

- 1.51 x and y are distinguishable by L if some string z exists whereby exactly one of the strings xz and yz is a member of L .
otherwise $xz \in L$ whenever $yz \in L$ we say that x and y are indistinguishable by L
$$x \equiv_L y$$

Show \equiv_L is an equivalence relation

We have to show \equiv_L is

- a) reflexive
- b) symmetric
- c) transitive

a) reflexive: $x \equiv_L x$ is true

For all strings z , xz is in L iff xz is in L

Hence $x \equiv_L x$ is true and also reflexive

b) Symmetric: $x \equiv_L y$ implies $y \equiv_L x$

if $x \equiv_L y$ is true then "for all z , xz is in L iff yz is in L " which also means "for all z , yz is in L iff xz is in L "

Hence, $y \equiv_L x$ is true and \equiv_L is symmetric

c) transitive: if $a \equiv_L b$, and $b \equiv_L c$ then $a \equiv_L c$

The above states "for all z , az is in L iff bz is in L and for all z , bz is in L iff cz is in L "

Therefore "for all z , az is in L iff cz is in L "

$a \equiv_L c$ is true. Hence, \equiv_L is transitive

Since we proved \equiv_L is reflexive, symmetric, transitive then \equiv_L is an equivalence relation.

□

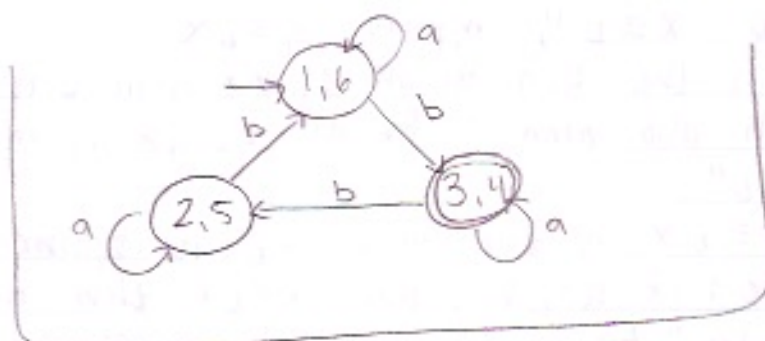
1: Mark all pairs where $P \in F$ and $Q \in F$
 only if unmarked
 2: $[J(P,x), J(Q,x)]$ is marked then mark $[P,Q]$
 0: Combined unmarked pairs

A)

	x	y
1	6	3
2	5	6
3F	4	5
4F	3	2
5	2	1
6	1	4

	1	2	3	4	5	6
1						
2	2					
3	1	1				
4	1	1	0			
5	2	0	1	1		
6	0	2	1	1	2	

$(2,1) \xrightarrow{a} (6,5)$
 $\xrightarrow{b} (3,6) \checkmark$
 $(1,5) \xrightarrow{a} (6,2)$
 $\xrightarrow{b} (3,1) \checkmark$
 $(1,6) \xrightarrow{a} (6,1) \star$
 $\xrightarrow{b} (3,4) \star$
 $(5,6) \xrightarrow{a} (2,1)$
 $\xrightarrow{b} (1,4)$
 $(4,3) \xrightarrow{a} (4,3) \star$
 $\xrightarrow{b} (5,2) \star$
 $(2,5) \xrightarrow{a} (5,2) \star$
 $\xrightarrow{b} (6,1) \star$
 $(2,6) \xrightarrow{a} (5,1)$
 $\xrightarrow{b} (6,4) \checkmark$
 $(3,4) \quad (2,5) \quad (1,6)$



B)

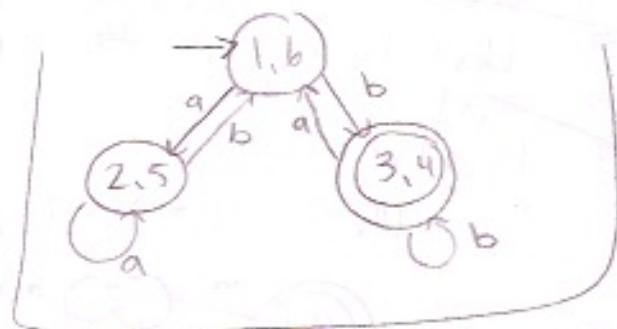
	x	y
1	2	3
2	5	6
3F	1	4
4F	6	3
5	2	1
6	5	4

	1	2	3	4	5
1					
2	2				
3	1	1			
4	1	1	0		
5	2	0	1	1	
6	0	2	1	1	2

$(2,1) \xrightarrow{a} (2,5)$
 $\xrightarrow{b} (3,6) \checkmark$
 $(3,4) \xrightarrow{a} (1,6) \star$
 $\xrightarrow{b} (4,3) \star$
 $(1,5) \xrightarrow{a} (2,2)$
 $\xrightarrow{b} (3,1) \checkmark$

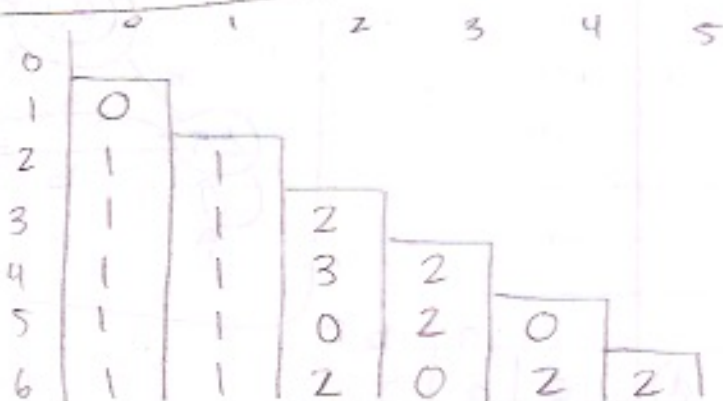
$(2,5) \xrightarrow{a} (5,2) \star$ $(1,6) \xrightarrow{a} (2,5) \star$ $(6,2) \xrightarrow{a} (5,5)$
 $\quad \quad \quad \xrightarrow{b} (6,1) \star$ $\quad \quad \quad \xrightarrow{b} (3,4) \star$ $\quad \quad \quad \xrightarrow{b} (6,4)$

$(6,5) \xrightarrow{a} (2,5) \checkmark$ $(3,4) (2,5) (1,6)$
 $\quad \quad \quad \xrightarrow{b} (1,4)$



c)

	a	b
0 F	3	2
1 F	3	5
2	2	6
3	2	1
4	5	4
5	5	3
6	5	0



$(1,0) \xrightarrow{a} (3,3)$ $(3,2) \xrightarrow{a} (2,2)$ $(2,4) \xrightarrow{a} (2,5)$
 $\quad \quad \quad \xrightarrow{b} (2,5)$ $\quad \quad \quad \xrightarrow{b} (6,1) \checkmark$ $\quad \quad \quad \xrightarrow{b} (6,4)$

$(3,4) \xrightarrow{a} (2,5)$ $(2,5) \xrightarrow{a} (2,5)$ $(3,5) \xrightarrow{a} (2,5)$
 $\quad \quad \quad \xrightarrow{b} (1,4) \checkmark$ $\quad \quad \quad \xrightarrow{b} (6,3)$ $\quad \quad \quad \xrightarrow{b} (1,3) \checkmark$

$(4,5) \xrightarrow{a} (5,5)$ $(2,6) \xrightarrow{a} (2,5)$ $(6,3) \xrightarrow{a} (2,5)$
 $\quad \quad \quad \xrightarrow{b} (4,3)$ $\quad \quad \quad \xrightarrow{b} (6,0) \checkmark$ $\quad \quad \quad \xrightarrow{b} (1,0)$

$(4,6) \xrightarrow{a} (5,5)$ $(6,5) \xrightarrow{a} (5,5)$
 $\quad \quad \quad \xrightarrow{b} (4,0) \checkmark$ $\quad \quad \quad \xrightarrow{b} (3,0) \checkmark$

3rd
pass:

$(0,1)$ $(2,5)$ ~~$(2,4)$~~ $(3,6)$ ~~$(4,5)$~~

$(0,1) \xrightarrow{a} (3,3)$
 $\xrightarrow{b} (2,5)$

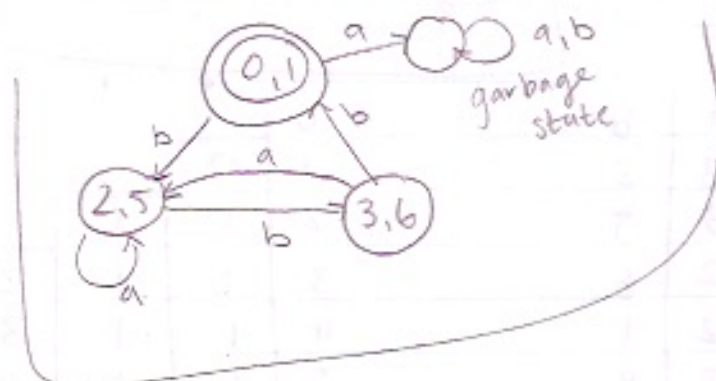
~~$(2,5) \xrightarrow{a} (2,5)$
 $\xrightarrow{b} (6,3)$~~ ✓

~~$(2,4) \xrightarrow{a} (2,5)$
 $\xrightarrow{b} (6,4)$~~

$(3,6) \xrightarrow{a} (2,5)$
 $\xrightarrow{b} (1,0)$

~~$(4,5) \xrightarrow{a} (4,5)$
 $\xrightarrow{b} (4,3)$~~ ✓

$(0,1)$ $(2,5)$ $(3,6)$



d)

	a	b
0	3	5
1	2	4
2	6	3
3	6	6
4F	0	2
5F	1	6
6	2	6

	0	1	2	3	4	5
0						
1	0					
2	2	2				
3	2	2	0			
4	1	1	1	1		
5	1	1	1	1	0	
6	2	2	0	0	1	1

$(4,5) \xrightarrow{a} (0,1) \star$
 $\xrightarrow{b} (2,6) \star$

$(1,0) \xrightarrow{a} (3,2) \star$
 $\xrightarrow{b} (5,4) \star$

$(0,2) \xrightarrow{a} (3,6)$
 $\xrightarrow{b} (5,3) \checkmark$

$(1,2) \xrightarrow{a} (2,6)$
 $\xrightarrow{b} (4,3) \checkmark$

$(0,3) \xrightarrow{a} (3,6)$
 $\xrightarrow{b} (5,6) \checkmark$

$(1,3) \xrightarrow{a} (2,6)$
 $\xrightarrow{b} (4,6) \checkmark$

$(3,2) \xrightarrow{a} (6,6) \star$
 $\xrightarrow{b} (3,6) \star$

$(0,6) \xrightarrow{a} (3,2)$
 $\xrightarrow{b} (5,6) \checkmark$

$(1,6) \xrightarrow{a} (2,2)$
 $\xrightarrow{b} (4,6) \checkmark$

$(2,6) \xrightarrow{a} (6,2) \star$ $(3,6) \xrightarrow{a} (6,2) \rightarrow$
 $\quad \quad \quad \xrightarrow{b} (3,6) \star$ $\quad \quad \quad \xrightarrow{b} (6,6) \star$

~~$(0,1)$~~ ~~$(2,3)$~~ ~~$(4,5)$~~ ~~$(2,6)$~~ $(3,6)$

