CS524 – Problem Set #3

Due Date: September 29, 2023, **9:00AM**

Instructions for Handing In Homework

Formulate the following problems in GAMS and solve them. You should hand in one zip file containing exactly 6 files with the following names: `hw3-1.gms, hw3-1.lst, hw3-2.gms, hw3-2.lst, hw3-3.gms, hw3-3.lst`. Be sure to follow the instructions for displaying the appropriate solution values at the bottom of your GAMS files. If comments are requested, then include these in the gms file between ontext and offtext compile time directives. Try to limit the length of your listing file (after debugging of course) by using the `limrow = 0, limcol = 0, solprint = off` directives (for which some small credit will be given).

1 Student finances

Andrea Student has $400 in cash from Christmas presents on Dec 31, 2022. At the beginning of January, February, March, and April, 2023, she receives certain revenues from her parents and pays certain college bills, as indicated in the table below. Any money left over can be invested (immediately) for one month at an interest rate of 0.25%/month, for two months at 0.35%/month, for 3 months at 0.4%/month, or for 4 months at 0.6%/month. Investment is compounded each month and paid out at term.

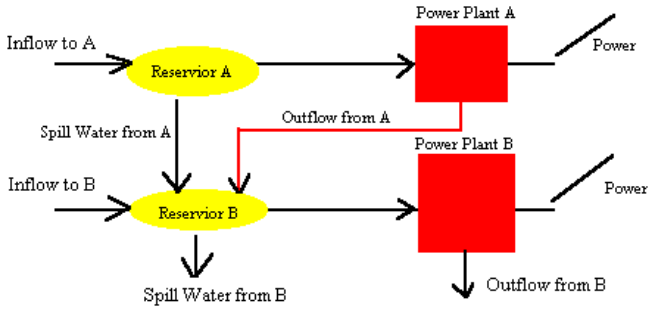
Month	Revenues	Bills
January	600	700
February	800	500
March	250	600
April	300	250

1.1 Problem

Determine the investment strategy that maximizes A. Student's cash on hand at the beginning of May. (Of course, "cash-on-hand" cannot include money that is currently tied up in investments.) Model should be called `finco` and you should display `finalCash.l` containing the cash on hand in May.

2 Hydro Planning

The General Eccentric Power and Lighting Company has a system consisting of two dams and their associated reservoirs and power plants on a river. The important flows of power and water are shown in the following diagram:



In the following table, all quantities measuring water are in units of 1000 acre-feet (KAF's). Power is measured in megawatt hours (MWH's).

	A	B	Units
Storage Capacity	2000	1500	KAF
Minimum allowable level	1200	800	KAF
Predicted inflow:			
March	200	40	KAF
April	130	15	KAF
March 1 level	1900	850	KAF
Water-Power Conversion	400	200	MWH/KAF
Power Plant Capacity	60,000	35,000	MWH/month

Power can be sold at $5.00 per MWH for up to 50,000 MWH each month, and excess power above that figure can be sold for $3.50 per MWH. Assume flow rates in and out through the power plants are constant within the month.

2.1 Problem

Suppose there is a spillway by each reservoir that allows water to spill out (at any level) and bypass the relevant power plant. A consequence of these assumptions is that the maximum and minimum water-level constraints need to be satisfied only at the end of the month. Formulate a linear program to maximize the amount of money General Eccentric receives for the power it sells during the months of March and April, given the constraints.

3 Minimax and absolute errors

The set of six equations in four variables (1)—(6) does not have a unique solution.[1]

$$\begin{aligned} 8x_1 - 2x_2 + 4x_3 - 9x_4 &= 17 && (1)\\ x_1 + 6x_2 - x_3 - 5x_4 &= -16 && (2)\\ x_1 - x_2 + x_3 &= 7 && (3)\\ x_1 + 2x_2 - 7x_3 + 4x_4 &= -15 && (4)\\ x_3 - x_4 &= 6 && (5)\\ x_1 + x_3 - x_4 &= 0 && (6) \end{aligned}$$

For each equation i, and values of variables $x = (x_1, x_2, x_3, x_4)$, let e_i be the absolute difference (error) between the left hand side and the right hand side. For example, for $i = 2$ and $x = (-5, 3, 1, 4)$, the error is

$$e_2 = |(1)(-5) + 6(3) - (1)(1) - 5(4) - (-16)| = |8| = 8.$$

3.1 Problem

Let I index constraints and J index variables and write a linear programming instance that will minimize the total absolute error:

$$\sum_{i \in I} e_i$$

Solve this instance with GAMS. Display both the (minimum) total absolute deviation (in a parameter named `TotalDevSmall` and the values of x that achieve this, in a parameter named `xValSmall(J)`. For example, your code may look like this:

```
parameters TotalDevSmall, xValSmall(J);

TotalDevSmall = ztotdev.L ;
xValSmall(J) = x.L(J);

display TotalDevSmall, xValSmall;
```

3.2 Problem

For the same instance, write a linear programming instance that will minimize the maximum error in any one equation. Namely find values of x that will

$$\min \max\{e_1, e_2, e_3, e_4, e_5, e_6\}.$$

Create your instance in GAMS and solve it. What is the minimum max-error that can be achieved? Display this value in a parameter called `MinMaxDevSmall`. For example, your code may look like this:

```
parameters MinMaxDevSmall;

MinMaxDevSmall = zminmax.L;
display MinMaxDevSmall;
```

[1] Most six equations with four variables don't.