I have read and agree to the collaboration policy. Lynne Diep.

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Homework Heavy Grading
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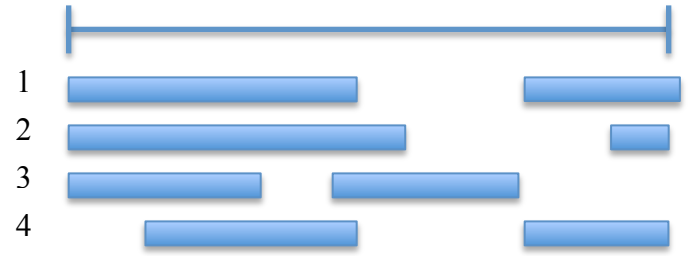
Homework 2-3
Due: May 1, 2017

Problem 3:
Your efficient greedy algorithm should take input a list of pairs of times (ai , bi) for i = 1 to n
(a) Consider the greedy algorithm that selects nurses by repeatedly choosing the nurse who will be there for the longest time among the periods not covered by previously selected nurses. Give an example showing that this algorithm does not always find the smallest set of nurses.
(b) Present an algorithms that outputs a smallest subset of nurses that can cover the entire duration of Mr. Banks' stay at the hospital or say that no such subset exists.
(c) Prove that your algorithm is correct.
(d) State its running time.

Solution:
a.

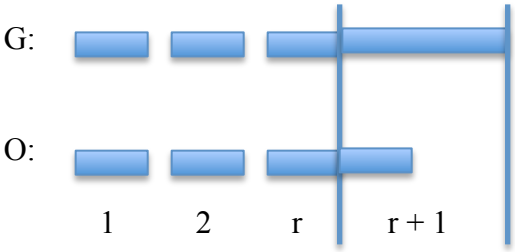


Ideally we would want to choose nurse 2,4,2 in that order from left to right. However, we cannot choose nurse 2 twice because we cannot choose previously selected nurses. So, using longest interval does not always find the smallest set of nurses.

b. Algorithm –
SelectNurses([(S_1, F_1), (S_2, F_2), … (S_n, F_n)])
Sort intervals by start time $S_1 \leq S_2 \leq \cdots \leq S_n$
A ← ∅ // nurses selected so far
if nurses have the same earliest start time {
nurse = nurse with earliest start time and longest interval
}
else {
nurse = nurse with earliest start time
}
A← A ∪ {nurse}
for nurse 1 … n

if ($F_{nurse} > S_{next\ nurse}$ or $F_{nurse} == S_{next\ nurse}$) {
    nurse = nurse with latest finish time
}
if (more than 1 "next nurses" have latest finished time) {
    nurse = nurse with latest finish time & shortest interval
}
A← A ∪ {nurse}
return A

c. Prove algorithm is correct –
Theorem: Greedy is optimal
Proof (by contradiction):
S = set of optimal solutions
Assume greedy is not optimal
$I_1 \ldots I_k$ = set of nurses selected by greedy
$J_1 \ldots J_m$ = optimal solution
$I_1 = J_1, \ldots, I_r = J_r$ for largest possible r



By the greedy algorithm, i(r+1) has the latest finish time so i(r+1) has a later finish time than j(r+1).

If r < k
    Replace j(r+1) with i(r+1)
By definition, k nurses are not optimal
    So k nurses > m optimal nurses

By contradiction, greedy scheduled less nurses than the optimal solution number of nurses. Which means greedy is better than optimal; hence greedy is the optimal solution as claimed.

d. Running time – O(n log n)