NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Discrete Mathematics

MH1812

Topic 2.1 - Propositional Logic I

Dr. Gary Greaves

Topic Overview

What's in store…

- **P**roposition and Paradox
- **L**ogical Operators
- **D**e Morgan's Laws
- **C**ontradiction and Tautology
- **E**quivalent Expressions

By the end of this lesson, you should be able to…

- Explain what is a proposition and a paradox.
- Use logical operators to combine statements.
- Apply De Morgan’s Laws.
- Explain what is a contradiction and a tautology.
- Identify equivalent expressions.
- Demonstrate that two expressions are equivalent.

Proposition and Paradox

Proposition and Paradox: Logic

- Accepted rules for making precise statements

- Logic for computer science:
  - Programming
  - Artificial intelligence
  - Logic circuits
  - Database

- Logic:
  - Represents knowledge precisely
  - Helps to extract information (inference)

Proposition and Paradox: Proposition

A **proposition** is a declarative statement that is either **true** or **false**.

Examples of propositions

- “$1 + 1 = 2$”. . . True
- “$1 + 1 > 3$”. . . False
- “Singapore is in Europe.”. . . False

```
gap> (5>3);
true
gap> (1>3);
false
gap>
```

Proposition and Paradox: Proposition

A **proposition** is a declarative statement that is either **true** or **false**.

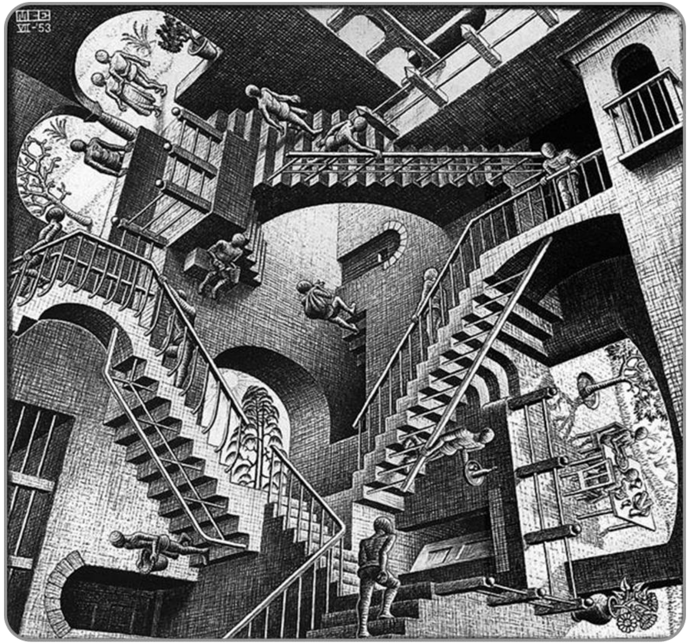
Examples that are not propositions

- "$1 + 1 > x$". . . X
- "What a great book!". . . X
- "Is Singapore in Asia?". . . X

Proposition and Paradox: Paradox

A declarative statement that **cannot** be assigned a **truth value** is called a **paradox**.

- A paradox is not a proposition.
- E.g., the liar paradox:
  “This statement is false”.



Relativity Lattice (M.C. Escher)

Retrieved Feb 19, 2018 from https://www.wikiart.org/en/m-c-escher/relativity-lattice

Logical Operators

Logical Operators: Symbolic Logic

- Use symbols to represent statements (both have the same truth values)

- Use logical operators to combine statements:

Compound Propositions **=** **Propositions Combined with Logical Operator(s)**

Logical Operators: Three Basic Operators

$\wedge$ Conjunction (and)

Disjunction (or) $\vee$

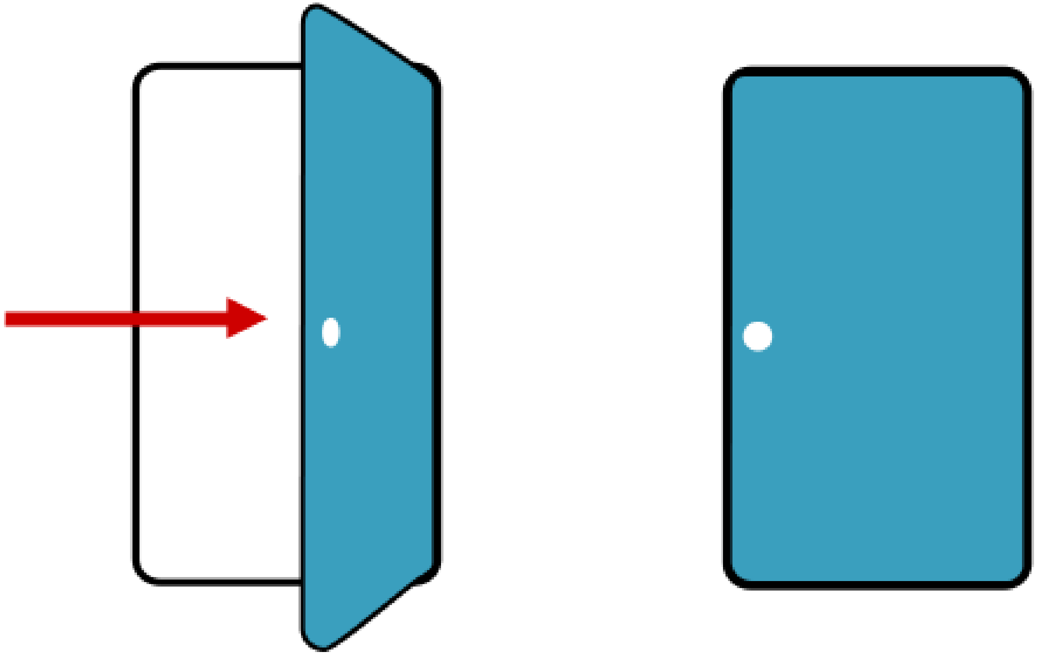
$\neg$ Negation (not, alternatively $\sim$)

Logical Operators: Negation

- Negation (not) of p: $\neg p$ ($\sim p$ is also used)

p	$\neg p$
T	F
F	T

Truth Table



p: You may enter

$\neg p$: You may not enter

Logical Operators: Disjunction

∨

- Disjunction (or) of p with q: $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$q \vee p$
T
T
T
F

Truth Table

True when "at least one" of them is true

$$p \vee q \equiv q \vee p$$

i.e., operator ∨ commutes

Means "equivalent"

```
gap>
gap> (5>3) or (1>5);
true
gap>
```

Logical Operators: Conjunction

$\wedge$

- Conjunction (**and**) of p with q: $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$q \wedge p$
T
F
F
F

Truth Table

True only when "both" of them are true

$\wedge$ is also commutative: $p \wedge q \equiv q \wedge p$

```
gap> (5>3) and (7>5);
true
gap>
gap>
gap> (5>3) and (1>5);
false
```

De Morgan’s Laws

De Morgan's Laws: Definition

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

p q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T T	F	F	T	F	F
T F	F	T	F	T	T
F T	T	F	F	T	T
F F	T	T	F	T	T

Augustus De Morgan
(1806 - 1871)

Augustus De Morgan by Sophia Elizabeth De Morgan under WikiCommons (PD-US)

Contradiction and Tautology

Contradiction and Tautology: Definition

A compound proposition that is always false is called a **contradiction**.

Example

This course is easy "and" this course is not easy.

$$p \wedge (\neg p) \equiv \mathrm{F}$$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Contradiction and Tautology: Definition

A compound proposition that always gives a true value is called a **tautology**.

Example

$$p \vee (\neg p) \equiv \mathrm{T}$$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Always true!

Equivalent Expressions

Equivalent Expressions: Bob and Alice

1. Alice is not married but Bob is not single.

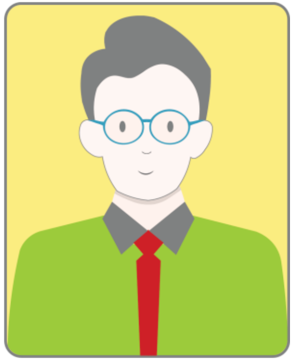
$$\neg h \wedge \neg b$$

2. Bob is not single and Alice is not married.

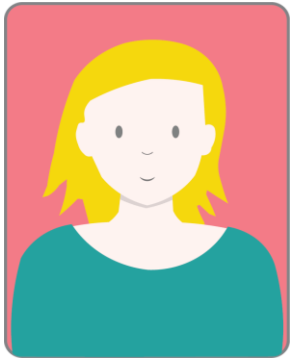
$$\neg b \wedge \neg h$$

3. Neither Bob is single nor Alice is married.

$$\neg(b \vee h)$$



Bob



Alice

These three statements are equivalent.

$$\neg h \wedge \neg b \equiv \neg b \wedge \neg h \equiv \neg(b \vee h)$$

Equivalent Expressions: The Statements

These three statements are equivalent:

$$\neg h \wedge \neg b \equiv \neg b \wedge \neg h \equiv \neg(b \vee h)$$

b h	$\neg b$	$\neg h$	$b \vee h$	$\neg h \wedge \neg b$	$\neg b \wedge \neg h$	$\neg(b \vee h)$
T T	F	F	T	F	F	F
T F	F	T	T	F	F	F
F T	T	F	T	F	F	F
F F	T	T	F	T	T	T

Topic Summary

Let's recap…

- We have covered:
  - Proposition (Compound Propositions)
  - Paradox
  - Contradiction
  - Tautology
  - Equivalent Expressions

- Basic logical operators (and De Morgan's laws):
  - Negation
  - Conjunction
  - Disjunction