I have read and agree to the collaboration policy. Lynne Diep.

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Homework Heavy Grading
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CMPS 102: HW 1-2
Due: April 21, 2017

Problem 2:
Rank the following functions by increasing order of growth, that is, find an arrangement g1, ... of the functions satisfying g1(n) = O(g2(n)), g2(n) = O(g3(n)), ... Break the functions into equivalence classes so that f and g are in the same class if and only if f(n) = Θ(g(n)).

$a = \ln(\ln n)$ $b = n \log n$ $c = 14\log_3 n$ $d = \sum_{i=5}^{n} \frac{(i+1)}{2}$ $e = \log^2(n)$

$f = n^2$ $g = \sum_{i=1}^{n} \left(\frac{1}{2}\right)^i$ $h = \log(n!)$ $i = 3^n$

$j = n^{\log 7}$ $k = \sum_{i=1}^{n} 3^i$ $l = 2^{\log^2(n)}$ $m = 2^{\log n}$ $n = n!$

$o = n$ $p = 2^{\log_4 n}$ $q = \sqrt{n}$ $r = \log(n^2)$ $s = 4^{\log n}$

$t = \left(\frac{5}{4}\right)^n$

Information:

1. The growth of exponential function is larger than any polynomial function.
2. The base of a logarithm is asymptotically irrelevant, but the base in an exponential function and the degree of a polynomial does matter.
3. $a^{\log_b c} = c^{\log_b a}$
4. $d = (1/4)(n^2 + 3n - 28)$
5. $g = 1-2^{-n}$
6. $k = (3/2)(3^n-1)$

Part a):
Order of growth (best to worst):
gacreqmohbptldfsjikn

Part b):
For each of the following statements, decide whether it is always true, never true, or sometimes true for asymptotically nonnegative functions f and g.

i. $f(n) + g(n) = \Omega(\max(f(n), g(n)))$

Always true.

Since f(n) and g(n) are asymptotically nonnegative there exists n_o such that $f(n) \geq 0$ and $g(n) \geq 0$ for all $n \geq n_o$. For any n, h(n) is larger than f(n) and g(n). For all $n \geq n_o$, $0 \leq f(n) \leq h(n)$ and $0 \leq g(n) \leq h(n)$. Adding them together, results in $0 \leq f(n)+g(n) \leq 2(h(n))$. Ultimately, the result is:

$$\max(f(n),g(n)) \leq f(n)+g(n) \leq 2\max(f(n),g(n))$$

Which means $f(n) + g(n) = \Omega(\max(f(n), g(n)))$ as claimed.

ii. $f(n) = \omega(g(n))$ and $f(n) = O(g(n))$

Never true.

In order for $f(n) = \omega(g(n))$, $f(n) = \Omega(g(n))$ and f(n) does not equal $\Theta(g(n))$. If $f(n) = O(g(n))$, then f(n) must equal g(n) in order for $f(n) = \Omega(g(n))$ to be true. However, that means $f(n) = \Theta(g(n))$, which goes against little omega notation. Thus, $f(n) = \omega(g(n))$ and $f(n) = O(g(n))$ is never true.

iii. Either $f(n) = O(g(n))$ or $f(n) = \Omega(g(n))$ or both

Always true.

If f(n) is asymptotically less than or equal to g(n), then $f(n) = O(g(n))$. If f(n) is asymptotically greater than or equal to g(n), then $f(n) = \Omega(g(n))$. If $f(n) = g(n)$, then both are true. Depending on what f(n) is, one part of the statement is true, hence the “or”, so overall, the whole statement is always true.