NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014-2015

MH1812 - Discrete Mathematics

November 2014   TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. (20 marks)

(a) Let A and B be two sets. Write the definition of $A \neq B$ (in terms of predicate logic).

(b) Show that
$$(A \neq B) \rightarrow \neg((A \cup B) \subseteq (A \cap B)).$$

(c) Deduce that
$$((A \cup B) \subseteq (A \cap B)) \rightarrow (A = B).$$

QUESTION 2. (25 marks)

(a) Consider the set S of 3×3 matrices with binary coefficients, that is the coefficients are integers modulo 2.

  (i) Compute $|S|$.

  (ii) Consider the subset R of S formed by 3×3 matrices with binary coefficients such that they are equal to their transposes:
  $$R = \left\{ M \in S, \ M = M^T \right\}.$$

  Compute $|R|$.

(b) Consider the 3×3 real matrix
$$M = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 0 \\ x & 0 & 2 \end{pmatrix}.$$

Compute the value(s) of x for which M is invertible, in which case(s), compute M^{-1}.

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QUESTION 3. (25 marks)

(a) Compute the real part and the imaginary part of the following complex number:
$$\frac{5 - 3i}{1 + i}.$$

(b) Let z be a complex number. Prove that the real part of z is given by $\frac{z + \bar{z}}{2}$. Give a formula to express the imaginary part of z in terms of z and $\bar{z}$.

(c) Define a relation R on the set of complex numbers by
$$zRw \iff |z| \leq |w|.$$

Is this relation R a partial order? Justify your answer.

QUESTION 4. (20 marks)

(a) Let A_5 denote the set of integers modulo 5, and A_8 denote the set of integers modulo 8. Consider the maps $f_1 : A_8 \to A_8$, $f_2 : A_5 \to A_5$ given by

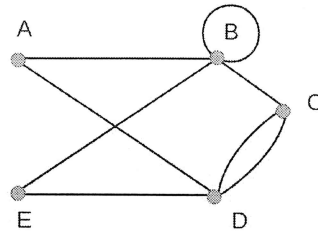$$\begin{aligned} f_1(x) &= 2x \mod 8, \\ f_2(x) &= 4x + 1 \mod 5. \end{aligned}$$

(i) Is the map f_1 injective (one-to-one)? Justify your answer.

(ii) Is the map f_2 invertible? If so, give its inverse. Justify your answer.

(b) Let B be a finite set. Let $f : B \to B$ be an injective map. Show that f is surjective.

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QUESTION 5. (10 marks)

Does the following graph contain an Euler circuit? Justify your answer.

END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.