随机过程

$$R_x(\tau) = E\{x(t)x^*(t+\tau)\} = E\{x^*(t-\tau)x(t)\}$$

$$P_x(f) = \int_{-\infty}^{\infty} R_x(\tau)e^{-j2\pi f\tau}d\tau$$

参数估计

Crame-Rao界 $Var[\theta-\hat{\theta}] \geq \left\{E\left\{\left[\frac{\partial \ln p(r/\theta)}{\partial\theta}\right]^2\right\}\right\}^{-1}$

或 $\geq \left\{-E\left\{\left[\frac{\partial^2 \ln p(r/\theta)}{\partial\theta^2}\right]\right\}\right\}^{-1}$

当且仅当 $\frac{\partial \ln p(r/\theta)}{\partial\theta} = [\theta-\hat{\theta}]k(\theta)$

最小二乘 $\hat{\theta} = (H^TH)^{-1}H^Tr$

$E(n)=0 \quad E(nn^T)=R$ 则

$MSE=(H^TH)^{-1}H^TRH(H^TH)^{-1}$

加权最小二乘

$$\hat{\theta}_{lsw} = (H^TWH)^{-1}H^TWr$$

$$MSE=(H^TWH)^{-1}H^TWRWH(H^TWH)^{-1}$$

MAP方程 $\left[\frac{d\ln p(r/\theta)}{d\theta}+\frac{d\ln p(\theta)}{d\theta}\right]\Big|_{\theta=\hat{\theta}_{MAP}}=0$

ML方程 $\frac{d\ln p(r/\theta)}{d\theta}\Big|_{\theta=\hat{\theta}_{ML}}=0$

均值为0，方差为 σ^2 I 的高斯分布pdf $p(\theta)=\left[\frac{1}{2\pi\sigma^2}\right]^{\frac{N}{2}}\exp\left\{-\frac{\theta^2}{2\sigma^2}\right\}$

线性最小误差估计 $\hat{\theta}_{lms}=E(\theta)+cov(\theta,r)\left[Var[r]\right]^{-1}[r-E(r)]$

现代谱估计

直接法 $X_N(w) = \sum_{n=0}^{N-1} x(n) e^{-jnw}$

$$P_X(w) = \frac{1}{N} |X_N(w)|^2$$

间接法 $\hat{R}_X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n+k) x^*(n)$

$$P_X(w) = \sum_{k=-M}^{M} \hat{R}_X(k) e^{-jkw}$$

ARMA过程 $A(z)x(n) = B(z)e(n)$

$\swarrow$ AR $\qquad \searrow$ MA

$B=1 \quad AR(p) \quad IIR$

$A=1 \quad MA(q) \quad FIR$

ARMA过程功率谱密度 $P_X(w) = \sigma^2 \left. \frac{|B(z)|^2}{|A(z)|^2} \right|_{z=e^{jw}}$, $e(n) \sim N(0, \sigma^2)$

$\underbrace{\text{白噪声}}_{\sigma_v^2}$中的$\underbrace{AR(p)\text{过程}}_{\sigma^2}$是$\underbrace{ARMA(p,p)\text{过程}}_{\sigma_w^2}$.

$\sigma_w^2 = \sigma^2 + \sigma_v^2$

维纳滤波

$$H(s) = \frac{1}{G_r^+(s)} \left[\frac{G_{sr}(s)}{G_r^-(s)} \right]^+$$

$Var[\hat{s}(t+a)] = R_s(0) - \int_0^{\infty} \phi^2(t) dt$, $\quad \phi(t) = L^{-1}\left[\frac{G_{sr}(s)}{G_r^-(s)} \right]$

卡尔曼滤波

射影 $\hat{x} = proj(x|y) = Ex + P_{xy} P_{yy}^{-1} (y - Ey)$

一步预报 $\hat{y}(k|k-1) = proj(y(k) | y(1) \cdots y(k-1))$

新息序列 $\varepsilon(k) = y(k) - \hat{y}(k|k-1)$

递推射影公式 $proj(x | y(1) \cdots y(k))$

$$= proj(x | y(1) \cdots y(k-1)) + E[x(k)\varepsilon^T(k)]E[\varepsilon(k)\varepsilon^T(k)]^{-1}\varepsilon(k)$$

卡尔曼滤波递推公式

递推射影公式 $\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)\varepsilon(k+1)$

状态方程取射影 $\hat{x}(k+1|k) = \Phi\hat{x}(k|k)$

新息序列 $\varepsilon(k+1) = y(k+1) - H\hat{x}(k+1|k)$

增益阵 $K(k+1) = P(k+1|k)H^T[HP(k+1|k)H^T + R]^{-1}$

方差阵 $P(k+1|k) = \Phi P(k|k)\Phi^T + \Gamma Q \Gamma^T$

$P(k+1|k+1) = [I_n - K(k+1)H]P(k+1|k)$

初值 $\hat{x}(0|0) = \mu_0 \quad P(0|0) = P_0$