

NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER 1 EXAMINATION 2016-2017
MH1812 - Discrete Mathematics

November 2016

TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

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QUESTION 1.

(40 marks)

- (a) Let T be a tautology. Prove or disprove the following statement using logical equivalences:

$$[(p \wedge q) \rightarrow r] \leftrightarrow [p \rightarrow (q \rightarrow r)] \equiv T.$$

- (b) Consider the set S of integers which are multiples of 3, and the set T of integers which are multiples of 7, that is $S = \{\dots, -6, -3, 0, 3, 6, \dots\}$, and $T = \{\dots, -14, -7, 0, 7, 14, \dots\}$.
- (i) Is the set $S \cap T$ closed under addition? Justify your answer.
- (ii) Is the set $S \cup T$ closed under multiplication? Justify your answer.

QUESTION 2.

(30 marks)

- (a) Let A be the set of integers modulo 4. Compute the cardinality of the set

$$\{f : A \rightarrow A, f(x) = ax + b, \text{ for some } a, b \in A, f \text{ is injective (one-to-one)}\}.$$

- (b) Let A, B, C be sets. Prove or disprove the following set equality:

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

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QUESTION 3.

(20 marks)

Consider the set $S = \{1, \dots, n\}$, for some integer $n \geq 2$, and its power set $P(S)$. For $A, B \in P(S)$, define a relation R by $ARB \iff A \cap B = \emptyset$.

- (a) Is R reflexive?
- (b) Is R symmetric?
- (c) Is R antisymmetric?
- (d) Is R transitive?

Justify your answers.

QUESTION 4.

(10 marks)

Does the following graph shown on Figure 1 contain an Euler path? Justify your answer.

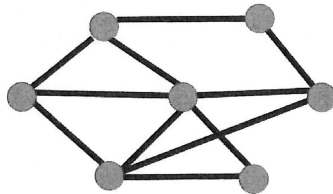


Figure 1: Graph

END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.