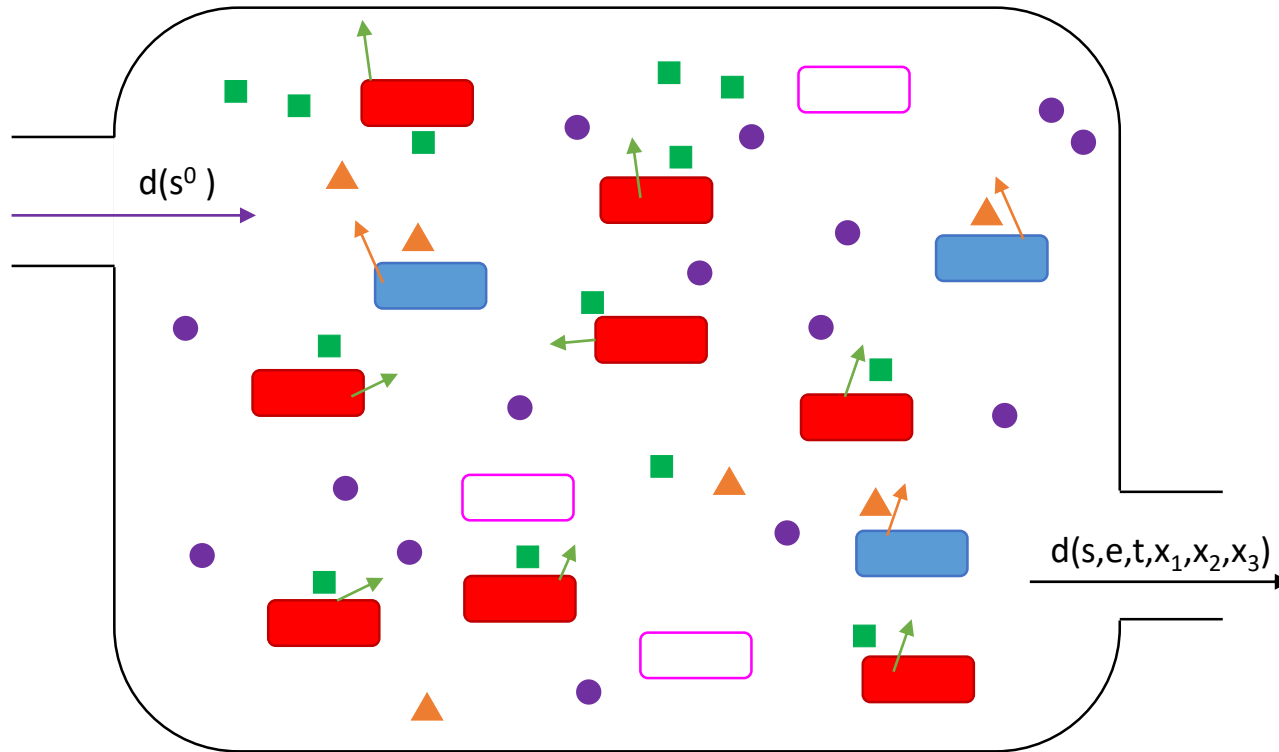


Policing as a mechanism to
avoid a tragedy of the commons
in *P. aeruginosa*

Bryan K. Lynn

Policing to avoid a tragedy of the commons



$$\dot{s} = d(s^0 - s) - (x_1 + x_2 + x_3)f(s, e)$$

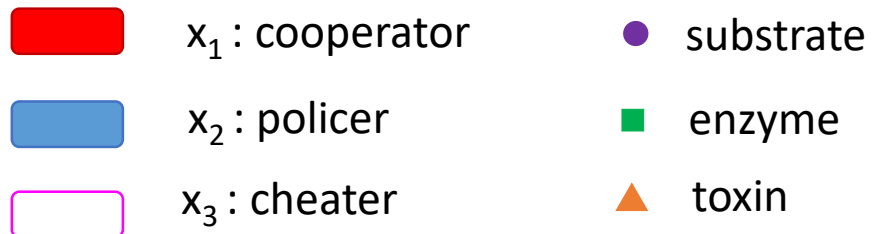
$$\dot{e} = q_1 x_1 f(s, e) - de$$

$$\dot{t} = q_2 x_2 f(s, e) - dt$$

$$\dot{x}_1 = x_1((1 - q_1)f(s, e) - d - k_1 t)$$

$$\dot{x}_2 = x_2((1 - q_2)f(s, e) - d)$$

$$\dot{x}_3 = x_3(f(s, e) - d - k_3 t)$$



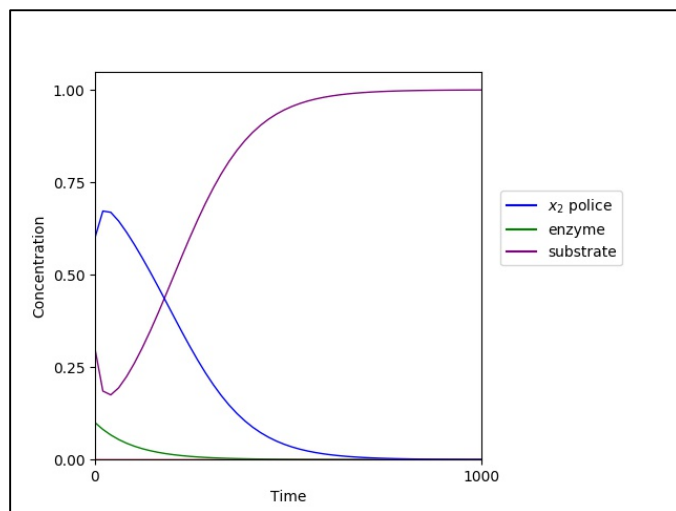
Police only

$$\dot{s} = d(s^0 - s) - (x_2)f(s, e)$$

$$\dot{e} = -de \rightarrow 0$$

$$\dot{t} = q_2 x_2 f(s, e) - dt$$

$$\dot{x}_2 = x_2((1 - q_2)f(s, e) - d)$$



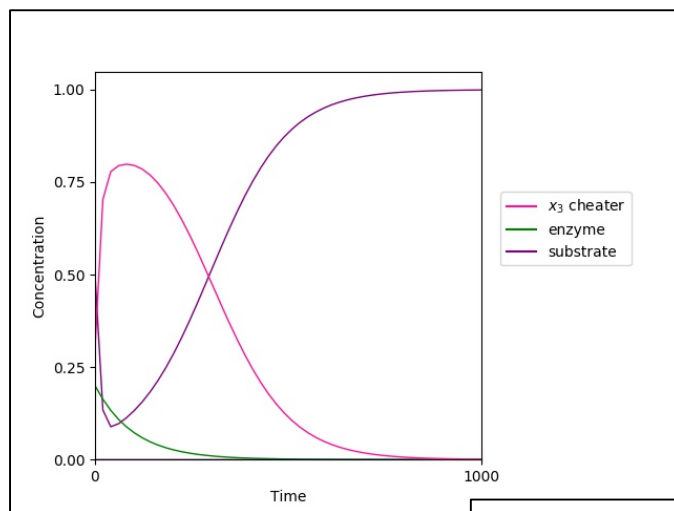
Cheater only

$$\dot{s} = d(s^0 - s) - (x_3)f(s, e)$$

$$\dot{e} = -de \rightarrow 0$$

$$\dot{t} = -dt \rightarrow 0$$

$$\dot{x}_3 = x_3(f(s, e) - d)$$



Police and Cheater

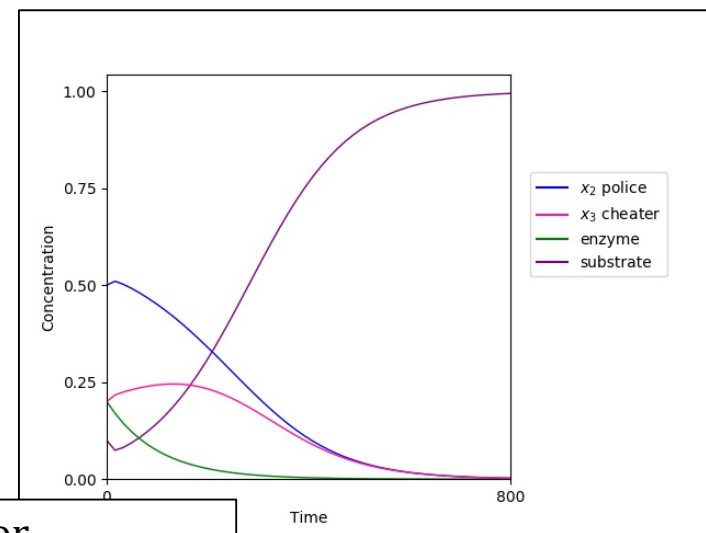
$$\dot{s} = d(s^0 - s) - (x_2 + x_3)f(s, e)$$

$$\dot{e} = -de \rightarrow 0$$

$$\dot{t} = q_2 x_2 f(s, e) - dt$$

$$\dot{x}_2 = x_2((1 - q_2)f(s, e) - d)$$

$$\dot{x}_3 = x_3(f(s, e) - d - k_3 t)$$



Analysis says, for

$$x_1 = 0 \Rightarrow e = 0$$

$$e = 0 \Rightarrow x_2, x_3 = 0$$

Thus a tragedy should occur.

Cooperator only

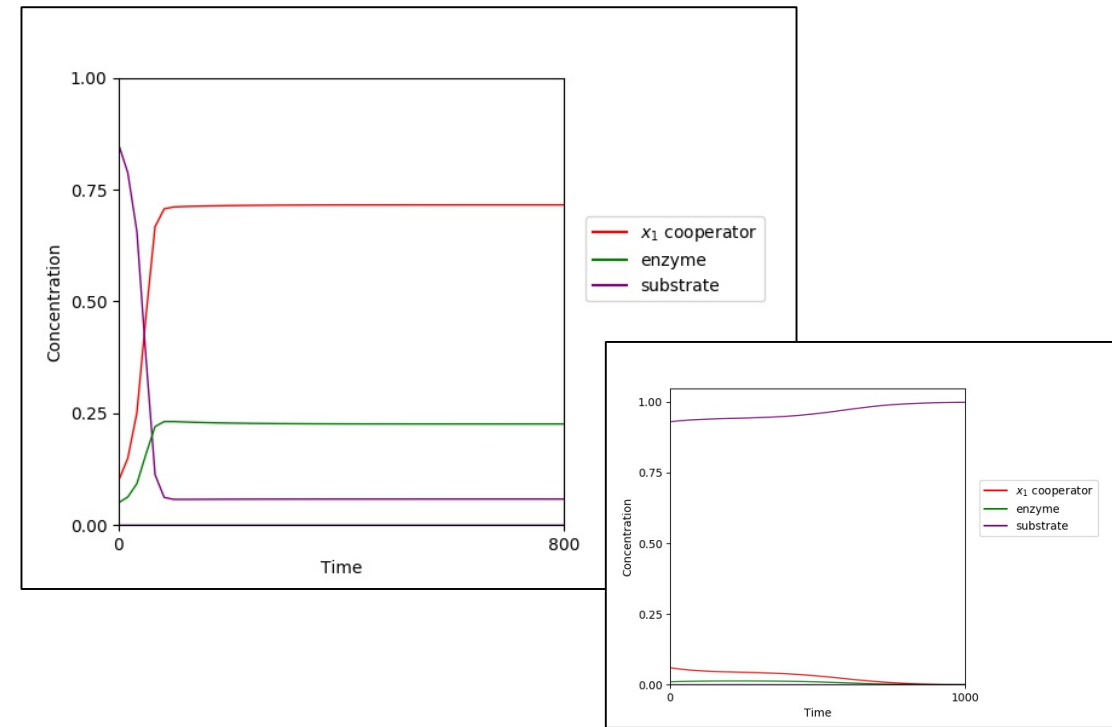
$$\dot{s} = d(s^0 - s) - (x_1)f(s, e)$$

$$\dot{e} = q_1 x_1 f(s, e) - de$$

$$\dot{t} = -dt \rightarrow 0$$

$$\dot{x}_1 = x_1((1 - q_1)f(s, e) - d)$$

Analysis says,
If $f(s, e) = \frac{d}{1 - q_1}$ is sufficiently small, there will be three steady state solutions.
Two stable, one unstable.



Cooperator and Cheater

$$\dot{s} = d(s^0 - s) - (x_1 + x_3)f(s, e)$$

$$\dot{e} = q_1 x_1 f(s, e) - de$$

$$\dot{t} = -dt \rightarrow 0$$

$$\dot{x}_1 = x_1((1 - q_1)f(s, e) - d)$$

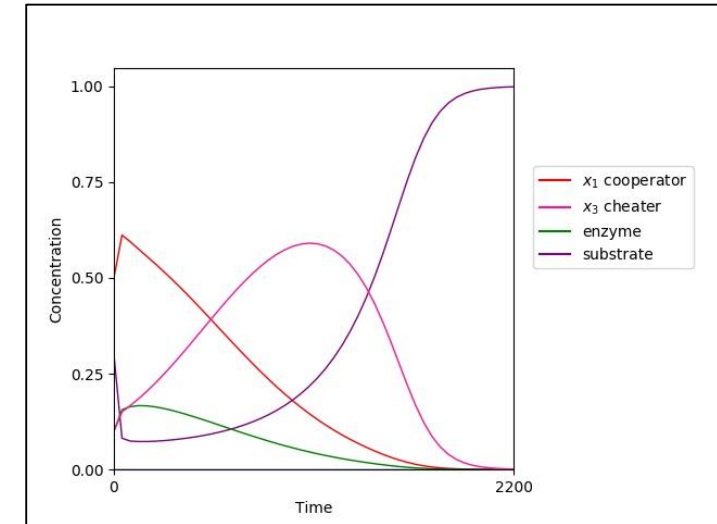
$$\dot{x}_3 = x_3(f(s, e) - d)$$

Analysis says,

Requires

$$f(s, e) = d = \frac{d}{1 - q_1}$$

Not possible for $q_1 > 0$.
Thus a tragedy should occur.



Cooperator and Police

$$\dot{s} = d(s^0 - s) - (x_1 + x_2)f(s, e)$$

$$\dot{e} = q_1 x_1 f(s, e) - de$$

$$\dot{t} = q_2 x_2 f(s, e) - dt$$

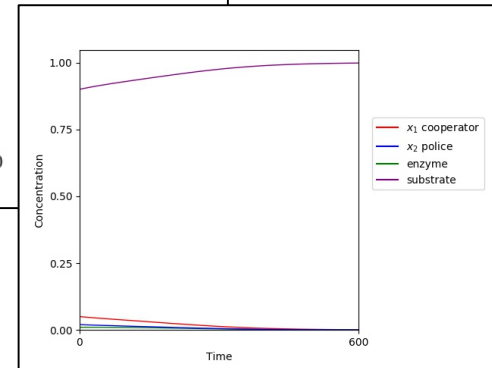
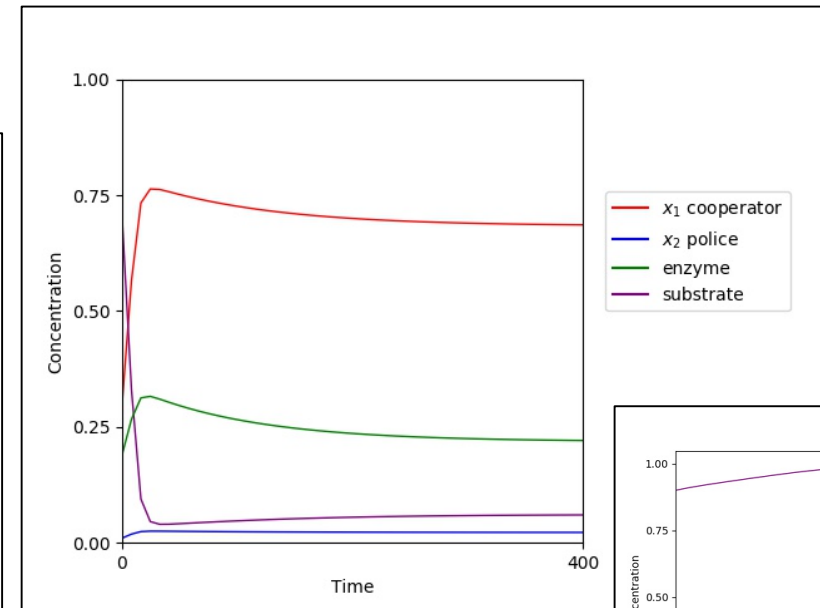
$$\dot{x}_1 = x_1((1 - q_1)f(s, e) - d - k_1 t)$$

$$\dot{x}_2 = x_2((1 - q_2)f(s, e) - d)$$

Analysis says,

if $f(s, e) = \frac{d}{1 - q_2}$ is
sufficiently small and
 $q_2 > q_1$

Two additional steady states;
one stable, one unstable.



Full system

$$\dot{s} = d(s^0 - s) - (x_1 + x_2 + x_3)f(s, e)$$

$$\dot{e} = q_1 x_1 f(s, e) - de$$

$$\dot{t} = q_2 x_2 f(s, e) - dt$$

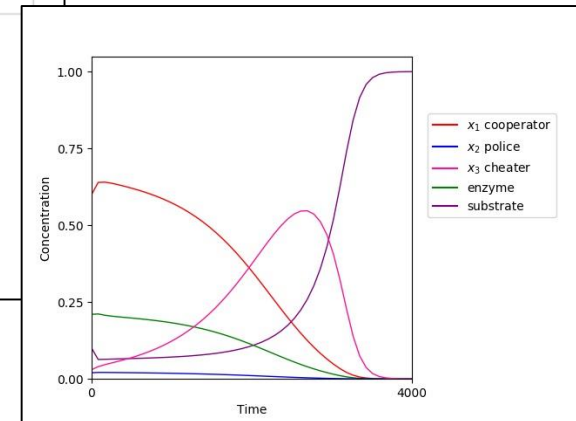
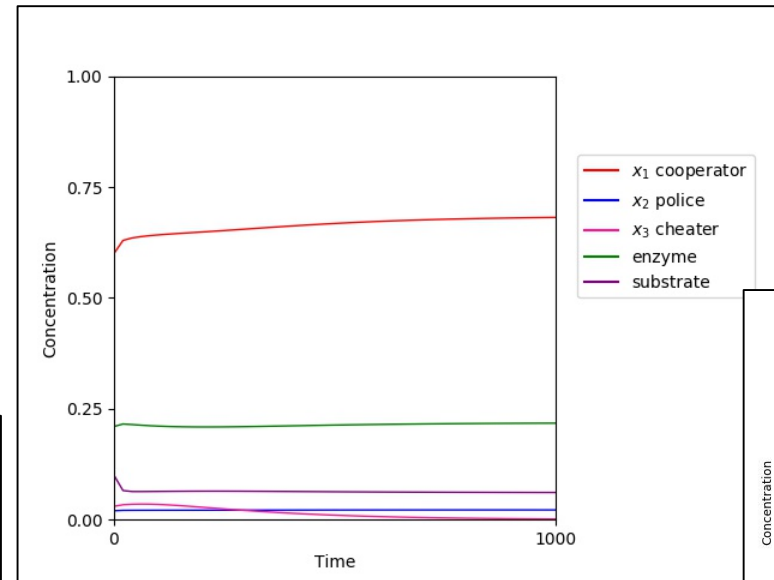
$$\dot{x}_1 = x_1((1 - q_1)f(s, e) - d - k_1 t)$$

$$\dot{x}_2 = x_2((1 - q_2)f(s, e) - d)$$

$$\dot{x}_3 = x_3(f(s, e) - d - k_3 t)$$

Analysis says,

No classical solutions.



Useful things I learned

- The power of doing mathematical analysis before numerical simulations.
- Coding best practices:
 - So, maybe I'm finally convinced of the importance of commenting code and not using silly variable names.
- Having a workflow (AKA a roadmap of goals) increased my productivity by narrowing my focus to the immediate steps.
- Sometimes what you set out to prove is different from what you find.

Reproducibility

- Actually commented my code for once.
- Readme file:
 - explains the function of each piece of code
 - describes all other non-figure files
 - records system for naming files
- Text files with each figure:
 - Records the initial conditions and parameter values associated with each figure

Next Steps

- Finalize rigorous analysis of what occurs at steady states when cheater introduced.
- Finish (carefully) writing the analysis
 - LaTeX!
- Write the paper
- Take the nap