I have read and agree to the collaboration policy. Lynne Diep.

Lynne Diep
Homework Heavy Grading
Collaborators: none

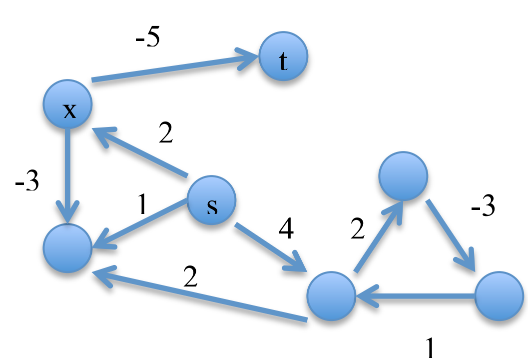
Homework 4-3
Due: June 5, 2017

a) Proof:
A max flow is acyclic and has positive edges.
For max flow, an s-t cut is a min cut
    Because if v(f) = cap(A,B) then f is a max flow and (A,B) is a min cut
    v(f) = cap(A,B) is true because of weak duality.
By definition and property of min cut:
    No cycles are possible across the s-t min cut
For cycles within s and t:
    They don't contribute to max flow, hence they are not part of max flow.
    By conservation of flow, flow exiting cycle $\leq$ flow going in cycle
Thus, every flow f has at least one corresponding acyclic flow that has the same value, due to the definition and property of min cut.

b) Proof by induction –
Claim: P(n) – an acyclic flow is finite combination of n s-t path flows.
BC: P(1) – which means the acyclic flow is composed of 1 path flow. Trivially the acyclic flow equals path flow, so P(n). Thus, base case holds true for P(1).
IH: For an acyclic flow with k s-t paths, P(k) is true with k > 1
IS: In an acyclic flow with k + 1 s-t paths, we remove a path flow, p
The new flow of size k - (AF – P) is still an acyclic flow
The above statement is true because:
1. the flow still has no cycles since no new edges are added or removed.
2. Of the conservation of flow the previously positive edges will not become negative

By the IH, the acyclic flow is a finite combination of k s-t paths (combination of path flows).

c)



Consider the path s-x, x-t:
The path from s to t does not give a positive value, thus it is not a path flow.