NANYANG TECHNOLOGICAL UNIVERSITY

AY2017-2018 Special Term I

MH1812– Discrete Mathematics

June 2018

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **Ten (10)** questions and comprises **Five (5)** printed pages.
2. Students are required to answer **ALL** questions. Each question carries 10 marks.
3. Students are to write the answers for each question on a new page.
4. A list of formulae is given on Page 5.
5. This is a **CLOSED BOOK** examination.
6. Candidates may use calculators. However, they should lay out systematically the various steps in the workings.

Question 1 (10 Marks)

(a) Let n be a positive integer. Suppose $n^2 + 1$ is even. Prove by contradiction that n is odd.

(b) Is the converse of (a) true? Justify your answer.

Question 2 (10 Marks)

Prove by induction that for all natural number n,

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

Question 3 (10 Marks)

There are 6 boys and 5 men waiting for their turn in a barber shop. Two particular boys are A and B, and one particular man is Z. There is a row of 11 seats for the customers. Find the number of ways of arranging them in each of the following cases:

(a) A and B are at two ends;

(b) no two of A, B and Z are adjacent.

Question 4 (10 Marks)

The sequence $\{a_n\}$ is defined recursively as follows:

$$a_0 = 1 \text{ and } a_n = a_{n-1} + n \text{, for } n \geq 1.$$

Using the backtracking method to find an explicit formula for a_n.

You may use the formula

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

Question 5 (10 Marks)

Solve the following recurrence relation using the characteristic equation

$$a_n = 3a_{n-1} - 2a_{n-2}$$

with initial conditions $a_1 = 5$ and $a_2 = 3$.

Question 6 (10 Marks)

(a) Using membership table, show that $A \cap \overline{B} = A - B$.

(b) Let $U = \{a, b, c, d\}$ and A, B, C be subsets of U . Give a counter example to show that $A \cup (B - C) = (A \cup B) - (A \cap C)$ is false.

Question 7 (10 Marks)

The relation R is defined on the set $\mathbb{Z}^*$ of all nonzero integers by

$$aRb \leftrightarrow ab > 0 \ .$$

Prove that R is an equivalence relation. What are the equivalence classes?

Question 8 (10 Marks)

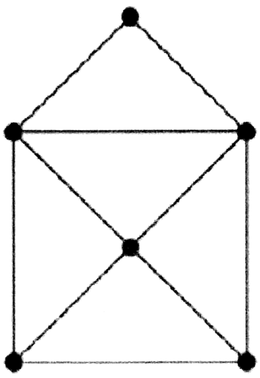
In each of the following cases, either construct a bijection (one -to-one and onto function) from the set X to the set Y if it exits, or explain why such a bijection cannot exit.

(a)
$X =$ set of integers; $Y =$ set of integers that are multiples of 5.

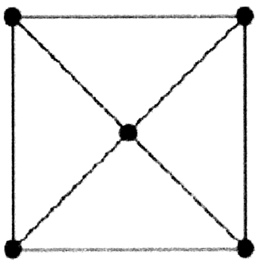
(b)
X = set of positive integers;
Y = set of integers < -2000.

Question 9 (10 Marks)

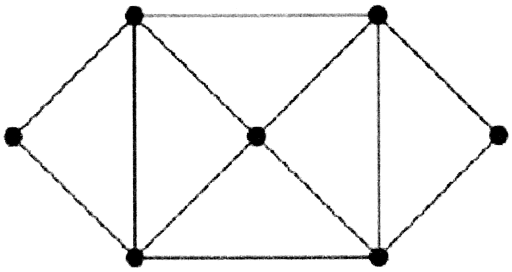
Which of the graphs below have an Euler path? Which have an Euler circuit? Justify your answers.



Graph 1



Graph 2



Graph 3

Question 10 (10 Marks)

Fibonacci number is defined by the recurrence relation

$$F_0 = 0, F_1 = 1,\ F_n = F_{n-1} + F_{n-2}\ ,\ n = 2,3,\dots$$

The number of binary strings of length n that contains no adjacent 0's is F_{n+2} .

(a) What is the number of binary strings of length n that contains no adjacent 1's? Justify your answer.

(b) What is the number of binary strings of length n that contains no adjacent 0's and the last digit is 1? Justify your answer.

(c) What is the number of binary strings of length n that contains no adjacent 0's and the last digit is 0? Justify your answer.

– END OF PAPER –

List of useful Formulae

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Fibonacci sequence, 0,1,1,2,3,5,8,13,21,34,55,.... .

$$F_n=F_{n-1}+F_{n-2} \text{ with } F_0=0,\ F_1=1 .$$

The number of binary strings of length n that contains no adjacent 0's is F_{n+2} .

A linear homogeneous relation of degree 2 is of the form

$$a_n=c_1a_{n-1}+c_2a_{n-2} .$$

The characteristic equation of the above equation is

$$x^2=c_1x+c_2 \text{ i.e., } x^2-c_1x-c_2=0 .$$

Suppose the characteristic equation has two distinct roots s_1 , s_2 .

Then $a_n=u(s_1)^n+v(s_2)^n$.

Suppose that the characteristic equation has only ONE root s .

Then $a_n=u\ s^n+vn\ s^n$.

where u,v are determined by initial conditions.

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.