NUMERICAL CALCULATION OF THE BASE INERTIAL PARAMETERS OF ROBOTS

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ABSTRACT

This paper presents a new approach to the problem of determining the minimum set of inertial parameters of robots. The calculation is based on numerical QR and SVD factorizations and on scaling procedure of matrices. It proceeds in two steps :
- at first the number of base parameters is determined,
- then a set of base parameters is determined by eliminating some standard parameters which are regrouped to some others in linear relations.

Different models, linear in the inertial parameters are used: a complete dynamic model, a simplified dynamic model and an energy model.
The method is general, it can be applied to open loop, or graph structured robots. The algorithms are easy to implement. An application for the PUMA 560 robot is given.

1-Introduction

Exact dynamic model of robot is required to control or simulate its motion. The model is characterized by ten inertial parameters per link, which are called the standard inertial parameters.
Because of the redundancy of this representation, there is an infinite sets of standard parameters which satisfies the dynamic model. In order to reduce the computational cost of the dynamic model and to facilitate the identification process, a minimum set of inertial parameters, which are called also base parameters, must be used to determine the dynamic model [1,...,6].
Dynamic model, using Newton Euler or Lagrange, or energy formulation model, linear in the inertial parameters, can be used to study this problem.
From a linear algebra point of view, this problem appears to be a rank deficiency problem.
The study is carried out in two steps:
- find the rank of a linear system, this gives the number of base parameters.
- choose the base parameters from the standard ones by eliminating some of them which are regrouped to the others in linear relations, in this step the regrouping relations will be determined.

Previous work:
Several authors have studied the problem using two principal approaches:
i- Symbolic approach:
- In [1, 2, 3, 4] a case by case method using the symbolic expression of the dynamic model have been presented,
- In [5, 6], we have presented a general and direct method to determine most of the set of minimum inertial parameters,of serial or tree structured robots, using the energy formulation.
- At the same time similar results concerning the special case of robots whose successive axes are perpendicular or parallel have been given by Mayeda et al.[7, 8].

ii- Numerical approach:
-Atkenson et all [9] used ridge regression and singular value decomposition, SVD, to solve the rank deficiency problem. The dynamic model using Newton Euler formulation was used.
- Sheu and Walker [10] used SVD also on the energy model.
These two methods did not give explicitely the minimum set of inertial parameters, nor the linear relations which define them.
In this paper we propose to use the QR decomposition, which serve equally well as the SVD and at less cost [11, p11-23], and the SVD itself to solve the deficiency problem. We find the number of base parameters and we give a method to define them. Then we give an algorithm to compute their numerical values from those of the standard parameters.
The method has been applied to 3 linear models:
Dynamic model, a simplified dynamic model, and an energy model.
An example treating the PUMA 560 robot type is given. The results are the same as given by our symbolic approach.

2- Dynamic model

2-1 Standard inertial parameters

The system to be considered is an open loop structure, simple or tree structure, mechanism. The description of the system will be carried out by the use of the modified Denavit and Hartenberg notation [12, 13]. The system is composed of n joints and n+1 links, link 0 is the base while link n is the end effector. A coordinate frame j is assigned fixed with respect to link j.
The ten standard inertial parameters of link j are composed of the elements of:
- ${}^jJ_j$ the inertia matrix of link j about the origin of frame j, referred to frame j,
- ${}^jMS_j$ the first moments of link j about the origin of frame j, referred to frame j,
- M_j the mass of link j.
Let:

$$ {}^jJ_j = \begin{bmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{bmatrix} \tag{1} $$

$$ {}^jMS_j = [\, MX_j \;\; MY_j \;\; MZ_j]^T \tag{2} $$

So the classical inertial parameters of link j will be represented by the vector ${}^jX_j$, where:

$$ {}^jX_j = [\, XX_j \;\; XY_j \;\; XZ_j \;\; YY_j \;\; YZ_j \;\; ZZ_j \;\; MX_j \;\; MY_j \;\; MZ_j \;\; M_j]^T \tag{3} $$

The inertial parameters of the robot will be represented by the vector **X** containing the inertial parameters of all the links. Thus the dimension of X, denoted by c, is equal to 10n.

$$ X = [\, {}^1X^{1T} \;\; {}^2X^{2T} \;\; \ldots \;\; {}^nX^{nT}]^T \tag{4} $$

2-2 Energy formulation of the dynamic model

It is known that the total energy H is linear in the inertial parameters.

$$H(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{h}\ \mathbf{X} = \sum h_i X_i \tag{5}$$

$\mathbf{q}$, $\dot{\mathbf{q}}$ are the nx1 vectors of joint positions and velocities,
H is the total energy, given as:
H = E + U, where :

$E(\mathbf{q},\dot{\mathbf{q}})$ is the kinetic energy,
$U(\mathbf{q})$ is the potential energy,

$\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ is a 1xc row vector.

Using a sequence of r samples $(\mathbf{q}, \dot{\mathbf{q}})_{(i)}$, i=1, ..., r, we define the matrix $\mathbf{H}$:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(1) \\ \dots \\ \mathbf{h}(r) \end{bmatrix} \tag{6}$$

2-3 Lagrangian formulation of the dynamic model

From the Lagrangian equation we obtain a dynamic model linear in the inertial parameters:

$$\Gamma = \mathbf{d}\ \mathbf{X} = \sum \mathbf{d}_{:,i} X_i \tag{7}$$

where:

$\mathbf{d}$ is a nxc matrix function of $\mathbf{q}$, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$ and the constant geometric parameters,
$\mathbf{d}_{:,i}$ is the ith column of $\mathbf{d}$,
Γ is the nx1 vector of joint torques.

Using a sequence of e samples $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})_{(i)}$, i=1, ..., e, we define the rxc matrix $\mathbf{D}$:

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}(1) \\ \dots \\ \mathbf{d}(e) \end{bmatrix} \tag{8}$$

We define also a simpler matrix $\mathbf{D}_0$, obtained from $\mathbf{D}$ by putting $\dot{\mathbf{q}} = \mathbf{0}$, thus:

$$\mathbf{D}_0 = \mathbf{D}(\mathbf{q}, \mathbf{0}, \ddot{\mathbf{q}}) \tag{9}$$

The study of the minimum inertial parameters can be carried out by studying the dependence of the symbolic expressions of the elements of the columns $\mathbf{d}_{:,i}$, [1], or of the functions h_i, [5, 6]. Numerically this is equivalent to study the space span by the columns of a matrix $\mathbf{W}$ which can be taken as $\mathbf{D}$ or $\mathbf{D}_0$, or $\mathbf{H}$. The use of $\mathbf{D}$ or $\mathbf{D}_0$, or $\mathbf{H}$ is mathematically equivalent, but this doesn't implies the numerical equivalence because of different scalings of these 3 matrices.
The proprieties to study are:
- the rank b of $\mathbf{W}$ which is the dimension of the space and the number of base parameters,
- the choice of c-b columns to be deleted and of b independent columns constituing a base of the space and which define the base parameters,
- the determination of c-b linear relations between the base columns and the deleted ones,
- the determination of the values of the b base parameters from those of the standard parameters.

We propose to use a QR and a SVD decomposition of $\mathbf{W}$ to solve these problems.

3- Base parameters by QR decomposition

3-1 QR decomposition

The rxc matrix $\mathbf{W}$ can be decomposed into the following form using QR decomposition[11,Sect. 9, 14,Sect.3]:

$$\mathbf{Q}^T \mathbf{W} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{10}$$

$\mathbf{Q}$ is a rxr orthogonal matrix:

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{11}$$

$\mathbf{I}$ is the identity matrix.
$\mathbf{R}$ is upper triangular.

3-2 Rank deficiency:QR with pivoting, QRΠ

If $\mathbf{W}$ is rank deficient (b<c) then $\mathbf{W}$ hasn't a unique QR decomposition. However, there is a permutation matrix Π (i.e an identity matrix with its columns permuted) such that $\mathbf{W}\,\Pi$ has a unique decomposition [11, 15 Sect.6-4]:

$$\mathbf{Q}^T \mathbf{W}\,\Pi = \begin{bmatrix} \mathbf{R1} & \mathbf{R2} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{12}$$

Where:
$\mathbf{Q}$ is orthogonal,
$\mathbf{R1}$ is a bxb upper triangular and regular matrix,
$\mathbf{R2}$ is a bx(c-b) matrix.
The strategy of pivoting is made to classify the diagonal elements $|R_{ii}|$ of $\mathbf{R1}$ in nonincreasing order.
The numerical rank is defined with a tolerance $\tau \neq 0$, because of roundoff errors[16 Sect.2].

$$\text{Rank}(\mathbf{W}) = b \quad \Leftrightarrow \quad |R_{ii}| \leq \tau, \text{ for } i=b+1, \dots, c \tag{13}$$

τ can be chosen as [11 p.9-25]:

$$\tau = r.\varepsilon.R_{11} \tag{14}$$

Where ε is the machine precision.
When the values R_{ii} are clustered in two groups, one near R_{11} and the other less than τ, there is no problem to determine the rank of $\mathbf{W}$. This always has been the case in our experiments.
QR with pivoting gives the rank of $\mathbf{W}$ which is equal to the number of base parameters.

3-3 Choice of the columns to be deleted

The choice of c-b columns to be deleted is not unique. As we did in our previous symbolic approach [5, 6], we choose to eliminate columns $\mathbf{W}_{:,j}$ with the larger subscript such that the parameters of link j will be regrouped on those of links j-1,...,0. The solution is very easy to find from a QR decomposition of $\mathbf{W}$ without pivoting. The c-b diagonal elements $|R_{ii}| \leq \tau$ give the subscripts i of the columns $\mathbf{W}_{:,i}$ to be deleted which are the subscripts of the standard parameters to be regrouped.
We deduce the subscripts of the b base parameters.

3-4 Linear relations between the columns of W

From the previous step we deduce a permutation matrix $\mathbf{P}$ such that:

$$\mathbf{P}^T \mathbf{X} = \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \end{bmatrix} \tag{15}$$

$\mathbf{X2}$ is a c-b vector of standard parameters to be regrouped on the b standard parameters of the bx1 vector $\mathbf{X1}$.
Permuting the columns of $\mathbf{W}$ we obtain:

$$\mathbf{W}\,\mathbf{P} = [\ \mathbf{W1}\ \ \mathbf{W2}\ ] \tag{16}$$

A QR decomposition of $\mathbf{W}\,\mathbf{P}$ gives :

$$[\ \mathbf{W1}\ \ \mathbf{W2}\ ] = [\ \mathbf{Q1}\ \ \mathbf{Q2}\ ] \begin{bmatrix} \mathbf{R1} & \mathbf{R2} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} = [\ \mathbf{Q1.R1}\ \ \mathbf{Q1.R2}\ ] \tag{17}$$

Where $\mathbf{R1}$ is a bxb regular matrix. Then it comes:

W1 = **Q1.R1** and **W2** = **Q1.R2**
We deduce the relation:

$$\mathbf{W2} = \mathbf{W1}\,\mathbf{R1}^{-1}\,\mathbf{R2} \qquad (18)$$

which expresses the c-b columns of **W2** as linear combinations of the b independent columns of **W1**.
The zero rows of $\mathbf{R1}^{-1}\,\mathbf{R2}$ correspond to the independent columns of **W P**.

3-5 Explicit relations for the base parameters

Let **XR** be any solution which assumes the invariancy of the model :

$$\mathbf{W\,X} = \mathbf{W\,P\,P^T X} = \mathbf{W\,P\,P^T XR}$$

with :

$$\mathbf{P^T X} = \begin{bmatrix} \mathbf{XR1} \\ \mathbf{XR2} \end{bmatrix}, \quad \mathbf{P^T X} = \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \end{bmatrix} \qquad (19)$$

Rewritting the invariancy :

$$[\ \mathbf{W1}\ \ \mathbf{W2}\ ] \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \end{bmatrix} = [\ \mathbf{W1}\ \ \mathbf{W2}\ ] \begin{bmatrix} \mathbf{XR1} \\ \mathbf{XR2} \end{bmatrix} \qquad (20$$

we obtain:

$$\mathbf{XR1} = \mathbf{X1} + \mathbf{R1}^{-1}\,\mathbf{R2}\,(\ \mathbf{X2} - \mathbf{XR2}\ )$$

There are infinity of values of **XR1**, depending on arbitrary values in **XR2**, which satisfy the equation (20).
A basis solution, named **XB**, corresponds to **XB2=0**, which eliminates the c-b columns of **W2**.
The final relation which gives the numerical values of the b base parameters **XB1** from those of the c standard parameters is given by:

$$\mathbf{P^T XB} = \begin{bmatrix} \mathbf{XB1} \\ \mathbf{XB2} \end{bmatrix} = \begin{bmatrix} \mathbf{X1} + \mathbf{R1}^{-1}\,\mathbf{R2}\,\mathbf{X2} \\ \mathbf{0} \end{bmatrix}$$

This choice permits to reduce the computational cost of the dynamic model using a customized method [1, 2, 3].

4- Base parameters using SVD decomposition

4-1 Singular value decomposition SVD

Another efficient method to solve rank deficiency is to use a SVD decomposition of **W**. It is known that a rxc matrix **W** can be factorized by the following expression [14, 15, 17]:

$$\mathbf{U^T W} = \mathbf{S\,V^T} \qquad (21)$$

$$\mathbf{S} = \begin{bmatrix} \Sigma \\ \mathbf{0} \end{bmatrix}$$

Where:
U is a rxr orthogonal matrix,
V is a cxc orthogonal matrix,
Σ is a cxc diagonal matrix whose elements σ_i are in nonincreasing order: $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_c \geq 0$.
The elements σ_i are the singular values of **W**.

4-2 Rank deficiency

The numerical rank of **W** can be deduced from the singular values of **W**.
If σ_{b+1} is less than a tolerance τ, then $\sigma_{b+1}, \ldots, \sigma_c$ are considered to be zero and rank(**W**) = b, with respect to τ.
τ can be chosen as[11 p.11-20]:
$\tau = r.\varepsilon.\sigma_1$

4-3 Choice of the columns to be deleted

When **W** is rank deficient, the SVD takes the form:

$$\mathbf{W}\ [\ \mathbf{V1}\ \ \mathbf{V2}\ ] = \mathbf{U} \begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad (22)$$

V1 is a cxb matrix, **V2** is a cx(c-b) matrix,
Σ is a bxb diagonal matrix, b=rank(**W**), b<c.
From (22), it comes:

$$\mathbf{W\,V2} = \mathbf{0} \qquad (23)$$

The columns of **V2** define the c-b linear relations between the columns of **W**.
Absolute independent columns of **W** correspond to zero rows of **V2**.
As in section 3-3 the problem is to choose c-b dependent columns $\mathbf{W}_{:,j}$ of largest subscript to be deleted.
From the expression (23), we have:

$$\mathbf{W\,X} = \mathbf{W}\,(\ \mathbf{X} + \mathbf{V2}\,\mathbf{X_a}\ ) \qquad (24)$$

We define:

$$\mathbf{XR} = \mathbf{X} + \mathbf{V2}\,\mathbf{X_a} \qquad (25)$$

where $\mathbf{X_a}$ is a (c-b)x1 arbitrary vector.
There are an infinity of values of **XR** that keep invariant the linear system: **W X** = **W XR**.
We are looking for a solution of the form (Eq. 19), (Eq. 20), given by the permutation matrix **P**. We define the permutation of rows of **V2** as :

$$\mathbf{P^T V2} = \begin{bmatrix} \mathbf{V21} \\ \mathbf{V22} \end{bmatrix}$$

V21 is a bxc-b matrix, **V22** is a c-bxc-b matrix .
From (Eq. 25), we obtain :

$$\mathbf{P^T XR} = \mathbf{P^T X} + \mathbf{P^T V2\,X_a}$$

$$\begin{bmatrix} \mathbf{XR1} \\ \mathbf{XR2} \end{bmatrix} = \begin{bmatrix} \mathbf{X1} \\ \mathbf{X2} \end{bmatrix} + \begin{bmatrix} \mathbf{V21} \\ \mathbf{V22} \end{bmatrix} \mathbf{X_a} \qquad (26)$$

A base solution **XB** is obtained with a vector $\mathbf{X_b}$, such that :

$$\mathbf{XB1} = \mathbf{X1} + \mathbf{V21}\,\mathbf{X_b}$$

$$\mathbf{XB2} = \mathbf{X2} + \mathbf{V22}\,\mathbf{X_b} = \mathbf{0} \qquad (27)$$

$\mathbf{X_b}$ is solution of the linear system :

$$\mathbf{V22}\,\mathbf{X_b} = -\,\mathbf{X2} \qquad (28)$$

P must be chosen for **V22** to be regular.
Starting from the last row of **V2** we extract the first regular (c-b)x(c-b) matrix, which gives the subscript of the columns $\mathbf{W}_{:,j}$ to be deleted and defines the matrix **P**.

4-4 Explicit relations for the base parameters

From (Eq. 27) and (Eq. 28), the numerical values of the base parameters are given by:

$$\mathbf{P^T XB} = \begin{bmatrix} \mathbf{XB1} \\ \mathbf{XB2} \end{bmatrix} = \begin{bmatrix} \mathbf{X1} - \mathbf{V11}\,\mathbf{V22}^{-1}\,\mathbf{X2} \\ \mathbf{0} \end{bmatrix} \qquad (29)$$

5- Application

5-1 A serial link manipulator

The PUMA 560 manipulator is chosen as an example of a 6 joints serial link manipulator, Figure 1.
The geometric parameters are given in table 1.
There are 60 standards inertial parameters.
The values of the matrices $\mathbf{d}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$, $\mathbf{d}_0(\mathbf{q},\ddot{\mathbf{q}})$, $\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})$ can be calculated directly numerically. We have preferred to make use of the symbolic expressions, automatically computed using the software package SYMORO[18].
There are 11 functions h_i which are constant and which correspond to 11 zero columns of **d** or $\mathbf{d_0}$.
These columns and the corresponding parameters are eliminated because they have no effect on the dynamic model.
So **W** has c=49 columns.

Table 1 : The geometric parameters of the PUMA Robot

j	σ_j	α_j	d_j	θ_j	r_j
1	0	0	0	θ_1	0
2	0	-90	0	θ_2	0
3	0	0	D3	θ_3	R3
4	0	-90	D4	θ_4	R4
5	0	90	0	θ_5	0
6	0	-90	0	θ_6	0

D3=0.4318m. R3=0.150m. D4=0.0203m. R4=0.4331m.

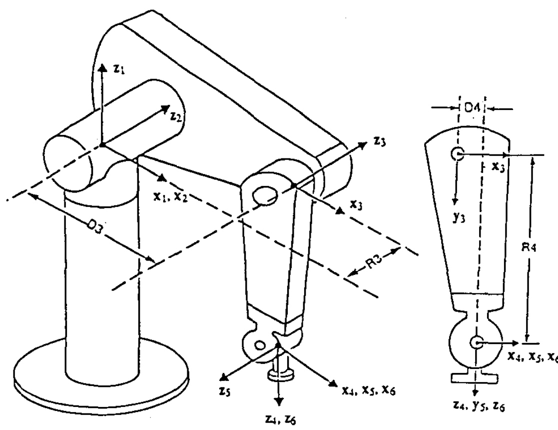

Figure 1: PUMA 560

5-1-1 Scaling of W

The sequence of samples $(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})_{(i)}$, i=1, ..., e or $(\mathbf{q},\dot{\mathbf{q}})_{(i)}$ i=1, ..., r can be generated by random sequences.
To study the rank deficiency we can choose r=c=49.
To avoid numerical difficulties during SVD or QR decomposition we propose to look for a sequence of samples which scales **W**.
This procedure is generally not necessary using double precision, but can be usefull to treat particular robots having a large number of parameters, as graph structured robots.
In our problem, the elements of **W** are computed to high accuracy with only the rounding errors. In this case the error in an element of **W** is proportionnal to its size[11 p.I-11], and the strategy reduces to scale **W** so that all its elements are roughly equal. The criterion is :

$$J = \lambda_1 \|\mathbf{W}\|_F + \lambda_2 S \quad , \quad S = |W_{ij}|_{max} / |W_{ij}|_{min} \tag{30}$$

$|W_{ij}|_{max}$ and $|W_{ij}|_{min}$ are the maximum and minimum absolute values of the elements W_{ij} of **W**, and $|W_{ij}|_{min} \neq 0$.
$\|.\|_F$ is the Frobenius norm :

$$\|\mathbf{W}\|_F = \Big(\sum_{i=1}^{r} \sum_{j=1}^{c} W_{ij}^2 \Big)^{1/2} \tag{31}$$

λ_1 and λ_2 are two weighting scalars.
The non linear optimization problem is characterized by a large number of degrees of freedom n_d. Using the matrix **H**, with r=c=49, we have $n_d = (2*6-1)*49 = 539$.
Also, symbolic expressions of the derivative of J cannot be obtained.
Owing to these constraints the numerical method developed by Powell [19] and implemented in the subroutine VA04 of the software package Harwell[20] is adequate for this problem.
It is a gradient conjugate type method which approaches the conjugate directions by an iterative procedure.
Because of the large number of variables the algorithm fails for the 49x49 matrix **H** of the PUMA.
An alternative is to find r sequences of optimum points for the scaling of the 1xc row matrix **h**. In fact, we just want a solution which gives an acceptable scaling value of **h** (around 10). Starting the optimization from r different random sequences for the n_d variables we obtain r acceptable scaled matrices **h** which are concatenated to give the 49x49 matrix **H** with S = 150.
Using a random sequence without scaling we obtain a matrix **H** with a value of S about 10^8.

5-1-2 Results

Results are given using the previous 49x49 scaled **H** matrix and the QR decomposition, but similar results were obtained without scaling. For this example, scaling is not necessary.
Results were obtained with large facilities using the software CTRL-C [21].
Table 2 gives the diagonal elements of the matrix **R** resulting from the QRΠ decomposition. It can be seen that the elements R_{ii} are clustered in 2 separate groups with $\tau = 4.8\ 10^{-14}$.
So, there is no doubt about the value of rank(**H**) which equals 36.
We give the diagonal elements of the matrix **R** of the QR decomposition without pivoting. The subscript of the 13 elements R_{ii} which equal zero give the subscript of the columns $\mathbf{H}_{:,j}$ to be deleted, and define the permutation matrix **P**.
For each zero rows i of the matrix $\mathbf{R1}^{-1}\ \mathbf{R2}$ corresponds an independent functions h_i. A fast and necessary, not sufficient, test can be done by looking for the zero elements of the (36x1) vector **K** with :
$\mathbf{K} = \mathbf{R1}^{-1}\ \mathbf{R2}\ \mathbf{N}$
with **N** is (13x1) vector whose elements equal to 1.
So if $K_i = 0$, the function h_i may be independent. Then we look for the elements of the corresponding row of $\mathbf{R1}^{-1}\ \mathbf{R2}$ to get the final result (for i=20 and i=30, K_i=0, but the corresponding rows of $\mathbf{R1}^{-1}\ \mathbf{R2} \neq \mathbf{0}$).
We obtain 19 functions h_i which are independent of all the others, they correspond to the standard parameters affecting the dynamic model separately.
Table 3 gives the values of the standard and base parameters. Standard values come from [22].
We conclude that there are 36 base parameters.
11 standard parameters are eliminated because h_i is constant,
13 standard parameters are eliminated by regrouping to 17 others,
19 standard parameters are not changed.
These results agree completely with those obtained by our symbolic method given in table 4[23].
The same results were obtained using SVD. Without numerical problem we prefer to use QR method which is simpler than SVD and at less cost. But, in fact, the 2 methods are very easy to implement using CTRL-C package.
Also, the same results were obtained using matrices **D** or $\mathbf{D_0}$. The comparison concerns the scaling. Using the same random sequence we found that $\mathbf{D_0}$ ($S \cong 10^4$) is the best, then **D** ($S \cong 10^6$) and at last **H** ($S \cong 10^8$). But using double precision, the difference is not so dramatic and we prefer to use **H**, eventualy with a scaling as given in section 5-1, because of the simplicity of the function h_i.

Table 2: Rank (H) and choice of base parameters

QRΠ Diag(R)	QR Diag(R)	i
-7.1529D+01	-5.6383D+01	
-5.7220D+01	-1.8031D+01	1.
-5.4435D+01	4.1409D+01	2.
-5.3032D+01	5.7633D+01	3.
-4.8755D+01	-1.6877D-15	4.
-4.7501D+01	-4.4843D+01	5.
4.5015D+01	-3.3658D+01	6.
-4.3872D+01	4.6765D+01	7.
-4.3829D+01	-4.3737D+01	8.
4.2782D+01	1.8216D+01	9.
4.1539D+01	-3.2816D+01	10.
3.8367D+01	-3.5722D+01	11.
3.6727D+01	-1.1102D-16	12.
-3.5891D+01	-3.8852D+01	13.
-3.4315D+01	-2.4451D+01	14.
3.1576D+01	-4.6242D+01	15.
3.0323D+01	-3.8904D+01	16.
-2.9363D+01	-2.4864D-16	17.
-2.8518D+01	0.0000D+00	18.
-2.7669D+01	1.6256D+01	19.
-2.6973D+01	3.1457D+01	20.
-2.6062D+01	-2.4668D+01	21.
2.4508D+01	1.3463D-15	22.
-1.9042D+01	-3.0834D+01	23.
1.8496D+01	1.5640D+01	24.
-1.4360D+01	-3.2085D+01	25.
1.3583D+01	2.8199D+01	26.
-1.2946D+01	2.5019D-15	27.
-1.2721D+01	1.1114D-15	28.
1.1508D+01	1.1302D+01	29.
-1.0466D+01	-2.0340D+01	30.
9.4172D+00	1.0190D+01	31.
8.6742D+00	1.6071D-15	32.
6.1021D+00	-2.0295D+01	33.
-4.2379D+00	5.1346D+00	34.
-2.7200D+00	2.0802D+01	35.
2.3997D-15	2.6457D+01	36.
-1.7909D-15	-1.5690D-15	37.
-1.4615D-15	5.4843D-16	38.
1.0795D-15	6.1112D+00	39.
7.4554D-16	7.7551D+00	40.
7.3586D-16	7.5668D+00	41.
-7.1778D-16	-3.8436D-16	42.
4.7353D-16	-4.7694D+00	43.
4.3449D-16	-1.0537D+00	44.
2.3416D-16	1.2450D+01	45.
-1.3111D-16	3.6854D+00	46.
-2.9195D-32	-3.9697D-16	48.
0.0000D+00	1.6649D-16	49.

Table 3: Standard and Base parameters

X_i	i	K	$P^T X$	$P^T XB$
ZZ1	1.	3.1370	1.4900	4.2397
XX2	2.	-1.7458	0.9722	-1.6073
XY2	3.	0.0000	0.0000	0.0000
XZ2	4.	-0.6909	-0.2680	-0.6888
YZ2	6.	0.0000	0.0000	0.0000
ZZ2	7.	0.7458	5.3343	6.4623
MX2	8.	1.7272	1.2180	3.8304
MY2	9.	0.0000	0.0000	0.0000
XX3	10.	1.4277	0.0905	0.2996
XY3	11.	-0.0467	0.0000	-0.0107
XZ3	12.	0.0000	0.0000	0.0000
YZ3	14.	0.0000	-0.0047	-0.0047
ZZ3	15.	2.4302	0.1095	0.3331
MX3	16.	0.0609	0.0000	0.0254
MY3	17.	2.2993	0.3360	0.8618
XX4	20.	0.0000	0.0021	0.0001
XY4	21.	0.0000	0.0000	0.0000
XZ4	22.	0.0000	0.0000	0.0000
YZ4	24.	0.0000	0.0000	0.0000
ZZ4	25.	1.0000	0.0013	0.0014
MX4	26.	0.0000	0.0000	0.0000
MY4	27.	-1.0000	0.0000	0.0000
XX5	30.	0.0000	0.0001	0.0002
XY5	31.	0.0000	0.0000	0.0000
XZ5	32.	0.0000	0.0000	0.0000
YZ5	34.	0.0000	0.0000	0.0000
ZZ5	35.	1.0000	0.0004	0.0006
MX5	36.	0.0000	0.0000	0.0000
MY5	37.	1.0000	0.0000	0.0029
XX6	40.	-1.0000	0.0002	0.0000
XY6	41.	0.0000	0.0000	0.0000
XZ6	42.	0.0000	0.0000	0.0000
YZ6	44.	0.0000	0.0000	0.0000
ZZ6	45.	0.0000	0.0000	0.0000
MX6	46.	0.0000	0.0000	0.0000
MY6	47.	0.0000	0.0000	0.0000
YY2	5.		1.4514	0.0000
YY3	13.		0.0134	0.0000
MZ3	18.		0.0672	0.0000
M3	19.		4.8000	0.0000
YY4	23.		0.0021	0.0000
MZ4	28.		-0.0156	0.0000
M4	29.		0.8200	0.0000
YY5	33.		0.0001	0.0000
MZ5	38.		0.0000	0.0000
M5	39.		0.3400	0.0000
YY6	43.		0.0002	0.0000
MZ6	48.		0.0029	0.0000
M6	49.		0.0900	0.0000

5-2 Application to graph structured robots

Our method is applicable to the analysis of graph-structured mechanisms with explicit constraint relations as those given by parallelogram loops.
In the equivalent opened tree structure robot, we define :

$$q = [\ q_a^T \ \ q_p^T\ ]^T \ , \ \ \dot{q} = \left[\ \dot{q}_a^T \ \ \dot{q}_p^T\ \right]^T$$

q_a, $\dot{q}_a$, are the position and velocity vectors of active joints. These variables are independent and can be chosen as random sequences.

q_p, $\dot{q}_p$, are the position and velocity vectors of passive joints. These variables are calculated using the explicit constraint equations :

$$q_p = F(q_a)$$

$$\dot{q}_p = G(q_a)\,\dot{q}_a$$

$G(q_a)$ is the jacobian matrix of **F**.
This method has been applied to the 5 degrees of freedom HITACHI-HPR robot. Details for the description of this robot can be found in [24].
We have :

$$q_a = [\ \theta_1 \ \ \theta_3 \ \ \theta_4 \ \ \theta_6 \ \ \theta_7\ ]^T \ , \ \ q_p = [\ \theta_2 \ \ \theta_5\ ]^T$$

$$\begin{bmatrix} \theta_2 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} \theta_3-\theta_4 \\ 3\pi-\theta_4 \end{bmatrix}, \quad \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_5 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

The results completely agree with those obtained using the symbolic approach given in [24].

Table 4: Symbolic results

1-The parameters having no effect on the dynamic model are:
XX_1, XY_1, XZ_1, YY_1, YZ_1, MX_1, MY_1, MZ_1, M_1, MZ_2, M_2.
2-The parameters to be eliminated owing to the regrouping are:
YY_2, YY_3, M_3, MZ_3, YY_4, MZ_4, M_4, YY_5, M_5, MZ_5, YY_6, MZ_6, M_6 .
3- The regrouping relations are:

$ZZR_1 = ZZ_1 + YY_2 + YY_3 + 2\, R3\, MZ_3 + (R3^2+D3^2+D4^2)(M_4 + M_5 + M_6) + (R3^2+D3^2)M_3$

$XXR_2 = XX_2 - YY_2 - D3^2 (M_3 + M_4 + M_5 + M_6)$

$XZR_2 = XZ_2 - D3\, MZ_3 - R3D3(M_3 + M_4 + M_5 + M_6)$

$ZZR_2 = ZZ_2 + D3^2 (M_3 + M_4 + M_5 + M_6)$

$MXR_2 = MX_2 + D3\ (M_3 + M_4 + M_5 + M_6)$

$XXR_3 = XX_3 + YY_4 + 2R4\, MZ_4 + (R4^2-D4^2)(M_4 + M_5 + M_6) - YY_3$

$XYR_3 = XY_3 - D4\, MZ_4 - R4D4\, (M_4 + M_5 + M_6)$

$ZZR_3 = ZZ_3 + YY_4 + 2\, R4\, MZ_4 + (R4^2+D4^2)(M_4 + M_5 + M_6)$

$MXR_3 = MX_3 + D4\, (M_4 + M_5 + M_6)$

$MYR_3 = MY_3 + MZ_4 + R4\, (M_4 + M_5 + M_6)$

$XXR_4 = XX_4 + YY_5 - YY_4$

$ZZR_4 = ZZ_4 + YY_5$

$MYR_4 = MY_4 - MZ_5$

$XXR_5 = XX_5 + YY_6 - YY_5$

$ZZR_5 = ZZ_5 + YY_6$

$MYR_5 = MY_5 + MZ_6$

$XXR_6 = XX6 - YY6$

6- Conclusion

Two numerical methods are given to determine the set of base parameters of dynamic model of robots.

Starting from the standard inertial parameters it gives the number and also the choice and the numerical values of the base parameters.

The methods are based on QR or SVD decompositions.

The numerical method is universal, it can be applied to open loop, or graph structured robots.

It is very easy to implement using commercial software packages as CTRL-C for the numerical tools, and software SYMORO to obtain the symbolic expressions of h_i.

The only limitations could be from numerical problems.

We propose a scaling strategy to avoid these problems.

All the results agree with those of the symbolic method whose great advantages remain its insensitivity to numerical problems and the insight in physical meaning of regroupements.

Our final conclusion is to apply at first the symbolic general relations as given in [5, 6, 23, 24], to obtain most of the regrouped inertial parameters, and then to use the numerical algorithm on this reduced model to confirm the result or to find eventually supplementary regrouping relations.

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