NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2018–2019

MH1812 – Discrete Mathematics

May 2019 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Calculators are allowed.
6. Candidates should clearly explain their reasoning used in each of their answers.

QUESTION 1. **(25 marks)**

(a) Prove that $\neg(p \rightarrow q) \vee (p \wedge q) \equiv p$

(i) using a truth table;

(ii) using a sequence of logical equivalences.

(b) Consider the distinguishable permutations of the number 15052.

(i) How many are there in total?

(ii) How many are even?

(iii) How many are greater than 2019?

QUESTION 2. **(35 marks)**

(a) Consider the relation $R = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,3)\}$.

(i) Is R reflexive?

(ii) Is R symmetric?

(iii) Is R anti-symmetric?

(iv) Find the transitive closure of R.

(b) Let F be the set of all injective functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$, where $\mathbb{Z}$ denotes the integers. Define the functions $g, h : \mathbb{Z} \rightarrow \mathbb{Z}$ as $g(x) = 7x-2$ and $h(x) = x^2-5x$.

(i) Show that $g \in F$.

(ii) Is $h \in F$? Justify your answer.

(iii) Is g invertible? Justify your answer.

QUESTION 3. **(8 marks)**

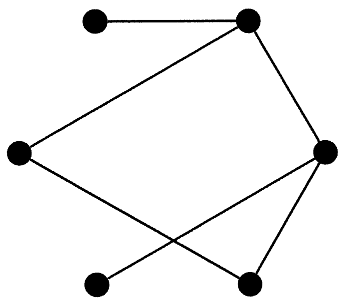
Solve the recurrence relation

$$a_0 = 1,\ a_1 = 8, \quad a_n = a_{n-1} + 6a_{n-2} \quad \text{for all } n \geqslant 2,$$

that is, write a_n in terms of n. Justify your answer.

QUESTION 4. **(17 marks)**

The graph X:



(a) Does the graph X have

(i) an Euler path?

(ii) a Hamiltonian path?

(iii) an Euler circuit?

Justify your answers.

(b) Does there exist an undirected simple graph (V, E) with $|V| = 5$ such that, for every three pairwise distinct vertices $u, v, w \in V$,

$$0 < |\{\{u, v\}, \{u, w\}, \{v, w\}\} \cap E| < 3?$$

If so, give an example of such a graph, otherwise explain why it cannot exist.

QUESTION 5. **(15 marks)**

(i) Prove that

$$\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1},$$

for all integers n satisfying $n \geqslant 1$.

(ii) Use part (i) to evaluate

$$\sum_{k=13}^{37} \frac{1}{(2k-1)(2k+1)}.$$

END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.