NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

MH1812 - Discrete Mathematics

November 2015 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(40 marks)**

(a) Prove or disprove the following statement using logical equivalences:

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q).$$

(b) Consider the domains $X = \{2, 3\}$ and $Y = \{6, 7, 8\}$, and the predicate

$$P(x, y) = \text{“}x \equiv y \mod 5\text{”}.$$

Decide what is the truth value of the statement

$$\neg(\exists x \in X,\ \forall y \in Y,\ P(x, y)).$$

(c) Decide whether the following argument is valid:

$p \wedge q$;
$p \rightarrow \neg r$;
$q \rightarrow \neg s$;
$\therefore \neg r \wedge \neg s$.

QUESTION 2. **(30 marks)**

(a) Let f be a function from the set A to the set B. Let S and T be subsets of A.

(i) Show that

$$f(S \cup T) = f(S) \cup f(T).$$

(ii) Prove or disprove that

$$f(S \cap T) = f(S) \cap f(T).$$

(b) Given two sets A and B, their symmetric difference $A \oplus B$ is the set containing those elements in either A or B, but not in both A and B. Suppose that $A \oplus B = C \oplus B$ for three sets A, B, C. Does it imply that $A = C$? Justify your answer.

QUESTION 3. **(15 marks)**

How many distinct reflexive relations are there on a set with n elements? Justify your answer.

QUESTION 4. **(15 marks)**

(a) Is the following graph shown on Figure 1 bipartite? Justify your answer.

(b) Does the following graph shown on Figure 1 contain an Euler path? Justify your answer.

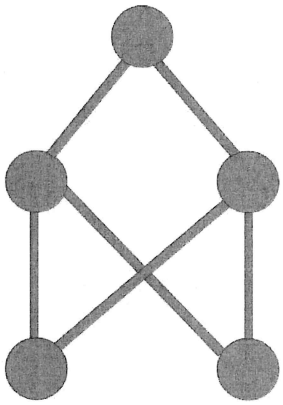

Figure 1: Graph

END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.