NANYANG TECHNOLOGICAL UNIVERSITY

SPECIAL TERM I EXAMINATION 2016-2017

MH1812 (P/T) - Discrete Mathematics

June 2017 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. (20 marks)

(a) Recall the definition of Euclidean division, show that the choice of quotient-remainder pair is unique. That is, given an integer m and a positive integer n, let (q_1, r_1) and (q_2, r_2) both be quotient-remainder pairs when dividing m with n, then we must have $q_1 = q_2$ and $r_1 = r_2$.

(b) Show that there is NO largest prime number. That is, given any prime number n, there is always a prime number greater than n. (Hint: Consider $M = n! + 1$. If M itself is prime, that is it. If M is not prime, there must be a prime factor p of M. Show that $p > n$.)

QUESTION 2. (25 marks)

(a) Copy the following truth table to your answer book and complete it.

p	q	r	$q \to r$	$p \wedge (\neg(q \to r))$	$r \vee \neg p$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

(b) Identify one row in the above truth table, which can be used as a counter-example for the FALSE equality $p \wedge (\neg(q \to r)) \equiv \neg(p \to r)$.

(c) Using the method of mathematical induction, show that

$$\sum_{k=0}^{n} k(k+1) = \frac{1}{3} n(n+1)(n+2),$$

for every non-negative integer n.

QUESTION 3. **(28 marks)**

(a) Let Q be a general set and $P(Q)$ be its power set. Suppose R and S are two relations on $P(Q)$, which are defined as $ARB \leftrightarrow A \subseteq B$ and $ASB \leftrightarrow A \cap B = \phi$.

(i) Is R a partial order? Justify your answer.

(ii) Show that S is symmetric but NOT an equivalence relation.

(b) Let Z_6 and Z_7 denote the sets of integers modulo 6 and 7, respectively. Define two functions f_1: $Z_6 \to Z_6$ and f_2: $Z_7 \to Z_7$, as $f_1(x) = (2x + 1) \bmod 6$ and $f_2(x) = (5x + 1) \bmod 7$.

(i) Show that f_1 is NOT injective.

(ii) Show that f_2 is invertible.

QUESTION 4. **(27 marks)**

For this question, intermediate workings will **NOT** be graded. You only need to present the final answer to each subquestion.

(a) Consider the equation $x_1 + x_2 + ... + x_r = n$, where n, r, x_1, x_2, ..., x_r are all positive and non-zero integers with $n > r$. Since we have more unknowns than equations, there are supposed to be more than one solutions.

(i) How many solutions are there when $n = 3$ and $r = 2$?

(ii) How many solutions are there when $n = 5$ and $r = 3$?

(iii) How many solutions are there when $n = 7$ and $r = 5$?

(iv) **MULTIPLE CHOICE**. How many solutions are there for general values of n and r?
A. $C(n-1, r-1)$. B. $C(n-1, r)$. C. $C(n, r-1)$. D. $C(n, r)$.

(b) A standard deck of 52 different poker cards consists of 4 suits (spade, heart, diamond, club) with 13 distinct cards of each suit. We randomly draw 5 cards from the deck, and let Ω denote the sample space of this random experiment. Let A denote the event that the 5 chosen cards are of a same suit.

(i) Calculate $|\Omega|$.

(ii) Calculate $|A|$.

(iii) Calculate the probability of event A. Leave your answer to 4 decimal places.

(c) Consider the complex number

$$z = \frac{7 - 5i}{1 - i}.$$

(i) Find the exponential form of z. Leave the value of argument of z in terms of the tan^{-1} function.

(ii) Find the rectangular form of $\bar{z}$, the conjugate of z.

END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.