

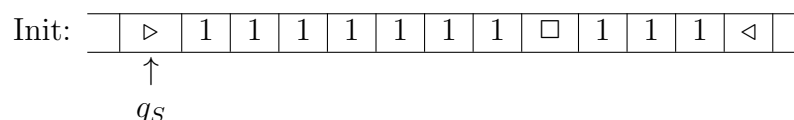
Lab08-Computational Complexity

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2019.

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- Design a one-tape TM M that computes the function $f(x, y) = x - y$, where x and y are positive integers ($x > y$). The alphabet is $\{1, 0, \square, \triangleright, \triangleleft\}$, and the inputs are x 1's, \square and y 1's. Below is the initial configuration for input $x = 7$ and $y = 3$. The result $z = f(x, y)$ should also be represented in the form of z 1's on the tape with the pattern of $\triangleright 111 \cdots 111 \triangleleft$.



- Please describe your design and then write the specifications of M in the form like $\langle q_s, \triangleright \rangle \rightarrow \langle q_1, \triangleright, R \rangle$. Explain the transition functions in detail.
 - Please draw the state transition diagram using Microsoft Visio.
 - Show briefly and clearly the whole process from initial to final configurations for input $x = 7$ and $y = 3$.
- What is the “certificate” and “certifier” for the following problems?
 - PARTITION*: Given a finite set A and a size $s(a) \in \mathbb{Z}$ for each $a \in A$, is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)$?
 - CLIQUE*: Given a graph $G = (V, E)$ and a positive integer $K \leq |V|$, is there a subset $V' \subseteq V$ with $|V'| \geq K$ such that every two vertices in V' are joined by an edge in E ?
 - ZERO-ONE INTEGER PROGRAMMING*: Given an integer $m \times n$ matrix A and an integer m -vector b , is there an integer n -vector x with elements in the set $\{0, 1\}$ such that $Ax \leq b$?

Solution.

(a) certificate: a partition of set A .

certifier: compute the some of each part and check if they are the same.

(b) certificate: a subset of V

certifier: check if the subset contains at least k elements and there is an edge between each pair of the nodes in the subset.

(c) certificate: an assignment of 0,1 to n -vector x

certifier: compute Ax and check if each element of it is smaller than the corresponding element in b . \square

- SUBSET SUM*: Given a finite set A , a size $s(a) \in \mathbb{Z}$ for each $a \in A$ and an integer B , is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = B$?

KNAPSACK: Given a finite set A , a size $s(a) \in \mathbb{Z}$ and a value $v(a) \in \mathbb{Z}$ for each $a \in A$ and integers B and K , is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) \leq B$ and $\sum_{a \in A'} v(a) \geq K$?

- Prove $PARTITION \leq_p SUBSET SUM$.

Proof.

Instance of PARTITION: A finite set A and a size $s(a) \in \mathbb{Z}$, there is a subset $A' \subseteq A$ that satisfies $\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)$.

Instance of SUBSETSUM: The same set A with the instance of PARTITION and let B be the half of the sum of the elements in A .

If A' satisfies the requirement of the instance of PARTITION, we can easily know that it also satisfies the requirement of the instance of SUBSETSUM and vice versa. \square

(b) Prove $SUBSET\ SUM \leq_p KNAPSACK$.

Proof.

Instance of PARTITION: A finite set A and a size $s(a) \in \mathbb{Z}$ for each $a \in A$ and an integer B . A subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = B$.

Instance of KNAPSACK: The same set with set in the instance of PARTITION, and the same integer B . Let value of every element in A be 1 and $K = B$.

If A' satisfies the requirement of the instance of SUBSETSUM, we can easily know that it also satisfies the requirement of the instance of KNAPSACK and vice versa. \square

4. 3-SAT: Given a set U of variables, a collection C of clauses over U such that each clause $c \in C$ has $|c| = 3$, is there a satisfying truth assignment for C ?

Prove $3\text{-SAT} \leq_p CLIQUE$.

Proof.

Given an instance of 3-SAT, we construct an instance (G, k) that has a subset equal or larger than k .

G contains 6 vertices for each clause, one for each literal. Initially, each pair of vertices is connected. Then we only remain on vertex between different clause. Also, the connection between x_k and \bar{x}_k in the same clause.

If we can find a subset of size k in G satisfies the requirement of CLIQUE, then we can assign true to each element in the subset so that ϕ is true which satisfies 3-SAT. Also we can prove this statement vice versa. \square

5. Algorithm class is a democratic class. Denote class as a finite set S containing every students. Now students decided to raise a student union $S' \subseteq S$ with $|S'| \leq K$.

As for the members of the union, there are many different opinions. An opinion is a set $S_o \subseteq S$. Note that number of opinions has nothing to do with number of students.

The question is whether there exists such student union $S' \subseteq S$ with $|S'| \leq K$, that S' contains at least one element from each opinion. We call this problem *ELECTION* problem, prove that it is NP-complete.

Proof.

Given all opinions and a student union $S' \subseteq S$, we can check if S' contains at least one element from each opinion one by one. So we can certificate it in polynomial time and prove it's a NP problem.

From the course we know that 3-SAT is a NP-complete problem and $3\text{-SAT} \leq_p VERTEX\text{-COVER}$. Therefore *VERTEX - COVER* is a NP-complete problem.

We claim that *VERTEX-COVER* problem can reduce to *ELECTION* problem. Given an arbitrary graph, let each vertex be a student and each edge be an opinion. Two corresponding vertices of the edge is the elements of the opinion.

To check if there is a subset whose size is equal or smaller than k that covers all edges, we only have to check if there is a student union whose size is equal or smaller than k that contains at least one element from each opinion. Therefore, $VERTEX-COVER \leq_p ELECTION$. So *ELECTION* is np-complete. \square

Remark: You need to include your .pdf and .tex files in your uploaded .zip file.