Discrete Mathematics
MH1812

Topic 3.3 - Predicate Logic III
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Topic Overview
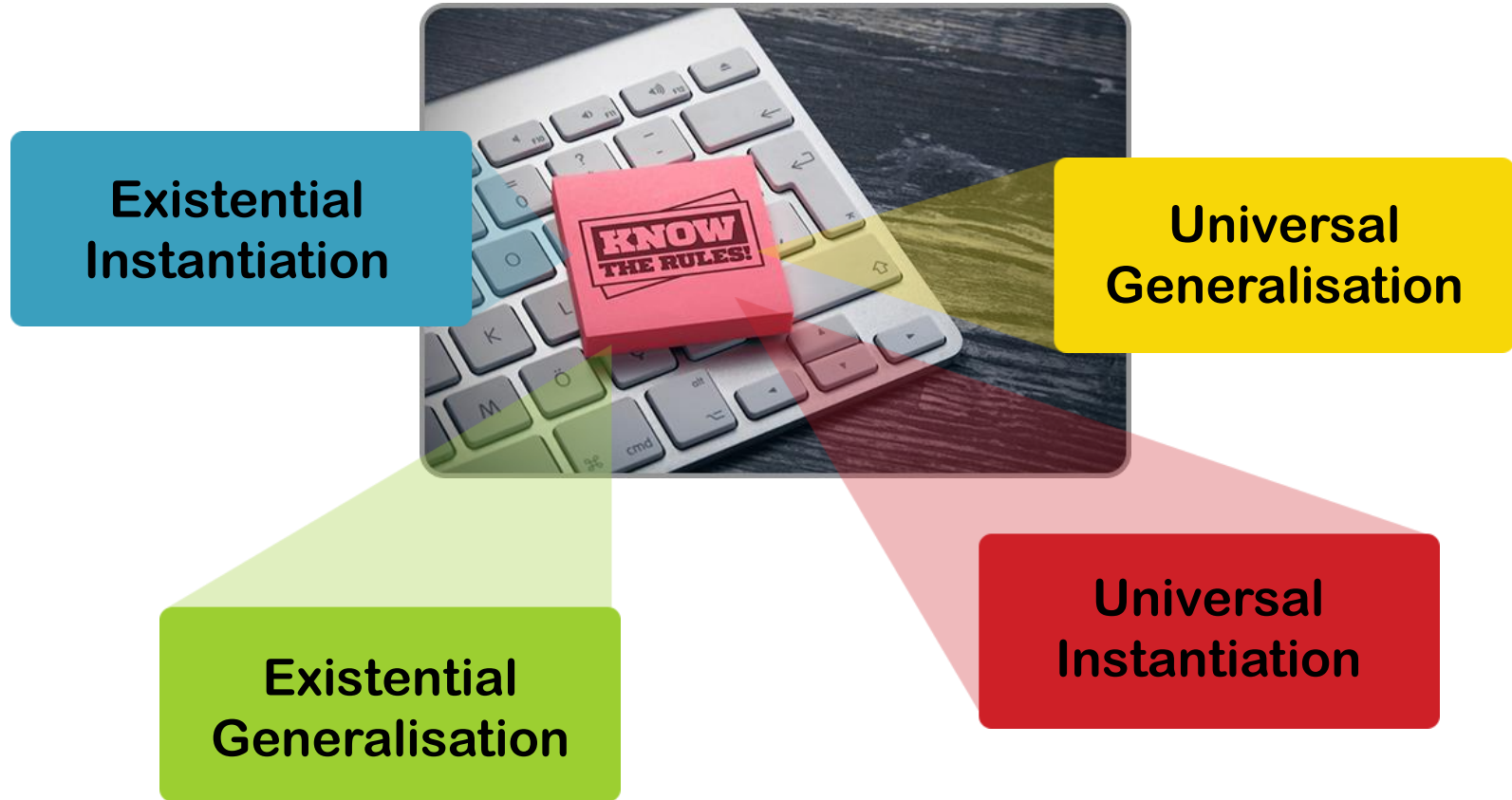
What's in store…

Basic Inference Rules

∀
∃

By the end of this lesson, you should be able to…

Apply inference rules to verify an argument.

Basic Inference Rules

Basic Inference Rules

Existential Instantiation

Universal Generalisation

Existential Generalisation

Universal Instantiation

Basic Inference Rules: Universal Generalisation

∀
∃

$P(c)$ for **any arbitrary** c from the domain D.
∴ $\forall x \in D, P(x)$

x^2 is non-negative

- $P(x)$ = "x^2 is non-negative"

- $P(c)$ for an arbitrary real c
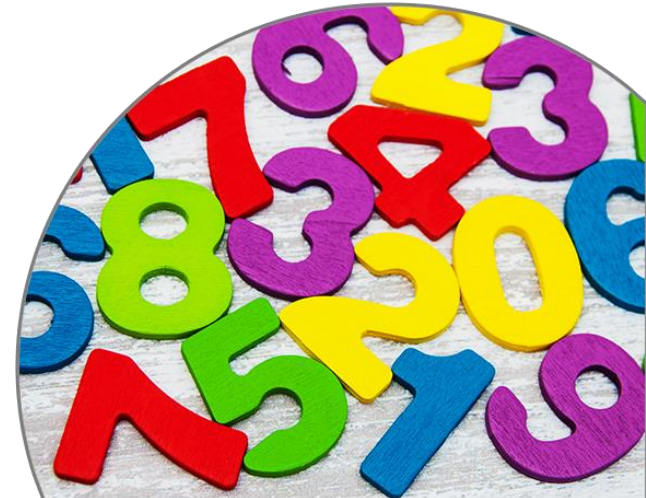
- Therefore $P(x)$ for all x in $\mathbb{R}$

Basic Inference Rules: Universal Generalisation

Domain = $\mathbb{R}$

$P(x)$ = "x^2 **is non-negative**"

1	$P(c)$ **for an arbitrary real** c	Hypothesis
2	$\forall x \in \mathbb{R}, P(x)$	**Universal Generalisation on 1**

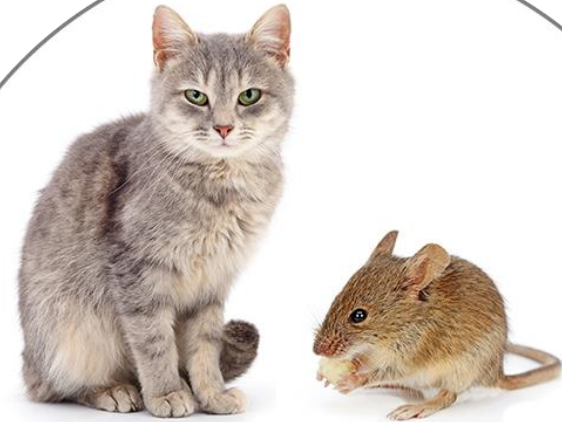# Basic Inference Rules: Universal Instantiation

$\forall x \in D, P(x)$

$\therefore P(c)$

where c is any element of the domain D.

Tom and Jerry

- **No cat can catch Jerry.**

- **Tom is a cat.**

- **Therefore, Tom cannot catch Jerry.**

Basic Inference Rules: Universal Instantiation

D = {all animals}

Cat(x) = x is a Cat

Catch(x) = x can catch Jerry

1	$\forall x \in D, [\text{Cat}(x) \rightarrow \neg \text{Catch}(x)]$	Hypothesis
2	Cat(Tom)	Hypothesis
3	Cat(Tom) $\rightarrow \neg$ Catch(Tom)	Universal Instantiation on 1
4	$\neg$ Catch(Tom)	Modus Ponens on 2 and 3

Basic Inference Rules: Existential Generalisation

∀
∃

$P(c)$
∴ $\exists x \in D, P(x)$

for c **some** specific element of the domain D.

Selling Stocks

If everyone is selling stocks,
then someone is selling stocks.

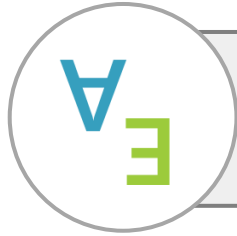# Basic Inference Rules: Existential Generalisation

D = **{all people}**

Sell(x) = "x **is selling stocks**"

$\forall x \in D$, Sell(x) $\rightarrow \exists x \in D$, Sell($x$)

1	$\forall x \in D$, Sell(x)	**Hypothesis**
2	Sell(c)	**Universal Instantiation on 1**
3	$\exists x \in D$, Sell(x)	**Existential Generalisation on 2**

Basic Inference Rules: Existential Instantiation

∀
∃

$\exists x \in D,\ P(x)$
$\therefore\ P(c)$ **for some** c in the domain D.

Final Exam

- If any student scores > 80 in the final exam, then s/he receives an A.

- There are students who score > 80 in the final exam.

- Therefore, there are students who receive an A.

Basic Inference Rules: Existential Instantiation

D = **{all students}**

$A(x)$ = **"x receives an A"**

$M(x)$ = **"x scores > 80 in the final exam"**

1	$\forall x \in D, [M(x) \rightarrow A(x)]$	**Hypothesis**
2	$\exists x \in D, M(x)$	**Hypothesis**
3	$M(c)$	**Existential Instantiation on 2**
4	$M(c) \rightarrow A(c)$	**Universal Instantiation on 1**
5	$A(c)$	**Modus Ponens on 4 and 3**
6	$\exists x \in D, A(x)$	**Existential Generalisation on 5**

Topic Summary

Let's recap…

- **More inference rules to verify arguments**