

NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER 2 EXAMINATION 2014-2015
MH1812 - DISCRETE MATHEMATICS

April 2015

TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

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QUESTION 1.

(a) Compute $3^{2015} \bmod 7$. **(7 marks)**

(b) Let i denote the imaginary unit of complex numbers, i.e., $i = \sqrt{-1}$. Find all roots of the following equation and express them with i : **(8 marks)**

$$x^2 + 2x + 5 = 0$$

(c) Write the following system of linear equations in a matrix form and solve it using Gaussian elimination. **(15 marks)**

$$\begin{cases} x_1 - x_2 + 2x_3 = 11 \\ x_1 + x_2 + x_3 = 8 \\ 2x_1 - 3x_2 = 1 \end{cases}$$

QUESTION 2.

(a) Find the solution of the recurrence relation, $a_n = 5a_{n-1} - 6a_{n-2}$, with $a_1 = 8$ and $a_2 = 20$. **(10 marks)**

(b) A deck of 52 different cards consists of 4 suits (spade, heart, diamond, club) with 13 distinguished cards each. A “Flush” refers to 5 cards with the same suit, what is the probability for a randomly chosen 5 cards from one deck to be a Flush? **(10 marks)**

(c) Prove by mathematical induction that

$$\sum_{k=1}^{2^n} \frac{1}{k} \geq 1 + \frac{n}{2}$$

for all integer $n \geq 1$. (note: $\sum_{k=1}^{2^n} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{2^{n-1}} + \frac{1}{2^n}$) **(10 marks)**

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QUESTION 3.

- (a) Prove, for any two sets A and B , **(10 marks)**

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

- (b) In a party of 6 people, any two people can be either strangers to (they meet for the first time) or friends with (they met before) the other. Prove, among these 6 people, there exist at least 3 people such that they are all friends, or they are all strangers. **(10 marks)**

QUESTION 4.

- (a) Let \mathbb{Z}^+ be the set of positive integers, i.e. $\{1, 2, 3, \dots\}$, and let R be the relation defined by “ $x, y \in \mathbb{Z}^+$ and $(x, y) \in R \Leftrightarrow x^2 | y$ ”. Is R a partial order? Is R an equivalence relation? Justify your answer. **(10 marks)**
- (b) Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y . Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$. **(10 marks)**

END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.