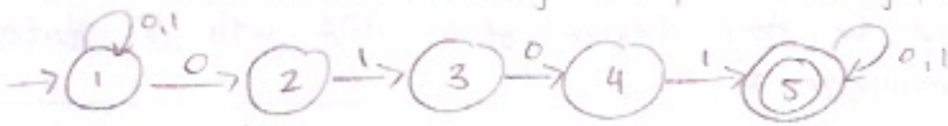
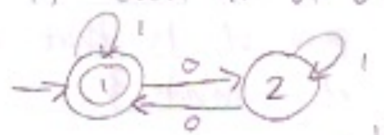
Lynne Diep
lytdiep@ucsc.edu
CMPS 130
HW # 3

1.7 b) Exercise 1.6c with 5 states
{w | w contains the substring 0101, w = x0101y, with 5 states}

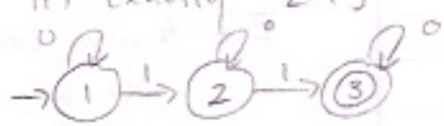


c) Exercise 1.61 with 6 states
{w | w contains an even # of 0's, or contains exactly 2 1's}

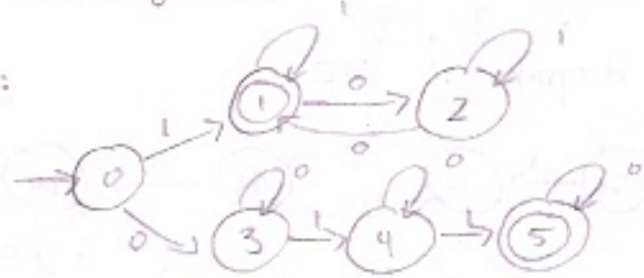
i) even # of 0's



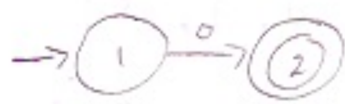
ii) exactly 2 1's



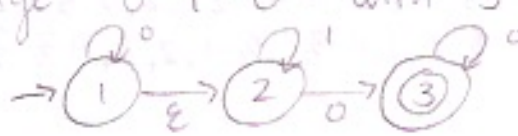
combine:



d) language {0} with 2 states



e) language $0^* 1^* 0^+$ with 3 states



g) language {ε} with one state

h) language 0^* with one state

1.13 F language over $\{0,1\}$ that do not contain a pair of 1s that are seperated by an odd # of symbols

Give the state diagram of a DFA with 5 states that recognize F

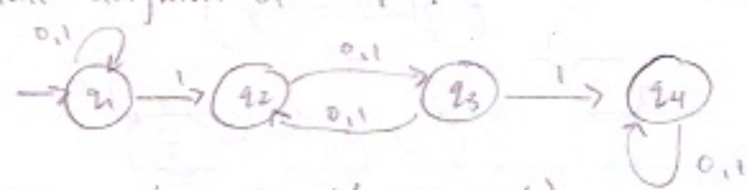
$F = \{w | w$ doesn't contain a pair of 1s that are seperated by an odd number of symbols$\}$

$\Sigma = \{0,1\}$

$\bar{F} = \{w | w$ contains a pair of 1s that are seperated by an odd number of symbols $\}$

$\Sigma = \{0,1\}$

state diagram of $N_{\bar{F}}$:



DFA $M_{\bar{F}} = (Q', \Sigma, \delta', q_0', F')$

equals

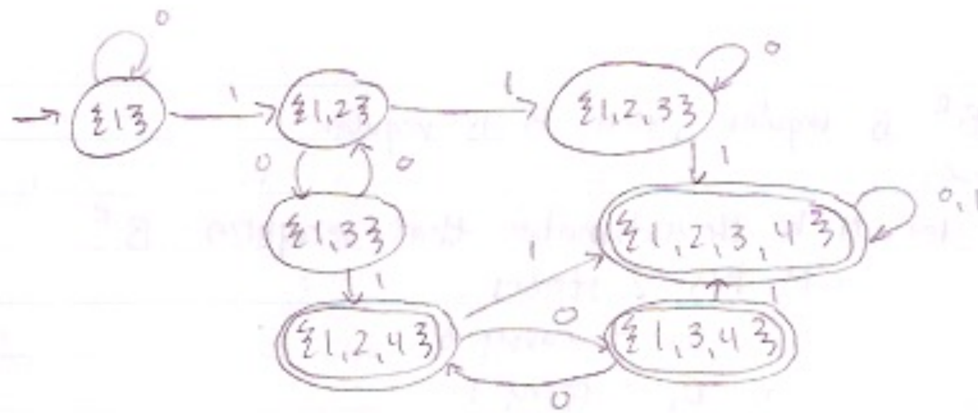
$N_{\bar{F}} = (Q, \Sigma, \delta, q_0, F)$

$Q = \{ \{1\}, \{2\} \{3\} \{4\} \{1,2\} \{1,3\} \{1,4\}$
$\{2,3\} \{2,4\} \{3,4\} \{1,2,3\} \{1,3,4\}$
$\{2,3,4\} \{1,2,4\} \{1,2,3,4\} \}$

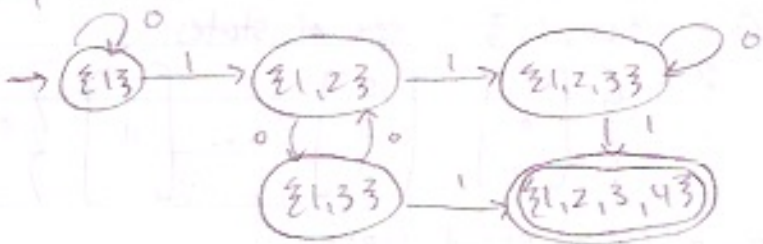
δ	0	1
$\{1\}$	$\{1\}$	$\{1,2\}$
$\{1,2\}$	$\{1,3\}$	$\{1,2,3\}$
$\{1,3\}$	$\{1,2\}$	$\{1,2,4\}$
$\{1,2,3\}$	$\{1,2,3\}$	$\{1,2,3,4\}$
$\{1,2,4\}$	$\{1,3,4\}$	$\{1,2,3,4\}$
$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$
$\{1,3,4\}$	$\{1,2,4\}$	$\{1,2,3,4\}$

$q_0' = \{1\}$

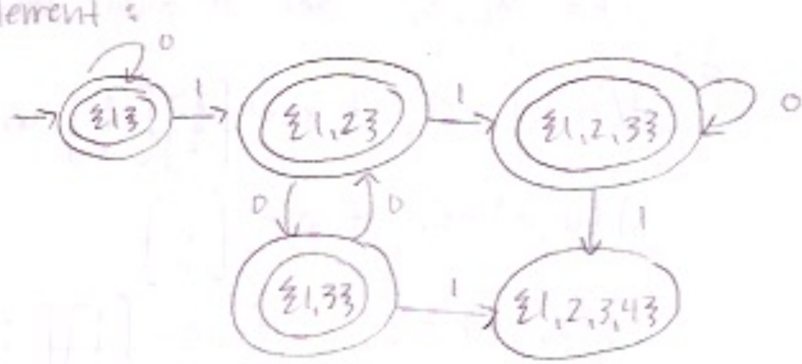
$F' \{ \{1,2,4\} \{1,3,4\}$
$\{1,2,3,4\} \}$



Simplify DFA:



Take complement:



1.32 Show that B is regular

Given:

- $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

- string of symbols in Σ_3 gives 3 rows of 0s and 1s
- each row to be binary number
- $B = \{ w \in \Sigma_3^* \mid$ the bottom row of w is the sum of the top two rows $\}$

We know that regular languages are closed under reversal

Prove B^R is regular, then B is regular

Proof:

let M be the automaton that recognizes B^R

- M has 2 states

C_0 - carry 0

C_1 - carry 1

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{C_0, C_1\}$ = set of states

$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \cdots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ = set of alphabets

$q_0 = C_0$ = start state

$F = \{C_0\}$ = set of final states

δ:

$\delta(C_0, a) = C_0$ if $a = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\delta(C_0, a) = C_1$ if $a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\delta(C_1, a) = C_1$ if $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\delta(C_1, a) = C_0$ if $a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

All other arrows go to trap state

Thus we defined a automaton M to recognize B^R

Therefore, B^R is regular, which makes B regular □

1.36 $B_n = \{a^k \mid k$ is a multiple of $n\}$ Show that for each $n \geq 1$, the language B_n is regular

Proof.

Suppose $k = ni$, where i = any positive integer

$i = 1$ and $n = 1$

$B_1 = \{a^k\} = \{a^{ni}\} = \{a^{[1]}\} = \{a\}$

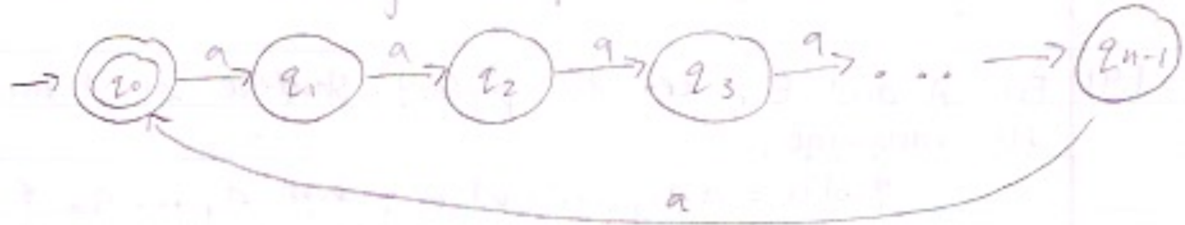
Let $i=1$ and $n=2$

$B_2 = \{a^k\} = \{a^{ni}\} = \{a^2\} = \{aa\}$

Let $i=1$ and $n=3$

$B_3 = \{a^k\} = \{a^{ni}\} = \{a^3\} = \{aaa\}$

Automaton of the regular expression:



q_0 is the initial state, and q_1, q_2, q_3, q_{n-1} are the subsequent states.

The language B_n is the regular language. By the closure property, the specific expression is a regular expression when the value of n is greater than or equal to 1.

Assume B_n is regular -

Union B_1 and B_2 results in the third string, which is a regular expression.

Additionally, if user applies any property of closure, then result is regular.

Hence, the language of B_n is regular.

1.37 Let $C_n = \{x \mid x$ is a binary number that is a multiple of $n\}$. Show that for each $n \geq 1$, the language C_n is regular

- language is regular if recognized by DFA

So, lets construct a DFA:

- Assume $n=3$
- If binary number divided by three, and has remainder zero, accept.
- Else, reject
- Read a digit, shift left on position
- If digit $=0$, value is $2x \pmod 3$

• If digit = 1, new value is $(2x+1) \pmod 3$

DFA:

Since C_n is recognized by a DFA, the language is regular □.

1.41 For A and B, let the perfect shuffle of A and B be the language,

$\{w \mid w = a_1 b_1 \dots a_k b_k$, where $a_1 \dots a_k \in A$ and $b_1 \dots b_k \in B$, each $a_i, b_i \in \Sigma\}$

Show that the class of regular languages is closed under perfect shuffle

Claim – If A and B are regular, then perfect shuffle of A and B are regular

Proof:

A is regular if recognized by automation

1) Let $M_1 = (Q_1, \Sigma, \delta_1, S_1, F_1)$ be a DFA recognized by A
2) Let $M_2 = (Q_2, \Sigma, \delta_2, S_2, F_2)$ DFA recognized by B
3) Then $M = (Q, \Sigma, \delta, \delta_0, F)$ DFA recognizes perfect shuffle of A and B. Defined:

a) $Q = Q_1 \times Q_2 \times \{1, 2\}$: set of possible states that should match with M

b) $S_0, \{S_1, S_2, 1\}$: S_1 initial state in M_1, S_2 " " M_2
M starts with 1

c) $F = \{(q_1, q_2, 1) \mid q_1 \in F_1 \cap q_2 \in F_2\}$: set of final state

d) δ is given as follows current state of M_1 is q_1, M_2 is q_2, and next character to read M_1.
When a is the next character, change current state of 1 to $\delta_1(q_1, a)$. State 2 does not change

$\delta((q_1, q_2, 1), a) = (\delta_1(q_1, a), q_2, 2)$ $b=1$
$\delta((q_1, q_2, 2), b) = (q_1, \delta_2(q_2, b), 1)$ $b=2$

We proved M to recognize the perfect shuffle of A and B. Thus A and B is regular language. Finally, class of regular languages are closed under perfect shuffle

$\square$