MH1812 Tutorial
Chapter 1: Elementary Number Theory

Q1: Show that 2 is the only prime number which is even.

Q2: Show that if n^2 is even, than n is even, for n an integer.

Q3: The goal of this exercise is to show that $\sqrt{2}$ is irrational. We provide a step by step way of doing so.

1. Suppose by contradiction that $\sqrt{2}$ is rational, that is $\sqrt{2} = \frac{m}{n}$, for m and n integers with no common factor. Show that m has to be even.

2. Compute m^2, and deduce that n has to be even too, a contradiction.

Q4: Show the following two properties of the integers modulo n:

1. $(a \bmod n) + (b \bmod n) \equiv (a + b) \bmod n$.

2. $(a \bmod n) \cdot (b \bmod n) \equiv (a \cdot b) \bmod n$.

Q5: Compute the addition table and the multiplication tables for integers modulo 4.

Q6: Show that $\frac{p(p+1)}{2} \equiv 0 \pmod{p}$ for p an odd prime.

Q7: Consider the following sets S, with respective operator Δ.

1. Let S be the set of rational numbers R, and Δ be the multiplication. Is S closed under Δ? Justify your answer.

2. Let S be the set of natural numbers N, and Δ be the subtraction. Is S closed under Δ? Justify your answer.

3. Let S be the set of irrational numbers I, and Δ be the addition. Is S closed under Δ? Justify your answer.