I have read and agree to the collaboration policy. Lynne Diep.

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Homework Heavy Grading
Collaborators: None

CMPS 102: HW 1-4
Due: April 21, 2017

Problem 4:
They want to find out who owns the median book of the joint book collection, which has 2n books. In this joint book collection, the median would be the n-th book among the union of the 2n alphabetically sorted books

They manage to find out who owns the median in Θ(log n).

What algorithm did they use? Prove that this algorithm is correct. Find the recurrence relation and show that it resolves to Θ(log n).

Answer –
The median can be obtained recursively.

Let the joint book collection be a sorted array A.
Pick the median of the sorted array A. This is just Θ (1) time as the median is the n/2th element in the sorted array.
Compare median of A, call it a*, with median of B, call it b*. Now we have 2 cases.
Case 1:
a* < b* - in this case, elements in B$[\frac{n}{2} \dots n]$ are also greater than a*. That means the median cannot lie in either A$[1 \dots \frac{n}{2}]$ or B$[\frac{n}{2} \dots n]$. So we can eliminate these away and recursively solve a subproblem with A$[\frac{n}{2} \dots n]$ and B$[1 \dots \frac{n}{2}]$.
Case 2:
a* > b* - in this case, we can still eliminate A$[\frac{n}{2} \dots n]$ and B$[1 \dots \frac{n}{2}]$ and solve a smaller subproblem recursively.

In both of these cases, our subproblem sizes are reduced in half, and we spend constant time to compare the medians of A and B (Jack's and Anthony's book collection).

So the recurrence relation would be T(n) = T(n/2) + Θ(1), which has a solution of T(n) = Θ(log n) as claimed.