

NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER 1 EXAMINATION 2018-2019
MH1812 - DISCRETE MATHEMATICS

December 2018

TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

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QUESTION 1.

- (a) Prove that $\neg p \rightarrow \neg q$ and its inverse are not logically equivalent. **(10 marks)**
- (b) Prove that $(q \wedge (p \rightarrow \neg q)) \rightarrow \neg p$ is a tautology using propositional equivalence and the laws of logic. **(10 marks)**

QUESTION 2.

Prove that $\sum_{j=n}^{2n-1} (2j+1) = 3n^2$ for all positive integers n . **(12 marks)**

QUESTION 3.

Find the solution to the recurrence relation $a_n = a_{n-1} + 2n + 1$ with $a_0 = 2$. **(10 marks)**

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QUESTION 4.

- (a) x_1, x_2, \dots, x_k are positive integers such that $\sum_{i=1}^k x_i = n$, for some positive integers k, n and $n \geq k$. How many distinct tuples of (x_1, x_2, \dots, x_k) are there?
(6 marks)
- (b) How many distinct tuples of (x_1, x_2, \dots, x_k) are there for the above question if x_1, x_2, \dots, x_k are non-negative integers, rather than positive integers ?
(4 marks)
- (c) How many bit strings contain exactly 5 '0's and 9 '1's if every '0' must be immediately followed by a '1' ?
(4 marks)

QUESTION 5.

Prove by the method of membership table that

$$\overline{(A - B) \cup (B - A)} = (A \cap B) \cup (\overline{A} \cap \overline{B}).$$

(14 marks)

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QUESTION 6.

Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and define the relation R as follows: $\forall x, y \in A$, $x R y$ iff $3^x \equiv 3^y \pmod{5}$.

- (a) Prove R is an equivalence relation. **(8 marks)**
- (b) List all the equivalence classes and all the elements in each class. **(8 marks)**

QUESTION 7.

Define a function $f : D \rightarrow \mathbb{Z}$ by $f(x) = x^2 + 5$, where $D = \{-4, -3, -2, -1, 0\}$.

- (a) Find the range of the function. **(8 marks)**
- (b) Find f^{-1} . **(6 marks)**

END OF PAPER

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Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.