CMPS 130
HW #7
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Use Myhill - Nerode Thm to prove 1.29 (a, c) and 1.46 (b, c) are regular

1.29 a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Consider $S = \{0^i \mid i \geq 1\}$ we can see that the set is infinite

Any 2 member of S have the form $x = 0^c$ and $y = 0^d$ where $c \neq d$

String $z = 1^c 2^c$ distinguishes them

$xz = 0^c 1^c 2^c \in A_1$ $yz = 0^d 1^c 2^c \notin A_1$

$0^c \cancel{R_{A_1}} 0^d$ if $c \neq d$

So, A_1 is not regular by MHT

c) $A_3 = \{a^{2^n} \mid n \geq 0\}$ Here, a^{2^n} means a string of 2^n a's

Consider $S = \{0^{2^i} \mid i \geq 1\}$ we can see that the set is infinite.

Any 2 members of S have the form $x = 0^{2^c}$ and $y = 0^{2^d}$ in which $c \neq d$.

String $z = 0^{2^c}$ distinguishes them

We consider 2 cases

1) $c < d$

2) $c > d$

1) $c < d$

$2^c < 2^d$

$2^c < 2^d < 2^d + 2^c < 2^d + 2^d = 2 \cdot 2^d = 2^{d+1}$

We want to show $0^d < 0^{(2^d + 2^c)} < 0^{2^{d+1}}$

$xz = 0^{2^c} 0^{2^c} = 0^{2^{c+1}} \in A_3$

$yz = 0^{2^d} 0^{2^c} = 0^{2^c + 2^d} \notin A_3$

2) $c > d$

$2^c > 2^d$

$2^d < 2^c < 2^d + 2^c < 2^c + 2^c < 2^{c+1}$

$yz = 0^{2^d} 0^{2^d} = 0^{2^d} + 0^{2^d} = 0^{2^{d+1}} \in A_3$

$xz = 0^{2^c} 0^{2^d} = 0^{2^c + 2^d} \notin A_3$

Hence, A_3 is not regular by the MHT

1.46 b) $\{0^m 1^n \mid m \neq n\} = A_2$

Consider the set $S = \{0^i \mid i \geq 1\}$ we can see this set is infinite

Any 2 members of S have the form $x = 0^c$ and $y = 0^d$ in which $c \neq d$

String $z = 1^c$ distinguishes them

$xz = 0^c 1^c \in A_2$ $\quad yz = 0^d 1^c \notin A_2$

$0^c 1^c \cancel{R_{A_2}} 0^d 1^c$

Hence A_2 is not regular by MHT

c) $\{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\} = A_3$

Consider the set $S = \{0^i 1 \mid i \geq 1\}$ we can see this set is infinite

Any 2 members of S have the form $x = 0^c 1$ and $y = 0^d 1$ in which $c \neq d$

String $z = 0^c 1$ distinguishes them

$xz = 0^c 1 0^c 1 \in A_3$ $\quad yz = 0^d 1 0^c 1 \notin A_3$

$0^c 1 \cancel{R_{A_3}} 0^d 1$

Hence A_3 is not regular by MHT

3 Prove any language of your choice is regular with MHT. Could you do the proof with Pumping Lemma?

The language $L = \{0^n \mid n \text{ is even}\}$ is regular

Proof: Consider the finite list of subsets of Σ^*: $A_1 = L$, $A_2 = \{0^n \mid n \text{ is odd}\}$. Claim the members of A_1 are all equivalent to each other, because for any 2 members, $0^i, 0^j$ where $i \neq j$, and both are even, it is the case that $0^i R_L 0^j$ since for any $z \in \Sigma^*$, say 0^m, both $0^i 0^m$ and $0^j 0^m$ are in L if m is even and both are not in L if M is odd. The members of A_2 are all equivalent by a similar argument with the roles of even and odd are switched. Clearly, every string in Σ^* is either odd or even in length, so $A_1 \cup A_2 = \Sigma^*$ $\square$

However, you cannot do the proof with the Pumping lemma because the lemma is used to prove that a language is not regular.

4 Give a CFG for the language $\{x \in \{a,b\}^* \mid x \neq ww$ for some $w \in \{a,b\}^*\}$

Basically, a set of strings that are not in the form of ww

$S \to TR \mid RT \mid T \mid R$
$T \to aTa \mid aTb \mid bTa \mid bTb \mid a$
$R \to aRa \mid aRb \mid bRa \mid bRb \mid b$

Exercise 2.3, 2.4

2.3
$$\left.\begin{array}{l} R \to XRX \mid S \\ S \to aTb \mid bTa \\ T \to XTX \mid X \mid \varepsilon \\ X \to a \mid b \end{array}\right\} G$$

a) variables : R, S, T, X
b) terminals : a, b
c) start variable : $R \to XRX \mid S$
d) 3 strings in L(G):

1) $R \to S$
$R \to aTb$
$R \to a\varepsilon b$
$R \to ab$

2) $R \to S$
$R \to bTa$
$R \to b\varepsilon a$
$R \to ba$

3) $R \to S$
$R \to aTb$
$R \to aXb$
$R \to aab$

$\boxed{ab,\ ba,\ aab}$

e) 3 strings not in $L(G)$: aba, b, ε
f) $T \Rightarrow aba$: false
g) $T \Rightarrow^* aba$: true
h) $T \Rightarrow T$: false
i) $T \Rightarrow^* T$: true
j) $XXX \Rightarrow^* aba$: true
k) $X \Rightarrow^* aba$: false
l) $T \Rightarrow^* XX$: true
m) $T \Rightarrow^* XXX$: true
n) $S \Rightarrow^* \varepsilon$: false
o) $L(G)$: $L(G)$ is formed over terminal symbols a and b, and consists of all strings that are not palindromes

2.4 Σ is $\{0,1\}$
a) $\{w \mid w$ contains at least 3 1s$\}$
$$S \to P1P1P1$$
$$P \to 0P \mid 1P \mid \varepsilon$$
b) $\{w \mid w$ starts and ends with the same symbol$\}$
$$S \to 0P0 \mid 1P1 \mid 0 \mid 1$$
$$P \to 0P \mid 1P \mid \varepsilon$$
c) $\{w \mid$ the length of w is odd$\}$
$$S \to 0 \mid 1 \mid 0S0 \mid 1S1 \mid 0S1 \mid 1S0$$
d) $\{w \mid$ the length of w is odd and its middle symbol is 0$\}$
$$S \to 0 \mid 0S0 \mid 1S0 \mid 1S1 \mid 0S1$$
e) $\{w \mid w = w^R$, w is a palindrome$\}$
$$S \to 0 \mid 1 \mid 0S0 \mid 1S1 \mid \varepsilon$$
f) empty set
$$S \to S$$