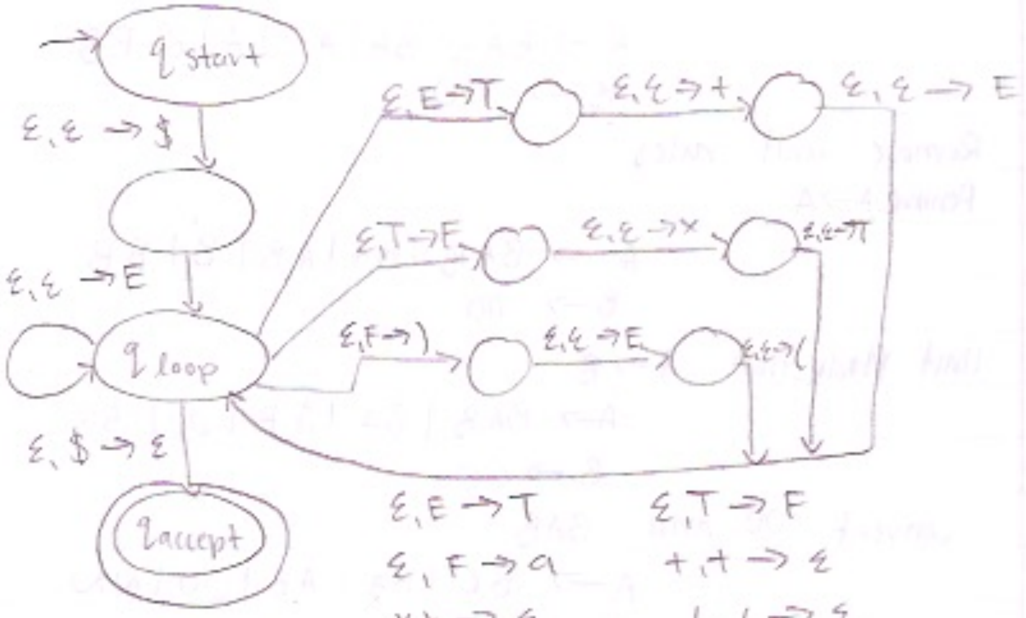
Lynne Diep
CMPS 130
HW # 8

2.11 Convert the CFG G_4 to an equivalent PDA

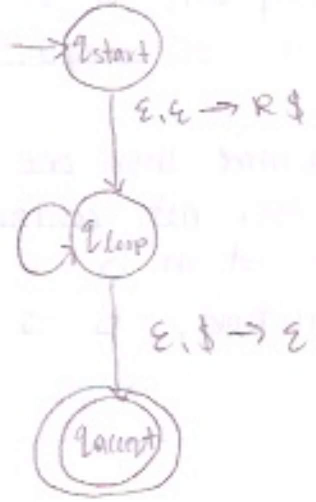
$E \to E+T \mid T$
$T \to T \times F \mid F$
$F \to (E) \mid a$



$\varepsilon, E \to T$ $\quad \varepsilon, T \to F$
$\varepsilon, F \to a$ $\quad +, + \to \varepsilon$
$\times, \times \to \varepsilon$ $\quad (, (\to \varepsilon$
$),) \to \varepsilon$ $\quad a, a \to \varepsilon$

2.12 Convert CFG G to PDA

$R \to XRX \mid S$
$S \to aTb \mid bTa$
$T \to XTX \mid X \mid \varepsilon$
$X \to a \mid b$



$\varepsilon, R \to S$
$\varepsilon, R \to XRX$
$\varepsilon, S \to aTb$
$\varepsilon, S \to bTa$
$\varepsilon, T \to XTX$
$\varepsilon, T \to X$
$\varepsilon, T \to \varepsilon$
$\varepsilon, X \to a$
$\varepsilon, X \to b$

$a, a \to \varepsilon$
$b, b \to \varepsilon$

2.14 CFG to Chomsky normal form

$$A \to BAB \mid B \mid \varepsilon$$
$$B \to 00 \mid \varepsilon$$

Remove rules containing ε

$$A \to BAB \mid BA \mid AB \mid A \mid B \mid BB$$
$$B \to 00$$

Remove unit rules

Remove $A \to A$

$$A \to BAB \mid BA \mid AB \mid B \mid BB$$
$$B \to 00$$

Unit Production $A \to B$

$$A \to BAB \mid BA \mid AB \mid 00 \mid BB$$
$$B \to 00$$

Convert 00 and BAB

$$A \to BC \mid BA \mid AB \mid 00 \mid NN$$
$$C \to AB$$
$$B \to NN$$
$$N \to 0$$

2.30 $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Let $B = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$ and p be the pumping length

let $s = 1^p 0 1^p 0^p$ and show it cannot be pumped

Consider s in the form $s = uvxyz$

1) if both v and y contain at most one type of alphabet symbol, the string will be of form uv^2xy^2z which runs of 0's and 1's of unequal length.

So s is not in B

2) if v and y contain more than one type of alphabet symbol then uv^2xy^2z does not contain symbols in correct order. So s is not in B

By pumping lemma contradiction, B is not context free

b) $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$

Let $B = \{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$ and p be the pumping length

Let $S = 0^p \# 0^{2p} \# 0^{3p}$ and show it cannot be pumped

Consider S in the form $S = uvxyz$

v and y don't contain $\#$, so xv^2wxy^2z contains more than 2 $\#$'s. Divide S into 3 segments: $0^p, 0^{2p}, 0^{3p}$, it is not contained within v or y. So, xv^2wy^2z is not in B. By pumping lemma contradiction, B is not context free

c) $\{w \# t \mid w$ is a substring of t, where $w, t \in \{a, b\}^*\}$

Let $B = \{w \# t \mid w$ is a substring of t, where $w, t \in \{a, b\}^*\}$ and p be the pumping lemma

Let $S = a^p b^p \# a^p b^p$ and show it cannot be pumped

Consider S in the form $S = uvxyz$

1) v and y don't contain $\#$, so uv^0xy^0z don't contain $\#$, thus S is not in B

2) uv^2xy^2z is longer on the left side if v and y are non-empty and occur on the left side of $\#$. S is not in B

3) uv^2xy^2z is longer on the right if " " right side of $\#$. S is not in B

S cannot be pumped, so B is context free

d) $\{t_1 \# t_2 \# \dots \# t_k \mid k \geq 2$, each $t_i \in \{a,b\}^*$, and $t_i = t_j$ for some $i \neq j\}$

Let B = above and p be pumping length

Let $S = a^p b^p \# a^p b^p$ and $S = uvxyz$

1) v and y don't contain $\#$, so uv^0xy^0z don't contain $\#$

2) if only one of v or y is nonempty we can treat them if both occurred on the same side

3) Both v and y occur on left side of $\#$, uv^2xy^2z is longer on left side

4) Both v and y occur on right side of $\#$, uv^2xy^2z is longer on right side

S cannot be pumped, so B is context free

2.31 B = palindromes over $\{0,1\}$ containing equal # of 0s and 1s
Prove by contradiction - B is a context free language
By pumping lemma, there is a number p where any string, s, in B of length (at least) p, then s may be divided into 5 segments. $s = uvxyz$
let $s = 0^p 1^{2p} 0^p$
1) when v and y contain only one type of symbol then v and y don't contain both 0s and 1s.
So, uv^2xy^2z can't contain equal numbers of 0's and 1's
2) when either v or y contains more than one type of symbol uv^2xy^2z contain equal #s but is not a palindrome.
So, both cases are contradictions, so B is not context free

2.32 $\Sigma = \{1,2,3,4\}$ and $C = \{w \in \Sigma^* \mid$ in w, the # of 1's equal the # of 2s, the # of 3s equal the # of 4s$\}$
Show C is not context free
Consider C being context free with pumping length p
let $s = 1^p 3^p 2^p 4^p \in C$ with $|s| > p$
let s be in the form uvxyz
1) if vxy contains 1, then $uv^2xy^2z \notin C$ since C cannot have the same number of 1s and 2s
2) " 2, "
vxy cannot contain any 1s
3) vxy contains a 3, $uv^2xy^2z \notin C$ since it cannot be same number of 3s and 4s. vxy cannot contain 4s
4) " 4, "
vxy cannot contain any 3s
So, cases are contradictions, so C is not context free

2.35 Let G be a CFG in Chomsky that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, L(G) is infinite

Since G is a CFG in Chomsky, then every derivation can generate 2 non-terminals at most. This means an internal node can have 2 children in any parse tree using G. Every parse tree with height k has at most $2^k - 1$ internal nodes. With G having some string with a derivation having at least 2^b steps, then there are at least 2^b internal nodes in the parse tree. The height is b+1, at least, so that means there are b+1 variables. By using pigeonhole principle, there is a variable occuring at least twice. Thus, we can use pumping lemma proof to construct many strings which are all in L(G), thus L(G) is infinite.