MH1812 – Additional Exercises for Final Examination

Note: the final exam may or may not be related with the questions here.

NTU, AY15/16 S2 April 2016

Q1: Find a positive integer x in the range $[1, 6]$ for each of the following:

I. $2^x \equiv 1 \mod 7$

II. $3^x \equiv 1 \mod 7$

III. $4^x \equiv 1 \mod 7$

IV. $5^x \equiv 1 \mod 7$

V. $6^x \equiv 1 \mod 7$

Q2: (optional) Is it generally true that $x^{p-1} \mod p = 1$, for p a prime number and x a positive integer with $\gcd(x, p) = 1$?

Q3: Use results of (Q1.), compute $5^{1024} \mod 7$.

Q4: Let the set $S = \{1, 2, \dots, 6\}$, is S closed under operator Δ=“multiplication modulo 7”, i.e., $x \Delta y = x \cdot y \mod 7$?

Q5: Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

I. the positive real numbers.

II. the integers.

III. the nonzero real numbers.

Q6: Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion“We will be home by sunset.”

Q7: Determine, using truth table, whether the following statement is valid

$$
\begin{array}{rcl}
p & \rightarrow & q \vee \neg r \\
q & \rightarrow & p \wedge r \\
\therefore p & \rightarrow & r
\end{array}
$$

Q8: Prove by mathematical induction that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n+1)}=\frac{n}{n+1}$$

for all integers $n \geq 1$.

Q9: Prove by mathematical induction that

$$2^n < (n+1)!$$

for all integer $n \geq 2$.

Q10: Prove by mathematical induction that

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{n}} > \sqrt{n}$$

for all integers $n \geq 2$.

Q11: Let relation R defined on set $A = \{a, b, c, d\}$ and $R = \{(a, b), (b, a), (b, c), (c, d)\}$, find R^{-1}, R^2 (note: $R^2 = R \circ R$), R^3 and the transitive closure R^t. Is R^t an equivalent relation or a partial order ?

Q12: Prove using both set identities and membership table that

$$(A - B) \cap (C - B) = (A \cap C) - B$$

Q13: Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

Q14: Let $f, g : \mathbb{R} \to \mathbb{R}$, and $f(x) = x - 3$ and $g(x) = x^3 + 10$, find $f^{-1} \circ g^{-1}$ and $(g \circ f)^{-1}$, what do you find ?

Q15: Find the Euler path, Euler circuit, Hamilton path, and Hamilton circuit of the following graphs (with a and d as starting and ending points for paths), if any.