Policing as a mechanism to avoid a tragedy of the commons in *P. aeruginosa*

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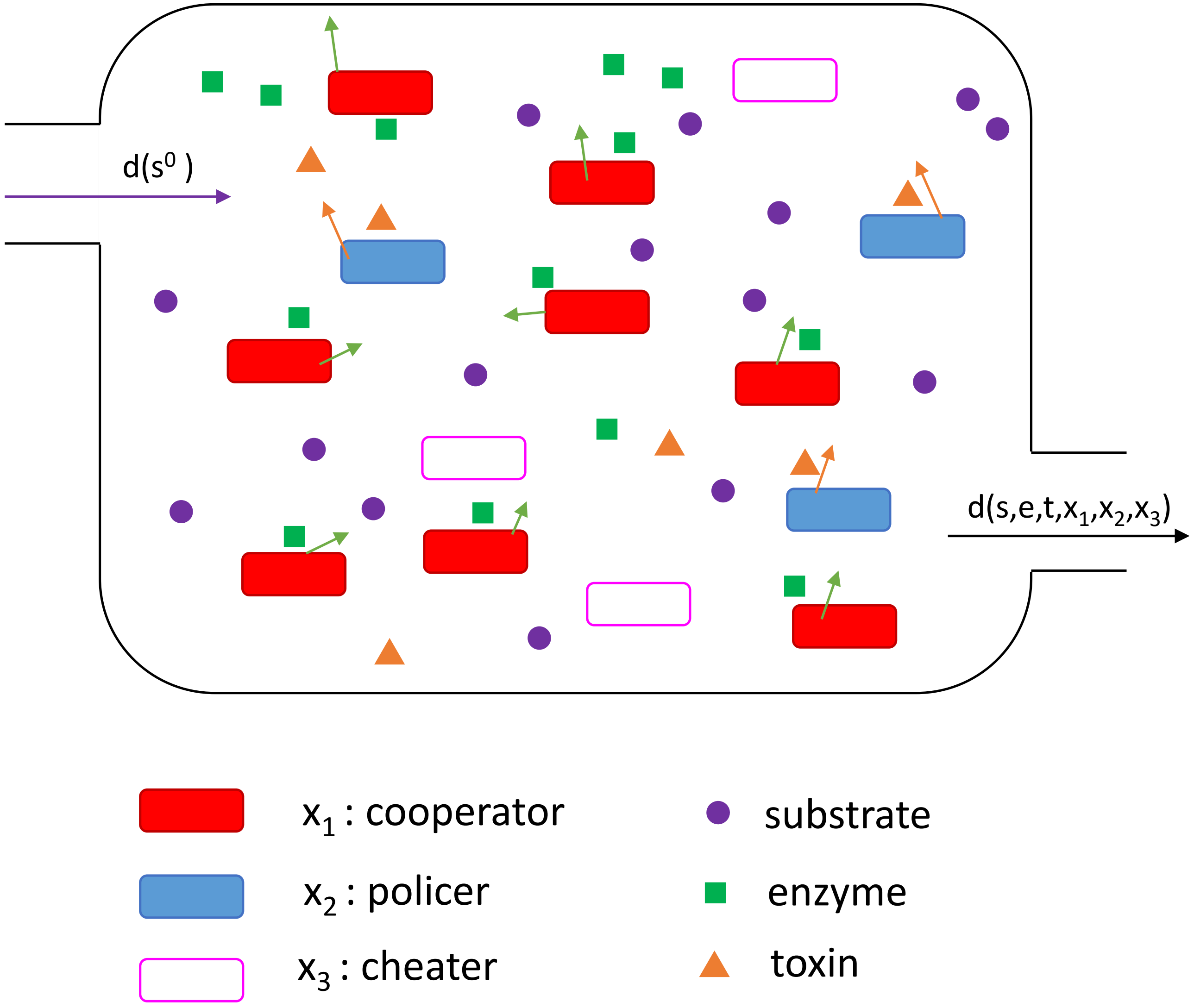
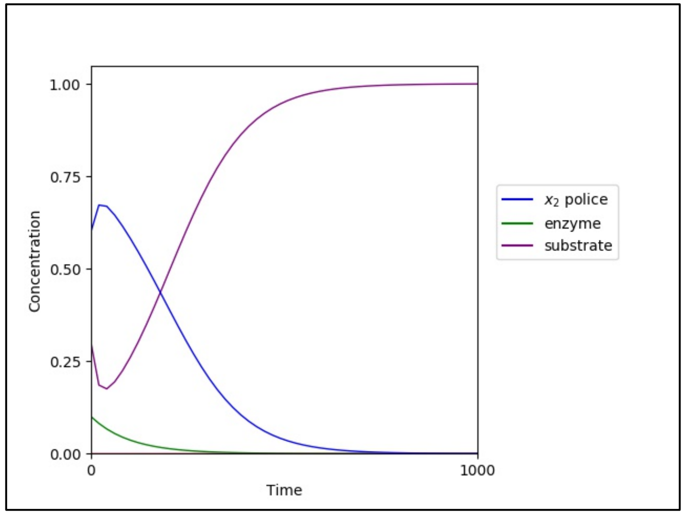
Policing to avoid a tragedy of the commons



$$\dot{s} = d(s^0 - s) - (x_1 + x_2 + x_3)f(s,e)$$
$$\dot{e} = q_1 x_1 f(s,e) - de$$
$$\dot{t} = q_2 x_2 f(s,e) - dt$$
$$\dot{x_1} = x_1((1-q_1)f(s,e) - d - k_1 t)$$
$$\dot{x_2} = x_2((1-q_2)f(s,e) - d)$$
$$\dot{x_3} = x_3(f(s,e) - d - k_3 t)$$

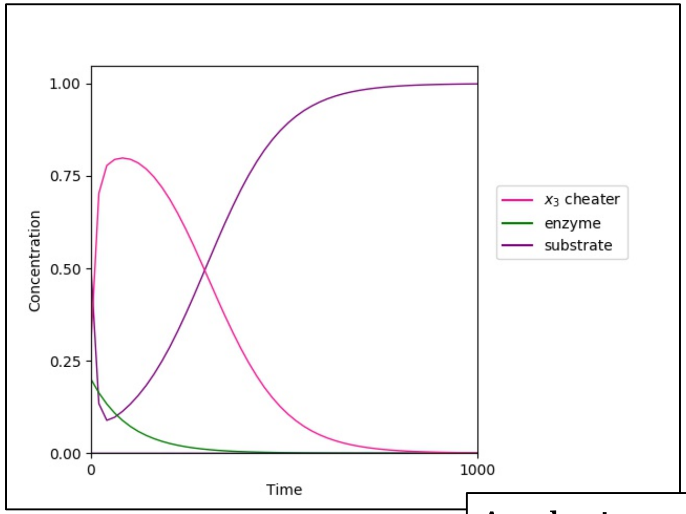
Police only

$$\dot{s} = d(s^0 - s) - (x_2)f(s,e)$$
$$\dot{e} = -de \to 0$$
$$\dot{t} = q_2 x_2 f(s,e) - dt$$
$$\dot{x_2} = x_2((1 - q_2)f(s,e) - d)$$



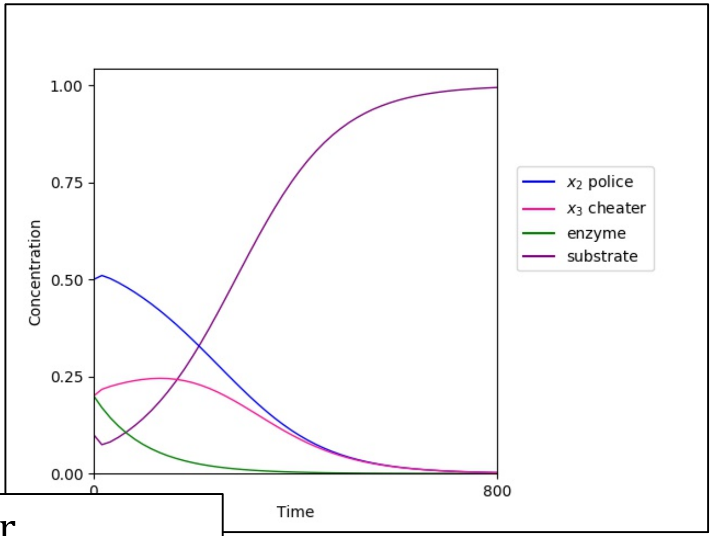
Cheater only

$$\dot{s} = d(s^0 - s) - (x_3)f(s,e)$$
$$\dot{e} = -de \to 0$$
$$\dot{t} = -dt \to 0$$
$$\dot{x_3} = x_3(f(s,e) - d)$$



Police and Cheater

$$\dot{s} = d(s^0 - s) - (x_2 + x_3)f(s,e)$$
$$\dot{e} = -de \to 0$$
$$\dot{t} = q_2 x_2 f(s,e) - dt$$
$$\dot{x_2} = x_2((1 - q_2)f(s,e) - d)$$
$$\dot{x_3} = x_3(f(s,e) - d - k_3 t)$$



Analysis says, for

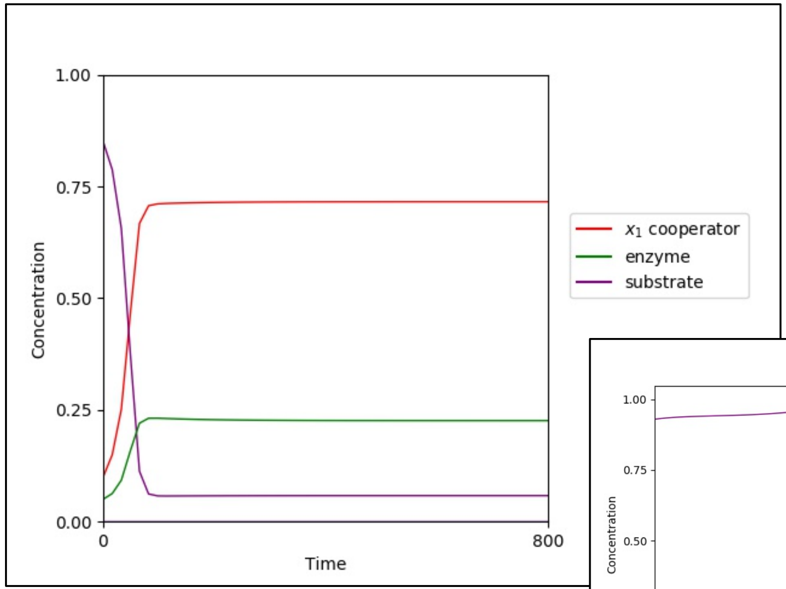
$$x_1 = 0 \Rightarrow e = 0$$
$$e = 0 \Rightarrow x_2, x_3 = 0$$

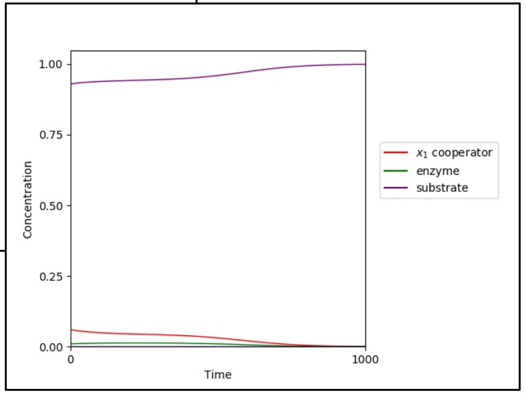
Thus a tragedy should occur.

Cooperator only

$$\dot{s} = d(s^0 - s) - (x_1)f(s,e)$$
$$\dot{e} = q_1 x_1 f(s,e) - de$$
$$\dot{t} = -dt \to 0$$
$$\dot{x}_1 = x_1((1 - q_1)f(s,e) - d)$$

Analysis says,
If $f(s,e) = \frac{d}{1-q_1}$ is sufficiently small, there will be three steady state solutions.
Two stable, one unstable.





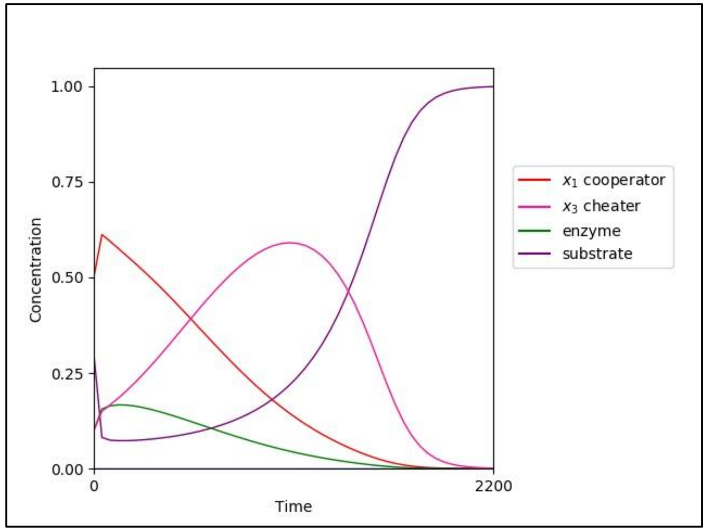
Cooperator and Cheater

$$\dot{s} = d(s^0 - s) - (x_1 + x_3)f(s,e)$$
$$\dot{e} = q_1 x_1 f(s,e) - de$$
$$\dot{t} = -dt \to 0$$
$$\dot{x}_1 = x_1((1 - q_1)f(s,e) - d)$$
$$\dot{x}_3 = x_3(f(s,e) - d)$$

Analysis says,

Requires
$$f(s,e) = d = \frac{d}{1 - q_1}$$

Not possible for $q_1 > 0$.
Thus a tragedy should occur.



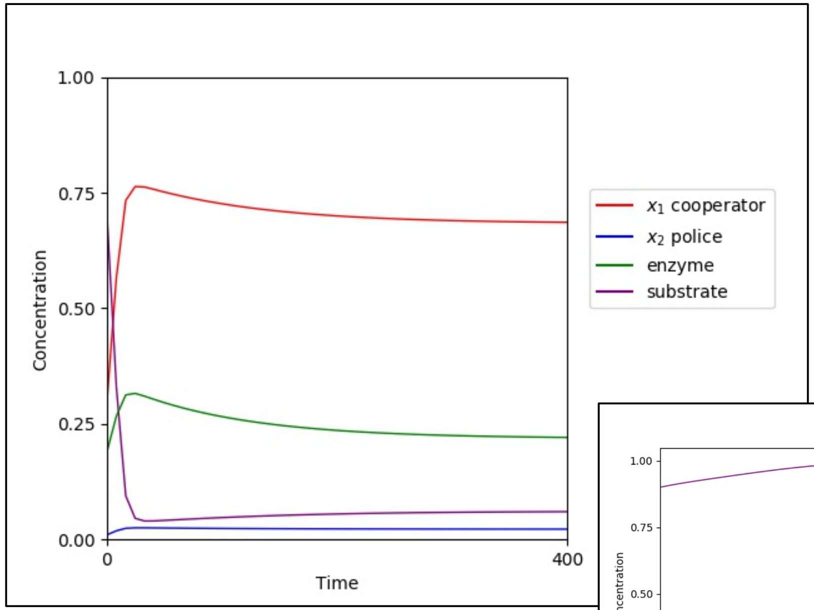
Cooperator and Police

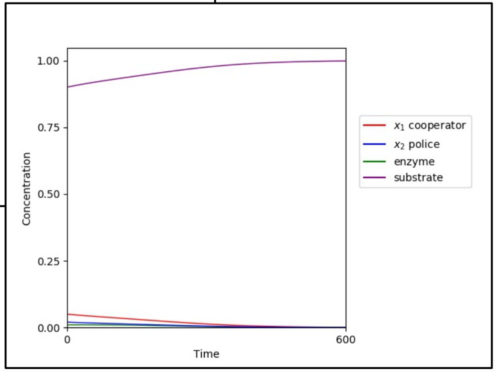
$$\dot{s} = d(s^0 - s) - (x_1 + x_2)f(s,e)$$
$$\dot{e} = q_1 x_1 f(s,e) - de$$
$$\dot{t} = q_2 x_2 f(s,e) - dt$$
$$\dot{x}_1 = x_1((1-q_1)f(s,e) - d - k_1 t)$$
$$\dot{x}_2 = x_2((1-q_2)f(s,e) - d)$$

Analysis says,

if $f(s,e) = \frac{d}{1-q_2}$ is sufficiently small and $q_2 > q_1$

Two additional steady states; one stable, one unstable.



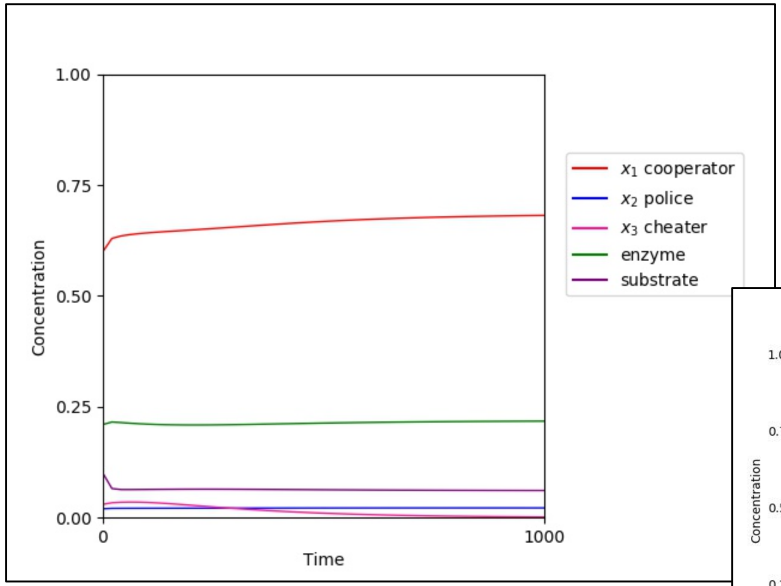


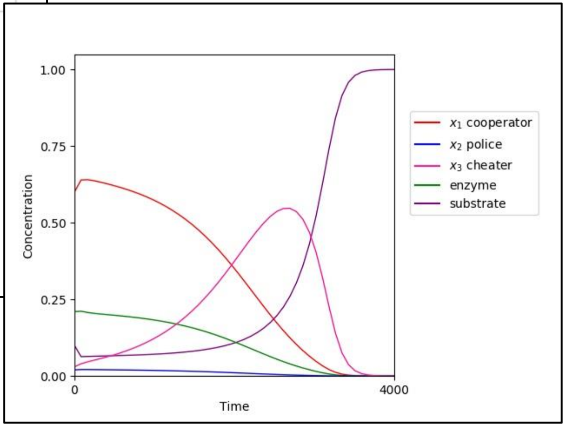
Full system

$$\dot{s} = d(s^0 - s) - (x_1 + x_2 + x_3)f(s,e)$$
$$\dot{e} = q_1 x_1 f(s,e) - de$$
$$\dot{t} = q_2 x_2 f(s,e) - dt$$
$$\dot{x}_1 = x_1((1-q_1)f(s,e) - d - k_1 t)$$
$$\dot{x}_2 = x_2((1-q_2)f(s,e) - d)$$
$$\dot{x}_3 = x_3(f(s,e) - d - k_3 t)$$

Analysis says,

No classical solutions.





Useful things I learned

- The power of doing mathematical analysis before numerical simulations.
- Coding best practices:
  - So, maybe I'm finally convinced of the importance of commenting code and not using silly variable names.
- Having a workflow increased my productivity by narrowing my focus to the immediate steps.
- Sometimes what you set out to prove is different from what you find.

Reproducibility

- Actually commented my code for once.
- Readme file:
  - explains the function of each piece of code
  - describes all other non-figure files
  - records system for naming files
- Text files with each figure:
  - Records the initial conditions and parameter values associated with each figure

Next Steps

- Finalize rigorous analysis of what occurs at steady states when cheater introduced.
- Finish (carefully) writing the analysis
  - LaTeX!
- Write the paper
- Take the nap