A model to study the impact of government-imposed social distancing on COVID-19 epidemic in Portugal

Clearing memory

In[900]:=
```
ClearSystemCache[]
ClearAll["Global`*"]
Clear["Subscript"]
Clear["Superscript"]
Clear["Subsuperscript"]
```

O estado de emergência: 18/03/2020, 17th day after 02/03/2020

In[905]:=
```
Emergencia = 17 / 365;
Hoje = 23 / 365;
```

Model equations

In[907]:=
```
eq[Reduction_][1] := S'[t] ==
  -S[t] ((β (σ IM[t] + IS[t]) If[t ≤ Emergencia, 1, If[t ≤ Hoje, 0.45, Reduction]]) /
     (NN[t] - IQ[t]))
eq[Reduction_][2] := EE'[t] ==
  S[t] ((β (σ IM[t] + IS[t]) If[t ≤ Emergencia, 1, If[t ≤ Hoje, 0.45, Reduction]]) /
      (NN[t] - IQ[t])) - α EE[t]
eq[Reduction_][3] := IM'[t] == p α EE[t] - γ1 IM[t]
eq[Reduction_][4] := IS'[t] == (1 - p) α EE[t] - ν IS[t]
eq[Reduction_][5] := IQ'[t] == ν IS[t] - γ2 IQ[t] - η IQ[t]
eq[Reduction_][6] := RM'[t] == γ1 IM[t]
eq[Reduction_][7] := RS'[t] == γ2 IQ[t]
eq[Reduction_][8] := DD'[t] == η IQ[t]
```

Numer of variables in the model (including deceased individuals)

In[915]:=
```
numvar = 8
eqs[Reduction_] := Table[eq[Reduction][i], {i, 1, numvar}]
lhs[Reduction_] := eqs[Reduction][[All, 1]];
rhs[Reduction_] := eqs[Reduction][[All, 2]];
TableForm[eqs[Reduction]]
```

Out[915]= 8

Out[919]//TableForm=

$$S'[t] == -\frac{\beta\,\mathrm{If}\left[t \le \frac{17}{365}, 1, \mathrm{If}[t \le \mathrm{Hoje}, 0.45, \mathrm{Reduction}]\right](\sigma\,IM[t] + IS[t])\,S[t]}{-IQ[t] + NN[t]}$$

$$EE'[t] == -\alpha\,EE[t] + \frac{\beta\,\mathrm{If}\left[t \le \frac{17}{365}, 1, \mathrm{If}[t \le \mathrm{Hoje}, 0.45, \mathrm{Reduction}]\right](\sigma\,IM[t] + IS[t])\,S[t]}{-IQ[t] + NN[t]}$$

$$IM'[t] == p\,\alpha\,EE[t] - IM[t]\,\gamma_1$$

$$IS'[t] == (1 - p)\,\alpha\,EE[t] - \nu\,IS[t]$$

$$IQ'[t] == -\eta\,IQ[t] + \nu\,IS[t] - IQ[t]\,\gamma_2$$

$$RM'[t] == IM[t]\,\gamma_1$$

$$RS'[t] == IQ[t]\,\gamma_2$$

$$DD'[t] == \eta\,IQ[t]$$

Model variables

In[920]:=
```
vars = {S[t], EE[t], IM[t], IS[t], IQ[t], RM[t], RS[t], DD[t]}
```

Out[920]= {S[t], EE[t], IM[t], IS[t], IQ[t], RM[t], RS[t], DD[t]}

Total population size N(t) is not constant due to disease-related mortality

In[921]:=
```
NN[t] = S[t] + EE[t] + IM[t] + IS[t] + IQ[t] + RM[t] + RS[t]
```

Out[921]= EE[t] + IM[t] + IQ[t] + IS[t] + RM[t] + RS[t] + S[t]

Epidemiological parameters of the model

Average contact rate (unique persons), 1/year

In[922]:=
```
AverageContactRate = c → 13.74 × 365
```

Out[922]= c → 5015.1

Relative infectivity of mildly infected

In[923]:=
```
RelativeInfectivity = σ → 0.5
```

Out[923]= σ → 0.5

1/latent period, 1/year

```
In[924]:= RateInfectiousnessOnset = α → 365 / 4
```

Out[924]= $\alpha \to \frac{365}{4}$

Proportion of mildly infected

```
In[925]:= ProportionMildSymptoms = p → 0.8
```

Out[925]= $p \to 0.8$

1/recovery period of mildly infected, 1/year

```
In[926]:= RecoveryRateMildSymptoms = γ1 → 365 / 7
```

Out[926]= $\gamma_1 \to \frac{365}{7}$

1/delay from onset of infectiousness to diagnosis for individuals with severe symptoms, 1/year

```
In[927]:= DiagnosisRate = ν → 365 / 5
```

Out[927]= $\nu \to 73$

1/delay from diagnosis to recovery for diagnosed unaware, 1/year

```
In[928]:= RecoveryRateSevereSymptoms = γ2 → 365 / 14
```

Out[928]= $\gamma_2 \to \frac{365}{14}$

Case fatality rate of unaware diagnosed

```
In[929]:= FatalityRate = f → 0.013
```

Out[929]= $f \to 0.013$

Disease-associated death rate of unaware diagnosed, 1/year

```
In[930]:= DeathRateDiagnosed = η → γ2 f / (1 - f) /. {RecoveryRateSevereSymptoms, FatalityRate}
```

Out[930]= $\eta \to 0.343393$

Basic reproduction number

```
In[931]:= BasicReproductionNumber = R0 → 6.45
```

Out[931]= $R_0 \to 6.45$

Probability of transmission per contact with infectious with severe symptoms

In[932]:=
```
TransmissionProbability = Solve[R0 == p β σ / γ1 + (1 - p) β / ν /. β → c ϵ, ϵ][[1, 1]] /.
   {ProportionMildSymptoms, AverageContactRate, RelativeInfectivity,
    RecoveryRateMildSymptoms, DiagnosisRate, BasicReproductionNumber}
```

$$R_0 == \frac{p\,\beta\,\sigma}{\gamma_1} + \frac{(1-p)\,\beta}{\nu}$$

Out[932]= ϵ → 0.123535

Transmission rate of infection via contact with infectious with severe symptoms, 1/year

In[933]:=
```
TransmissionRate = β → c ϵ /. {AverageContactRate, TransmissionProbability}
```

Out[933]= β → 619.539

Parameters of the model

In[934]:=
```
Parameters := {AverageContactRate, RelativeInfectivity, RateInfectiousnessOnset,
   ProportionMildSymptoms, RecoveryRateMildSymptoms, DiagnosisRate,
   RecoveryRateSevereSymptoms, FatalityRate, DeathRateDiagnosed,
   BasicReproductionNumber, TransmissionProbability, TransmissionRate}
```

Critical value for contact rate reduction where Reff < 1

In[935]:=

$$\left(1 - 1\Big/\left(\frac{p\,\beta\,\sigma}{\gamma_1} + \frac{(1-p)\,\beta}{\nu}\right) \text{ /. Parameters}\right) 100$$

Out[935]= 84.4961

Solving differential equations

Start time, year
The first day of the simulation (initial condition) corresponds to 02/03/2020, when 2 COVID-19 cases were confirmed

In[936]:= $t_{start} = 1 / 365$

Out[936]= $\frac{1}{365}$

End time, year
The simulation runs for 365 days

In[937]:= $t_{end} = 365 / 365;$

Total population size at the beginning of an outbreak

In[938]:= $Ntot = 10.2 \times 10^6$

Out[938]= 1.02×10^7

Initial number of infected individuals

In[939]:=
```
InfInit = 12
```

Out[939]= 12

Initial conditions

In[940]:=
```
ics = Table[ic[i], {i, 1, numvar}];

ic[1] = (Ntot - 13.74 InfInit - InfInit - 2) == vars[[1]] /. {t → t_start}
ic[2] = 13.74 InfInit == vars[[2]] /. {t → t_start}
ic[3] = 0.8 InfInit == vars[[3]] /. {t → t_start}
ic[4] = 0.2 InfInit == vars[[4]] /. {t → t_start}
ic[5] = 2 == vars[[5]] /. {t → t_start}
ic[6] = 0 == vars[[6]] /. {t → t_start}
ic[7] = 0 == vars[[7]] /. {t → t_start}
ic[8] = 0 == vars[[8]] /. {t → t_start}
```

Out[941]= $1.01998 \times 10^{7} == S\left[\frac{1}{365}\right]$

Out[942]= $164.88 == EE\left[\frac{1}{365}\right]$

Out[943]= $9.6 == IM\left[\frac{1}{365}\right]$

Out[944]= $2.4 == IS\left[\frac{1}{365}\right]$

Out[945]= $2 == IQ\left[\frac{1}{365}\right]$

Out[946]= $0 == RM\left[\frac{1}{365}\right]$

Out[947]= $0 == RS\left[\frac{1}{365}\right]$

Out[948]= $0 == DD\left[\frac{1}{365}\right]$

Solution

In[949]:=
```
solution[Reduction_, Parameters_] :=
  NDSolve[Join[eqs[Reduction], ics] /. Parameters, vars, {t, t_start, t_end}];
```

Computing peak number of confirmed cases

In[950]:=
```
Peak[Reduction_, Parameters_] :=
 Max[Flatten[Table[Evaluate[IQ[t] /. First@solution[Reduction, Parameters]],
    {t, t_start, t_end, 1 / 365}]]]
PeakBaseline = Peak[1, Parameters]
Peak75 = Peak[0.25, Parameters]
Peak90 = Peak[0.1, Parameters]
Peak90 0.3
```

Out[951]= 894 160.

Out[952]= 220 209.

Out[953]= 5842.14

Out[954]= 1752.64

Computing time until the peak number of confirmed cases since 02/03/2020 (days)

In[955]:=
```
PeakTiming[Reduction_, Parameters_] := Ordering[
    Flatten[Table[Evaluate[IQ[t] /. First@solution[Reduction, Parameters]],
      {t, tstart, tend, 1 / 365}]], -1][[1]]
PeakTimingBaseline = PeakTiming[1, Parameters]
PeakTiming75 = PeakTiming[0.25, Parameters]
PeakTiming90 = PeakTiming[0.1, Parameters]
```

Out[956]= 55

Out[957]= 124

Out[958]= 38

Data for confirmed cases - deaths - recoveries
Data is split in 02/03/2020-18/03/2020 and 19/03/2020-now
Source https://covid19.min-saude.pt/ponto-de-situacao-atual-em-portugal/

In[959]:=
```
DataBefore =  {{1 / 365, (2 - 0)}, {2 / 365, (4 - 0)}, {3 / 365, (6 - 0)},
    {4 / 365, (9 - 0)}, {5 / 365, (13 - 0)}, {6 / 365, (21 - 0)}, {7 / 365, (30 - 0)},
    {8 / 365, (39 - 0)}, {9 / 365, (41 - 0)}, {10 / 365, (59 - 0)}, {11 / 365, (78 - 0)},
    {12 / 365, (112 - 0)}, {13 / 365, (169 - 1)}, {14 / 365, (245 - 2)},
    {15 / 365, (331 - 3)}, {16 / 365, (448 - 3 - 1)}, {17 / 365, (642 - 3 - 2)}};
DataAfter = {{18 / 365, (785 - 3 - 3)}, {19 / 365, (1020 - 5 - 6)},
    {20 / 365, (1280 - 5 - 12)}, {21 / 365, (1600 - 5 - 14)},
    {22 / 365, (2060 - 14 - 23)}, {23 / 365, (2362 - 30 - 22)}};
```

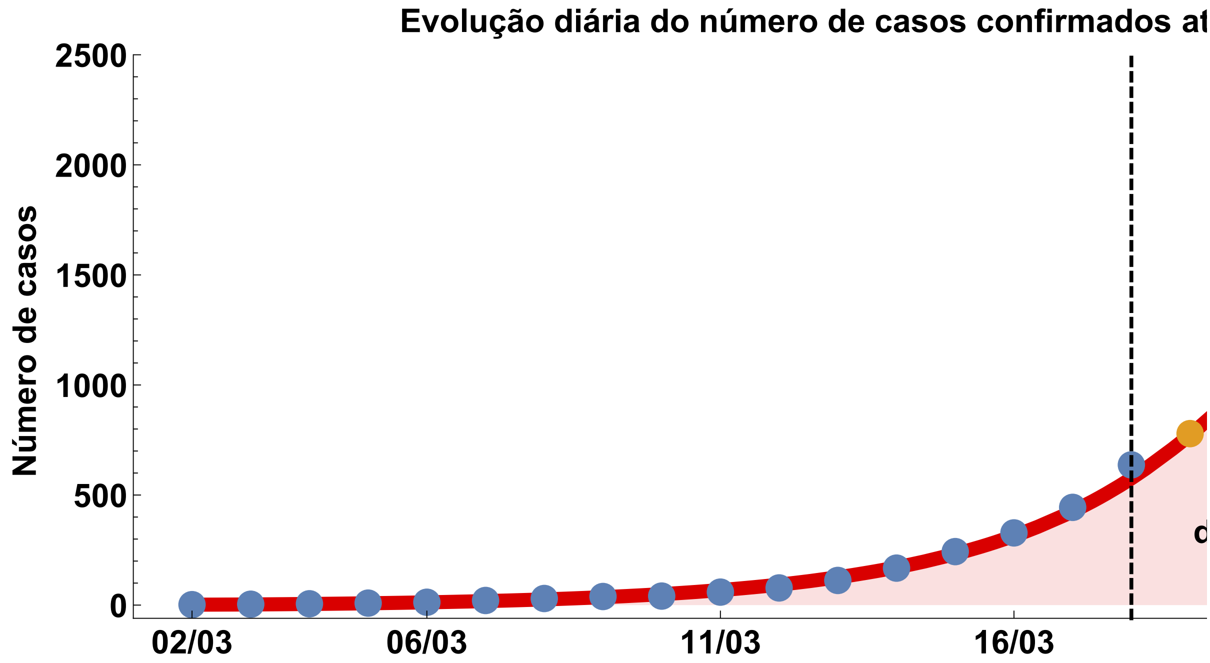
Plotting numero de casos confirmados

In[1042]:=
```
ymax = 2500;
ymin = -60;
tmax = 24 / 365;
tmin = tstart - 1 / 365;
LabelBaseline = "26/04";
Label75 = "04/07";
Label90 = "09/04";


fig1 =
 Table[Show[Plot[{Evaluate[IQ[t] /. solution[1, Parameters]]}, {t, tstart, tend},
     AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None, Filling → Axis,
     PlotRange → {{tmin, tmax}, {ymin, ymax}}, AxesOrigin → {0, 0}, Frame →
       {{True, False}, {True, False}}, FrameStyle → Directive[Black, 17, Bold],
     PlotStyle → {Thickness[0.01], RGBColor[217 / 255, 0, 0]},
```

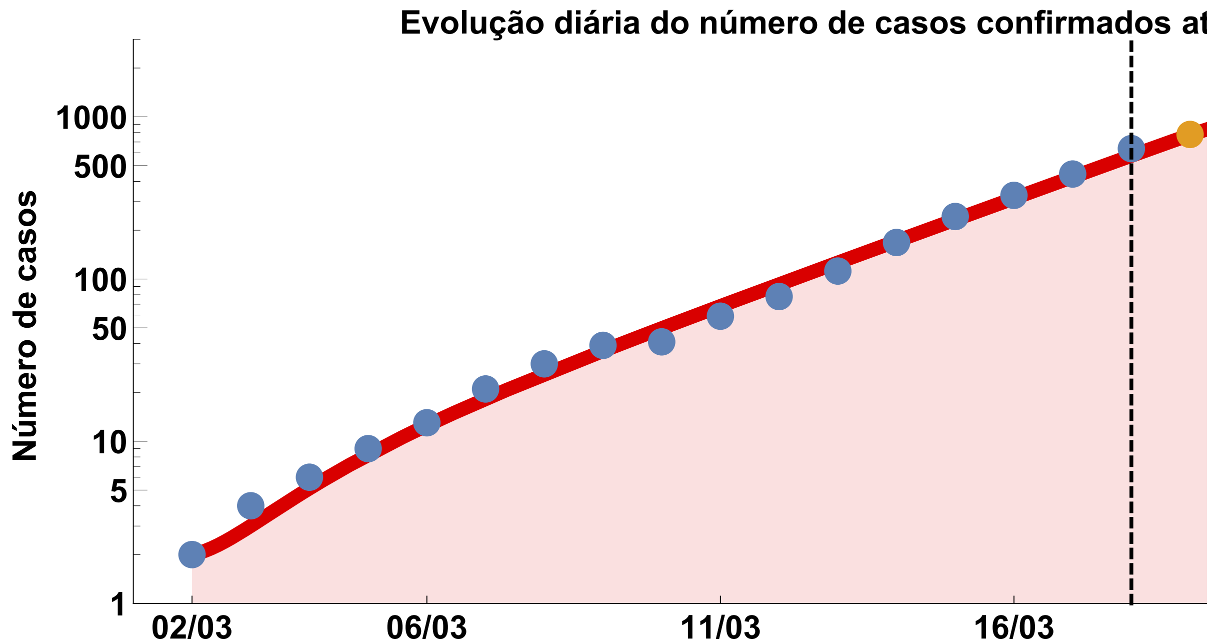
```
      FillingStyle -> Directive[Opacity[0.125]],
      FrameLabel → {{"Número de casos", None}, {None, None}},
      PlotLabel → Style["Evolução diária do número de casos confirmados ativos",
        17, Black, Bold],
      FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"}, {5 / 365, "06/03"},
          {10 / 365, "11/03"}, {15 / 365, "16/03"}, {20 / 365, "21/03"}}, None}}],
     ListPlot[{DataBefore, DataAfter}],
     Graphics[{Black, Dashed, Thick,
       Line[{{Emergencia, ymin}, {Emergencia, ymax}}]}],
     Graphics[{Red, Line[{{PeakTimingBaseline / 365, ymin},
          {PeakTimingBaseline / 365, PeakBaseline}}]}], Graphics[{Red,
       Line[{{tmin, PeakBaseline}, {PeakTimingBaseline / 365, PeakBaseline}}]}],
     Graphics[Text[StyleForm["O estado\nde emergência", FontSize → 17,
        Bold, FontColor → Black], {20 / 365, 400}]]], {i, 1, Length[vars]}][[1]]

ymax = 3000;
ymin = 1;
tmax = 24 / 365;
tmin = tstart - 1 / 365;

fig2 =
 Table[Show[LogPlot[{Evaluate[IQ[t] /. solution[1, Parameters]]}, {t, tstart, tend},
      AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None, Filling → Axis,
      PlotRange → {{tmin, tmax}, {ymin, ymax}}, AxesOrigin → {0, 0}, Frame →
       {{True, False}, {True, False}}, FrameStyle → Directive[Black, 17, Bold],
      PlotStyle → {Thickness[0.01], RGBColor[217 / 255, 0, 0]},
      FillingStyle -> Directive[Opacity[0.125]],
      FrameLabel → {{"Número de casos", None}, {None, None}},
      PlotLabel → Style["Evolução diária do número de casos confirmados ativos",
        17, Black, Bold],
      FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"}, {5 / 365, "06/03"},
          {10 / 365, "11/03"}, {15 / 365, "16/03"}, {20 / 365, "21/03"}}, None}}],
     ListLogPlot[{DataBefore, DataAfter}],
     Graphics[{Black, Dashed, Thick,
       Line[{{Emergencia, 0}, {Emergencia, ymax}}]}],
     Graphics[{Red, Line[{{PeakTimingBaseline / 365, ymin},
          {PeakTimingBaseline / 365, PeakBaseline}}]}], Graphics[{Red,
       Line[{{tmin, PeakBaseline}, {PeakTimingBaseline / 365, PeakBaseline}}]}],
     Graphics[Text[StyleForm["O estado\nde emergência", FontSize → 17,
        Bold, FontColor → Black], {20 / 365, 400}]]], {i, 1, Length[vars]}][[1]]

ymax = 950 000;
ymin = -25 000;
tmax = 240 / 365;
tmin = tstart - 7 / 365;

fig3 = Table[Show[Plot[{Evaluate[IQ[t] /. solution[1, Parameters]],
       Evaluate[IQ[t] /. solution[0.25, Parameters]]}, {t, tstart, tend},
      AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None,
      Filling → Axis, PlotRange → {{tmin, tmax}, {ymin, ymax}},
      AxesOrigin → {0, 0}, Frame → {{True, False}, {True, False}},
      FrameStyle → Directive[Black, 17, Bold],
      PlotStyle → {{Thickness[0.01], RGBColor[217 / 255, 0, 0]},
        {Thickness[0.01], RGBColor[26 / 255, 94 / 255, 214 / 255]}},
      FillingStyle -> Directive[Opacity[0.125]],
      FrameLabel → {{"Número de casos", None}, {None, None}}, PlotLegends →
       Placed[{Table[Style[Row[{label}], Black, 17, "Text", Bold], {label,
           {"Sem distanciamento social", "Redução de contactos em 75%"}}]},
        {Scaled[{0, 0.75}], {-1.25, 0.6}}], PlotLabel →
```

```
          Style["Evolução diária do número de casos confirmados ativos",
           17, Black, Bold],
         FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"},
              {PeakTimingBaseline / 365, LabelBaseline},
              {Emergencia, "18/03"}, {PeakTiming75 / 365, Label75}}, None}}],
       ListPlot[{DataBefore, DataAfter}],
       Graphics[{Black, Dashed, Thick,
         Line[{{Emergencia, ymin}, {Emergencia, ymax}}]}],
       Graphics[{RGBColor[217 / 255, 0, 0], Line[{{PeakTimingBaseline / 365, ymin},
            {PeakTimingBaseline / 365, PeakBaseline}}]}],
       Graphics[{RGBColor[217 / 255, 0, 0], Line[{{tmin, PeakBaseline},
            {PeakTimingBaseline / 365, PeakBaseline}}]}],
       Graphics[{RGBColor[26 / 255, 94 / 255, 214 / 255],
         Line[{{PeakTiming75 / 365, ymin}, {PeakTiming75 / 365, Peak75}}]}],
       Graphics[{RGBColor[26 / 255, 94 / 255, 214 / 255], Line[{{tmin, Peak75},
            {PeakTiming75 / 365, Peak75}}]}]], {i, 1, Length[vars]}][[1]]

ymax = 6500;
ymin = -150;
tmax = 140 / 365;
tmin = tstart - 4 / 365;

fig4 =
  Table[Show[Plot[{Evaluate[IQ[t] /. solution[0.1, Parameters]]}, {t, tstart, tend},
       AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None, Filling → Axis,
       PlotRange → {{tmin, tmax}, {ymin, ymax}}, AxesOrigin → {0, 0}, Frame →
        {{True, False}, {True, False}}, FrameStyle → Directive[Black, 17, Bold],
       PlotStyle → {{Thickness[0.01], RGBColor[28 / 255, 162 / 255, 0]}},
       FillingStyle -> Directive[Opacity[0.125]],
       FrameLabel →  {{"Número de casos", None}, {None, None}},
       PlotLegends → Placed[{Table[Style[Row[{label}], Black, 17, "Text", Bold],
            {label, {"Redução de contactos em 90%"}}]},
         {Scaled[{0, 0.75}], {-1.25, 0.6}}], PlotLabel →
          Style["Evolução diária do número de casos confirmados ativos",
           17, Black, Bold],
         FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"},
              {PeakTiming90 / 365, Label90}, {PeakTimingBaseline / 365, LabelBaseline},
              {Emergencia, "18/03"}, {PeakTiming75 / 365, Label75}}, None}}],
       ListPlot[{DataBefore, DataAfter}],
       Graphics[{Black, Dashed, Thick,
         Line[{{Emergencia, ymin}, {Emergencia, ymax}}]}],
       Graphics[{RGBColor[28 / 255, 162 / 255, 0],
         Line[{{PeakTiming90 / 365, ymin}, {PeakTiming90 / 365, Peak90}}]}],
       Graphics[{RGBColor[28 / 255, 162 / 255, 0], Line[{{tmin, Peak90},
            {PeakTiming90 / 365, Peak90}}]}]], {i, 1, Length[vars]}][[1]]

Export[StringJoin[
   "//Users//LynxGAV//Documents//Work//CoronaPortugal//Figures//Figure1",
   ".pdf"], fig1];
Export[StringJoin[
   "//Users//LynxGAV//Documents//Work//CoronaPortugal//Figures//Figure2",
   ".pdf"], fig2];
Export[StringJoin[
   "//Users//LynxGAV//Documents//Work//CoronaPortugal//Figures//Figure3",
   ".pdf"], fig3];
Export[StringJoin[
   "//Users//LynxGAV//Documents//Work//CoronaPortugal//Figures//Figure4",
   ".pdf"], fig4];
```

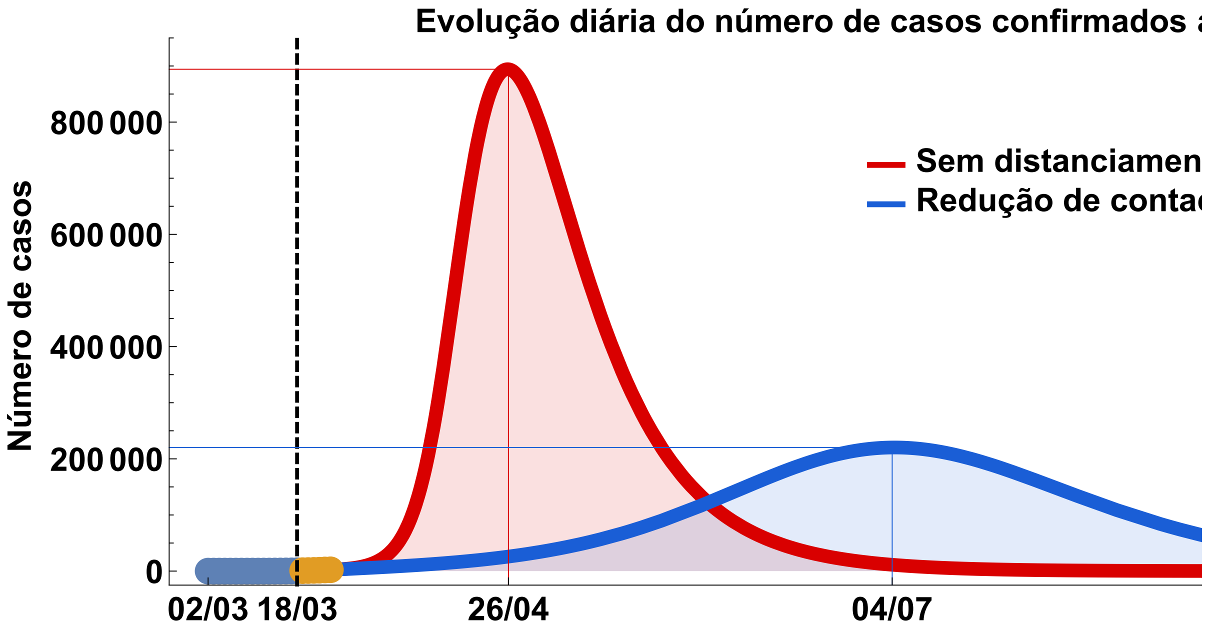
Out[1049]=



Out[1054]=



Out[1059]=



Out[1064]=