 \text{ true })$

$2 \times P(Z \geq |z| | H_0 \text{ true })$

```
> pnorm(z, lower.tail =
        FALSE)
> pnorm(z)

> 2*pnorm(abs(z),

        lower.tail = FALSE)
> 2*pnorm(-abs(z)) #option2
```

Step 5.1: decision

Reject H_0

We have enough evidence *in favor* of H_A

Fail to reject H_0

We **do not** have enough evidence *in favor* of H_A

Step 5.2: conclusion *in context*

Reject H_0

We have enough evidence that the true proportion of _____ is $>/</\neq p_0$.

Fail to reject H_0

We **do not** have enough evidence that the true proportion of _____ is $>/</\neq p_0$.

Let's practice!

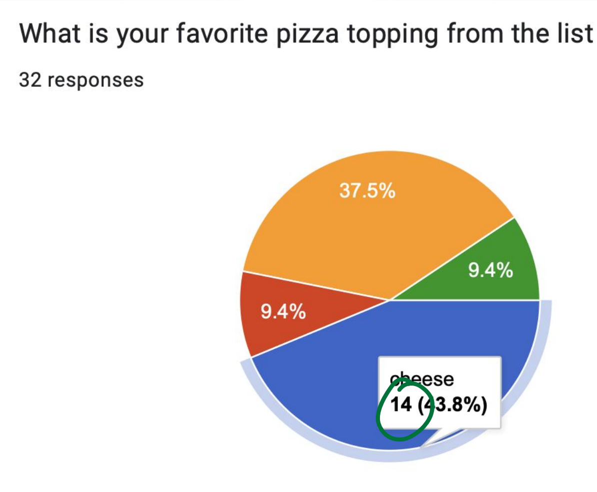
Suppose we want to know the true proportion of adults in the U.S. who prefer a plain cheese pizza. What do you think is the true proportion?

$p = 0.2$

Let's practice!

Suppose we want to know the true proportion of adults in the U.S. who prefer a plain cheese pizza. What do you think is the true proportion?

Your data from the beginning of the semester survey!



$\hat{p} = \frac{14}{32}$

(1) Hypotheses:

$H_0: \underline{\underline{p = 0.2}}$

$H_A: p \neq 0.2$

The true prop. of people living in U.S. who prefer cheese pizza is

- H_0: equal to 20%
- H_A: different from 20%.

(2) check conditions:

1. Independent
   - $n = 32$

   $n \times 10 = 320 < \overset{10\%}{\text{pop'n in U.S.}}$

2. $np_0 = 32 \times 0.2 = 6.4 < 10$

   $n(1-p_0) = 32 \times 0.8 = 25.6 \geq 10$ ✓

Not satisfied

:(

oh well

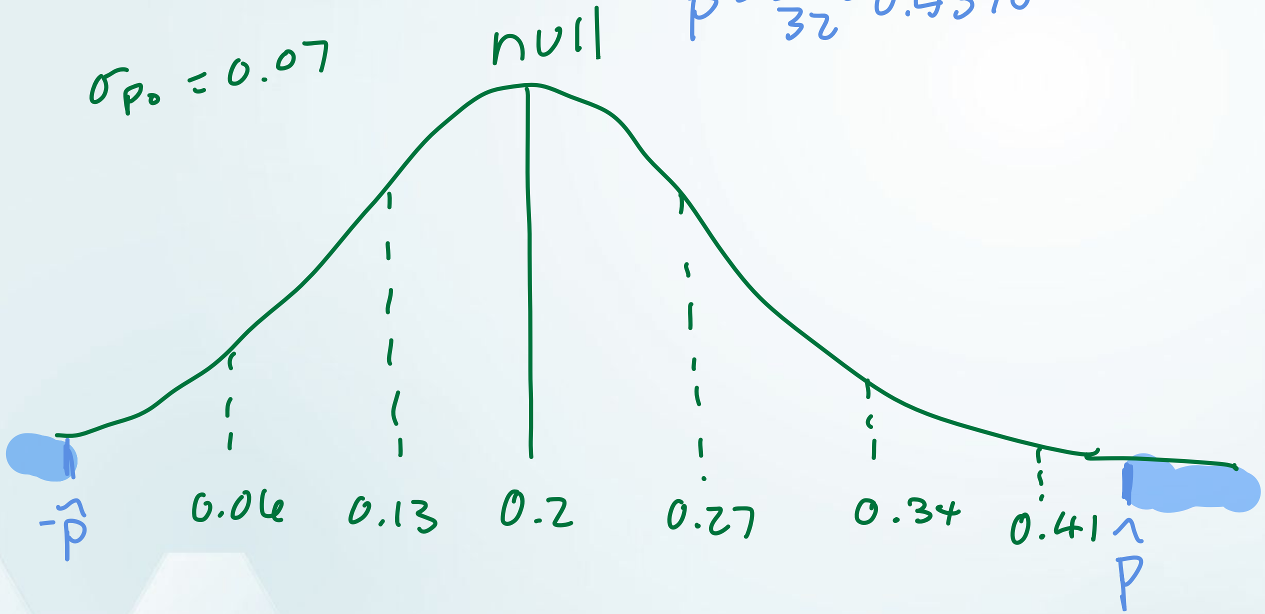
(3) Calculate test statistic

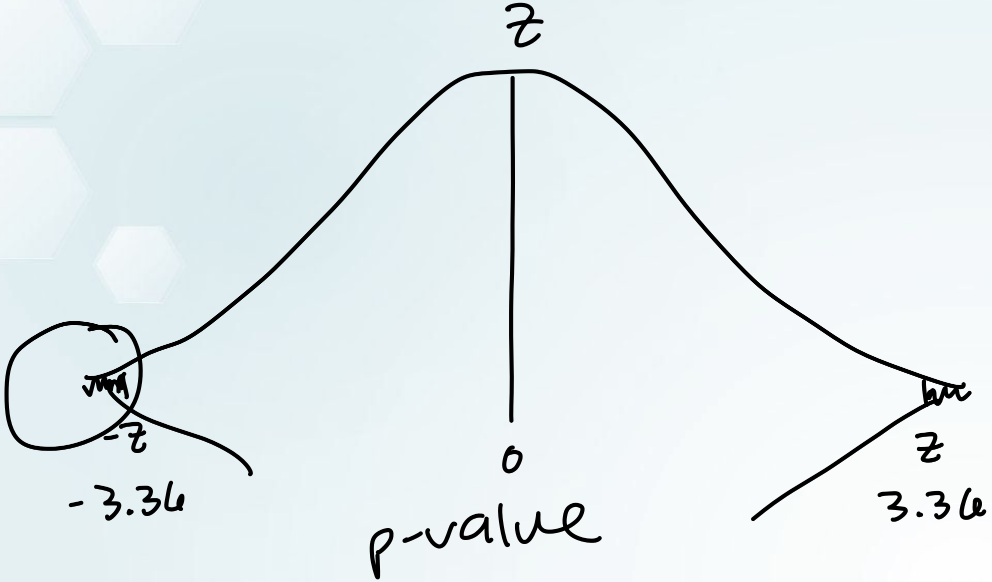
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{14}{32} - 0.2}{\sqrt{\frac{0.2(1-.2)}{32}}} = \boxed{3.3588}$$

(4) P-value:

$\hat{p} = \frac{14}{32} = 0.4375$

$\sigma_{p_0} = 0.07$





$2 * \text{pnorm}(-3.36)$

$= 0.0008$

(5) Decision & conclusion:

$\alpha = \text{threshold} = 0.05$

- Reject H_0 if p-val $< \alpha$ ✓
- Fail to reject H_0 if p-val $< \alpha$

We have enough evidence that the the prop. of people living in the U.S. who prefer cheese pizza is different than 20%.

CI version:

(1) Same $\quad H_A: \underline{\underline{p \neq p_0}}$

(2)

(3) calculate CI:

$\hat{p} = \frac{14}{32} \quad n = 32$

$z^* = 1.96$

$\alpha = 0.05$
significance level

$CL = 1 - \alpha$
$= 0.95$

confidence level

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\frac{14}{32} \pm 1.96 * \sqrt{\frac{\frac{14}{32}\left(1 - \frac{14}{32}\right)}{32}} = (0.2656, 0.6094)$$

Is p_0 inside the interval?

No: reject H_0

(4) Conclusion = " ↑ "

When we collect a **sufficiently large sample** of n **independent observations** from a population with mean μ and finite standard deviation σ, the sampling distribution of the sample mean will be nearly normal:

$$\bar{x} \dot{\sim} N\left(\mu_{\bar{x}} = \mu, \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\right)$$

$$E[\bar{x}] = \mu \qquad var[\bar{x}] = \frac{\sigma^2}{n}$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The Central Limit Theorem (CLT) for a mean

Conditions

Independent Observations

Two ways to satisfy:

1) Random sample
2) Sample size is less than 10% of population size

Normality

1) If $n \geq 30$
   - distribution of data has no extreme outliers
2) If $n < 30$
   - distribution of data has no outliers

3) is σ known?

Sampling Distribution Curve

In engineering, weights of people are considered so that airplanes and elevators aren't overloaded, chairs won't break. Men's weights are normally distributed with a mean of 173 lbs. and a standard deviation of 30 lbs.

1) What is the distribution for a one randomly selected man's weight

2) What is the probability a randomly selected man weighs more than 180 lbs.?

In engineering, weights of people are considered so that airplanes and elevators aren't overloaded, chairs won't break. Men's weights are normally distributed with a mean of 173 lbs. and a standard deviation of 30 lbs.

3) What is the distribution of the average men's weight if we are considering a SRS of 9 men? Draw it labeling the mean and +/- 2 standard deviations.

In engineering, weights of people are considered so that airplanes and elevators aren't overloaded, chairs won't break. Men's weights are normally distributed with a mean of 173 lbs. and a standard deviation of 30 lbs.

4) If 9 men are randomly selected (say to be in an elevator), what is the probability that their average weight is more than 180 lbs?
   a) Shade the area that represents the probability on the plot from part 3.
   b) Compute the probability using Z-scores