

NANYANG TECHNOLOGICAL UNIVERSITY  
SEMESTER 2 EXAMINATION 2015-2016  
MH1812 - DISCRETE MATHEMATICS

May 2016

TIME ALLOWED: 2 HOURS

---

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

MH1812

### QUESTION 1.

(a) Compute  $5^{2016} \bmod 7$ . (8 marks)

(b) Determine whether  $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$  is a tautology. (list all intermediate propositions if you choose to prove by truth table) (10 marks)

(c) How many solutions are there for the following equation

$$x_1 + x_2 + \cdots + x_r = n$$

with  $r, n, x_i$  positive integers for  $i = 1, 2, \dots, r$  and  $n \geq r$ . (10 marks)

(d) Let sets  $A = \{1, 3\}$ ,  $B = \{0, 2, 4\}$ , and  $P(x, y)$  denote “ $5 \mid (x + y)$ ”, i.e.,  $(x + y)$  is multiple of 5. Determine the truth value of the following and justify your answer: (12 marks)

- $\forall x \in A, \exists y \in B, P(x, y)$ .
- $\exists y \in B, \forall x \in A, P(x, y)$ .

### QUESTION 2.

(a) Find the solution of the recurrence relation,  $a_n = 4a_{n-1} - 4a_{n-2}$ , with  $a_1 = 2$  and  $a_2 = 8$ . (10 marks)

(b) Prove

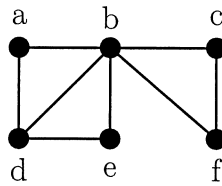
$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = n(n+1)(n+2)/3$$

for every positive integer  $n$ . (10 marks)

MH1812

**QUESTION 3.**

- (a) Let  $A, B$ , and  $C$  be sets, show  $(B - A) \cup (C - A) = (B \cup C) - A$ . **(10 marks)**
- (b) Refer to the graph below, find Euler Path, Euler Circuit and Hamilton Circuit if any, justify your answer if it does not exist. **(8 marks)**



**QUESTION 4.**

- (a) Let set  $A = \{a, b, c, d\}$  and relation  $R = \{(a, a), (a, b), (b, c), (c, d), (d, c)\}$ . **(12 marks)**
- Is  $R$  reflexive, symmetric, transitive ?
  - Find  $R^t$ , i.e., the transitive closure of  $R$ .
- (b) Let function  $f(x) = x^2 + 2x + 3$  with  $x$  being real numbers and  $x \leq -1$ , find **(10 marks)**
- the range of  $f$ .
  - the inverse function  $f^{-1}$ .

**END OF PAPER**

## **MH1812 DISCRETE MATHEMATICS**

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.