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CMPS 130
HW #5

1.29 a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

First assume that A_1 is regular. Let p be the pumping length given by pumping lemma. Let s be the string $0^p 1^p 2^p = s$

Since s is a member of A_1, and s is longer than p, the pumping lemma guarantees s is able to be split into 3 parts $s = xyz$ for any $i \geq 0$ the string xy^iz is in A_1

Consider 2 possibilities.

1) String y consists only of 0s, only of 1s, only of 2s. xy^2z won't have equal numbers of 0s, 1s, and 2s. Hence xy^2z isn't a member of A_1

2) String y consists of more than one kind of symbol. xy^2z have 0s, 1s, or 2s in a different order, hence xy^2z isn't a member of A_1

We have a contradiction, so A_1 isn't regular □

b) $A_2 = \{www \mid w \in \{a,b\}^*\}$

Assume A_2 is regular. P be the pumping length and string $s = a^p b a^p b a^p b$

$s \in A_2$ since $s = (a^p b)^3$ and $|s| = 3(p+1) \geq p$ so pumping lemma holds. Split string into 3 pieces, $s = xyz$ satisfying the conditions $xy^iz \in A_2$ for each $i \geq 0$, $|y| > 0$, $|xy| \leq p$

First p symbols of s are all a's, the condition implies x and y consists only of a's. $|y| > 0$ y has at least one a.

Thus we say : $x = a^j$ for $j \geq 0$
$y = a^k$ for $k \geq 1$
$z = a^m b a^p b a^p b$ for $m \geq 0$

$s = xyz$ we take the individual value of x, y, and z multiply them out to get $a^{j+k+m} b a^p b a^p b$ thus $j+k+m = p$.

Thus $xy^2z \notin A_2$ bc $k \geq 1$, we get a contradiction, hence A_2 is non regular

c) $A_3 = \{a^{2^n} \mid n \geq 0\}$

Assume A_3 is regular, let p be pumping length

string $s = a^{2^p}$.

Since s is a member of A_3, s is longer than p, pumping lemma guarantees s can be split into 3 parts $s = xyz$, thus we satisfy 3 conditions of lemma

3rd condition tells us that $|xy| \leq p$. Further, $p < 2^p$ and $|y| < 2^p$. Therefore $|xy^2z| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$. 2nd condition requires $|y| > 1$ so $2^p < |xy^2z| < 2^{p+1}$. Length of xy^2z cannot be power of 2, hence xy^2z is not a member of A_3. Which is a contradiction so A_3 is non-regular

1.30 The error in the proof is that "$s = 0^p1^p$ cannot be pumped"

However $s = 0^p1^p$ can be pumped because:

Assume $x = 0$, $y = 0$, $z = 0^{p-2}1^p$

Assumption x, y, z satisfies Pumping lemma

i) For $\forall i \geq 0$, $xy^iz = 00^i(0^{p-2}1^p)$

$= 0^{p-1+i}1^p \in 0^*1^*$

ii) $|y| = |0| = 1 > 0$

iii) $|xy| = |00|$

$= 2 < p$

Therefore, S can be pumped and 0^*1^* is regular

1.42 $\{w \mid w = a_1b_1 \ldots a_kb_k$, where $a_1 \ldots a_k \in A$ and $b_1 \ldots b_k \in B$, each $a_i, b_i \in \Sigma^*\}$

Prove regular languages under shuffle closed

Let $M_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$ be a DFA recognizing A and $M_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$ be a DFA recognizing B. The NFA N for the shuffle of A and B, simulate both M_A and M_B

We can mention definition of N

- $Q = (Q_A \times Q_B) \cup \{q_0\}$ where $Q_A \times Q_B$ tracks all possible current states of D_A and D_B, q_0 - state when nothing is read
- $q = q_0$
- $F = (F_A \times F_B) \cup \{q_0\}$ states N accepts the string if both D_A and D_B are in accept states or N is empty
- δ :
  - $\delta(q_0, \varepsilon) = (q_A, q_B)$ start state is q_0
    N can make D_A in q_A and D_B in q_B without reading anything
  - $(\delta_A(m, a), n) \in ((m, n), a)$ states if D_A is m, current state of D_B is n, then when the character a is read then change state of A to $\delta_A(m, a)$ while B is not changed
  - $(m, \delta_B(n, a)) \in \delta((m, n), a)$

1.46 Prove languages are NOT regular

a) $\{0^n 1^m 0^n \mid m, n \geq 0\} = L$

Assume L is regular and string $s = 0^P 1 0^P$. Divide string into x, y, z. So, $s = 0^P 1 0^P = xyz$ where P is pumping length
Assume $x = 0^{P-K}$, $y = 0^K$ and $z = 10^P$ where $K > 0$
Now, $xy^0z = 0^{P-K}(0^K)^0 10^P$
$= 0^{P-K} 10^P \notin L$

So, xy^0z does not belong to L. A contradiction, so L is nonregular

b) $\{0^m 1^n \mid m \neq n\} = L$

Assume L is regular, $s = 0^P 1^{P+1}$. Divide into xyz
So $s = 0^P 1^{P+1} = xyz \in L$ where P is pumping length
Assume $x = 0^{P-1}$, $y = 0^1$ and $z = 1^{P+1}$
Now, $xy^2z = 0^{P-1}(0^1)^2 1^{P+1}$
$= 0^{P+1} 1^{P+1} \notin L$

So, xy^2z does not belong to L. A contradiction, so L is nonregular

c) $\{w \mid w \in \{0,1\}^*$ not a palindrome $\} = L$

Assume L is regular. $\overline{L} = \{w \mid w \in \{0,1\}^*$ is a palindrome$\}$ is regular. $S = 0^p 1 0^p$, divide into xyz

So, $S = 0^p 1 0^p = xyz \in L$ where p is pumping length

Assume $x = 0^{p-k}$, $y = 0^k$, $z = 10^p$ where $k > 0$

Now $xy^0z = 0^{p-k}(0^k)^0 10^p$

$= 0^{p-k} 10^p \notin \overline{L}$

xy^0z does not belong in $\overline{L}$, so assumption is contradiction and L is nonregular

d) $\{wtw \mid w, t \in \{0,1\}^*\} = L$

Assume L is regular. $S = 0^p 1^p 0^p$, divide into xyz

So, $S = 0^p 1 0^p = xyz \in L$, where p is pumping length

Assume $x = 0^{p-k}$ $y = 0^k$ $z = 10^p$ where $k > 0$

Now, $xy^0z = 0^{p-k}(0^k)^0(10^p)$

$= 0^{p-k}(10^p) \notin L$

xy^0z doesn't belong in L, so contradiction. L is nonregular

1.47 Let $\Sigma = \{1, \#\}$ and $A = \{w \mid w = x_1 \# x_2 \# \ldots \# x_k, k \geq 0,$ each $x_i \in 1^*$ and $(i \neq j) \rightarrow (x_i \neq x_j)\}$

Let p be pumping lemma for A

let $u = 1^p \# 1^{p+1} \# \ldots \# 1^{2p}$

let $xyz = u$ such that $|y| > 0$ and $|xy| \leq p$

let $v = xy^2z$. Bc $1 \leq |y| \leq p$, we conclude $p+1 \leq (p+|y|) \leq 2p$ and $v_0 = v_{p+|y|}$ Thus $v \notin A$

A does not satisfy pumping lemma, so A is not regular

1.55 e) $(01)^*$

Let s be the string in the language. S could be ε but it can't be pumped so the length isn't 0. S could be 01 if we divide into $xyz \Rightarrow x = \emptyset$ $y = 01$ $z =$ everything after

So length is $\boxed{1}$

f) ε

minimum length is $\boxed{0}$

i) 1011

Divide into xyz → $x = 10$ $y = 1$ $z = \emptyset$

length $\boxed{3}$

j) ε^*

Divide into xyz → x = empty string

$y = (\varepsilon | 011)$

z = empty string

ε can't be pumped

length is $\boxed{1}$