Nonlinear Dynamics and Chaos (AMS 114/214)

Homework 1 - Due Monday Octorber 16th (2017) 11:59 PM

Please submit to CANVAS your homework in a PDF format (scan of your handwritten notes, compiled Latex source, exported Word document, etc ...). Also, please submit any code you develop.

Exercise 1 Consider the following nonlinear differential equations

$$\frac{dx}{dt} = \ln(x^2+1)\cos(x), \tag{1}$$

$$\frac{dx}{dt} = 2x + x^3 - x^5, \tag{2}$$

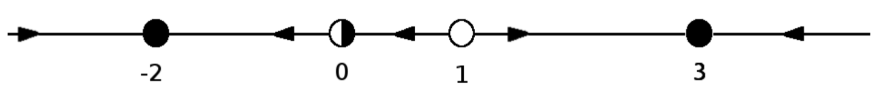
$$\frac{dx}{dt} = \sin(x)(x^2 - 5x + 6). \tag{3}$$

For each case, find all fixed points, discuss their stability by using the geometric approach (i.e., the plot of the velocity in the $(x, \dot{x})$ plane), and sketch the corresponding flow (vector field) on the real line. In addition, sketch the graph of the solution $x(t)$ versus t in each case and for different initial conditions x_0.

Exercise 2 Use linear stability analysis to classify the fixed points of equation (2).

Exercise 3 Set an arbitrary initial condition $x(0) \in \mathbb{R}$. Does the solution to equation (1) blow up in a finite time? Or it exists and it is unique for any finite $t \geq 0$ (global solution)? Justify your answer.

Exercise 4 Find a smooth velocity field $f(x)$ so that the phase portrait generated by the equation $dx/dt = f(x)$ is consistent with the following one:



Exercise 5 Find a potential $V(x)$ for the vector field defined by equation (3).

Exercise 6 For each of (a)-(d) below, find an equation $dx/dt = f(x)$, where $f \in C^1(\mathbb{R})$, satisfying the stated properties. If there are no examples, explain why not.

a Every real number is a fixed point.

b Every integer number is a fixed point, and there are no others.

c There are precisely two fixed points and they are both stable.

d There are no fixed points.

Exercise 7 Suppose that $x(t)$ and $y(t)$ represent the populations at time t of two species that reproduce exponentially fast, i.e. $dx/dt = Ax$, $dy/dt = By$ with $A > B > 0$ and $x(0), y(0) > 0$. Show that the function

$$N(t) = \frac{x(t)}{x(t) + y(t)} \tag{4}$$

satisfies the logistic equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right), \tag{5}$$

where K and r are suitable positive constants to be determined in terms of A and B.

Exercise 8 (Mandatory for 214, Extra Credit for 114) Write a Matlab or Octave code that computes numerically the flow map generated by equation (1), i.e., the surface $\hat{x}(t, x_0)$, where $t \in [0, 50]$ and $x_0 \in [-30, 30]$. Attach the computer code and the plot of the flow map to your submission. (Hint: you are welcome to use/modify the Matlab code uploaded in CANVAS).