NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2017–2018

MH1812 – Discrete Mathematics

May 2018 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Calculators are allowed.
6. Candidates should clearly explain their reasoning used in each of their answers.

QUESTION 1. (25 marks)

(a) Let $S = \{1, 2, 4\}$ and let P be the set of prime numbers. Determine the truth value of the following proposition:

$$\neg\left(\exists x \in S,\, \forall y \in S,\; x + y \notin P\right).$$

Justify your answer.

(b) Decide whether or not the following argument is valid:

$$\begin{aligned} &p \vee q;\\ &p \rightarrow s;\\ &q \rightarrow r;\\ &\neg r \vee p;\\ &\therefore s \end{aligned}$$

Justify your answer.

QUESTION 2. (25 marks)

(a) Let $S = \{1, 2, 3\}$. How many binary relations R on S are there such that

(i) R is reflexive?

(ii) R is symmetric?

(iii) R is an equivalence relation?

Justify your answers.

(b) Define the function $f : \mathbb{Q} \rightarrow \mathbb{Q}$ by $f(x) = \frac{2}{3}x + 5$.

(i) Prove that the function f is bijective.

(ii) What is the inverse of f?

QUESTION 3. (25 marks)

(a) Solve the recurrence relation

$$a_0 = 2,\ a_1 = 3, \quad a_n = 3a_{n-1} - 2a_{n-2} + 1 \quad \text{for all } n \geqslant 2,$$

that is, write a_n in terms of n. Justify your answer.

(b) Prove that, for all $n \in \mathbb{N}$,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4.$$

QUESTION 4. (25 marks)

(a) Let G be an undirected graph with n vertices. Find the minimum number of edges required such that

(i) G is connected;

(ii) G has a Hamiltonian circuit;

(iii) G has an Euler path.

Justify your answers.

(b) Does the graph X have

(i) an Euler path?

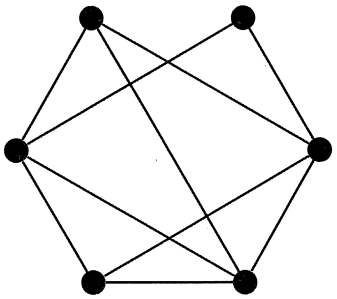
(ii) a Hamiltonian path?

(iii) an Euler circuit?

(iv) a Hamiltonian circuit?

Justify your answers.

The graph X:



END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.