Nonlinear Dynamical Systems and Chaos (AMS 114/214)

Homework 3 - Due Saturday Nov 4 (2017)

Exercise 1 Sketch the phase portrait (including nullclines) and classify the fixed points of the following linear dynamical systems:

$$\begin{cases} \dot{x}_1 = -x_1 + 2x_2 \\ \dot{x}_2 = -2x_1 - x_2 \end{cases} \qquad \begin{cases} \dot{x}_1 = x_1 + 4x_2 \\ \dot{x}_2 = x_2 \end{cases} \tag{1}$$

If the eigenvectors are real, include them in your sketch.

Exercise 2 Compute the analytical solution to the linear systems (1) for arbitrary initial conditions $(x_1(0), x_2(0)) = (x_{01}, x_{02})$. If the eigenvalues/eigenvectors are complex-valued, express the solution in terms of real-valued functions only. (Hint: use the Euler formula $e^{ix} = \cos(x) + i\sin(x)$)

Exercise 3 Show that any matrix in the form

$$\boldsymbol{A} = \begin{bmatrix} a & b & b \\ 0 & a & b \\ 0 & 0 & a \end{bmatrix}, \tag{2}$$

with a and b nonzero real numbers, has only one eigenvalue with algebraic multiplicity 3 and geometric multiplicity 1. Compute the fundamental matrix that defines the general solution of the three-dimensional linear system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ in the case $a = b = 1$.

Exercise 4 Consider the nonlinear dynamical system

$$\begin{cases} \dot{x} = y^2 - x \\ \dot{y} = x^2 - y \end{cases} \tag{3}$$

1. Plot the nullclines, determine the fixed points and classify them. Is linear analysis effective in this case? Justify your answer.
2. On the same plot with the nullclines sketch a plausible phase portrait. Make sure you also include the eigendirections you obtain from linear analysis at the fixed points, and label any heteroclinic orbit.

Exercise 5 (AMS 214 Mandatory, AMS 114 Extra Credit) Write a computer code that returns the phase portrait of the system (3) in the square domain $[-2, 2] \times [-2, 2]$ together with the nullclines. Include the plot of the phase portrait and your code in your CANVAS submission.