NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2016–2017

MH1812 – Discrete Mathematics

April 2017 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Calculators are allowed.
6. Candidates should clearly explain their reasoning used in each of their answers.

QUESTION 1. **(15 marks)**

Decide whether or not the following argument is valid:

$$\begin{aligned} &p \wedge q; \\ &r \to s; \\ &\neg r \to q; \\ &p \vee r; \\ &\therefore (p \vee q) \wedge r \end{aligned}$$

Justify your answer.

QUESTION 2. **(15 marks)**

Consider three sets S, T, and U where S is defined to be the set of all even integers, $T = \{n \in \mathbb{Z} \ : \ 3 \,|\, n\}$, and $U = \{n \in \mathbb{Z} \ : \ n \equiv 0 \pmod 6\}$.

(a) Prove the set equality $S \cap T = U$.

(b) Determine the truth value of the following proposition

$$\neg \left(\forall x \in U,\, \exists y \in T,\ x \cdot y \notin S\right),$$

where $\cdot$ denotes multiplication. Justify your answer.

QUESTION 3. **(30 marks)**

(a) Using the characteristic equation, solve the recurrence relation

$$a_0 = 2,\ a_1 = 3, \quad a_n = 7a_{n-1} - 12a_{n-2} \quad \text{for all } n \geqslant 2.$$

(b) Consider the recurrence relation given by the initial conditions $D_0 = 1$, $D_1 = 0$, and $D_n = (n-1)(D_{n-1} + D_{n-2})$ for all $n \geqslant 2$. Prove the equality

$$D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}.$$

QUESTION 4. **(25 marks)**

(a) Consider the relation R on the set of integers $\mathbb{Z}$ given by

$$aRb \iff b \equiv a^3 - a \pmod 3.$$

(i) Is R reflexive?

(ii) Is R symmetric?

(iii) Is R transitive?

Justify your answers.

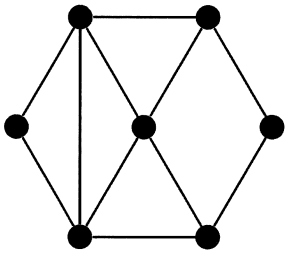
(b) Let $S = \{1, 2, \ldots, n\}$. Determine

(i) the cardinality of the set T of all functions $f : S \to S$?

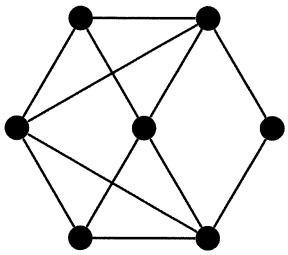
(ii) the cardinality of the set $U = \{f \in T \ : \ f \text{ is invertible}\}$?

QUESTION 5. **(15 marks)**

Consider the two graphs, X and Y, in Figure 1.



X



Y

Figure 1: The graphs X and Y.

(a) For each of the graphs X and Y

(i) determine whether or not it has an Euler path;

(ii) determine whether or not it has an Euler circuit;

(iii) determine whether or not it has an Hamilton circuit.

(b) Are the graphs X and Y isomorphic? Justify your answer.

END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.