I have read and agree to the collaboration policy. Lynne Diep.

Lynne Diep
Homework Heavy Grading
Collaborators: Sabrina Tsui

Homework 2-4
Due: May 1, 2017

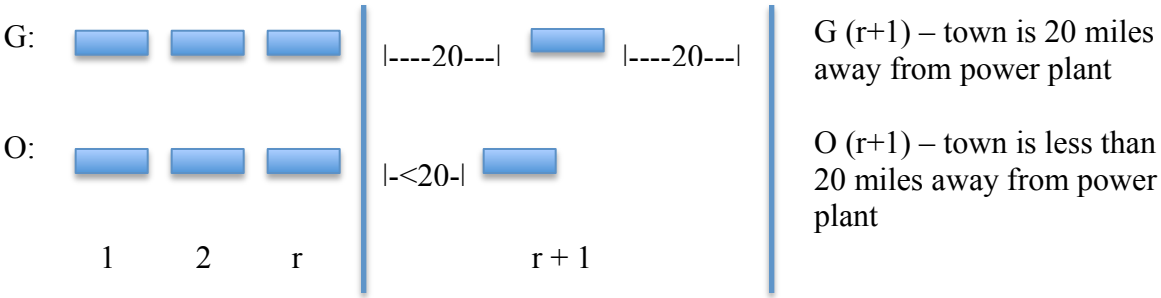
Problem 4:
The power plants can't power a town further than 20 miles away, due to their particular design - they can, however, give power to as many towns as they can reach. You want to build them along the river such that every town is within 20 miles of at least one of the plants.

Give an efficient algorithm that achieves this goal using the minimum number of power plants. Prove using greedy stays ahead strategy.

Solution:
Algorithm –
Start at the beginning of the river
    Find first town and move 20 miles along the river {
        Place power plant
        Move additional 20 miles along the river
        If there is a new town {
            Plant power plant
            Move additional 20 miles along the river
            }
        Else, move along the river until a new town is discovered {
            Plant power plant
            Move additional 20 miles along the river
            }
        Repeat until you reach end of river
    }

Prove using greedy stays ahead strategy:
Theorem – Greedy is optimal
Proof (by contradiction):
Suppose greedy is not optimal
    $P_1 \ldots P_k$ = power plants selected by greedy
    $P'_1 \ldots P'_m$ = optimal solution

    $P_1 = P'_1$, $P_2 = P'_2$, $\ldots P_r = P'_r$ for largest possible r



Greedy - P(r+1) is the plant that is 20 miles away from $house_h$ by the algorithm
Optimal – P'(r+1) places a plant < 20 miles away from $house_h$ by the algorithm

Contradiction: Because for r<k replace P'(r+1) with P(r+1). Since you get one more optimal strategy that was not in the original set of optimal strategies, we have a contradiction, thus greedy stays ahead.

Running Time: O(n)

Space Complexity: O(n)