# A model to study the impact of government-imposed social distancing on COVID-19 epidemic in Portugal

## Clearing memory

```
ClearSystemCache[]
ClearAll["Global`*"]
Clear["Subscript"]
Clear["Superscript"]
Clear["Subsuperscript"]
```

## O estado de emergência: 18/03/2020, 17th day after 02/03/2020

```
Emergencia = 17 / 365;
Hoje = 23 / 365;
```

## Model equations

eq[Reduction_][1] := S'[t] ==

$$-S[t]\,\frac{\beta\,(\sigma\,IM[t] + IS[t])\ \text{If}[t \le \text{Emergencia}, 1, \text{If}[t \le \text{Hoje}, 0.65, \text{Reduction}]]}{(NN[t] - IQ[t])}$$

eq[Reduction_][2] := EE'[t] == S[t]

$$\frac{\beta\,(\sigma\,IM[t] + IS[t])\ \text{If}[t \le \text{Emergencia}, 1, \text{If}[t \le \text{Hoje}, 0.65, \text{Reduction}]]}{(NN[t] - IQ[t])} - \alpha\,EE[t]$$

eq[Reduction_][3] := IM'[t] == $p\,\alpha\,EE[t] - \gamma_1\,IM[t]$

eq[Reduction_][4] := IS'[t] == $(1 - p)\,\alpha\,EE[t] - \nu\,IS[t]$

eq[Reduction_][5] := IQ'[t] == $\nu\,IS[t] - \gamma_2\,IQ[t] - \eta\,IQ[t]$

eq[Reduction_][6] := RM'[t] == $\gamma_1\,IM[t]$

eq[Reduction_][7] := RS'[t] == $\gamma_2\,IQ[t]$

eq[Reduction_][8] := DD'[t] == $\eta\,IQ[t]$

# Numer of variables in the model (including deceased individuals)

```
numvar = 8
eqs[Reduction_] := Table[eq[Reduction][i], {i, 1, numvar}]
lhs[Reduction_] := eqs[Reduction][[All, 1]];
rhs[Reduction_] := eqs[Reduction][[All, 2]];
TableForm[eqs[Reduction]]
```

8

$$S'[t] == -\frac{\beta\, \mathrm{If}\left[t \le \frac{17}{365}, 1, \mathrm{If}[t \le \mathrm{Hoje}, 0.65, \mathrm{Reduction}]\right] (\sigma\, \mathrm{IM}[t] + \mathrm{IS}[t])\, S[t]}{-\mathrm{IQ}[t] + \mathrm{NN}[t]}$$

$$\mathrm{EE}'[t] == -\alpha\, \mathrm{EE}[t] + \frac{\beta\, \mathrm{If}\left[t \le \frac{17}{365}, 1, \mathrm{If}[t \le \mathrm{Hoje}, 0.65, \mathrm{Reduction}]\right] (\sigma\, \mathrm{IM}[t] + \mathrm{IS}[t])\, S[t]}{-\mathrm{IQ}[t] + \mathrm{NN}[t]}$$

$$\mathrm{IM}'[t] == p\, \alpha\, \mathrm{EE}[t] - \mathrm{IM}[t]\, \gamma_1$$

$$\mathrm{IS}'[t] == (1 - p)\, \alpha\, \mathrm{EE}[t] - \nu\, \mathrm{IS}[t]$$

$$\mathrm{IQ}'[t] == -\eta\, \mathrm{IQ}[t] + \nu\, \mathrm{IS}[t] - \mathrm{IQ}[t]\, \gamma_2$$

$$\mathrm{RM}'[t] == \mathrm{IM}[t]\, \gamma_1$$

$$\mathrm{RS}'[t] == \mathrm{IQ}[t]\, \gamma_2$$

$$\mathrm{DD}'[t] == \eta\, \mathrm{IQ}[t]$$

# Model variables

```
vars = {S[t], EE[t], IM[t], IS[t], IQ[t], RM[t], RS[t], DD[t]}
```

{S[t], EE[t], IM[t], IS[t], IQ[t], RM[t], RS[t], DD[t]}

# Total population size N(t) is not constant due to disease-related mortality

```
NN[t] = S[t] + EE[t] + IM[t] + IS[t] + IQ[t] + RM[t] + RS[t]
```

EE[t] + IM[t] + IQ[t] + IS[t] + RM[t] + RS[t] + S[t]

# Epidemiological parameters of the model

## Average contact rate (unique persons), 1/year

```
AverageContactRate = c → 13.74 × 365
```

c → 5015.1

## Relative infectivity of mildly infected

```
RelativeInfectivity = σ → 0.5
```

σ → 0.5

## 1/latent period, 1/year

```
RateInfectiousnessOnset = α → 365 / 4
```

$\alpha \to \frac{365}{4}$

## Proportion of mildly infected

```
ProportionMildSymptoms = p → 0.8
```

p → 0.8

## 1/recovery period of mildly infected, 1/year

```
RecoveryRateMildSymptoms = γ1 → 365 / 7
```

$\gamma_1 \to \frac{365}{7}$

## 1/delay from onset of infectiousness to diagnosis for individuals with severe symptoms, 1/year

```
DiagnosisRate = ν → 365 / 5
```

ν → 73

## 1/delay from diagnosis to recovery for diagnosed unaware, 1/year

```
RecoveryRateSevereSymptoms = γ2 → 365 / 14
```

$\gamma_2 \to \frac{365}{14}$

## Case fatality rate of unaware diagnosed

```
FatalityRate = f → 0.013
```

f → 0.013

## Disease-associated death rate of unaware diagnosed, 1/year

```
DeathRateDiagnosed = η → γ2 f / (1 - f) /. {RecoveryRateSevereSymptoms, FatalityRate}
```

η → 0.343393

## Basic reproduction number

```
BasicReproductionNumber = R0 → 6.25
```

$R_0 \to 6.25$

### Probability of transmission per contact with infectious with severe symptoms

$$\text{TransmissionProbability} = \text{Solve}\left[R_0 == \frac{p\,\beta\,\sigma}{\gamma_1} + \frac{(1-p)\,\beta}{\nu}\ /.\ \beta \to c\,\epsilon,\ \epsilon\right][[1,1]]\ /.$$
$$\{\text{ProportionMildSymptoms, AverageContactRate, RelativeInfectivity, RecoveryRateMildSymptoms, DiagnosisRate, BasicReproductionNumber}\}$$

$\epsilon \to 0.119704$

### Transmission rate of infection via contact with infectious with severe symptoms, 1/year

```
TransmissionRate = β → c ϵ /. {AverageContactRate, TransmissionProbability}
```

$\beta \to 600.329$

## Parameters of the model

```
Parameters := {AverageContactRate, RelativeInfectivity, RateInfectiousnessOnset,
  ProportionMildSymptoms, RecoveryRateMildSymptoms, DiagnosisRate,
  RecoveryRateSevereSymptoms, FatalityRate, DeathRateDiagnosed,
  BasicReproductionNumber, TransmissionProbability, TransmissionRate}
```

## Critical value for contact rate reduction where Reff < 1

$$\left(1 - 1\Big/\left(\frac{p\,\beta\,\sigma}{\gamma_1} + \frac{(1-p)\,\beta}{\nu}\right)\ /.\ \text{Parameters}\right)\ 100$$

84.

## Solving differential equations

### Start time, year
The first day of the simulation (initial condition) corresponds to 02/03/2020, when 2 COVID-19 cases were confirmed

$t_{start} = 1/365$

$\frac{1}{365}$

### End time, year
The simulation runs for 365 days

$t_{end} = 365/365;$

### Total population size at the beginning of an outbreak

$\text{Ntot} = 10.2 \times 10^6$

$1.02 \times 10^7$

### Initial number of infected individuals

```
InfInit = 12
```

12

### Initial conditions

```
ics = Table[ic[i], {i, 1, numvar}];

ic[1] = (Ntot - 13.74 InfInit - InfInit - 2) == vars[[1]] /. {t → tstart}
ic[2] = 13.74 InfInit == vars[[2]] /. {t → tstart}
ic[3] = 0.8 InfInit == vars[[3]] /. {t → tstart}
ic[4] = 0.2 InfInit == vars[[4]] /. {t → tstart}
ic[5] = 2 == vars[[5]] /. {t → tstart}
ic[6] = 0 == vars[[6]] /. {t → tstart}
ic[7] = 0 == vars[[7]] /. {t → tstart}
ic[8] = 0 == vars[[8]] /. {t → tstart}
```

$1.01998\times10^{7} == S\left[\frac{1}{365}\right]$

$164.88 == EE\left[\frac{1}{365}\right]$

$9.6 == IM\left[\frac{1}{365}\right]$

$2.4 == IS\left[\frac{1}{365}\right]$

$2 == IQ\left[\frac{1}{365}\right]$

$0 == RM\left[\frac{1}{365}\right]$

$0 == RS\left[\frac{1}{365}\right]$

$0 == DD\left[\frac{1}{365}\right]$

### Solution

```
solution[Reduction_, Parameters_] :=
  NDSolve[Join[eqs[Reduction], ics] /. Parameters, vars, {t, tstart, tend}];
```

# Computing peak number of confirmed cases

```
Peak[Reduction_, Parameters_] :=
 Max[Flatten[Table[Evaluate[IQ[t] /. First@solution[Reduction, Parameters]],
    {t, tstart, tend, 1 / 365}]]]
PeakBaseline = Peak[1, Parameters]
Peak75 = Peak[0.25, Parameters]
Peak90 = Peak[0.1, Parameters]
Peak90 0.3
```

887 816.

```
200 044.

7275.95

2182.79
```

# Computing time until the peak number of confirmed cases since 02/03/2020 (days)

```
PeakTiming[Reduction_, Parameters_] := Ordering[
   Flatten[Table[Evaluate[IQ[t] /. First@solution[Reduction, Parameters]],
      {t, tstart, tend, 1 / 365}]], -1][[1]]
PeakTimingBaseline = PeakTiming[1, Parameters]
PeakTiming75 = PeakTiming[0.25, Parameters]
PeakTiming90 = PeakTiming[0.1, Parameters]
```

```
55

124

39
```

# Data for confirmed cases - deaths - recoveries Data is split in 02/03/2020-18/03/2020 and 19/03/2020-now Source https://covid19.min-saude.pt/ponto-de-situacao-atual-em-portugal/

```
DataBefore =  {{1 / 365, (2 - 0)}, {2 / 365, (4 - 0)}, {3 / 365, (6 - 0)},
    {4 / 365, (9 - 0)}, {5 / 365, (13 - 0)}, {6 / 365, (21 - 0)}, {7 / 365, (30 - 0)},
    {8 / 365, (39 - 0)}, {9 / 365, (41 - 0)}, {10 / 365, (59 - 0)}, {11 / 365, (78 - 0)},
    {12 / 365, (112 - 0)}, {13 / 365, (169 - 1)}, {14 / 365, (245 - 2)},
    {15 / 365, (331 - 3)}, {16 / 365, (448 - 3 - 1)}, {17 / 365, (642 - 3 - 2)}};
DataAfter = {{18 / 365, (785 - 3 - 3)}, {19 / 365, (1020 - 5 - 6)},
    {20 / 365, (1280 - 5 - 12)}, {21 / 365, (1600 - 5 - 14)},
    {22 / 365, (2060 - 14 - 23)}, {23 / 365, (2362 - 30 - 22)}};
```

# Plotting numero de casos confirmados

```
ymax = 2500;
ymin = -60;
tmax = 24 / 365;
tmin = tstart - 1 / 365;
LabelBaseline = "26/04";
Label75 = "04/07";
Label90 = "10/04";
```

```
fig1 =
 Table[Show[Plot[{Evaluate[IQ[t] /. solution[1, Parameters]]}, {t, tstart, tend},
     AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None, Filling → Axis,
     PlotRange → {{tmin, tmax}, {ymin, ymax}}, AxesOrigin → {0, 0}, Frame →
       {{True, False}, {True, False}}, FrameStyle → Directive[Black, 17, Bold],
     PlotStyle → {Thickness[0.01], RGBColor[217 / 255, 0, 0]},
```

```
      FillingStyle -> Directive[Opacity[0.125]],
      FrameLabel → {{"Número de casos", None}, {None, None}},
      PlotLabel → Style["Evolução diária do número de casos confirmados ativos",
        17, Black, Bold],
      FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"}, {5 / 365, "06/03"},
          {10 / 365, "11/03"}, {15 / 365, "16/03"}, {20 / 365, "21/03"}}, None}}],
     ListPlot[{DataBefore, DataAfter}],
     Graphics[{Black, Dashed, Thick,
       Line[{{Emergencia, ymin}, {Emergencia, ymax}}]}],
     Graphics[{Red, Line[{{PeakTimingBaseline / 365, ymin},
          {PeakTimingBaseline / 365, PeakBaseline}}]}], Graphics[{Red,
       Line[{{tmin, PeakBaseline}, {PeakTimingBaseline / 365, PeakBaseline}}]}],
     Graphics[Text[StyleForm["O estado\nde emergência", FontSize → 17,
        Bold, FontColor → Black], {20 / 365, 400}]]], {i, 1, Length[vars]}][[1]]

ymax = 3000;
ymin = 1;
tmax = 24 / 365;
tmin = tstart - 1 / 365;

fig2 =
 Table[Show[LogPlot[{Evaluate[IQ[t] /. solution[1, Parameters]]}, {t, tstart, tend},
      AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None, Filling → Axis,
      PlotRange → {{tmin, tmax}, {ymin, ymax}}, AxesOrigin → {0, 0}, Frame →
       {{True, False}, {True, False}}, FrameStyle → Directive[Black, 17, Bold],
      PlotStyle → {Thickness[0.01], RGBColor[217 / 255, 0, 0]},
      FillingStyle -> Directive[Opacity[0.125]],
      FrameLabel → {{"Número de casos", None}, {None, None}},
      PlotLabel → Style["Evolução diária do número de casos confirmados ativos",
        17, Black, Bold],
      FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"}, {5 / 365, "06/03"},
          {10 / 365, "11/03"}, {15 / 365, "16/03"}, {20 / 365, "21/03"}}, None}}],
     ListLogPlot[{DataBefore, DataAfter}],
     Graphics[{Black, Dashed, Thick,
       Line[{{Emergencia, 0}, {Emergencia, ymax}}]}],
     Graphics[{Red, Line[{{PeakTimingBaseline / 365, ymin},
          {PeakTimingBaseline / 365, PeakBaseline}}]}], Graphics[{Red,
       Line[{{tmin, PeakBaseline}, {PeakTimingBaseline / 365, PeakBaseline}}]}],
     Graphics[Text[StyleForm["O estado\nde emergência", FontSize → 17,
        Bold, FontColor → Black], {20 / 365, 400}]]], {i, 1, Length[vars]}][[1]]

ymax = 900 000;
ymin = -25 000;
tmax = 240 / 365;
tmin = tstart - 7 / 365;

fig3 = Table[Show[Plot[{Evaluate[IQ[t] /. solution[1, Parameters]],
       Evaluate[IQ[t] /. solution[0.25, Parameters]]}, {t, tstart, tend},
      AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None,
      Filling → Axis, PlotRange → {{tmin, tmax}, {ymin, ymax}},
      AxesOrigin → {0, 0}, Frame → {{True, False}, {True, False}},
      FrameStyle → Directive[Black, 17, Bold],
      PlotStyle → {{Thickness[0.01], RGBColor[217 / 255, 0, 0]},
        {Thickness[0.01], RGBColor[26 / 255, 94 / 255, 214 / 255]}},
      FillingStyle -> Directive[Opacity[0.125]],
      FrameLabel → {{"Número de casos", None}, {None, None}}, PlotLegends →
       Placed[{Table[Style[Row[{label}], Black, 17, "Text", Bold], {label,
           {"Sem distanciamento social", "Redução de contactos em 75%"}}]},
        {Scaled[{0, 0.75}], {-1.25, 0.6}}], PlotLabel →
```

```
    Style["Evolução diária do número de casos confirmados ativos",
     17, Black, Bold],
   FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"},
       {PeakTimingBaseline / 365, LabelBaseline},
       {Emergencia, "18/03"}, {PeakTiming75 / 365, Label75}}, None}}],
  ListPlot[{DataBefore, DataAfter}],
  Graphics[{Black, Dashed, Thick,
    Line[{{Emergencia, ymin}, {Emergencia, ymax}}]}],
  Graphics[{RGBColor[217 / 255, 0, 0], Line[{{PeakTimingBaseline / 365, ymin},
      {PeakTimingBaseline / 365, PeakBaseline}}]}],
  Graphics[{RGBColor[217 / 255, 0, 0], Line[{{tmin, PeakBaseline},
      {PeakTimingBaseline / 365, PeakBaseline}}]}],
  Graphics[{RGBColor[26 / 255, 94 / 255, 214 / 255],
    Line[{{PeakTiming75 / 365, ymin}, {PeakTiming75 / 365, Peak75}}]}],
  Graphics[{RGBColor[26 / 255, 94 / 255, 214 / 255], Line[{{tmin, Peak75},
      {PeakTiming75 / 365, Peak75}}]}]], {i, 1, Length[vars]}][[1]]

ymax = 8000;
ymin = -180;
tmax = 140 / 365;
tmin = t_start - 4 / 365;

fig4 =
 Table[Show[Plot[{Evaluate[IQ[t] /. solution[0.1, Parameters]]}, {t, t_start, t_end},
    AspectRatio → 0.4, ImageSize → 800, PlotRangePadding → None, Filling → Axis,
    PlotRange → {{tmin, tmax}, {ymin, ymax}}, AxesOrigin → {0, 0}, Frame →
     {{True, False}, {True, False}}, FrameStyle → Directive[Black, 17, Bold],
    PlotStyle → {{Thickness[0.01], RGBColor[28 / 255, 162 / 255, 0]}},
    FillingStyle -> Directive[Opacity[0.125]],
    FrameLabel → {{"Número de casos", None}, {None, None}},
    PlotLegends → Placed[{Table[Style[Row[{label}], Black, 17, "Text", Bold],
        {label, {"Redução de contactos em 90%"}}]},
      {Scaled[{0, 0.75}], {-1.25, 0.6}}], PlotLabel →
     Style["Evolução diária do número de casos confirmados ativos",
      17, Black, Bold],
    FrameTicks → {{Automatic, None}, {{{1 / 365, "02/03"},
        {PeakTiming90 / 365, Label90}, {PeakTimingBaseline / 365, LabelBaseline},
        {Emergencia, "18/03"}, {PeakTiming75 / 365, Label75}}, None}}],
   ListPlot[{DataBefore, DataAfter}],
   Graphics[{Black, Dashed, Thick,
     Line[{{Emergencia, ymin}, {Emergencia, ymax}}]}],
   Graphics[{RGBColor[28 / 255, 162 / 255, 0],
     Line[{{PeakTiming90 / 365, ymin}, {PeakTiming90 / 365, Peak90}}]}],
   Graphics[{RGBColor[28 / 255, 162 / 255, 0], Line[{{tmin, Peak90},
       {PeakTiming90 / 365, Peak90}}]}]], {i, 1, Length[vars]}][[1]]

Export[StringJoin[
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   ".pdf"], fig1];
Export[StringJoin[
   "//Users//LynxGAV//Documents//Work//CoronaPortugal//Figures//Figure2",
   ".pdf"], fig2];
Export[StringJoin[
   "//Users//LynxGAV//Documents//Work//CoronaPortugal//Figures//Figure3",
   ".pdf"], fig3];
Export[StringJoin[
   "//Users//LynxGAV//Documents//Work//CoronaPortugal//Figures//Figure4",
   ".pdf"], fig4];
```
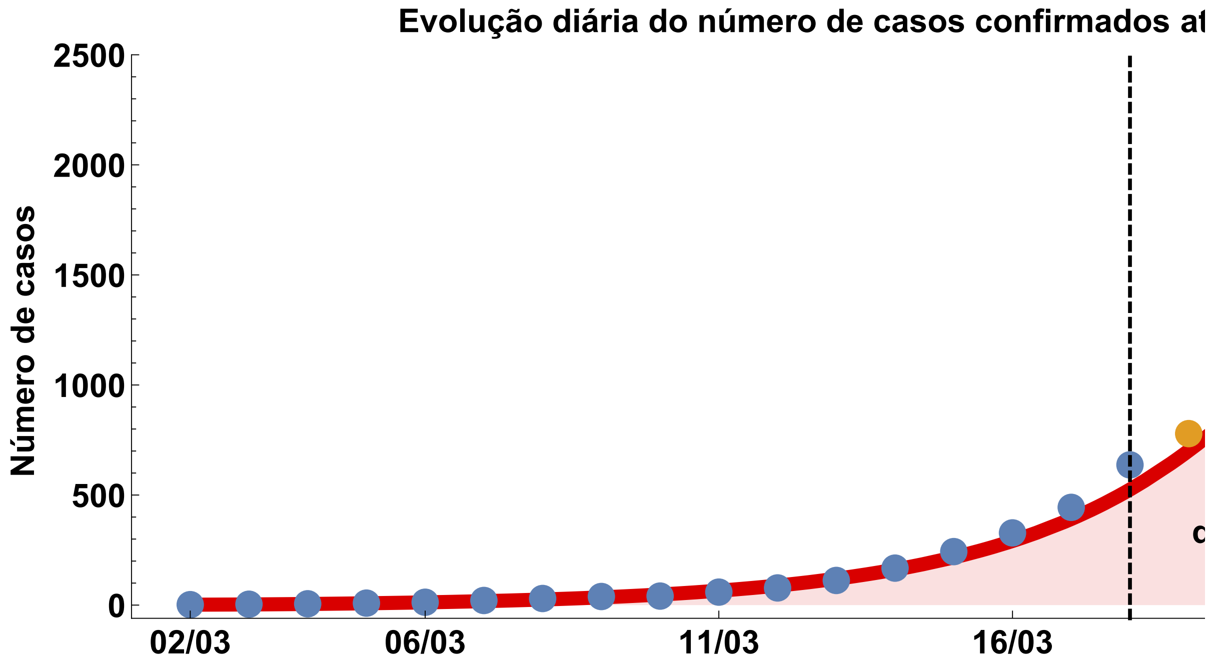
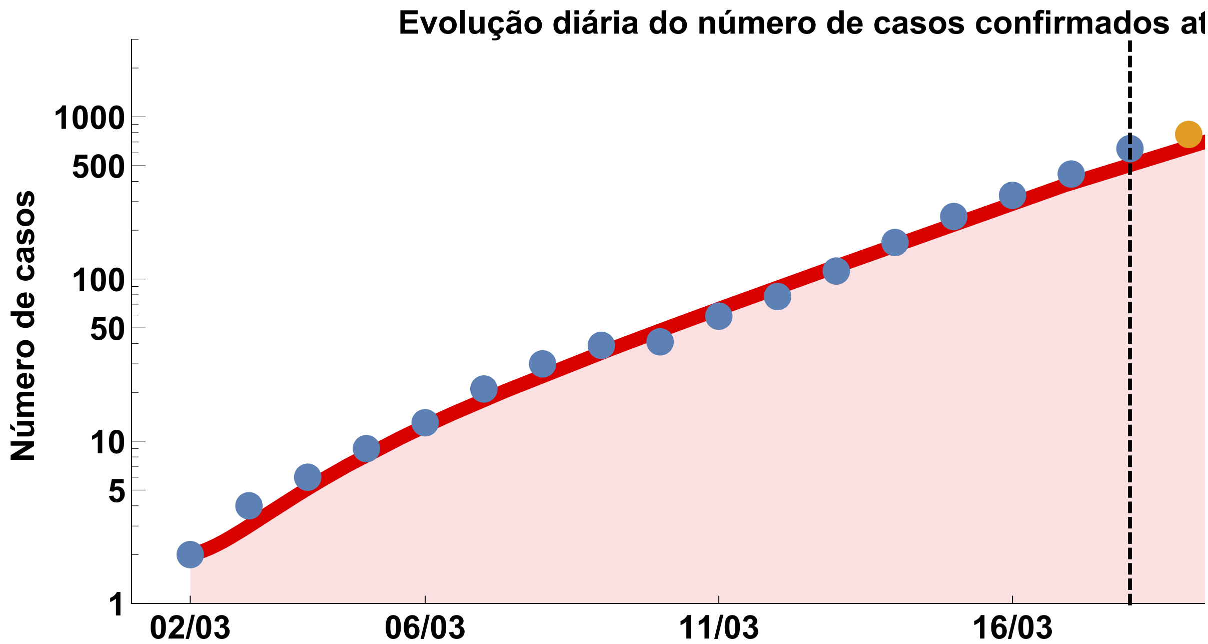
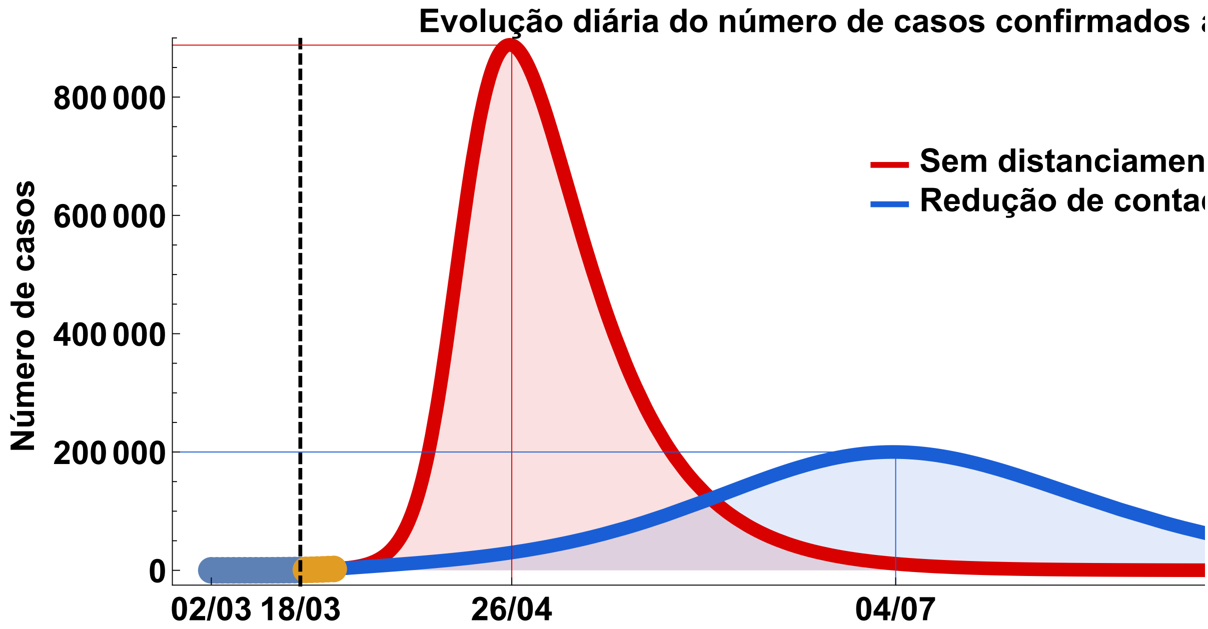
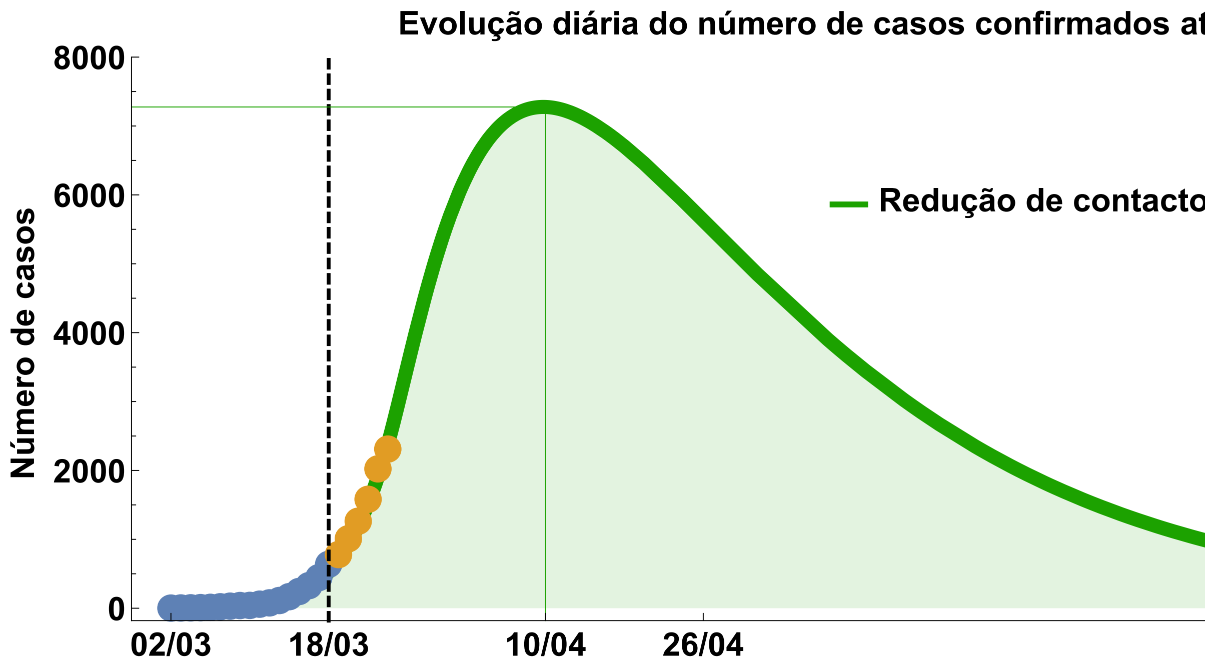


Evolução diária do número de casos confirmados at
Número de casos
2500
2000
1500
1000
500
0
02/03
06/03
11/03
16/03


Evolução diária do número de casos confirmados at
Número de casos
1000
500
100
50
10
5
1
02/03
06/03
11/03
16/03


Evolução diária do número de casos confirmados a
800 000
600 000
400 000
200 000
0
Número de casos
Sem distanciamen
Redução de conta
02/03 18/03
26/04
04/07


Evolução diária do número de casos confirmados at
8000
6000
4000
2000
0
Número de casos
Redução de contacto
02/03
18/03
10/04
26/04