MH1812 – Additional Exercises for Final Examination 2

Note: the final exam may or may not be related with the questions here.

NTU, AY15/16 S2 April 2016

Q1: Let p be a prime number, set $A = \{1, 2, \ldots, p-1\}$, and function $f : A \to A$ defined by the rule "$f(x) = b \cdot x \bmod p$",

A) prove f is one-to-one correspondence for any $b \in A$.

B) prove there exists a number $c \in A$ such that $b \cdot c = 1 \bmod p$.

C) prove $f^{-1}(x) = c \cdot x \bmod p$.

D) when $p = 5$ and $b = 2$, find c.

E) Let $d \in A$ and function $g(x) = d \cdot x \bmod p$, prove $f \circ g$ is also one-to-one correspondence.

Q2: Let p and q be logics, prove $(p \wedge \neg q) \vee (p \wedge q) \equiv p$ with both logical equivalence and truth table.

Q3: Let p and q be sets, prove set identity $(p \cap \overline{q}) \cup (p \cap q) = p$.

Q4: Prove by mathematical induction that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$, for all integers $n \geq 1$.

Q5: Let $\mathbb{N}$ be the set of positive integers, and $|$ be the "divisibility" relation, i.e., $x \mid y$ means x divides y (in other words, y is multiple of x) for any $x, y \in \mathbb{N}$. Prove $|$ is a partial order on $\mathbb{N}$.

Q6: Let set $A = \{1, 2, \ldots, 2016\}$

A) How many numbers in A are multiple of 3?

B) How many numbers in A are multiple of 5?

C) How many numbers in A are multiple of 3 and 5?

D) How many numbers in A are multiple of 3, but not multiple of 5?

Q7: Let set $A = \{a, b, c, d\}$, and relation R defined on A as $\{(a, a), (a, b), (c, d)\}$, find $R^{-1} \circ R$, and determine whether it is reflexive.