Lynne Diep
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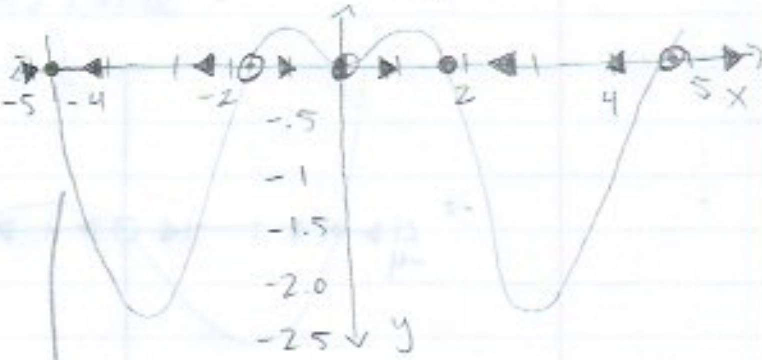
AMS 114

HW # 1

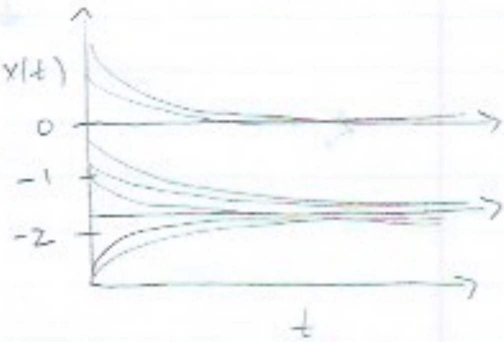
Exercise 1.1

1) $\frac{dx}{dt} = \ln(x^2+1)\cos(x)$ fixed points were $\frac{dx}{dt} = 0$

$\boxed{\text{fixed pts: } x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}}$



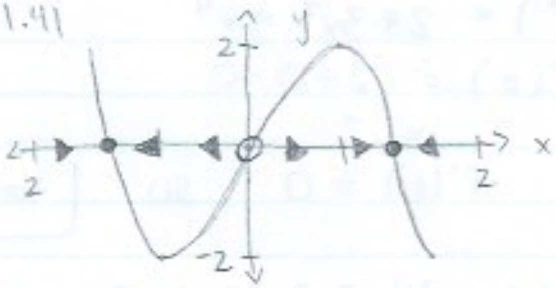
stability: $x = -\frac{3\pi}{2}$ stable
$x = -\frac{\pi}{2}$ unstable
$x = 0$ half stable
$x = \frac{\pi}{2}$ stable
$x = -\frac{3\pi}{2}$ unstable



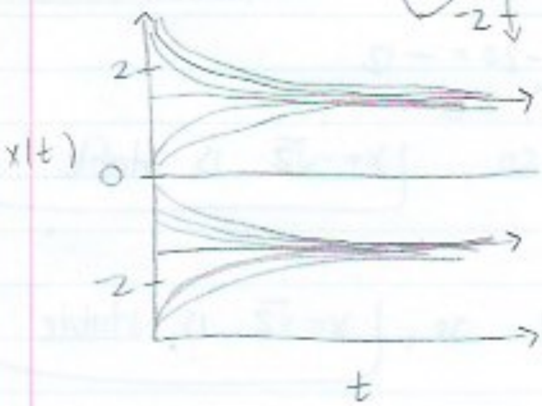
2) $\frac{dx}{dt} = 2x + x^3 - x^5$
$= -x(-2 - x^2 + x^4)$
$= -x(x^2 - 2)(x^2 + 1)$

$\boxed{\text{fixed pts: } x = 0, \pm\sqrt{2}}$

$\sqrt{2} \approx 1.41$



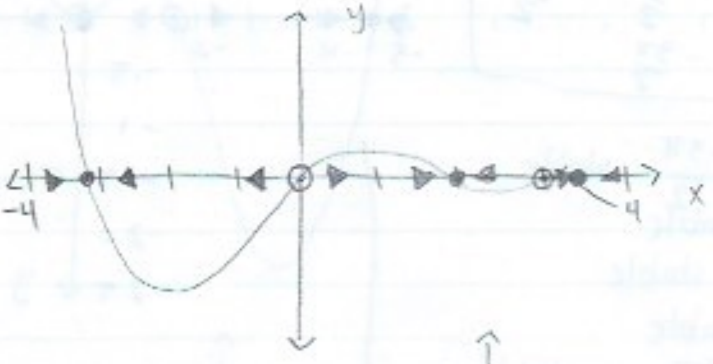
$x = 0$ unstable
$x = -\sqrt{2}$ stable
$x = \sqrt{2}$ stable



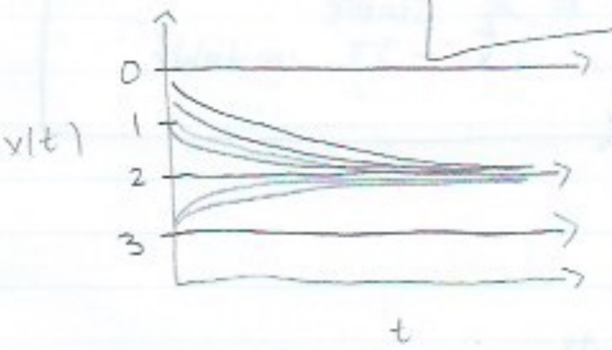
Fixed points: $x = 3, 2, 0, \pm\pi$

3) $\frac{dx}{dt} = \sin(x)(x^2 - 5x + 6)$

$= \sin(x)(x-3)(x-2)$



$x = -\pi$	stable
$x = 0$	unstable
$x = 2$	stable
$x = 3$	unstable
$x = \pi$	stable



Exercise 2

$\frac{dx}{dt} = 2x + x^3 - x^5$ fixed pts: $x = 0, \pm\sqrt{2}$

$f'(x^*) = 2 + 3x^2 - 5x^4$

$f'(0) = 2 + 0 - 0$

$= 2$

$f'(0) > 0$, so $\boxed{x = 0 \text{ is unstable}}$

$f'(-\sqrt{2}) = 2 + 6 - 20 = 8 - 20 = -12$

$f'(-\sqrt{2}) < 0$

so, $\boxed{x = -\sqrt{2} \text{ is stable}}$

$f'(\sqrt{2}) = 2 + 6 - 20 = -12$

$f'(\sqrt{2}) < 0$ so, $\boxed{x = \sqrt{2} \text{ is stable}}$

the solution

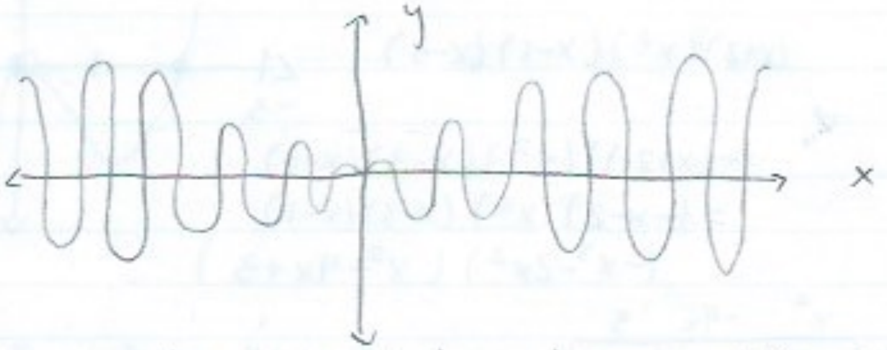
Exercise 3

$x(0) \in \mathbb{R}$ does $\frac{dx}{dt} = \ln(x^2+1)\cos(x)$ blow up? or exists and is unique?

By graphing the equation, the solution does not blow up over time. It exists and it is unique.

Here is the graph of the equation

* Coordinates not accurate



As the graph expands, the oscillations become stable in size, and do not increase or decrease.

So, the solution to equation (1) is:

exists and unique

Exercise 4

Find smooth velocity field $f(x)$

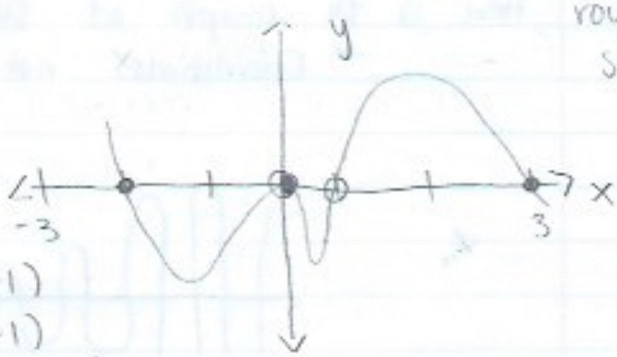
stable : $-2, 3$

fixed pts : $-2, 0, 1, 3$

unstable : 1

half stable : 0

rough sketch



$(x+2)(x^2)(x-1)(x-3)$

$(-(x+2))(x^2)(x-3)(x-1)$

$= (-x-2)(x^2)(x-3)(x-1)$

$(-x^3-2x^2)(x^2-4x+3)$

	x^2	$-4x$	3
$-x^3$	$-x^5$	$4x^4$	$-3x^3$
$-2x^2$	$-2x^4$	$8x^3$	$-6x^2$

$-x^5+2x^4+5x^3-6x^2 = f(x)$

$f'(x) = -5x^4+8x^3+15x^2-12x$

✓ half $f'(0) = 0+0+0-0 = 0 = 0$

✓ un $f'(1) = -5+8+15-12 = 6 > 0$

✓ stable $f'(3) = -405+216+135-36 = -90 < 0$

✓ stable $f'(-2) = -80-64+60+24 = -60 < 0$

$$\boxed{f(x) = (-(x+2))(x^2)(x-3)(x-1)}$$

Exercise 5

Find potential $V(x)$ for vector field by $\frac{dx}{dt} = \sin(x)(x^2 - 5x + 6)$

$$-\frac{dV}{dt} = \sin(x)(x^2 - 5x + 6)$$

$$\int dV = -\int (\sin(x)(x^2 - 5x + 6))\, dx$$

$$= -\int \sin(x) x^2 + \int \sin(x)(-5x) - \int \sin(x) 6$$

$$= -(2x\sin(x) - (x^2 - 2)\cos x) + (5\sin(x) - 5x\cos(x)) - (-6\cos x)$$

$$= -2x\sin(x) + (x^2 - 2)\cos x + 5\sin(x) - 5x\cos(x) + 6\cos(x)$$

$$V + C = -2x\sin(x) + (x^2 - 2)\cos(x) + 5\sin(x) - 5x\cos(x) + 6\cos(x)$$

let $C = 0$

$$V = -2x\sin(x) + (x^2 - 2)\cos(x) + 5\sin(x) - 5x\cos(x) + 6\cos(x)$$

$$\boxed{V(x) = -2x\sin(x) + (x^2 - 2)\cos(x) + 5\sin(x) - 5x\cos(x) + 6\cos(x) + C}$$

Exercise 6

Find equation for the following

a) every real number is a fixed point

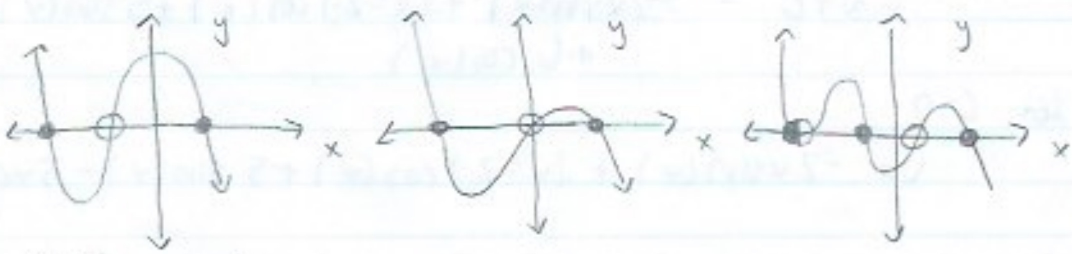
$$\frac{dx}{dt} = 0$$

b) Every integer number is a fixed pt, there are no others

$$\frac{dx}{dt} = \sin(\pi x)$$

c) 2 fixed points, both are stable

No examples because when drawing a graph, there has to be an unstable point in order to get another stable point. Example:



d) No fixed points

$$\frac{dx}{dt} = 1$$

Exercise 7

$\frac{dx}{dt} = Ax$ $\quad \frac{dy}{dt} = By$ with $A > B > 0$, and $x(0), y(0) > 0$

Show $N(t) = \frac{x(t)}{x(t) + y(t)}$ satisfies $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$

K and r are constants

$$\frac{dx}{dt} = Ax \rightarrow \frac{dx}{x} = A\,dt \rightarrow \ln x = At + K$$

$$\ln x(0) = K$$

$$\ln x = At + \ln x(0)$$

$$\rightarrow x(t) = x(0)e^{At}$$

$$\frac{dy}{dt} = By \rightarrow \frac{dy}{y} = B\,dt \rightarrow \ln y = Bt + K$$

$$\ln y(0) = K$$

$$\ln y = Bt + \ln y(0)$$

$$\rightarrow y(t) = y(0)e^{Bt}$$

$$N(t) = \frac{x(t)}{x(t)+y(t)}$$

$$= \frac{x(0)e^{At}}{x(0)e^{At} + y(0)e^{Bt}} = \frac{\cancel{x(0)e^{At}}\,(1)}{\cancel{x(0)e^{At}}\left[1 + \frac{y(0)}{x(0)}e^{(B-A)t}\right]}$$

$$= \frac{1}{1 + \frac{y(0)}{x(0)}e^{(B-A)t}} = \left(1 + \frac{y(0)}{x(0)}e^{(B-A)t}\right)^{-1}$$

$$\frac{dN(t)}{dt} = -\left(1 + \frac{y(0)}{x(0)}e^{(B-A)t}\right)^{-2}\left(\frac{y(0)}{x(0)}(B-A)\,e^{(B-A)t}\right)$$

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$