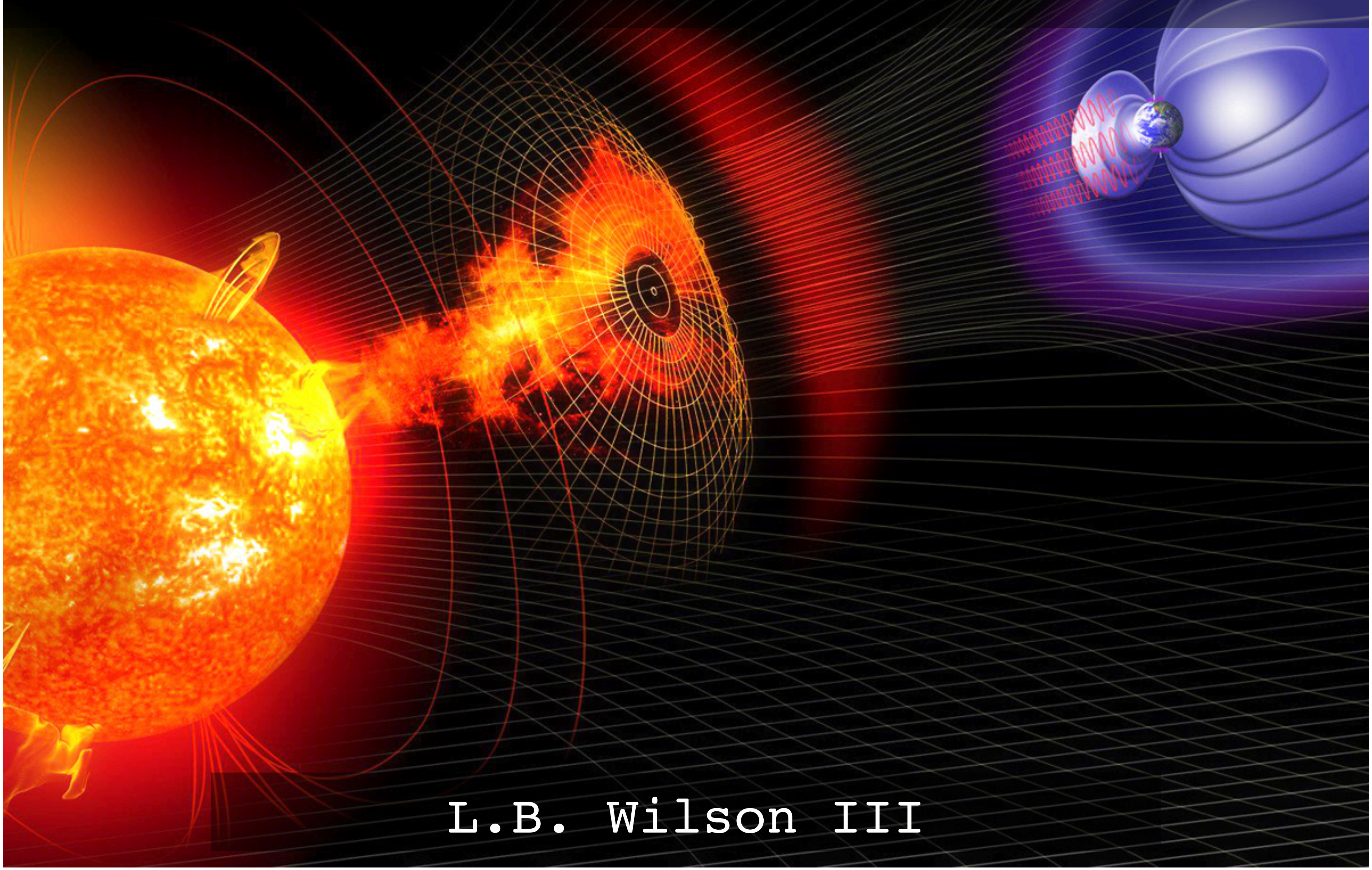
Collisionless Shock Waves: Critical Mach Numbers and Whistler Precursors



L.B. Wilson III

Introduction: Terminology

Q: What are shock waves?

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nonlinearly steepened wave
with energy dissipation
balancing wave steepening

stable discontinuity forms

Q: Wait a minute, why do Water Waves Break?

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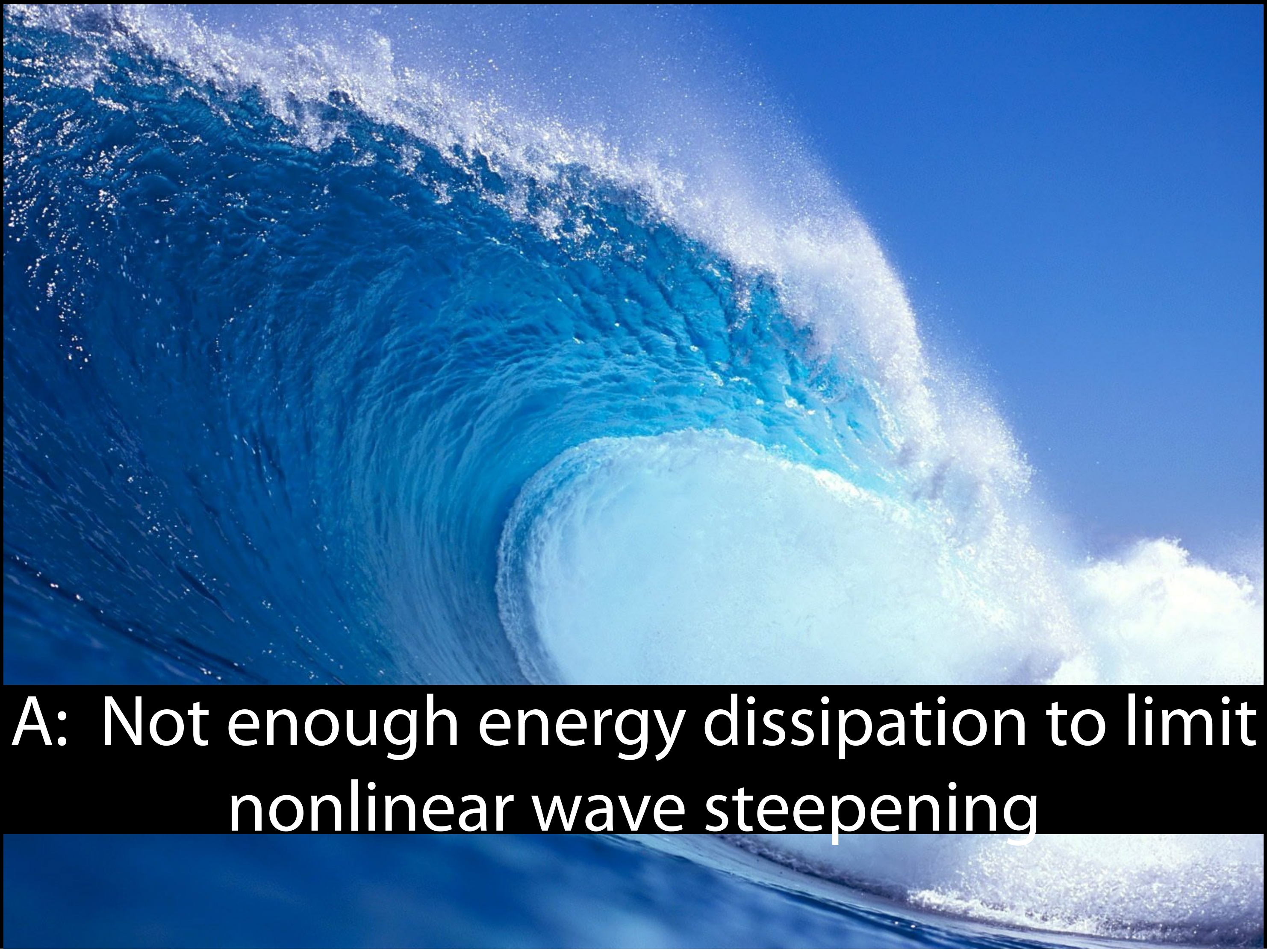

A: Not enough energy dissipation to limit nonlinear wave steepening

Q: What is nonlinear wave steepening?

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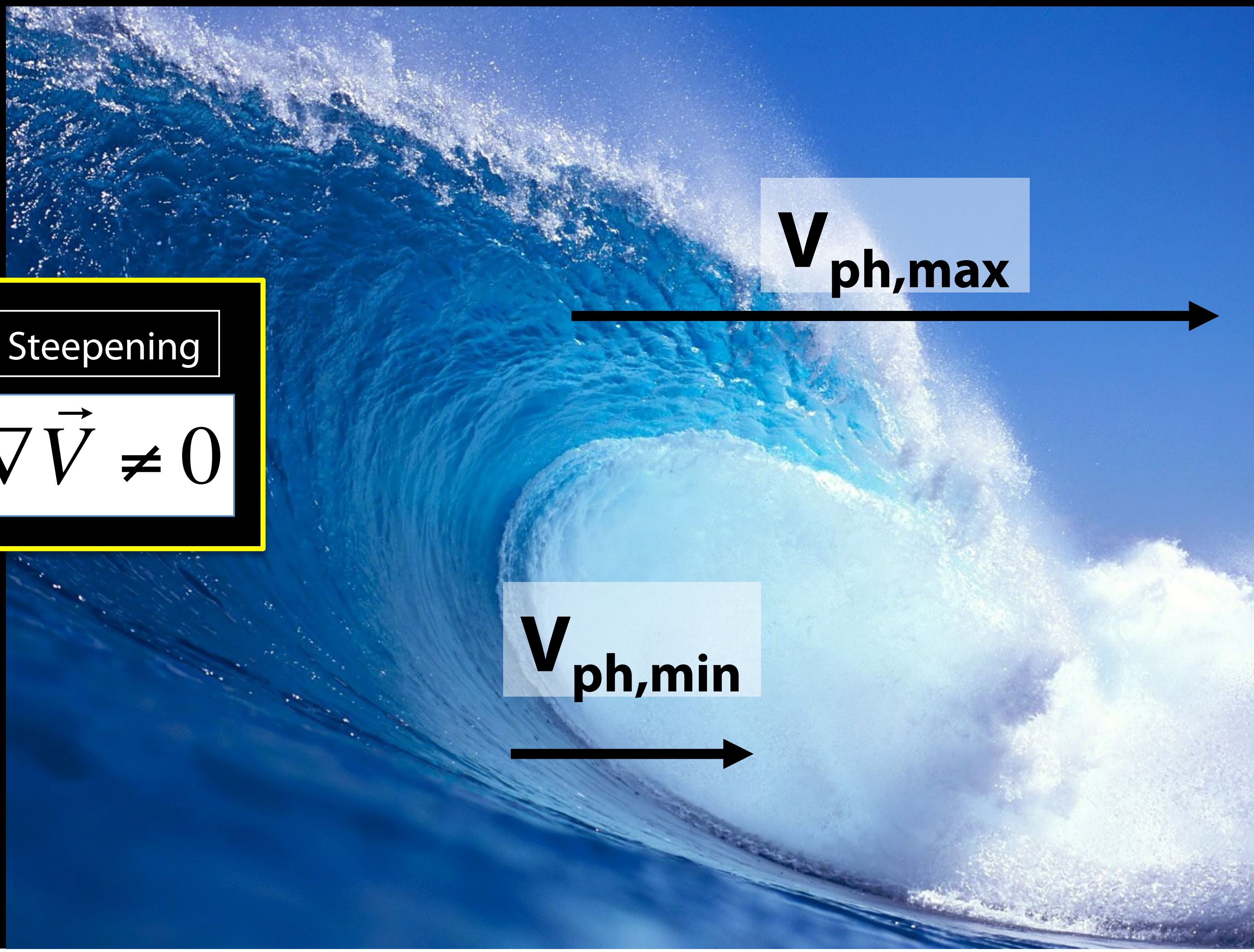
A: Amplitude dependence in phase velocity



Wave Steepening

$$\vec{V}\cdot\nabla\vec{V}\neq 0$$

Q: What is Energy Dissipation?

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A: Transformation of free energy from one form to another through an irreversible process

Q: Why is energy dissipation necessary?

Upstream

Transition Region

Downstream

High Ram Pressure
Low Thermal Pressure

Low Ram Pressure
High Thermal Pressure

$$KE_{up} - KE_{down} \neq 0$$

Kinetic energy must be transformed to some other form to conserve energy

What other forms?

Q: How is energy dissipated?

Upstream

Transition Region

Downstream

High Ram Pressure
Low Thermal Pressure

Low Ram Pressure
High Thermal Pressure

Collisional fluids (e.g. our atmosphere) → Binary Particle Collisions → Increase in random thermal energy (i.e. heat)

Q: What happens if there are no binary particle collisions?

Collisionless fluids
(e.g. solar wind)

dispersive effects
(e.g. wave radiates)

→ waves carry energy away from transition region (or ramp)

resistive effects
(e.g. waves scatter)

→ particle scattering reduces currents in ramp

viscous effects
(e.g. particles gyrate)

→ reflected particles reduce momentum of shock wave

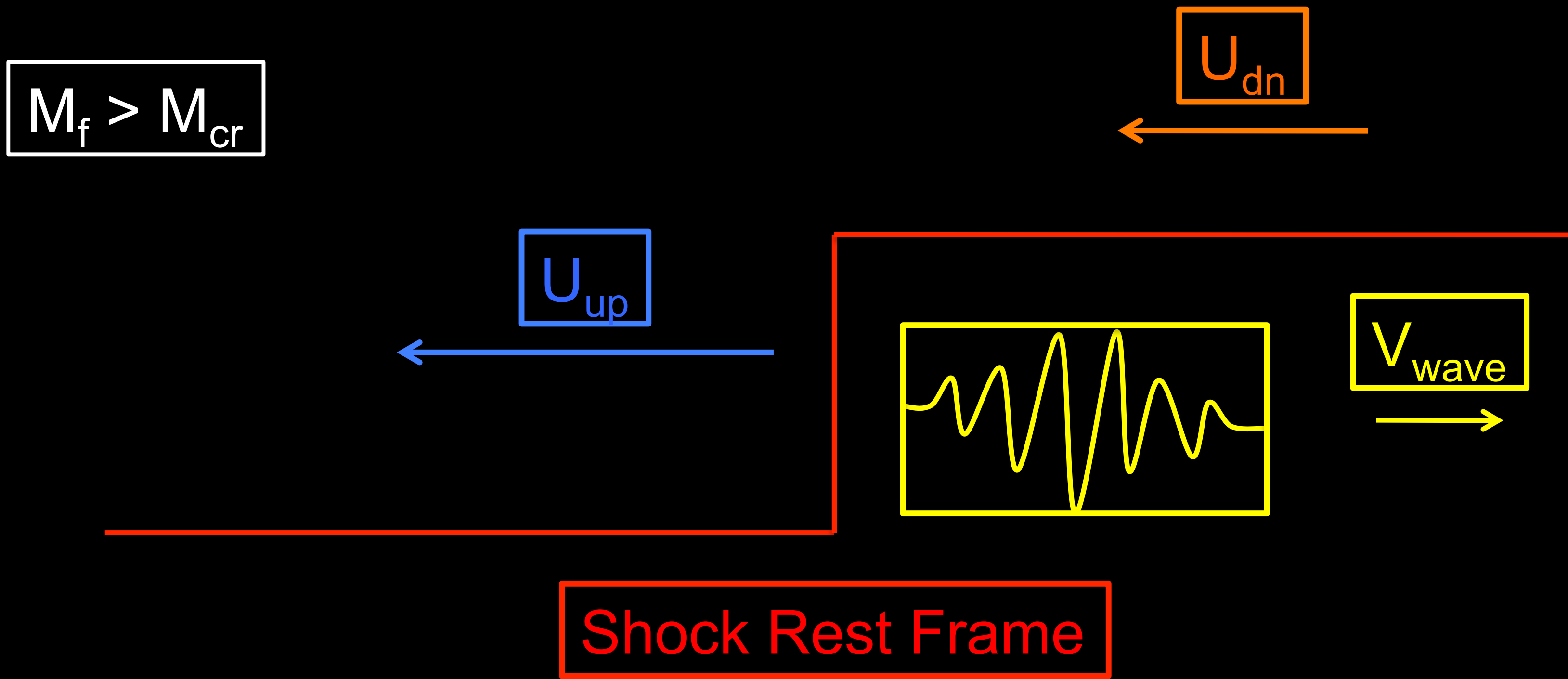
For more details, see:
[*e.g.* *Kennel et al.*, 1985; *Krasnosel'skikh et al.*, 2002]

Introduction: Shock Criticality

Critical Mach Numbers

- There are 4 critical Mach numbers used in space plasma physics:
  1) M_{cr} = 1st critical Mach # [e.g. *Edmiston and Kennel*, 1984]
  2) M_{w} = whistler critical Mach # [e.g. *Krasnosel'skikh et al.*, 2002]
  3) M_{gr} = whistler group critical Mach # [e.g. *Krasnosel'skikh et al.*, 2002]
  4) M_{nw} = nonlinear whistler critical Mach # [e.g. *Krasnosel'skikh et al.*, 2002]

1^{st} Critical Mach Number



$$M_{cr} = M_f \text{ at which point } |V_{wave}| \leq |\mathbf{U}_{dn}|$$

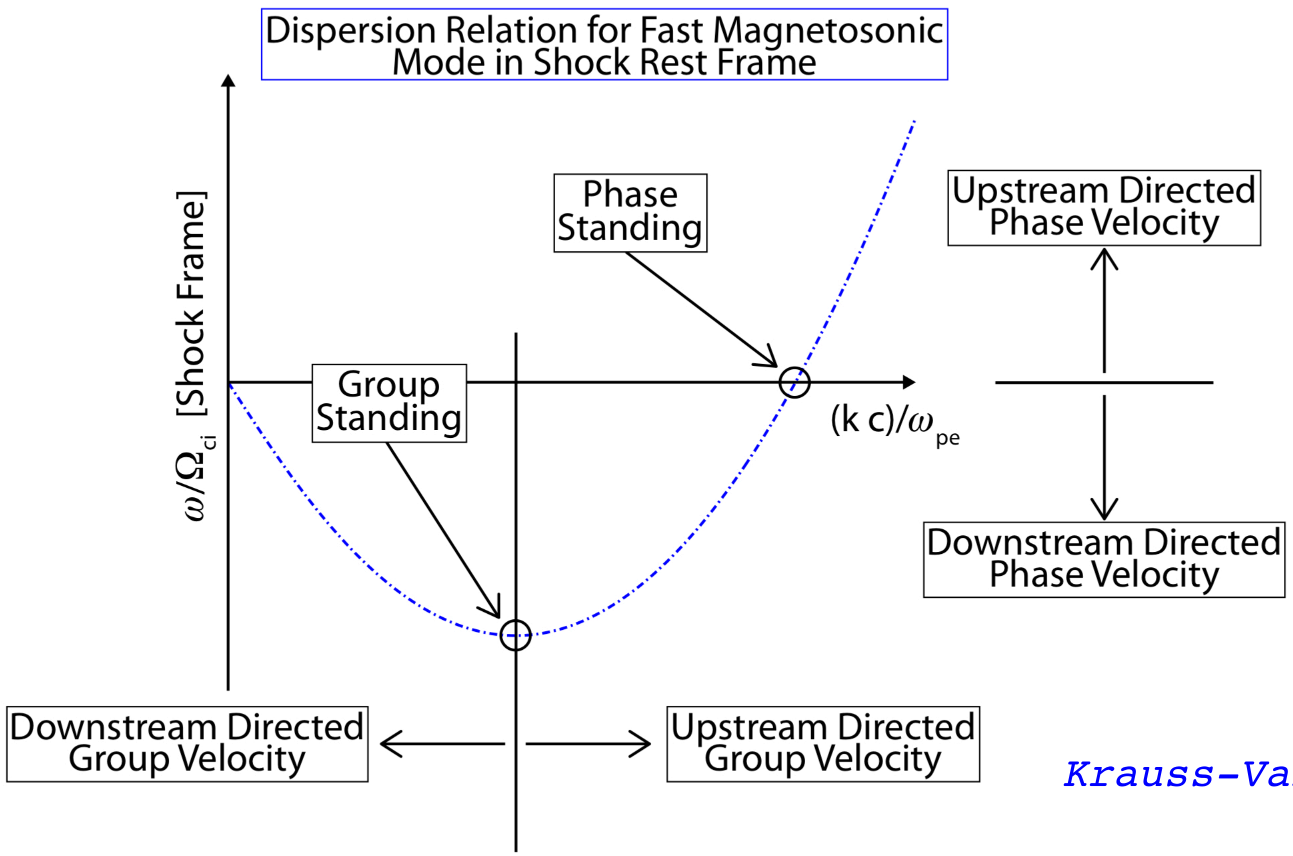
Magnetosonic-Whistler Dispersion Relations

$$\omega^2 \cong k^2 V_A^2 \left(1 + \cos^2\theta_{kB}\left(1 + \frac{k^2 c^2}{\omega_{pi}^2}\right)\right)$$

Doppler Shift to Shock Frame

$$\omega_{sh} = \omega_{rest} + \vec{k}\cdot\vec{V}_{trans}$$

$$\left\{\vec{V}_{trans} = \vec{V}_{sw} - V_{sh}\,\hat{n}\right\}$$



Krauss-Varban and Omidi, [1991]

Whistler Critical Mach Numbers

$$M_w = \frac{1}{2}\sqrt{\frac{m_i}{m_e}}\left|\cos\theta_{Bn}\right|$$

$$M_{gr} = \sqrt{\frac{27\,m_i}{64\,m_e}}\left|\cos\theta_{Bn}\right|$$

$$M_{nw} = \sqrt{\frac{m_i}{2\,m_e}}\left|\cos\theta_{Bn}\right|$$

M_w = max Mach number where **linear whistlers** can phase stand upstream of shock ramp

M_{gr} = max Mach number where **linear whistlers** can carry energy upstream away from shock ramp

M_{nw} = Mach number above which a **nonlinear whistler** wave becomes unstable to a gradient catastrophe [i.e., the wave breaks]

Krasnosel'skikh et al., [2002]

IP Shock RH Parameters

Date	Shock Normal [GSE]	V_{shn} (km/s)	θ_{Bn} (deg)	M_f	N_2/N_1
1997-12-10	<-0.90,+0.17,-0.40>	391 +/- 12	71 +/- 2	2.3 +/- 0.1	2.5 +/- 0.4
1998-08-26	<-0.66,+0.04,-0.75>	687 +/- 27	82 +/- 3	4.7 +/- 0.2	2.9 +/- 0.3
1998-09-24	<-0.91,-0.22,-0.34>	772 +/- 96	82 +/- 2	2.9 +/- 0.1	2.2 +/- 0.4
2000-02-11	<-0.87,-0.45,+0.22>	641 +/- 13	87 +/- 2	3.3 +/- 0.1	3.3 +/- 0.5

Shock parameters determined by: J. C. Kasper, Interplanetary Shock Database, Harvard Smithsonian Center for Astrophysics, Online: http://www.cfa.harvard.edu/shocks/

M_f = fast mode Mach number = U_{n1}/V_f
N_2/N_1 = shock compression ratio
θ_{Bn} = shock normal angle
V_{shn} = shock normal speed [SC frame]
V_f = fast mode speed
U_{n1} = upstream shock normal speed [shock frame]
C_s = sound speed = $(\gamma P/\rho)$
V_A = Alfvén speed = $B_o/(\mu_o M_i n_i)^{1/2}$

$$2V_f^2 = V_s^2 + \sqrt{V_s^4 - 4C_s^2 V_A^2 \cos^2\theta_{Bn}}$$

$$V_s^2 = \left(C_s^2 + V_A^2\right)$$

Note: 1 km/s ~ 39.2 femtoParsecs/microfortnight

IP Shock Mach Numbers

Date	M_f/M_{cr}	M_f/M_w	M_f/M_{gr}	M_f/M_{nw}
1997-12-10	1.1 +/- 0.2	0.3 +/- 0.0	0.3 +/- 0.0	0.2 +/- 0.0
1998-08-26	2.6 +/- 0.4	1.6 +/- 0.5	1.3 +/- 0.5	1.2 +/- 0.4
1998-09-24	1.3 +/- 0.2	1.0 +/- 0.2	0.8 +/- 0.2	0.7 +/- 0.2
2000-02-11	1.6 +/- 0.2	2.5 +/- 1.2	1.9 +/- 1.2	1.8 +/- 1.1

M_f = fast mode Mach number
M_{cr} = critical Mach number assuming γ = 5/3 [*Edmiston and Kennel*, 1984]
M_w = first whistler critical Mach [*Krasnosel'skikh et al.*, 2002]
M_{gr} = whistler group critical Mach [*Krasnosel'skikh et al.*, 2002]
M_{nw} = nonlinear whistler critical Mach [*Krasnosel'skikh et al.*, 2002]

Whistler Waves: Background

Whistler Mode Waves: Description

Whistler = generic term used to describe a broad range of waves with an even broader range of properties

Dispersive

$$n^2 = \frac{\omega_{pe}^2}{\omega\left(\Omega_{ce}\cos\theta - \omega\right)}$$

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m_e \varepsilon_o}}$$

$$\Omega_{ce} = \frac{eB_o}{m_e}$$

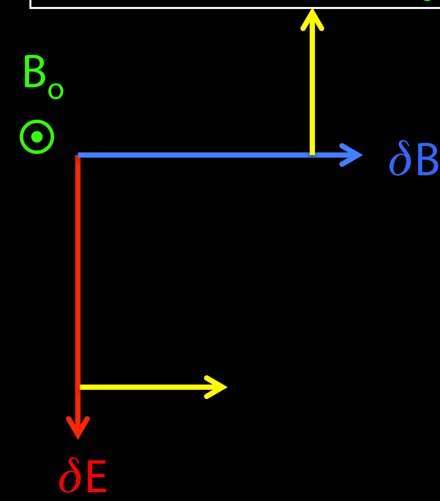
Broad Rand of Frequencies

$$\Omega_{ci} \le \omega \le \Omega_{ce}$$

Oblique Propagation

$$\theta_{kB} = \cos^{-1}\left(\frac{\hat{k}\cdot B_o}{\left|B_o\right|}\right)$$

Right-hand polarized (with respect to B_o)



Can interact with both electrons and ions by: (1) Landau resonance, (2) cyclotron resonance [*Kennel and Petschek*, 1966], (3) nonlinear trapping [*Kellogg et al.*, 2010], etc.

Whistler Mode Waves: Types

Note: Whistler mode waves can lie on the same branch of the dispersion curve as lower hybrid and magnetosonic waves

Magnetosphere Whistlers

Lightning Whistlers:
[*e.g.* *Storey*, 1953]

Chorus:
[*e.g.* *Burtis and Helliwell*, 1969]

Plasmaspheric Hiss:
[*e.g.* *Thorne et al.*, 1973]

Very Large Amplitude Radiation Belt Whistlers:
[*e.g.* *Cattell et al.*, 2008]

Solar Wind Whistlers

Whistler Precursors:
[*e.g.* *Fairfield et al.*, 1975]

"1 Hz Waves":
[*e.g.* *Hoppe et al.*, 1982]

Steepened ULF Foreshock Waves:
[*e.g.* *Hoppe et al.*, 1981]

Electromagnetic Lower Hybrid Waves:
[*e.g.* *Marsch and Chang*, 1983; *Coroniti et al.*, 1982]

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Whistler Waves: Precursors

Whistler Precursors: Basics

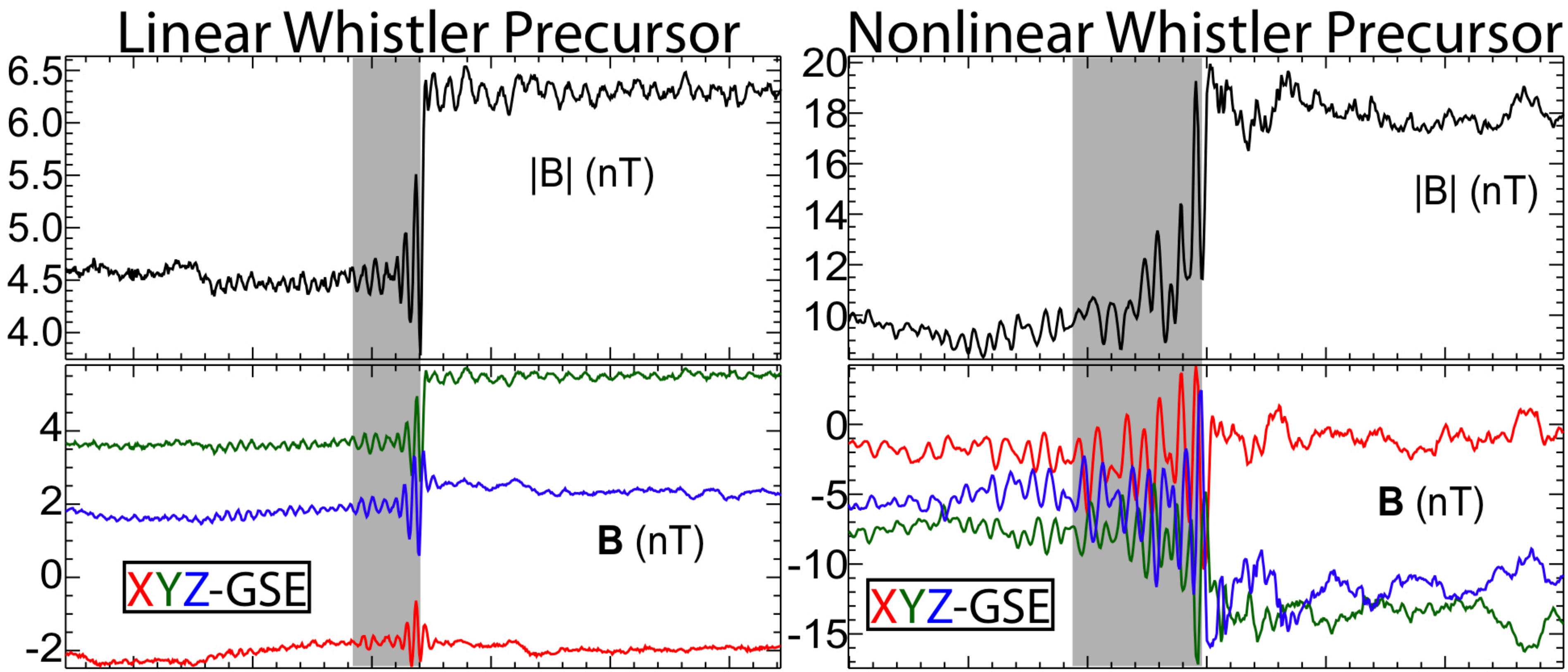
- They are observed as a wave train of whistler mode waves upstream of a shock ramp [e.g. *Greenstadt et al.*, 1975]
- They are thought to be produced by dispersive radiation from the steepened shock ramp [e.g. *Mellott and Greenstadt*, 1984]
- They have the same general properties as whistler mode waves, but are synonymous with magnetosonic [e.g. *Wu et al.*, 1983] and electromagnetic lower hybrid waves [e.g. *Marsch and Chang*, 1983]

Whistler Precursors: Wave Properties

- They propagate obliquely with respect to both B_o and the shock normal vector [e.g. *Wilson III et al.*, 2009; *Mazelle et al.*, 2010; *Sundkvist et al.*, 2012]

- They have rest frame frequencies of $f_{ci} \leq f \leq f_{lh}$ ($f_{lh}^2 = f_{ci} f_{ce}$) [e.g. *Greenstadt et al.*, 1981]

Whistler Precursors: Linear vs. Nonlinear



Whistler Precursor Generation:
Dispersion

Wave Steepening

$$\vec{V} \cdot \nabla \vec{V} \neq 0$$

For more details, see:
[*e.g. Kennel et al.*, 1985; *Krasnosel'skikh et al.*, 2002]

Whistler Precursor Generation:
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Gradient scale length of wave front decreases

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Wave couples to smaller and smaller wavelength, λ, modes

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Wave couples to smaller and smaller wavelength, λ, modes

$\omega/k \sim (1 + a\,k^2)^{1/2}$; thus smaller λ means larger phase velocity, V_{ph}

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Higher(Lower) $\omega(\lambda)$ propagate faster and outrun steepened wave front

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Higher(Lower) $\omega(\lambda)$ propagate faster and outrun steepened wave front

Wave train moves upstream carrying energy away from steepened wave

For more details, see:
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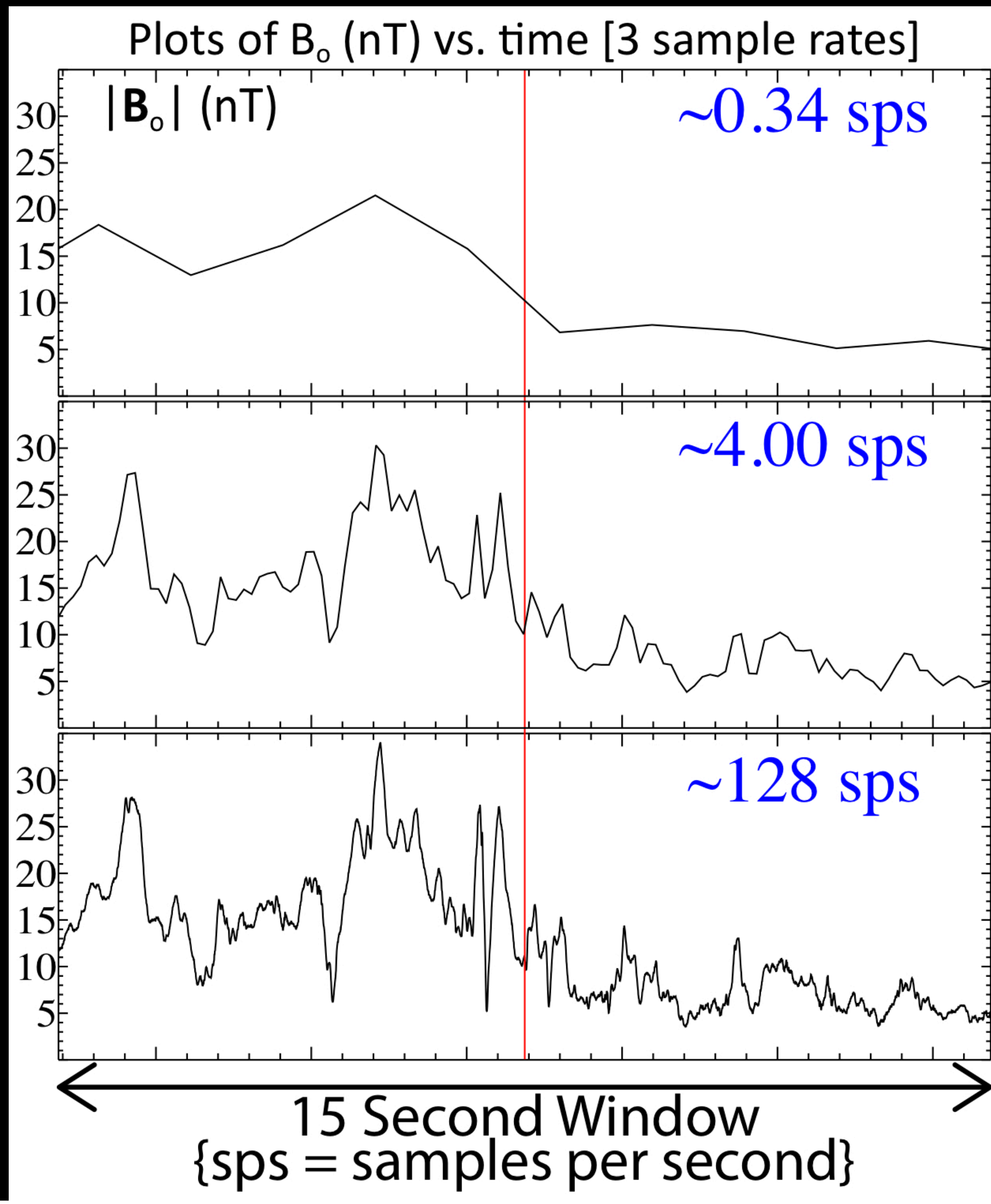
Whistler Precursors: Observations

Whistler Precursors: Need for high time resolution



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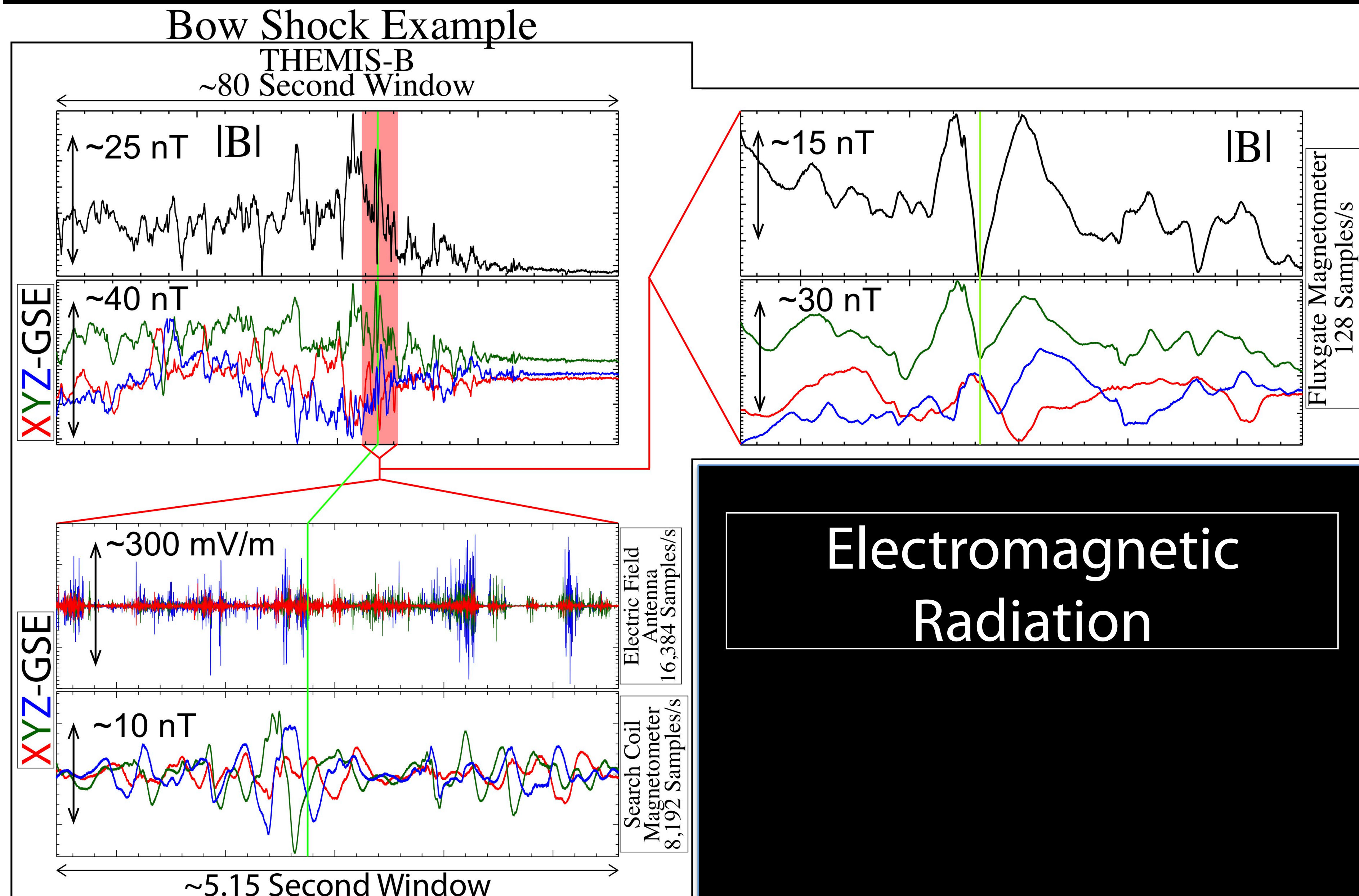
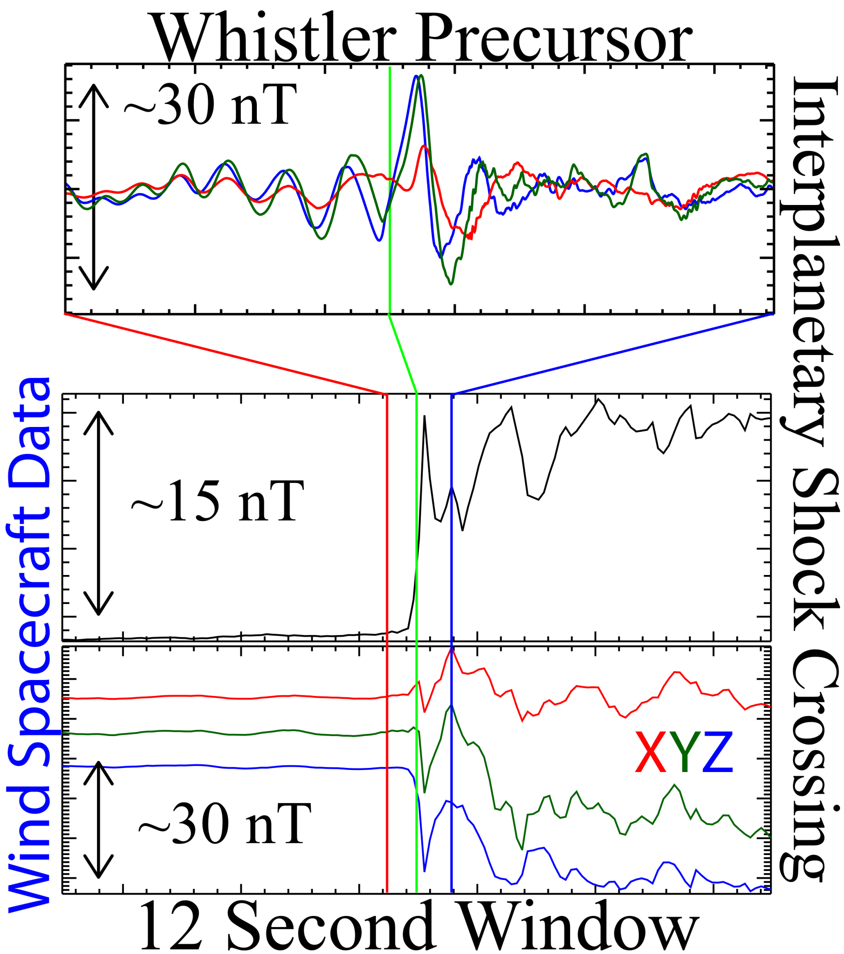


Electromagnetic
Radiation

Whistler Precursors: Observations



New Results from the Wind Spacecraft

A) Precursors have $\delta B/B_o \sim 2$
--> much larger than previous observations

B) Precursors observed upstream of shock with $M_f > M_{nw}$
--> waves source must be upstream of ramp

C) Precursor driven wave-particle effects observed
--> strong parallel electron acceleration and perpendicular ion heating

D) Fluxgate magnetometer undersamples shock
--> previous assumptions/observations of laminar shock crossings were undersampled
--> brings into question assumptions of planar and stationary shocks

NASA homepage news:
http://www.nasa.gov/mission pages/sunearth/news/riding-plasma-waves.html

Wilson III, L.B., A. Koval, A. Szabo, A. Breneman, C.A. Cattell, K. Goetz, P.J. Kellogg, K. Kersten, J.C. Kasper, B.A. Maruca, and M. Pulupa (2012), "Observations of Electromagnetic Whistler Precursors at Supercritical Interplanetary Shocks," *Geophys. Res. Lett.*, **39**, L08109, doi:10.1029/2012GL051581.

References

1. Burtis, W. J., and R. A. Helliwell (1969), Banded chorus - A new type of VLF radiation observed in the magnetosphere by OGO 1 and OGO 3., J. Geophys. Res., 74, 3002–3010, doi: 10.1029/JA074i011p03002.
2. Cattell, C., J. R. Wygant, K. Goetz, K. Kersten, P. J. Kellogg, T. von Rosenvinge, S. D. Bale, I. Roth, M. Temerin, M. K. Hudson, R. A. Mewaldt, M. Wiedenbeck, M. Maksimovic, R. Ergun, M. Acuna, and C. T. Russell (2008), Discovery of very large amplitude whistler-mode waves in Earth's radiation belts, Geophys. Res. Lett., 35, 1105–+, doi: 10.1029/2007GL032009.
3. Coroniti, F. V., C. F. Kennel, F. L. Scarf, and E. J. Smith (1982), Whistler mode turbulence in the disturbed solar wind, J. Geophys. Res., 87, 6029–6044, doi:10.1029/JA087iA08p06029.
4. Edmiston, J.P. and C.F. Kennel (1984), A parametric survey of the first critical Mach number for a fast MHD shock., *J. Plasma Phys.*, **32**, 429–441.
5. Fairfield, D. H., and W. C. Feldman (1975), Standing waves at low Mach number laminar bow shocks, J. Geophys. Res., 80, 515–522, doi:10.1029/JA080i004p00515.
6. Greenstadt, E. W., F. L. Scarf, C. T. Russell, V. Formisano, and M. Neugebauer (1975), Structure of the quasi-perpendicular laminar bow shock, J. Geophys. Res., 80, 502–514, doi: 10.1029/JA080i004p00502.
7. Greenstadt, E. W., R. W. Fredricks, F. L. Scarf, C. T. Russell, R. R. Anderson, and D. A. Gurnett (1981), Whistler mode wave propagation in the solar wind near the bow shock, J. Geophys. Res., 86, 4511–4516, doi:10.1029/JA086iA06p04511.
8. Hoppe, M. M., C. T. Russell, L. A. Frank, T. E. Eastman, and E. W. Greenstadt (1981), Upstream hydromagnetic waves and their association with backstreaming ion populations - ISEE 1 and 2 observations, J. Geophys. Res., 86, 4471–4492, doi: 10.1029/JA086iA06p04471.
9. Hoppe, M. M., C. T. Russell, T. E. Eastman, and L. A. Frank (1982), Characteristics of the ULF waves associated with upstream ion beams, J. Geophys. Res., 87, 643–650, doi: 10.1029/JA087iA02p00643.
10. Kellogg, P.J., C.A. Cattell, K. Goetz, S.J. Monson, and L.B. Wilson III (2010b), Electron trapping and charge transport by large amplitude whistlers, *Geophys. Res. Lett.*, **37**, 20,106, doi: 10.1029/2010GL044845.
11. Kennel, C.F., and H.E. Petscheck (1966), Limit on stably trapped particle fluxes, *J. Geophys. Res.*, **71**, 1–28.
12. Kennel, C.F., J.P. Edmiston, and T. Hada (1985), A quarter century of collisionless shock research, in *Collisionless Shocks in the Heliosphere: A Tutorial Review*, *Geophys. Monogr. Ser.*, **34**, edited by R. G. Stone and B. T. Tsurutani, pp. 1–36, AGU, Washington, D.C.
13. Krasnoselskikh, V.V., B. Lembège, P. Savoini, and V. V. Lobzin (2002), Nonstationarity of strong collisionless quasi-perpendicular shocks: Theory and full particle numerical simulations, *Phys. Plasmas*, **9**, 1192–1209, doi:10.1063/1.1457465.
14. Krauss-Varban, D., and N. Omidi (1991), Structure of medium Mach number quasi-parallel shocks - Upstream and downstream waves, *J. Geophys. Res.*, **96**, 17,715, doi: 10.1029/91JA01545.
15. Marsch, E., and T. Chang (1983), Electromagnetic lower hybrid waves in the solar wind, J. Geophys. Res., 88, 6869–6880, doi: 10.1029/JA088iA09p06869.
16. Mazelle, C., B. Lemb`ege, A. Morgenthaler, K. Meziane, T. S. Horbury, V. G´enot, E. A. Lucek, and I. Dandouras (2010), Self-Reformation of the Quasi-Perpendicular Shock: CLUSTER Observations, Twelfth International Solar Wind Conference, 1216, 471–474, doi:10.1063/1.3395905.
17. Mellott, M. M., and E. W. Greenstadt (1984), The structure of oblique subcritical bow shocks - ISEE 1 and 2 observations, J. Geophys. Res., 89, 2151–2161, doi:10.1029/JA089iA04p02151.
18. Storey, L. R. O. (1953), An Investigation of Whistling Atmospherics, Royal Society of London Philosophical TransactionsSeries A, 246, 113–141.
19. Sundkvist, D., V. Krasnoselskikh, S. D. Bale, S. J. Schwartz, J. Soucek, and F. Mozer (2012), Dispersive Nature of High Mach Number Collisionless Plasma Shocks: Poynting Flux of Oblique Whistler Waves, Phys. Rev. Lett., 108, 025,002, doi: 10.1103/PhysRevLett.108.025002.
20. Thorne, R. M., E. J. Smith, R. K. Burton, and R. E. Holzer (1973), Plasmaspheric Hiss, J. Geophys. Res., 78, 1581–1596, doi:10.1029/JA078i010p01581.
21. Wilson III, L. B., C. A. Cattell, P. J. Kellogg, K. Goetz, K. Kersten, L. Hanson, R. MacGregor, and J. C. Kasper (2007), Waves in Interplanetary Shocks: A Wind/WAVES Study, Phys. Rev. Lett., 99, 041,101–+, doi: 10.1103/PhysRevLett.99.041101.
22. Wilson III, L. B., C. A. Cattell, P. J. Kellogg, K. Goetz, K. Kersten, J. C. Kasper, A. Szabo, and K. Meziane (2009a), Low-frequency whistler waves and shocklets observed at quasi-perpendicular interplanetary shocks, J. Geophys. Res., 114, 10,106–+, doi:10.1029/2009JA014376.
23. Wilson III, L. B., C. A. Cattell, P. J. Kellogg, K. Goetz, K. Kersten, J. C. Kasper, A. Szabo, and M. Wilber (2010a), Large-amplitude electrostatic waves observed at a supercritical interplanetary shock, J. Geophys. Res., 115, 12,104–+, doi: 10.1029/2010JA015332.
24. Wilson III, L. B., A. Koval, A. Szabo, A. Breneman, C. A. Cattell, K. Goetz, P. J. Kellogg, K. Kersten, J. C. Kasper, B. A. Maruca, and M. Pulupa (2012c), Observations of Electromagnetic Whistler Precursors at Supercritical Interplanetary Shocks, Geophys. Res. Lett., 39, L08,109–+, doi: 10.1029/2012GL051581.
25. Wilson III, L. B., A. Szabo, A. Koval, C. A. Cattell, P. J. Kellogg, K. Goetz, A. W. Breneman, K. Kersten, J. C. Kasper, B. A. Maruca, and M. Pulupa (2013a), Electromagnetic waves and electron anisotropies downstream of supercritical interplanetary shocks, J. Geophys. Res., 118, 1–11, doi:10.1029/2012JA018167.
26. Wilson III, L. B., A. Koval, D. G. Sibeck, A. Szabo, C. A. Cattell, J. C. Kasper, B. A. Maruca, M. Pulupa, C. S. Salem, and M. Wilber (2013b), Shocklets, SLAMS, and field-aligned ion beams in the terrestrial foreshock, J. Geophys. Res., doi: 10.1029/2012JA018186, in press.
27. Wu, C. S., D. Winske, K. Papadopoulos, Y. M. Zhou, S. T. Tsai, and S. C. Guo (1983), A kinetic cross-field streaming instabil-ity, Phys. Fluids, 26, 1259–1267, doi:10.1063/1.864285.
28. Wu, C. S., D. Winske, M. Tanaka, K. Papadopoulos, K. Akimoto, C. C. Goodrich, Y. M. Zhou, S. T. Tsai, P. Rodriguez, and C. S. Lin (1984), Microinstabilities associated with a high Mach number, perpendicular bow shock, Space Sci. Rev., 37,63–109, doi:10.1007/BF00213958.