Warshall-Floyd Algorithm

Pseudocode and Past Assignment (Assignment 21)

For Reference

Warshall's Algorithm

Algorithm Warshall$(A[1..n, 1..n])$

// Input: The adjacent matrix $A[1..n, 1..n]$ of a digraph

//Output: The transitive closure of the digraph

$\mathbf{R}^{(0)} \leftarrow \mathbf{A}$

for $k \leftarrow 1$ **to** n **do**

for $i \leftarrow 1$ **to** n **do**

for $j \leftarrow 1$ **to** n **do**

$\mathbf{R}^{(k)}[i, j] \leftarrow \mathbf{R}^{(k-1)}[i, j]$ **or**

$\mathbf{R}^{(k-1)}[i, k]$ **and** $\mathbf{R}^{(k-1)}[k, j]$

return $\mathbf{R}^{(n)}$

Floyd's Algorithm & Analysis

Algorithm $\text{Floyd}(\mathbf{W}[1..n, 1..n])$

// Input: The weight matrix $\mathbf{W}[1..n, 1..n]$ of a digraph

//Output: The distance matrix of shortest paths' lengths

$\mathbf{D} \leftarrow \mathbf{W}$

for $k \leftarrow 1$ **to** n **do**

for $i \leftarrow 1$ **to** n **do**

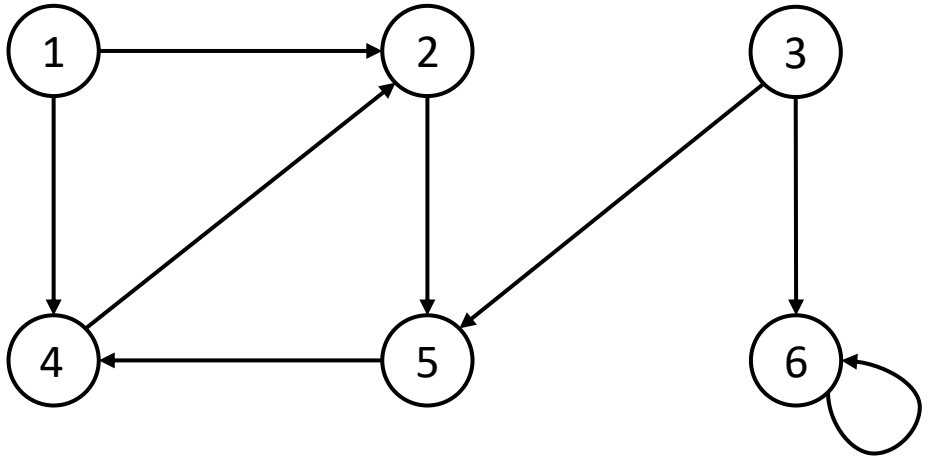
for $j \leftarrow 1$ **to** n **do**

$\mathbf{D}[i, j] \leftarrow \min\{\mathbf{D}[i, j], \mathbf{D}[i, k] + \mathbf{D}[k, j]\}$

return $\mathbf{D}$

Exercise

Consider the following digraph:



- construct the adjacency matrix
- construct the adjacency list
- apply Warshall's algorithm to find the transitive closure

Exercise

Solve the all-pairs shortest-path problem for the following digraph: