NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

MH1812 - DISCRETE MATHEMATICS

December, 2017 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1.

(a) Show $p \to (q \vee r) \equiv (p \wedge \neg q) \to r$. **(10 marks)**

(b) Let $X = \{1, 2, 3\}$ and let $\mathbb{P}(X)$ be the power set of X (the set of all subsets of X). A relation $\mathbf{R}$ is defined on $\mathbb{P}(X)$ as follows: for all $A, B \in \mathbb{P}(X)$, $A\mathbf{R}B$ if and only if the number of elements in A equals the number of elements in B.

(i) Show $\mathbf{R}$ is an equivalence relation on $\mathbb{P}(X)$. **(9 marks)**

(ii) List all the equivalence classes of $\mathbf{R}$. **(6 marks)**

QUESTION 2.

(a) Using the characteristic equation, solve the recurrence relation,

$$a_n = 7a_{n-1} - 10a_{n-2}$$

for $n \geq 2$, with $a_0 = 2, a_1 = 1$. **(10 marks)**

(b) Prove by mathematical induction that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

for all integers $n \geq 2$. **(15 marks)**

QUESTION 3.
Let $A = \{1, 2, 3, 4\}$, and define the functions $f, g : A \to A$ by the rules $f(x) = 3^x \bmod 5$ and $g(x) = 2^x \bmod 5$.

(a) Is f one-to-one? **(5 marks)**

(b) Is g onto? **(5 marks)**

(c) Find the composition $f \circ g$ of f and g. **(5 marks)**

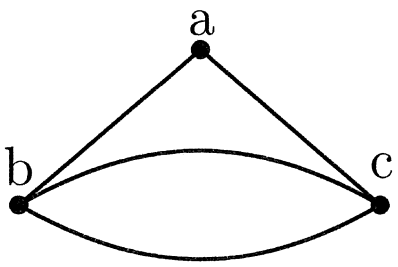
QUESTION 4.

(a) Let A, B, and C be sets, show $(B - A) \cap (C - A) = (B \cap C) - A$. **(10 marks)**

(b) From a group of 7 men and 6 women, 5 persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done? **(15 marks)**

QUESTION 5.

Refer to the graph below, find Euler Circuit and Hamilton Circuit if any, justify your answer if it does not exist. **(10 marks)**



END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.