Nonlinear Dynamical Systems and Chaos (AMS 114/214)

Homework 5 - Due Friday Dec 8th (2017)

Exercise 1 Consider the nonlinear dynamical system (in polar coordinates)

$$\begin{cases} \dot{r} = r - r^2 \\ \dot{\theta} = t \end{cases} \tag{1}$$

a) Let Σ be the positive real axis. Determine the Poincaré map from Σ into itself.

b) Show that the system has a unique periodic orbit and classify its stability by computing the Floquét multiplier.

Exercise 2 Consider the nonlinear dynamical system

$$\begin{cases} \dot{x} = xy - x^2 - x \\ \dot{y} = x^2 - y + \mu \end{cases} \tag{2}$$

a) Sketch the nullclines and determine the fixed points as a function of μ.

b) Find and classify the bifurcations that occur as μ varies.

c) (AMS 214 students) Plot several sections of the phase portrait before and after the bifurcation points. (You are allowed to use the Matlab code available in the course website.)

Exercise 3 Consider the predator-prey model

$$\begin{cases} \dot{x} = x^2(1 - x) - xy \\ \dot{y} = xy - \mu y, \quad \mu \geq 0 \end{cases} \tag{3}$$

where $x, y \geq 0$ are the dimensionless populations of preys and predators, respectively.

a) Determine and classify all fixed points as a function of μ.

b) Show that a Hopf bifurcation occurs at $\mu = 1/2$. At which point in the phase place?

c) (AMS 214 students) Plot several sections of the phase portrait for $\mu \in (0, 2)$. (You are allowed to use the Matlab code available in the course website.) Is the Hopf bifurcation at $\mu = 1/2$ subcritical or supercritical? What happens to the population of predators for $\mu > 1$?