I have read and agree to the collaboration policy. Lynne Diep.
Lynne Diep
Homework Heavy Grading
Collaborators: Sabrina

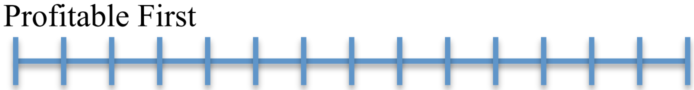
Homework 3-2
Due: May 22, 2017

a)
i. Next available location:

index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
watts	4	56	3	12	9	7	10	15	2	4	0	1	18	5	9

Starting at index 1, we have the value of watts as 4. Since we can only place the next power plant more than 5 miles away, we will miss putting a power plant at index 2, which has a watt value of 56. Due to this rule, we miss the optimal solution.

ii. Most Profitable First



index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
watts	3	10	81	7	101	3	4	120	38	3	4	1	82	12	3

By planting at the most profitable location, index 8, we get the value of watts as 120. However, we miss the plant locations at index 3 and 13. If we add the watt values at these indexes, the value is 163, which is larger than our "optimal solution". Thus placing the power plant at the most profitable location is not optimal.

b)
i. OPT(j) = value of optimal solutions to problem consisting of power plants 1,2,…,j
Case 1: OPT selects power plant j
Can't use incompatible power plants { P[j]+1, P[j]+2, …, j-1}
Most include optimal solutions to problem consisting of remaining compatible power plant 1, 2, …, p(j) ← previous best solution that is compatible with j
Case 2: OPT doesn't select power plant j
Must include optimal solutions to problem consisting of remaining compatible power plant 1, 2, …, j-1 ← previous best solution

ii. $$\text{OPT(j)} = \begin{cases} 0 & j = 0 \\ \max\left(w_j + OPT(p_j), OPT(j-1) \right) & otherwise \end{cases}$$
Correctness: Our intuition is that we go through each power plant, we don't skip any from j = 1 to n. For each power plant it considers, we choose best solution so far or will add and make solution better (case 1 and 2 from part i). The solution can only get better and will never become

worse. The best solution so far is case 2 from part i OR add power plant j if j is compatible and increase total watts (i.e. a better solution) – case 1.

iii. Use a data structure (like an array) that maps positions ($x_1 \ldots x_n$) to their corresponding wattages ($r_1 \ldots r_n$)
Let p(j) = largest index i < j. set power plant is at least 5 miles apart from j. Compute p(j) for each position and store in data structure p[]. Set maps positions to their corresponding p(j).
Define any array M[] that will keep track of max watts of each best solution (selection of power plant) up to j.

- initialize M[0] = 0
- for j from 1 to n
- recursively compute M[j] by either adding current power plant to previous most optimal solution that is compatible with current power plant or keep the previous optimal solution. i.e. recurrence relation

return M[n]

Time: O(n)
Space: O(3n) = O(n)
Correctness: Our intuition is that we go through each power plant, we don't skip any from j = 1 to n. For each power plant it considers, we choose best wattage so far or will add and make wattage better. The wattage can only get better and will never become worse.

iv. Use a data structure (like an array) that maps positions ($x_1 \ldots x_n$) to their corresponding wattages ($r_1 \ldots r_n$)
Let p(j) = largest index i < j. set power plant is at least 5 miles apart from j. Compute p(j) for each position and store in data structure p[]. Set maps positions to their corresponding p(j).
Define any array N[] that will keep track of max watts of each best solution (selection of power plant) up to j. And array N[] of lists that will keep track of each subset of power plants representing the best solution up to j.

- initialize N[0] = 0
- for j from 1 to n
- recursively compute M[j] by either adding current watts of power plant to watts of previous most optimal solution with current power plant or keep the watts of previous optimal solution.
- Compute N[j] by adding power plant to j to previous most optimal set of compatible power plants or keep previous set of optimal power plants. i.e. recurrence relation of $N - \max(j + N[p_j], N[j-1])$

Return N[n]

Time: O(n)
Space: O(4n) = O(n)
Correctness: Correctness: Our intuition is that we go through each power plant, we don't skip any from j = 1 to n. For each power plant it considers, we choose best power plant setting so far or will add and make power plant setting better. The power plant setting can only get better and will never become worse.