ISyE/CS524 – Problem Set #4

Due Date: October 13, 2023. 09.00AM.

Instructions for Handing In Homework

Formulate the following problems in GAMS and solve them. Please follow the instructions given in the problems closely. Submit this assignment electronically to canvas, one zip file containing the files that are outlined below. You should hand in exactly 6 files with the following names: `hw4-1.gms`, `hw4-2.gms`, `hw4-3.gms`, `hw4-1.lst`, `hw4-2.lst`, `hw4-3.lst`

1 I Like My Wine

I like to drink a bottle of wine from my wine cellar every week. Since I am a poor Professor, I have only $n = 60$ bottles of wine available. The current age of each bottle of wine a_i (in weeks) is known. Wine improves with age, and the pleasure I derive from drinking bottle i in week t as a function of its age $T_i = a_i + t$ (for $t \in \{0, 1, \ldots, n-1\}$) is given as

$$\rho_i(T_i) = b_i T_i^3 - c_i T_i$$

for some given constants b_i, c_i. The data a_i, b_i, and c_i can be found in `wine.gdx` as parameter `data` and incorporated via:

```
set type, i;
parameter data(type<,i<);
$gdxin wine.gdx
$load data
$gdxin
```

1.1 Problem
Formulate and solve a linear program that tells me how to consume my wine to maximize my overall pleasure[1]. Make sure the model name is `assign` and that you use the `option x:0.0.2; display x.l` to print which bottle I drink on which week.

2 Broom Rental

Oliver Wood retired from his career as keeper at Puddlemere United to start the Wood Broom Rental Company WBC, [2] and he needs your help. There is a fleet of 94 brooms that are distributed among 10 different locations. The (x, y) coordinates of each broom rental agency (in a grid based on kilometers[3] is given in Table 1, as are the number of brooms currently in each location and the number of brooms required for tomorrow's rentals. The cost of transporting a broom from one location to another is 0.5 galleon/km. Naturally brooms travel "as the crow flies," so distances between agencies are Euclidean.

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[1] I am hedonistic

[2] In the Wizarding World, brooms may be rented and used, much like cars, in our normal Muggle world.

[3] Wizards use the metric system

Place	x Coor.	y Coor.	Required Brooms	Current Brooms
Hogwarts	0	0	10	8
Godric's Hollow	20	20	6	13
Little Whinging	18	10	8	4
Shell Cottage	30	12	11	8
The Leaky Cauldron	35	0	9	12
Ollivander's	33	25	7	2
Zonko's Joke Shop	5	27	15	14
Dervish and Banges	5	10	7	11
Little Hangleton	11	0	9	15
Weasley's Wizard Wheezes	2	15	12	7

Table 1: Information About WBC

2.1 Problem

Write a *linear program* in GAMS that will determine the movement of all brooms to establish the required number of brooms at all agencies in a minimum cost manner. Ensure your linear program is in the form of a minimum cost network flow problem, and set up the GAMS file to use the appropriate CPLEX options so it solves as such.

Wood would like to know two things. First, he would like to know the minimum broom tranportation cost (in a GAMS parameter transportCost), as follows:

```
parameter transportCost ;
transportCost = cost.L;
display transportCost;
```

Second, according to the optimal transportation plan, he would like to know the set of all locations requiring extra brooms that do *not* receive any brooms from their closest location (in a set not_from_closest).

```
set not_from_closest(P);
option not_from_closest:0:0:1;
display not_from_closest;
```

Hints:

- Be sure to put quotation marks around elements of sets when defining them if there are spaces in the element names.

- GAMS has functions sqrt and sqr that may be useful.

- The second part will require a little bit of GAMS coding trickery.

3   Untied Airlines

Prof. Wright *hates* flying United airlines through O'Hare (ORD). This is a problem, as he is a sought-after lecturer who frequently must make trips from his home base in Madison (MSN) to San Francisco (SFO), Houston (IAH), Washington DC (DCA), and Orlando (MCO). If Prof. Wright flies United, he must travel through ORD, if he travels Delta he can choose to go via Detriot (DTW) or Minneapolis (MSP).

The travel times between various locations in minutes are:

```
MSN.ORD 22, MSN.DTW 65, MSN.MSP 46,
MSP.SFO 213, MSP.IAH 139, MSP.DCA 125, MSP.MCO 176,
ORD.SFO 247, ORD.IAH 124, ORD.DCA  82, ORD.MCO 135,
DTW.SFO 280, DTW.IAH 147, DTW.DCA  53, DTW.MCO 130
```

Delay times at ORD are approximately uniformly distributed between 0 and 3 hours, at DTW the delay time are are between 0 and 1.5 hours and between 0 and 2 hours at MSP.

We will assume that Prof. Wright makes 3 trips to each location every year. Prof. Wright is a notorious cheapskate, and lives for frequent flyer miles. So he would like to only fly one airline.

3.1   Problem
Should Prof. Wright switch to Delta? Justify your answer with a mathematical model and explain what your model does to deal with the uncertainty.

3.2   Problem
What if you add a constraint that he must always use the same hub?

3.3   Problem
What if he forgoes frequent flyer miles - which route should he then use for which flight?