

Lecture 16 **Errors & Powers**

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agenda

lecture	type I/II errors and power
review	review hypothesis tests for one and two proportions
activity	get into groups and work on a problem; share your results with the class
hw 6	Discuss expectations for hw 6

Significance Level α

We always set the significance level before performing a hypothesis test. What does it actually mean?

Decision Errors

$\alpha = 0.05$ means we are allowing for 5% error. Type I to be exact.

	<u>Test Conclusion</u>	
	Reject H_0	Fail to Reject H_0
<u>Truth</u>	H_0 true	Type I Error ☹️
	H_A true	☹️ Type II Error

Example

Which is worse: type I or type II?

Let H_0 = drug does not work vs. H_A : drug works

- Type I Error = $\text{Reject } H_0 \mid H_0 \text{ true}$ - we said drug works, but it actually doesn't
- Type II Error = $\text{Fail to reject } H_0 \mid H_A \text{ true}$
 - said drug doesn't work, but it actually does

p-values and α

 α

Significance Level

The maximum p-value for which we will reject the null hypothesis (a limit on the type I error rate)

- $\alpha = P(\text{Reject } H_0 | H_0 \text{ true})$



p

p-value

The probability of seeing something as or more extreme than what we saw in our sample, assuming the null hypothesis is true

- probability of committing a type I error in our data set

Errors

Type I Error

Reject H_0 | H_0 true

$$\alpha = P(\text{Reject } H_0 | H_0 \text{ true}) \\ = P(\text{Type I Error})$$

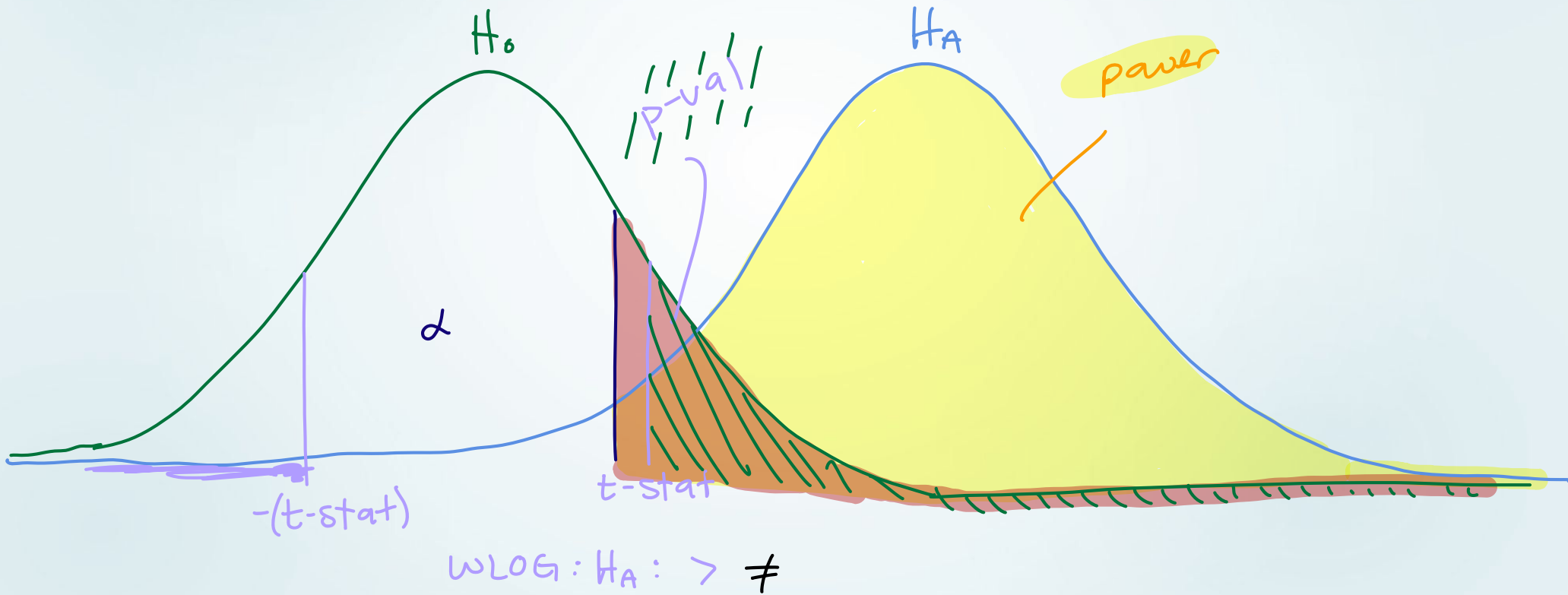
Type II Error

Fail to reject H_0 | H_A true

$$\beta = P(\text{fail to reject } H_0 | H_A \text{ true}) \\ = P(\text{Type II Error})$$

Power

$$\text{Power} = P(\text{Reject } H_0 \mid H_A \text{ true}) = 1 - \beta$$



A 2010 Pew Research foundation poll indicates that among 1,099 randomly selected college graduates, 33% watch The Daily Show. Meanwhile, 22% of the 1,110 randomly selected people with a high school degree but no college degree in the poll watch The Daily Show. Do the data provide convincing evidence of a difference between the proportions of college graduates and those with a high school degree or less who watch The Daily Show.

parameter: $P_1 - P_2 \rightarrow$ 1 = college grads
2 = HS grads

1) Write the hypotheses.

$$H_0: P_1 - P_2 = 0$$

$$H_A: P_1 - P_2 \neq 0$$

2) In the context of the problem, define the errors below

- Type I Error = we say $P_1 \neq P_2$, but actually $P_1 = P_2$
- Type II Error = we say $P_1 = P_2$, but actually $P_1 \neq P_2$

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2) Conditions:

- Independence

both groups randomly selected

- Normality

$$\left. \begin{array}{l} n_1 \hat{p}_1 = 1099 \times 0.33 = 362 \\ n_1 (1 - \hat{p}_1) = 1099 \times 0.67 = 1099 - 362 = 737 \\ n_2 \hat{p}_2 = 1110 \times 0.22 = 244 ; n_2 (1 - \hat{p}_2) = 864 \end{array} \right\} \geq 5$$

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3) Test statistic:

$$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} = 5.7710$$

4) P-value: ≈ 0 8×10^{-9}

5) Decision and conclusion in context: **Reject H_0**

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What type of error could we have made? Type I or Type II?

What is the probability that we made that error?

$$8 \times 10^{-9}$$

