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Dynamic Parameter Identification for the CRS A460 Robot

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Abstract— **Dynamic Parameter Identification is a useful tool for developing and evaluating robot control strategies. However, a multi degree of freedom robot arm has many parameters, and the process of determining them is challenging. Much research has been done in this area and experimental methods have been applied on several robot arms. To our knowledge, there is currently no set of inertial parameters, either by modelling or by estimation, available for the CRS A460/A465 arm, a popular laboratory table top robot. In this paper we review and compare a number of methods for dynamic parameter identification and for generating trajectories suitable for estimating the identifiable dynamic parameters of a given robot. We then present a step by step process for dynamic parameter identification of a serial manipulator, and demonstrate this process by experimentally identifying the dynamic parameters of the CRS A460 robot.**

I. Introduction

This paper describes the process, methodology and results for the identification of the dynamic parameters of the CRS A460 robot, a typical laboratory scale robot with a payload of 1kg, by applying a direct procedure [1]. The inertial parameters of manipulator loads and links mass, center of mass and movements of inertia are required in order to design model based controllers for high speed, accurate, robot motion and force interaction.

Robot manufacturers typically do not release such information and may not have complete information at hand. Since the PID controllers which are provided by the manufactures do not take link dynamics into account, there is also no inducement for them to determine these parameters.

Estimating the parameters by disassembling the robot and weighing and balancing the components is complex and time consuming. Another method would be to enter a computer model of the arm into a CAD/CAM database, but the accuracy of these models is not clear and would likely require at least some disassembly of the robot to produce an accurate model. Thus, dynamic parameter identification methods have gained importance for developing model based controllers.

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Some link inertial parameters may not be identifiable due to the manipulator's particular geometry. Reasons for unidentifiable parameters include restricted motion near the base and the lack of full force-torque sensing at each joint.

In order to identify the dynamic parameters of a robot arm the dynamic robot model is formulated as:

$$\tau = \phi(q, \dot{q}, \ddot{q})\theta, \tag{1}$$

where τ is the torque, $\phi(q, \dot{q}, \ddot{q})$ represents a nxr regressor matrix. The regressor matrix $\phi(q, \dot{q}, \ddot{q})$ depends on the joint angles, velocities, and accelerations. The dimension r indicates the number of robot parameters and n is the number of degrees of freedom of the robot. The r-vector, θ, contains the unknown inertial parameters. In order to determine the inertial parameters, a simple least-squares (LS) method can be applied. However, the regressor matrix ϕ retrieved from the kinematic calculation of the manipulator arm leads to a non-invertible matrix-product $\phi^T * \phi$ since it is not full rank.

Khosla [2] proposes a parameter categorization technique (also "robot model reduction" [3], [4]). By categorizing dynamic parameters, a minimum set of parameters affecting the equations of motion of an N-degrees-of-freedom (DOF) manipulator can be determined. Furthermore, this set of parameters is used to determine whether a given identification trajectory is persistently exciting, leading to a more robust estimation procedure. The maximum number of parameters to be estimated depends on the trajectory used and the kinematic structure of the manipulator.

The generation of trajectories that excite the robot dynamics is the main issue discussed by Armstrong in [3]. Analysis of two identification experiments described in [5] show that intuitively chosen trajectories are likely to provide poor excitation. Employing optimization to maximize excitation considerably improves the accuracy of the parameter estimates. In general, a good choice of trajectory results in good excitation and accurate estimates of the robot parameters. Parametric optimization in frequency domain was tried by Swevers *et al.* [4] with some success.

Swevers [4] claimed that existing excitation trajectory design do not consider the uncertainties of the measurements of the parameter estimates. Their approach differed from previous methods in the parametrization of the excitation trajectory and in the optimization criterion. The authors chose a finite fourier series for each joint to guarantee periodic excitation. Periodic excitation enables time-domain data averaging and estimation of the characteristics of the measurement noise which is useful for maximum-likelihood (ML) parameter estimation. Furthermore, the joint velocities and accelerations can be calculated in an analytic way from the measured position response. It also becomes possible to specify the bandwidth of the excitation trajectories. Instead of the often used condition number of the parameter estimation problem which only applies to a deterministic framework, the optimization criterion is the uncertainty on the estimated parameters or a lower bound for it. This second approach applies to the case at hand, namely a stochastic (errors in variables) framework. The authors show in simulations that, this criterion yields parameters estimates with smaller uncertainty bounds than trajectories optimized according to the classical criterion.

In the following section we will explain the methodology for dynamic parameter identification. Section 3 explains our implementation and Section 4 presents our experimental results. In Section 5 we discuss our approach and section 6 concludes the paper.

II. Methodology

Each rigid link is described by 10 inertial parameters: its mass, the position vector of its center of gravity with respect to the base frame scaled by the link mass, 3 inertia moments, and 3 inertia products. Thus, an n-link manipulator has $10n$ dynamic parameters to be estimated. The algorithm consists of the following several steps:

- Using the Newton-Euler formalism, generate a robot model that is linear in terms of the inertial parameters;
- reduce the inertial parameters set to a base set;
- determine the optimal trajectory parameters and optimize the excitation of the trajectories;
- estimate the link parameters using a standard least-squares procedure.

A. The Newton-Euler formulation

The Newton-Euler formulation computes the inverse dynamics in two sets of recursions: the forward and backward recursions. Considering the limited number of pages we do not present the Newton-Euler (NE) equations in full length. For the complete derivation of the dynamic robot model please see [6].

To achieve a linear formulation of the NE equations the inertias need to be expressed in their respective joint frame instead of their center of mass frame by using the parallel-axis theorem (Steiner's law) as mentioned in [5].

The actuating torque of every joint can then be described as follows:

$$\tau_i = \begin{pmatrix} (R_i^{i-1})^t (1-\sigma_i)\hat{k} \\ (R_i^{i-1})^t \sigma_i \hat{k} \end{pmatrix}^t \gamma_i. \tag{2}$$

where R_i^{i-1} is the orthogonal rotation matrix from link $i-1$ into link i, σ_i is 1 in case joint i is a rotational joint, else 0, and $\hat{k}$ is the standard unit vector for the z-axis. γ_i is the $6x1$ vector containing the force f_i and moment n_i exerted on link i by link $i-1$:

$$\gamma_i = \begin{pmatrix} f_i & n_i \end{pmatrix}^t = D_{i+1}^i \gamma_{i+1} + \Gamma_i, \tag{3}$$

where D_{i+1}^i is the $6x6$ *pseudo-rotation* matrix,

$$D_{i+1}^i = \begin{pmatrix} R_{i+1}^i & 0 \\ [p_i x] R_{i+1}^i & R_{i+1}^i \end{pmatrix}, \tag{4}$$

with $[p_i x]$ being the skew symmetric matrix of the vector p_i. Γ_i contains the net force and moment exerted on link i. The categorization of the dynamic parameters requires explicit expressions for the vector γ_i. Therefore the vector Γ_i is decomposed as the product $\Gamma_i = K_i * \theta_i$ of matrix K_i, which is a function of the kinematic parameters and the desired trajectory, and a vector θ_i whose elements are the dynamic parameters of the manipulator as described in [2].

Assuming that the vector of externally applied forces and moments is zero, with the above expansions the equation of γ_6 of a robot arm with 6 joints can be rewritten as:

$$\gamma_6 = \Gamma_6.$$

Implementing the backwards recursion for all remaining joints, we obtain the finalized linear model (1). Since, the regressor matrix for a 6 joint robot is a $6x60$ matrix, i.e., six equations and sixty unknowns, the torque of all joints must be sampled in at least 10 different manipulator configurations to solve for τ for all joints. Because of measurement noise more configurations are obviously even more desirable. The joint torques are inferred from the motor current. However, to infer the torques from the motor current a knowledge of the motor/torque constant is required. Since the motor constant of all joints might not always be known, experiments are required to experimentally determine this value, as described in Section 5.

The regressor matrix retrieved from this procedure is not invertible due to loss of rank from restricted degrees of freedom at the proximal links and linear dependencies between the columns of ϕ. Thus, in the following we describe a numerical procedure for the parameter categorization.

B. Model Reduction

The model reduction divides the inertial parameters into the three sets: identifiable, unidentifiable, and identifiable in linear combinations. To categorize the parameters the retrieved matrix-vector formulation of the dynamic robot model (1) is numerically analyzed, similarly to [7], in order to determine the linear dependencies of the columns of ϕ.

For this step let U, S, and D be the set of unidentifiable parameters, identifiable parameters, and parameters that are identifiable in linear combinations respectively. The final base set of identifiable parameters received from this procedure is called D_{BS}.

1) Choose N random sets of joint positions $q(k)$, velocities $\dot{q}$ and accelerations $\ddot{q}$ ($N \geq 10$).
2) Build for each of these sets k the matrix $\phi(q(k), \dot{q}(k), \ddot{q}(k))$ and combine them into one matrix

$$\phi_{tot} = \begin{pmatrix} \phi(q(1), \dot{q}(1), \ddot{q}(1)) \\ \phi(q(2), \dot{q}(2), \ddot{q}(2)) \\ \vdots \\ \phi(q(N), \dot{q}(N), \ddot{q}(N)) \end{pmatrix} = \begin{pmatrix} f_{c1} & f_{c2} & \cdots & f_{cr} \end{pmatrix}.$$

f_{ci} represents the i-th column of matrix ϕ_{tot}.

3) Calculate the rank r of ϕ_{tot}. The rank of ϕ_{tot} then determines the size of the base set D_{BS}, i.e. the number of linearly independent columns.
4) Eliminate the columns of ϕ_{tot} whose norms are equal to zero. The eliminated columns correspond to the set of unidentifiable parameters U.

Repeat step 5 for all columns f_{ci} of the reduced matrix ϕ_{tot} and their corresponding parameters θ_i.

5) Calculate the rank of the matrix ϕ_{tot} without the i-th column f_{ci} (rwi). If rwi is less than the rank r calculated above, then add θ_i to the set of identifiable parameters S. Furthermore, add θ_i to the base set D_{BS} and to the matrix L. Matrix L contains the linearly independent columns related to the parameters already present in subset D_{BS}. Else, check the linearity between f_{ci} and the rest columns as follows:
   - Calculate the rank of matrix L. Add f_{ci} to the matrix L.
   - If f_{ci} increases the rank of L, then add θ_i to D_{BS}.
   - Else, eliminate column f_{ci} from both matrix L and ϕ_{tot}. Add θ_i to the set of identifiable parameters in linear combinations D. Calculate the linear dependency between f_{ci} and the columns of L:

$$\begin{aligned} f_{ci} &= L\alpha_i \\ \alpha_i &= (L^tL)^{-1}L^t f_{ci}. \end{aligned}$$

   Vector α_i determines the linear dependencies between f_{ci} and the columns of L. Let D_{BSV} be the vector containing the parameters in D_{BS}. Then θ_i can be represented by a combination of the parameters in D_{BSV}.

$$\theta_i = D^t_{BSV} * \alpha_i.$$

At the end of this procedure matrix L is equal to the final reduced matrix ϕ_{tot}. Equation (1) with the new reduced matrix ϕ_{tot} is the basis for the optimization of the robot excitation and the estimation of the dynamic parameters.

Since the categorization of parameters is a function of the trajectory, it is possible to change the category of a parameter by selecting a different trajectory. If the complete set of dynamic parameters has been determined then it is not possible to upgrade the category to which a parameter belongs, but as we have discussed earlier the reverse is not true. It is possible to degrade a parameter's category by a suitable choice of trajectory.

C. Parametrization of the Robot Trajectories

For excitation trajectories we use the finite Fourier series suggested in [4], i.e., finite sums of harmonic sine and cosine functions. The angular position q_i for the ith joint can thus be written as:

$$q_i(t) = \sum_{l=1}^{N_i}\left(\frac{a_l^i}{\omega_f l}\sin\omega_f lt - \frac{b_l^i}{\omega_f l}\cos\omega_f lt\right) + q_{i0}, \quad \forall i \in [1\ldots n],$$

where n is the number of joints and ω_f is the fundamental pulsation of the Fourier series and thus the series have a time period $T_f = 2\pi/\omega_f$. The motion of each joint i depends on $2N_i + 1$ parameters, a_l^i, and b_l^i for $l = 1$ to N_i, which are the amplitudes of the sine and cosine functions, and q_{i0} which is the offset on the position trajectory. In order to preserve the periodicity of the overall robot excitation, the trajectory frequency is common for all joints. As already mentioned, it is now possible to average the motion and torque data and estimate the variance of the noise on these data using the following formulas in [6]. Time-domain data averaging improves the signal-to-noise ratio of the experimental data which is important because motor current (torque) measurements are very noisy. Furthermore, joint velocities and accelerations can be calculated from the measured response in an analytic way.

D. Optimization of the Parameterized Robot Excitation Trajectories

Under the assumption the measured joint angles are free of noise, the covariance matrix of the estimated model parameters equals

$$(\phi^t_{tot}\Sigma^{-1}\phi_{tot})^{-1}. \tag{5}$$

The above expression depends on the exact joint angles, velocities and accelerations which correspond to the designed excitation trajectory. Thus, the optimization of the inertial parameter covariance matrix as a function of the trajectory parameters δ does not require the knowledge of the exact inertial parameter vector θ. Since the covariance matrix of the Maximum-Likelihood-Estimation converges asymptotically to the Cramér-Rao lower bound, the inverse of the *Fisher information matrix*, only the knowledge of the exact model parameter vector θ is required. However, an exact model parameter vector is not known. Initial experimental data, can be obtained from a robot excitation that has been optimized according to the condition number of matrix ϕ_{tot}. In the Implementation section we will describe our method to obtain a "good initial guess".

In an iterative procedure the Cramér-Rao lower bound is minimized as a function of the trajectory parameters δ, resulting in a new excitation trajectory. Robot excitation and parameter estimation can be repeated until convergence occurs. Since the covariance matrix or its Cramér-Rao lower bound can not be optimized in matrix sense, we use a representative scalar measure suggested by Ljung [8], the *d-optimality* criterion: $-\log\det M$, with M the covariance matrix or its Cramér-Rao lower bound. This scalar measure is beneficial because its minimum is independent of the scaling of the parameters and it also has a physical interpretation: the determinant of M is related to the volume of the highest probability density region for the parameters.

E. Link Estimation of the Robot parameters

Due to model reduction the regressor matrix is now invertible; i.e., we can proceed with a standard least-squares estimation procedure. The procedure can be used to average many data points and compensate for noise. The optimization of the parameterized robot excitation trajectories gave us the most optimal coefficients for the finite fourier series. With these parameters the optimal joint trajectories, respectively the regressor matrix can be calculated. As previously mentioned, at least 10 different manipulator configurations are necessary to solve for τ for all joints. Trajectories with more than 10 time instants will give better and more reliable results. The regressor matrices at time instants $t_1, \ldots, t_N$ are combined in one large matrix ϕ_{tot}. Equation (1) can thus be solved as follows:

$$\theta = (\phi_{tot}^t \Sigma^{-1} \phi_{tot})^{-1} \phi_{tot}^t \Sigma^{-1} \tau. \tag{6}$$

In the following section we describe the implementation for the link estimation.

III. IMPLEMENTATION

The identification procedure is implemented off-line since there is no need to perform these calculations online. The implementation contains several steps which are described in their appropriate order in the following. Since the problem we are looking at is an optimization problem, first it is necessary to make a good initial guess for the trajectory parameters. Our approach estimates the fourier series parameters and the robot parameters simultaneously by minimizing the global optimality criterion (d-optimality criterion). The model parameters θ are estimated from the data measured during a robot excitation experiment. The data are sequences of joint angles and motor currents from which a sequence of joint velocities, accelerations, and motor torques are calculated.

A. Initial trajectory parameters

A good initial guess of the trajectory parameters should not create joint trajectories that violate the physical robot limits. Here it should be noted that the initial trajectory, or respectively any trajectory should not cause the robot collide with itself or its environment.

In order to generate good initial values for the trajectory parameters we use a simple least-squares method. Since, for the CRS A460 robot, the joint velocity constraints (specified by the robot manufacturer) tend to be the limiting constraints on the allowable trajectories, the joint velocity constraint equation is taken as the basis equation for all joints. We reformulate the finite fourier series for the joint angular velocity, shown in Section 2, as a single matrix-vector equation:

$$A\delta_i = \dot{q}_i, \quad \forall i \in [1 \ldots P], \tag{7}$$

where:

$$\begin{aligned}
\dot{q}_i &= \begin{pmatrix} \dot{q}_i(1) & \cdots & \dot{q}_i(M) \end{pmatrix}^t \\
\delta_i &= \begin{pmatrix} a_1^i & b_1^i & \cdots & a_{N_i}^i & b_{N_i}^i & q_0^i \end{pmatrix}^t \\
A &= \begin{pmatrix} \frac{s(t_1,1)}{\omega_f} & \frac{c(t_1,1)}{\omega_f} & \cdots & \frac{s(t_1,N_i)}{\omega_f N_i} & \frac{c(t_1,N_i)}{\omega_f N_i} & 1 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ \frac{s(t_M,1)}{\omega_f} & \frac{c(t_M,1)}{\omega_f} & \cdots & \frac{s(t_M,N_i)}{\omega_f N_i} & \frac{c(t_M,N_i)}{\omega_f N_i} & 1 \end{pmatrix}
\end{aligned}$$

TABLE I

FOURIER SERIES PARAMETERS FOR THE TRAJECTORIES USED IN THE EXPERIMENTS

	Joint 1	Joint 2	Joint 3	Joint 4	Joint 5	Joint 6
a	0.156	0.064	0.634	-0.197	0.499	0.435
	-0.478	-0.335	-0.421	-0.282	-0.135	0.111
	0.078	0.451	0.216	0.173	-0.112	-0.245
	-0.388	0.292	-0.310	0.154	-0.263	-0.162
	-0.070	1.046	-0.357	-0.095	-0.086	-0.442
b	0.088	-0.125	-0.359	0.408	0.552	-0.017
	0.253	0.292	0.112	-0.714	0.085	-0.178
	-0.207	-0.369	-0.128	0.267	-0.184	0.340
	0.549	0.557	-0.069	0.291	-0.065	-0.552
	0.150	0.964	0.183	0.751	0.116	-0.089
$\mathbf{q_0}$	0	1.813	-1.184	0.087	-0.415	0.124

with $s(t,l) = \sin \omega_f lt$ and $c(t,l) = \cos \omega_f lt$. By using a random set of 100 possible velocities we ensure that a wide range of exciting position and acceleration trajectories in space are obtained. The coefficient vector δ_i is then determined by the following equation:

$$\delta_i = (A^t A)^{-1} A^t \dot{q}. \tag{8}$$

In order to check the trajectories on possible violations of the constraints, the position, velocity, and acceleration trajectories are evaluated. This is discussed later in this section.

B. Optimization

For the optimization part we used the constrained minimization function “fmincon” of the Optimization Toolbox of Matlab. This function uses a sequential quadratic programming method. The Matlab function requires: (1) a Matlab function calculating the value of the d-optimality criterion given a set of trajectory parameters, (2) a Matlab function calculating a value which is negative if all constraints are satisfied and positive if one of them is violated. If knowledge of the possible boundaries on the trajectory parameters is available, it is also possible to use unconstrained optimization functions such as “fminsearch” or “fminunc”. Due to the optimized starting values for the trajectory parameters the optimization was completed after 3 iterations. Thus, it was possible to quickly retrieve the values for the inertia values. The finally retrieved coefficients for the most optimal design of the trajectories for the robot are presented in Table I.

The generated joint trajectories are presented in Fig.(1).

IV. EXPERIMENTS

In order to validate the above introduced methodologies and run the experiments, the CRS A460 Open Architecture Controller [9], [10] was used.

A. Measurement of the Robot Motor Electrical Constants

As explained in Section 2, both the torque and the regressor matrix can be experimentally estimated from the robot. However the torque is often not directly available. It can be received from a multiplication of the current and the torque constant. But the torque constant being an important parameter in modeling and controlling a robot axis

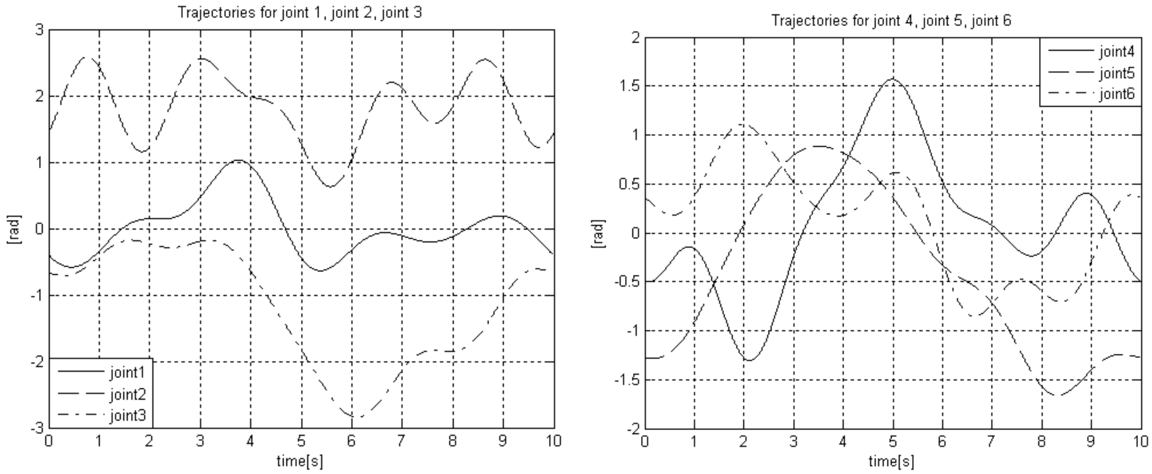

Fig. 1. Trajectories for joints 1,2,3,4,5,6

may vary considerably from the manufacturer's specification. Therefore it might be necessary to measure the torque constant. For the CRS A460 robot the torque constants of joint 4,5, and 6 were known. However, to be able to compare our results, we determined all the torque constants. For this in situ measurement we used a simple experimental method based on the equivalence of the motor torque and back EMF (*electro-motive force* measured in Volts) constants developed by Corke [11]. For further details concerning the characteristics of the robot we refer to the CRS manual [12].

B. Velocities and Acceleration

Since the trajectory is designed to be smoothly continuous, no filtering is done to calculate the derivatives of the input signal. First order difference equations are used. The derivative calculations on encoder inputs, however, are filtered because the joint angle signal is quantized due to the finite resolution of the encoders. One can note that, we are not analytically determining the data sequence for the joint velocities and joint accelerations, but we are using the experimental values.

C. Analysis of the Results

For the estimation results 10000 data points were sampled. The manipulator executes 6 sets of finite fourier series described in Section 2 with the following starting and ending points in degrees:

Joint 1: -23.04, -22.99
Joint 2: 82.94, 82.86
Joint 3: 38.01, -38.00
Joint 4: -28.08, -28.07
Joint 5: -73.00, -72.99
Joint 6: 20.64, 20.66

Mean values of joint positions, velocities, accelerations and currents of 18 periods were calculated and used to retrieve the dynamic parameters. As already mentioned in the introduction, we intend to identify the 60 dynamic parameters of the robot arm. The results of our experiments are that 9 dynamic parameters are not identifiable, 15 parameters are identifiable only in linear combinations of the identifiable parameters, and 36 parameters are identifiable. However, a small number of parameters (10 out of 51 combined parameters, but only 4 out of the 36 identifiable parameters) do not have reasonable values [13], [14]. The remaining 32 identified parameters are presented in Table II. By comparison, in [4] only 15 out of the 30 parameters of the axis 1,2 and 3 are identified, and 7 out of the identified ones are not valid.

Our exciting joint trajectories were optimized based on the condition number, since, contrary to statements in [4], [7] there was a small improvement in the condition number over the d-optimality criterion. The number of invalid values increases the more the condition number increases. Because of the past and existing research on dynamic parameter identification we were not expecting to identify all of the parameters. The missing parameters seem not to have any effect on the control of the arm, particularly the masses of link 4, 5 and 6. If we install the robot on a mobile base, we might also be able to identify also the 9 unidentifiable parameters of axis 1.

V. Discussion And Future Work

It may not be possible to experimentally identify all parameters of a dynamic model and identification of reduced models and direct measurement of some parameters should be considered. It should be mentioned that the following sources of error may exist:

- Sensor Error: The ultimate source of error is the random noise inherent in the sensing process itself. The noise level in position sensing is probably negligible and can be further reduced with a model-based filter such as the Extended Kalman Filter.
- Unmodeled dynamics: Flexibility in the robot joints and load might be one source of unmodeled structural dynamics. Another source of greater concern is the potential compliance of the force sensor itself. To

TABLE II

IDENTIFIABLE DYNAMIC PARAMETERS

I_{yy1}	4.5870	I_{xx4}	1.1344
m_2	17.3190	I_{xy4}	0.2872
$m2*rcx_2$	-6.4811	I_{xz4}	0.6251
$m2*rcy_2$	1.5141	I_{yz4}	0.0593
$m2*rcz_2$	-0.7113	$m5*rcx_5$	-0.8743
I_{xy2}	0.2604	$m5*rcz_5$	-0.2604
I_{yz2}	1.4234	I_{xy5}	-0.5673
$m3*rcx_3$	-1.1787	I_{xz5}	0.0145
$m3*rcz_3$	1.4229	I_{yz5}	0.7508
I_{xx3}	1.5213	$m6*rcx_6$	0.7508
I_{yy3}	1.0473	$m6*rcy_6$	-0.0221
I_{xy3}	-1.0187	I_{xx6}	0.6623
I_{xz3}	-0.2617	I_{zz6}	0.4579
I_{yz3}	-0.9041	I_{xy6}	0.5240
$m4*rcx_4$	-0.1602	I_{xz6}	0.6552
$m4*rcy_4$	-1.2604	I_{yz6}	0.3980

(masses are expressed in kg, first order moments in kgm, inertia moments and products in kgm^2)

avoid exciting these unmodeled dynamics smooth robot trajectories should be chosen. This way the continuity of velocities and accelerations can be maintained. A possible approach is to damp out the vibrations mechanically by introducing some form of energy dissipation into the structure tuned to permit measurement at desired frequencies. Much more relevant are the friction and the effects of the rotors of the motors. We are currently continuing work to extend our model to include the joint friction as well.

- Torque Constant: A systematic error in the unknown motor torque constants K_T, might be reflected as well in the dynamically estimated parameters.

VI. CONCLUSIONS

This paper presented a clear and complete development of the full procedure for estimating the dynamic parameters of any multi-link robot structure. We were able to determine how well the inertial parameters can be estimated through the presented approach, given sources of noise in sensors and in signal processing and limitations in robot position, velocity and acceleration. Furthermore, we also presented a model reduction and parameter classification for the CRS A460/465 robot. We characterized which link inertial parameters can be separately identified or need to be identified in linear combinations, given limitations on sensing and restrictions on movement near the manipulator base. Using optimally designed joint trajectories we were able to identify the dynamic parameters for the CRS A460/465 arm. In our experiments we observed that the optimization based on the condition number had advantages over the d-optimality criterion. Further investigations are necessary to prove the validity of this statement in general for dynamic parameter identification experiments.

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REFERENCES

[1] F. Benimeli, V. Mata, and F. Valero, "A Comparison between direct and indirect dynamic parameter identification methods in industrial robots," *Robotica*, vol. 24, no. 5, pp. 579–590, 2006.

[2] P. K. Khosla, "Categorization of Parameters in the Dynamic Robot Model," *IEEE Trans. Robot. Automat.*, vol. 5, no. 3, 1989.

[3] B. Armstrong, "On Finding Exciting Trajectories for Identification Experiments Involving Systems with Nonlinear Dynamics," *Int. J. Robot. Res.*, vol. 8, no. 6, pp. 28–48, 1989.

[4] J. Swevers, C. Gansemann, D. B. Tkel, J. D. Schutter, and H. Brussel, "Optimal Robot Excitation and Identification," *IEEE Trans. Robot. Automat.*, vol. 13, no. 5, pp. 730–740, 1997.

[5] C. G. Atkeson, C. H. An, and J. M. Hollerbach, "Estimation of Inertial Parameters of Manipulator Loads and Links," *Int. J. Robot. Res.*, vol. 5, no. 3, pp. 109–119, 1986.

[6] K. Radkhah, D. Kulic, and E. Croft, "Identifying the Dynamic Parameters of the CRS A465 Arm," Industrial Automation Laboratory, University of British Columbia, Tech. Rep., 2006. [Online]. Available: www.mech.ubc.ca/~caris/publication/techrep/TechRepKR08-2007b.pdf

[7] J. Swevers, C. Ganseman, J. D. Schutter, and H. V. Brussel, "Additional Remarks related to the Paper: Optimal Robot Excitation and Identification," K.U. Leuven, Div. P.M.A., B3001 Herverlee, Belgium," Tech. Rep. 95R65, 1995.

[8] L. Ljung, in *System Identification: Theory for the User*. Englewood Cliffs, NJ: Prentice-Hall, 1987.

[9] J. Ho, "Open Architecture Controller for the CRS A465 Robot Arm," Industrial Automation Laboratory, University of British Columbia, Tech. Rep., 2003.

[10] D. Meger, "CRS A465 Open Architecture Controller: Software User's Guide and Software Design Document," Industrial Automation Laboratory, University of British Columbia, Tech. Rep., 2003.

[11] P. I. Corke, "In situ Measurement of Robot Motor Electrical Constants," *Robotica*, vol. 23, no. 14, pp. 433–436, 1996.

[12] *A465 Robot Arm User Guide for Use with a C500C Controller*, UMI-33-465, CRS Robotics Cooperation, 2000.

[13] K. Yoshida and W. Khalil, "Verification of the Positive Definiteness of the Inertial Matrix of Manipulators Using Base Inertial Parameters," *Int. J. Robot. Res.*, vol. 19, no. 5, pp. 498–510, 2000.

[14] V. Mata, F. Benimeli, N. Farhat, and A. Valera, "Dynamic Parameter Identification in Industrial Robots Considering Physical Feasibility," *Journal of Advanced Robotics*, vol. 19, no. 1, pp. 101–120, 2005.