NANYANG TECHNOLOGICAL UNIVERSITY

CE1007/CZ1007 DATA STRUCTURES

Review: **Binary Trees**

College of Engineering

School of Computer Science and Engineering

LAB TEST 2

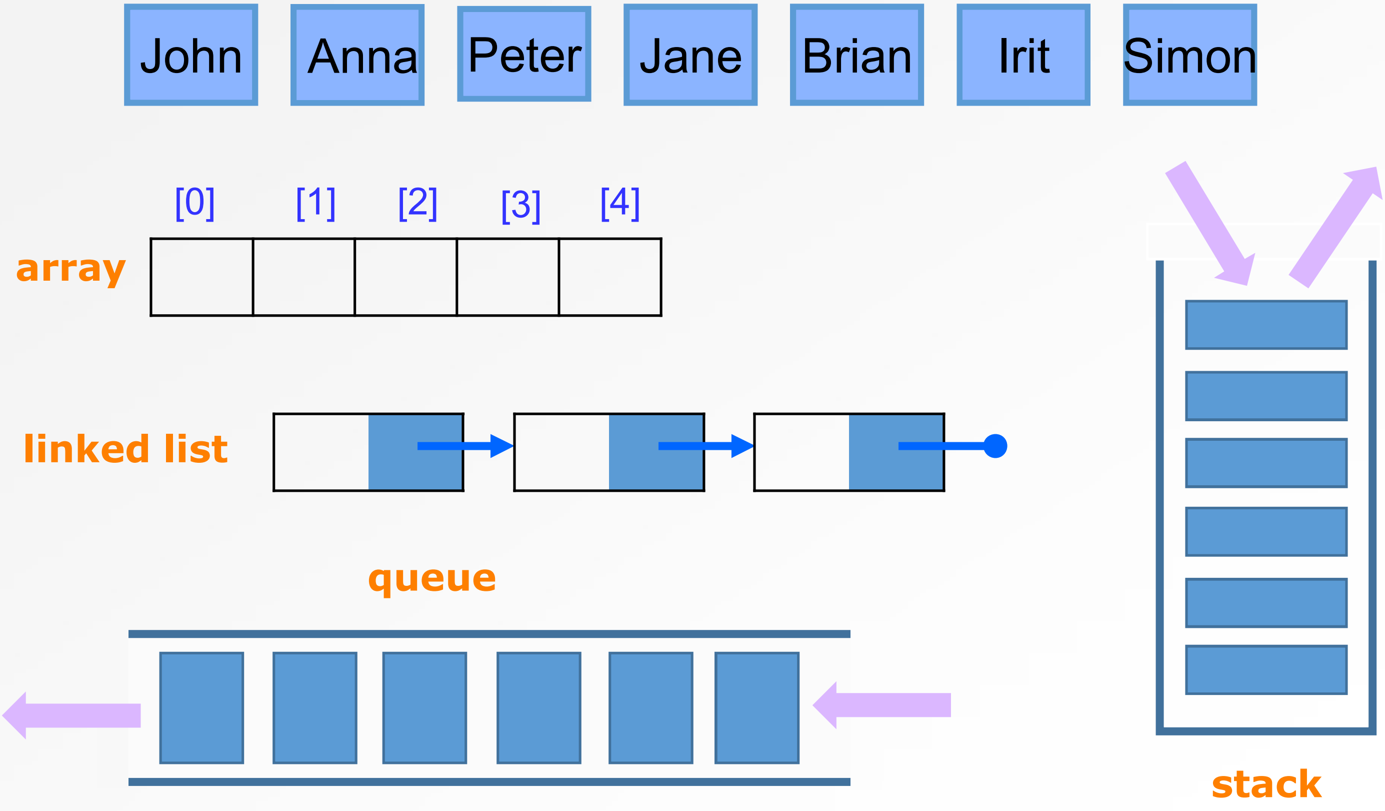
- Date : **18th Nov 2019**
- Time : **1pm – 3pm**
- Venue : TBA
- Seating Arrangement : TBA
- Topics (100 marks):
  - Linked List
  - Stack & Queue
  - Binary Tree
  - Binary Search Tree
- Overall weightage : 40%

OUTLINE

- **Non-linear data structures**
- Tree data structure
  - Binary trees
- Implement binary tree nodes in C
- Binary Tree Traversal
- Tree traversal order
  - Pre-order
  - In-order
  - Post-order
- Application examples
  - Count nodes in a binary tree
  - Find grandchild nodes
  - Calculate height of every node
- Level-by-level traversal
- Preorder traversal with a stack

LINEAR DATA STRUCTURE

- Array, linked list, queue, stack



John Anna Peter Jane Brian Irit Simon

[0] [1] [2] [3] [4]

array

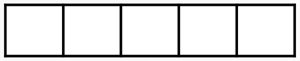
linked list

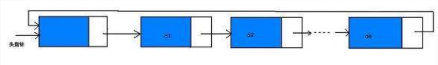
queue

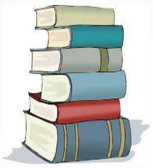
stack

DATA STRUCTURES SO FAR...

- Linear
  - Items all arranged one after another
  - Random access
    - Arrays
  - Sequential access
    - Linked list
  - Limited-access sequential
    - Stacks
    - Queues



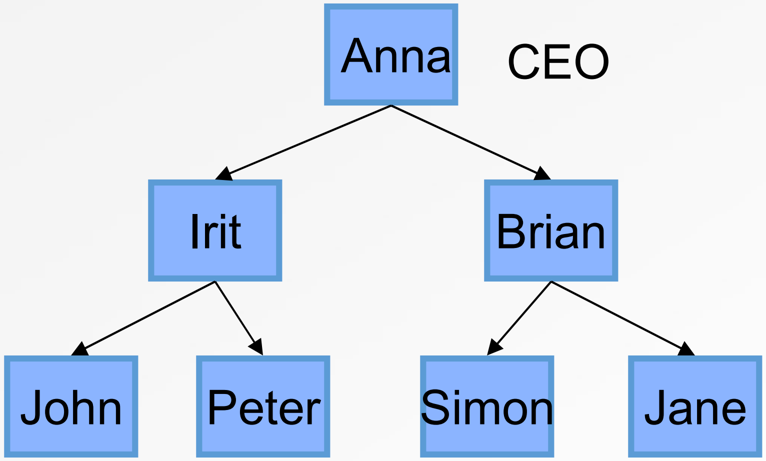




- Used them to store lists of numbers, lists of people, lists of moves, etc
  - Linear data

NON-LINEAR DATA STRUCTURE

- Suppose you have a set of names



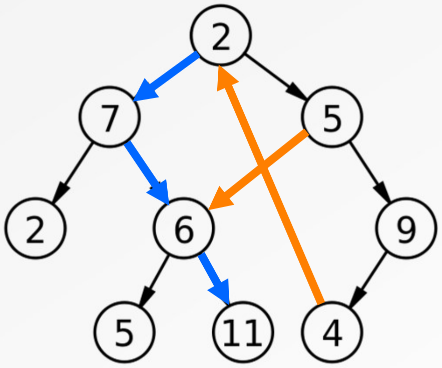
Tree

- Company organization

Not good to use linear data structure to store <u>hierarchical relationships</u>

TREE DATA STRUCTURE

- Still using nodes + links representation
- New idea:
  - Each node can have links to more than one other node
  - No loop



Observe that:

- **If we follow one path of a tree, we get a linked list**

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  - **Binary trees**
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  - Pre-order
  - In-order
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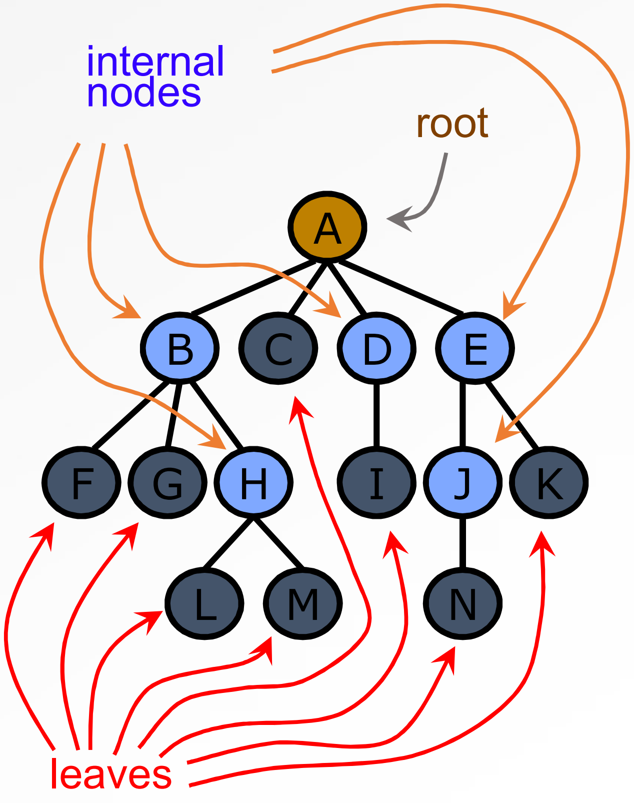
TREE DATA STRUCTURE

- A tree is composed of nodes
- Each node contains a value
- Types of nodes
  - **Root**: only one in a tree, has no parent.
  - Internal (non-leaf): Nodes with children are called **internal nodes**
  - Leaf: nodes without children are called **leaves**

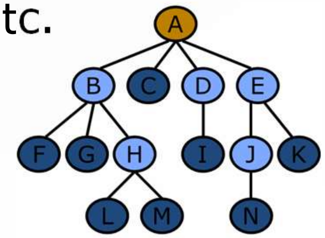


WHY TREES?

- Model layouts with hierarchical relationships between items
  - Chain of command in the army
  - Personnel structure in a company
  - (**Binary** tree structure is limited because each node can have **at most two children**)

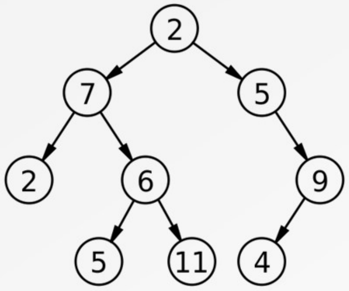
- Tree structures also allow us to
  - Some problems require a tree structure: some games, most optimization problems, etc.
  - Allow us to do the following very quickly:

    (we'll see that in the following lectures)
    - **Search for a node with a given value**
    - **Add a given value to a list**
    - **Delete a given value from a list**

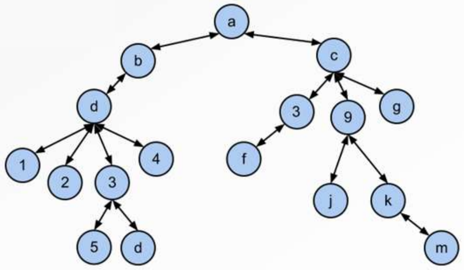


TREE DATA STRUCTURE

- Tree data structure looks like… a tree:
  - Only one root node (no nodes points to it)
  - Each node branches out to some number of nodes
  - Each node has only one "parent" node – the node pointing to it (except the root node)



Binary tree



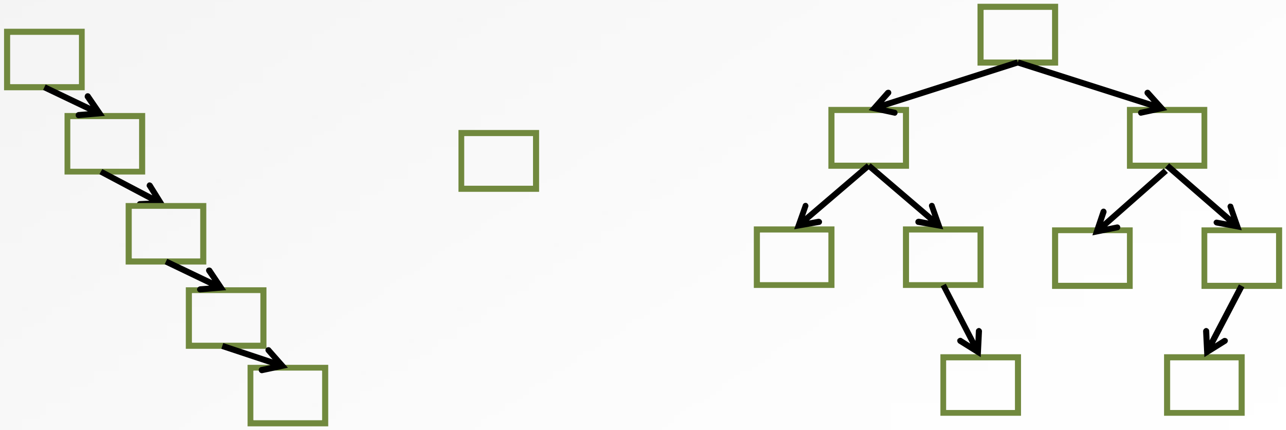
General tree

- General tree
  - Each node can have links to any number of other nodes

- **<u>Binary</u> tree (we'll work with this in our course)**
  - **Each node can have links to at most two other nodes**

POSSIBLE TREE CONFIGURATIONS

- Has to do with balance of a tree



OUTLINE

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- **Implement binary tree nodes in C**
- Binary Tree Traversal
- Tree traversal order
  - Pre-order
  - In-order
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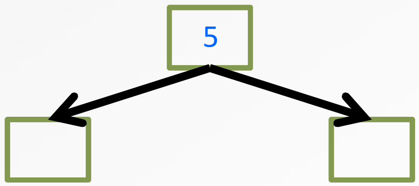
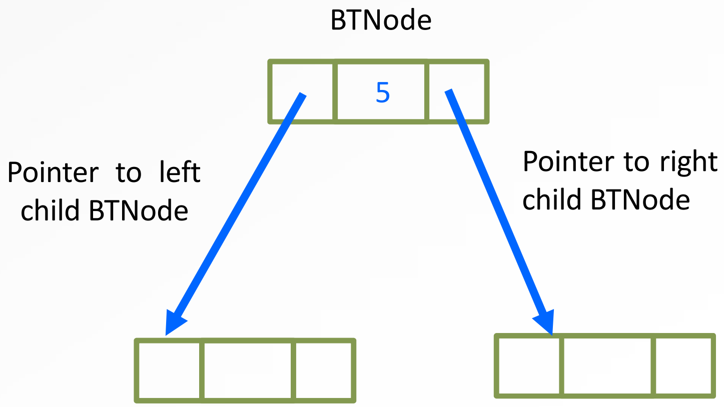
IMPLEMENTATION

- Recall implementation of LinkedList
  - Node has link to **at most <u>one</u>** other node
  - Defined a ListNode with one **`next`** pointer and a data **`item`**

```
typedef struct _listnode{
    int item;
    struct _listnode *next;
}ListNode;
```

- BinaryTree
  - Node has link to **at most <u>TWO</u>** other nodes
  - Define a BTNode with
    - Two pointers
    - A data item



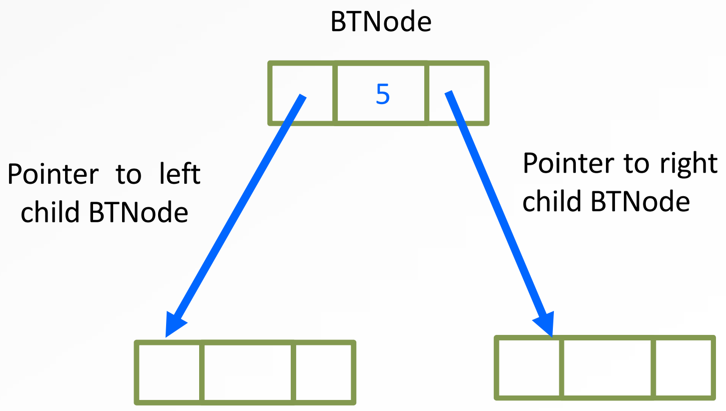


BTNode

- Start with a simple BTNode that stores an integer
  - The type of item can be character, string, or structure, etc.

```
typedef struct _btnode{
    int item;
    struct _btnode *left;
    struct _btnode *right;
} BTNode;
```

```
typedef struct _listnode{
   int item;
   struct _listnode *next;
}ListNode;
```



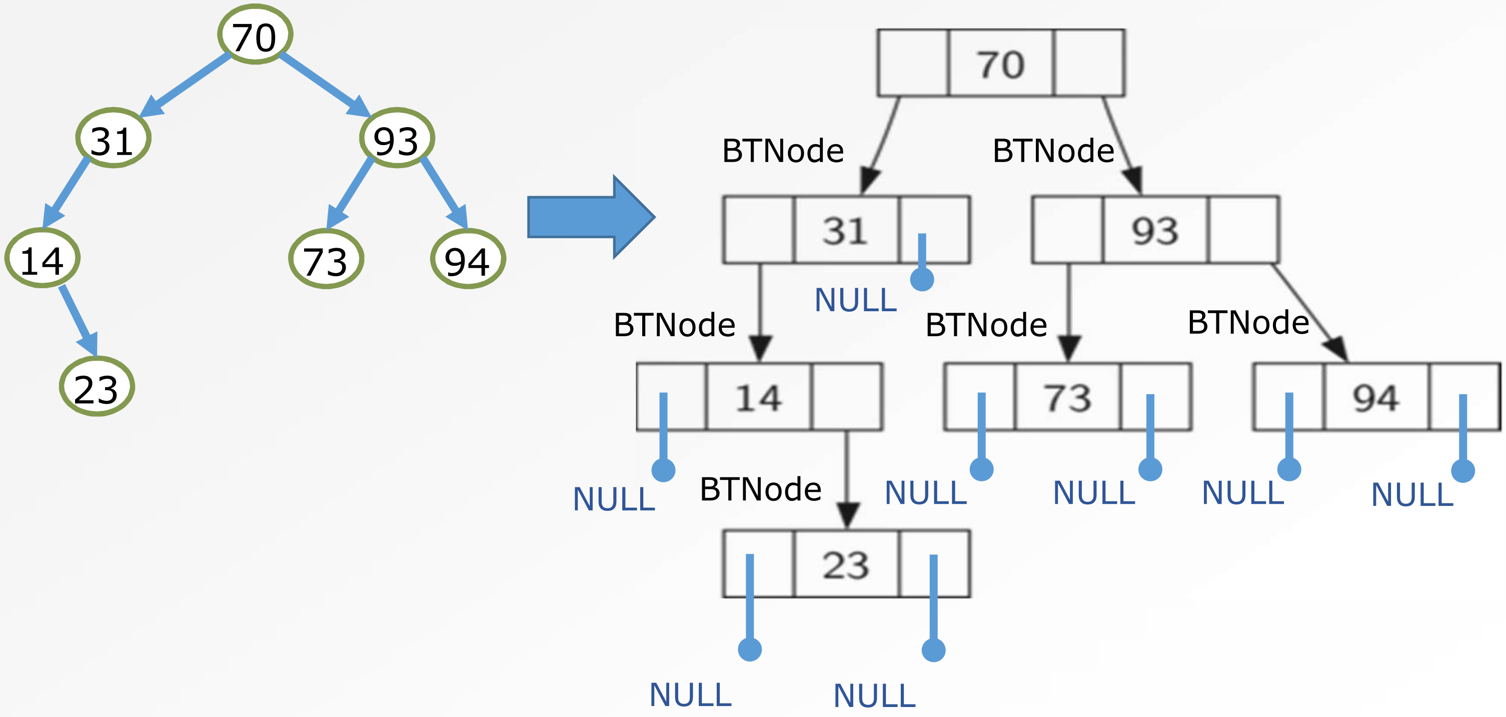
BTNode

5

Pointer to left child BTNode

Pointer to right child BTNode

EXAMPLE BINARY TREE



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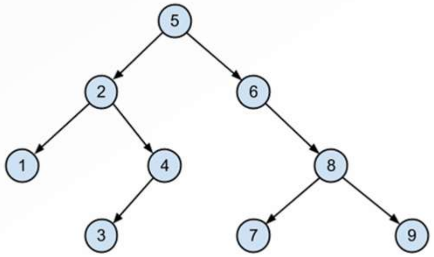
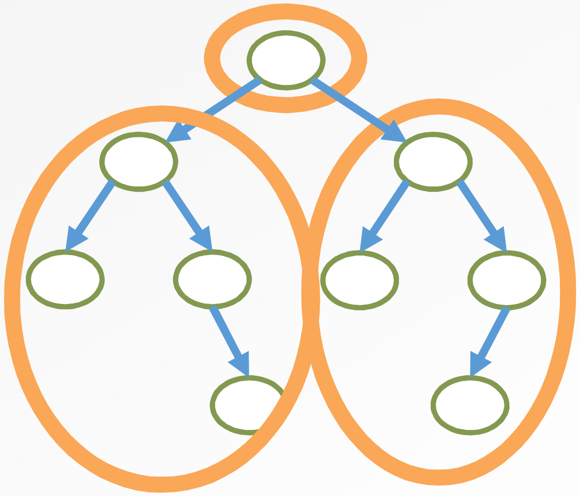
TREE TRAVERSAL

- Given a linear data structure and a particular item, very obvious what the "next" item is
  - Each node has an obvious "previous" and "next" node
- Trees are non-linear structures
  - How to extract data from a binary tree?
  - What is the traversal sequence?
    left/left/left, then left/left/right, then...?
- Need a systematic way to visit every node in the tree
  - Clearly defined steps
  - **No repeated** visits to nodes



TREE TRAVERSAL

- Why is this important?
  - Tree traversal is foundation for many functions
- Very common function template:



Traverse tree

- At each node, perform some operation

- Example task: count # of nodes in a tree

At every node N, size of that subtree

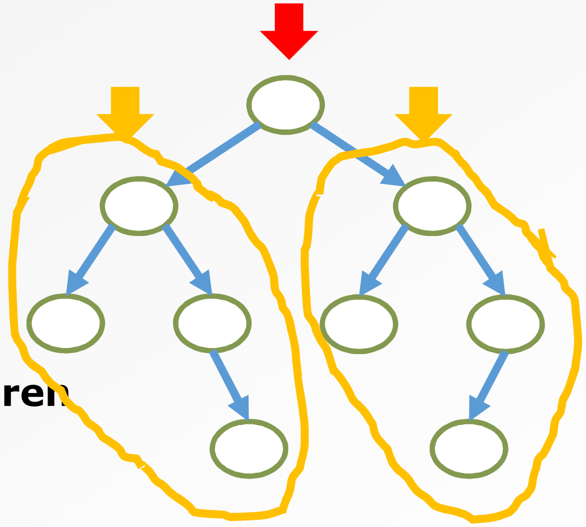
= size of N's left subtree

+ size of N's right subtree

+ N itself

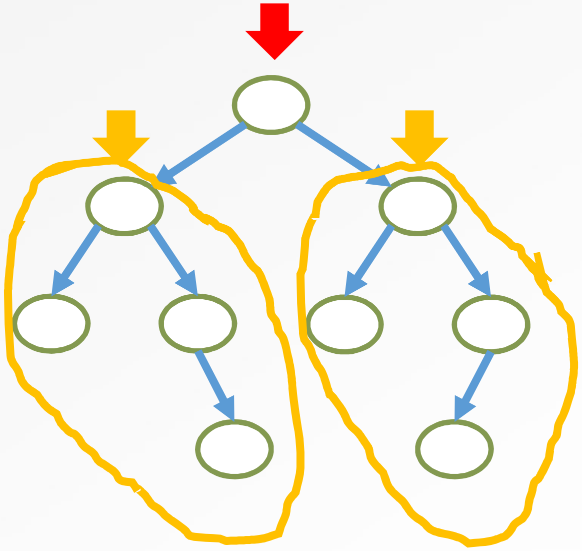
TREE TRAVERSAL

- Tree traversal is recursive
  - Recursion: is the process of repeating items in a self-similar way; divide a problem into several similar sub-problems.
  - **At each node**
    - **Visit the node and both children**
- Initial case + repeating case
  - **(Visit root) + (visit children)**
- When combined, guarantees that all nodes will be visited once and only once



TREE TRAVERSAL PROCESS

```
TreeTraversal(Node N):
    Visit N;
    If (N has left child)
        TreeTraversal(LeftChild);
    If (N has right child)
        TreeTraversal(RightChild);
    Return; // return to parent
```



TREE TRAVERSAL TEMPLATE #1

Pseudocode

```
TreeTraversal(Node N):
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        TreeTraversal(RightChild);
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```
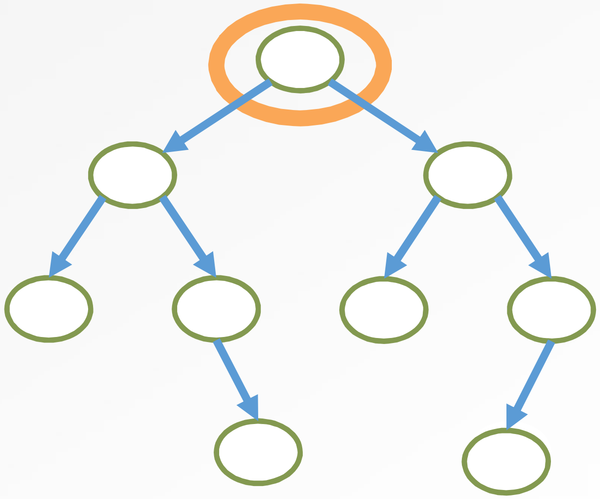
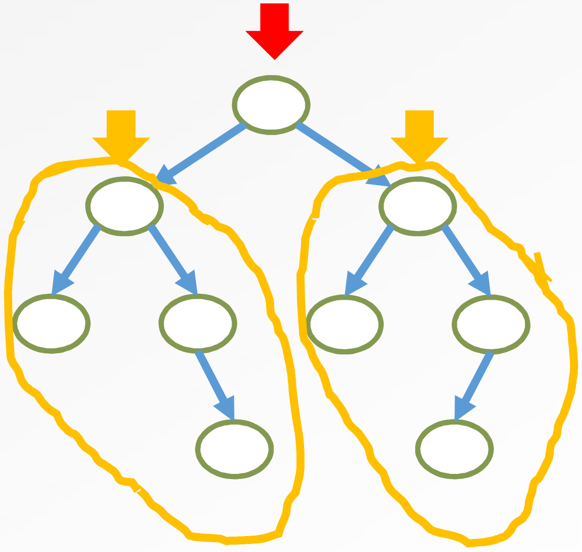



In main(), call TreeTraversal(root)

TREE TRAVERSAL TEMPLATE #2

- Current function:
  - Need to check for existence of left and right children before following them

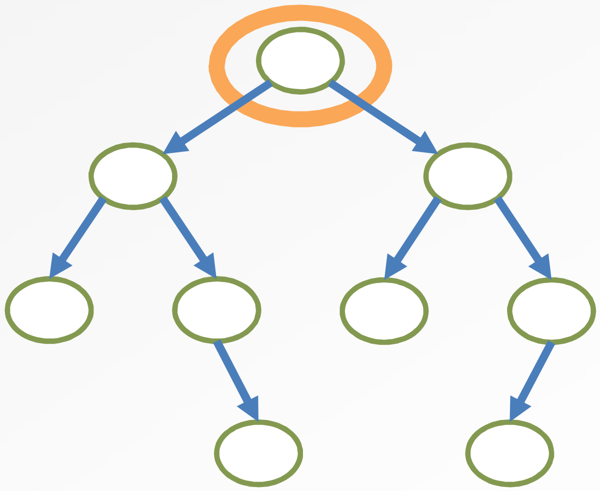
- New version:
  - Always follow links to children
  - Then check if the link is **NULL**
  - In other words, not actually pointing at a BTNode



TREE TRAVERSAL TEMPLATE #2

Pseudocode

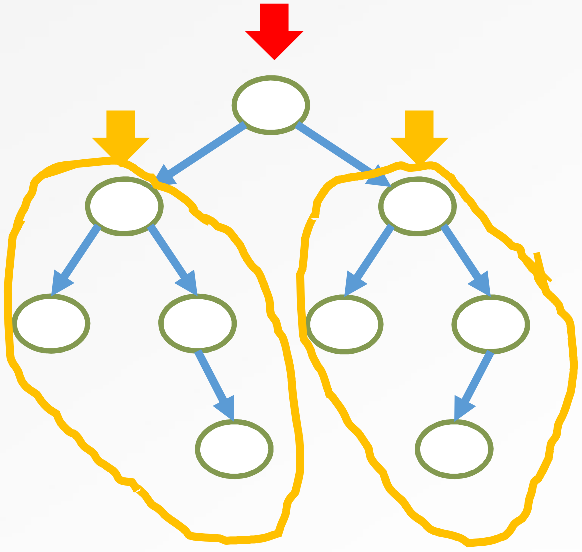
```
TreeTraversal2(Node N):
    If N==NULL return;
    Visit N;
    TreeTraversal2(LeftChild);
    TreeTraversal2(RightChild);
    Return; // return to parent
```



In main(), call TreeTraversal2(root)

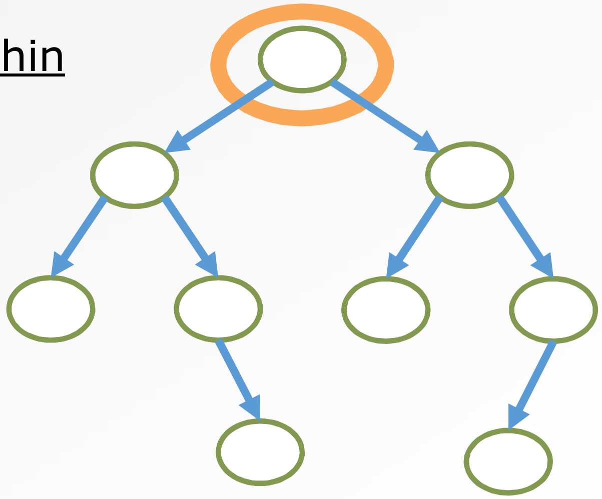
TreeTraversal2() IMPLEMENTATION

```
Void TreeTraversal2(BTNode *cur){

     If (cur == NULL) return;

     PrintNode(cur); // visit cur
     TreeTraversal2(cur->left);
     TreeTraversal2(cur->right);
}
```



TREETRAVERSAL() FEATURES

- Recursive
  - TreeTraversal() is called <u>from within its own body</u>
  - initial call TreeTraversal(root)



- Depth-first
  - The traversal goes as <u>deep</u> as possible before backtracking and going sideways
  - Not level-by-level! (that is called breadth-first)

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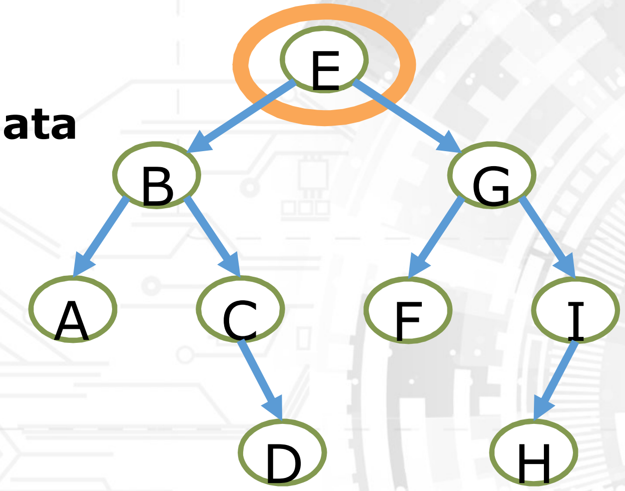
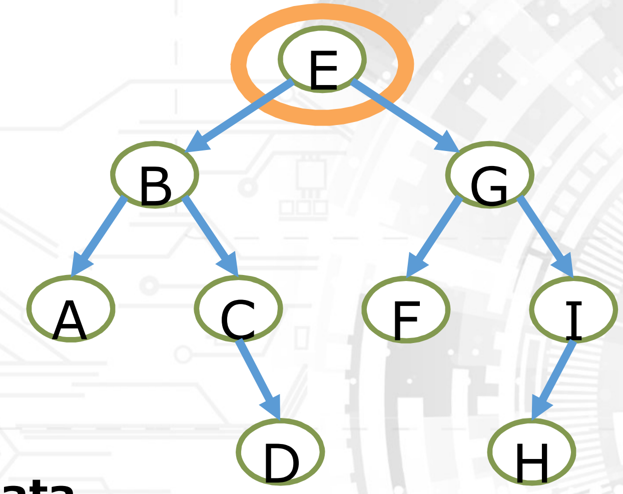
THREE “STANDARD” WAYS TO TRAVERSAL

- Pre-order
  - **Process the current node’s data**
  - **Visit the left child subtree**
  - **Visit the right child subtree**
- In-order
- Post-order



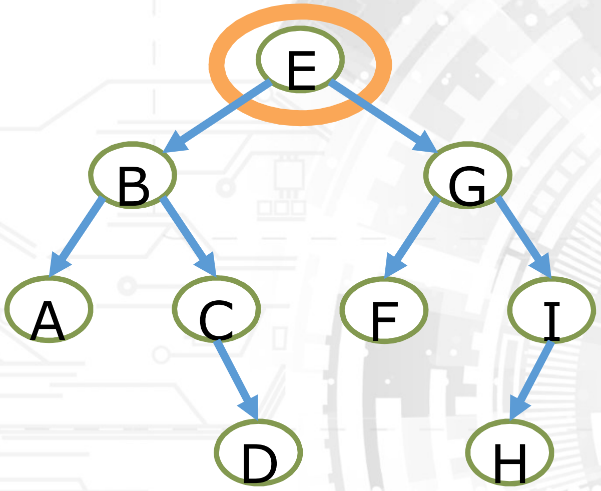
THREE "STANDARD" WAYS TO TRAVERSAL

- Pre-order
  - Process the current node's data
  - Visit the left child subtree
  - Visit the right child subtree
- **In-order**
  - **Visit the left child subtree**
  - **Process the current node's data**
  - **Visit the right child subtree**
- Post-order



THREE "STANDARD" WAYS TO TRAVERSAL

- Pre-order
  - Process the current node's data
  - Visit the left child subtree
  - Visit the right child subtree
- In-order
  - Visit the left child subtree
  - Process the current node's data
  - Visit the right child subtree
- **Post-order**
  - **Visit the left child subtree**
  - **Visit the right child subtree**
  - **Process the current node's data**



TREE TRAVERSAL - PRINT

- Recall the TreeTraversal() template (TT) – **Pre-order** :
  - Simple task at each node: <u>print out</u> data in that node

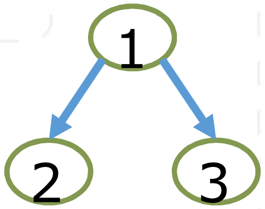
```
void TreeTraversal(BTNode *cur){
    if (cur == NULL)
        return;

    //  Do something with the current node's data

    TreeTraversal(cur->left); //Visit the left child node
    TreeTraversal(cur->right);//Visit the right child node
}
```

TREE TRAVERSAL - PRINT

- Recall the TreeTraversal() template (TT) – **Pre-order** :
    - Simple task at each node: print out data in that node

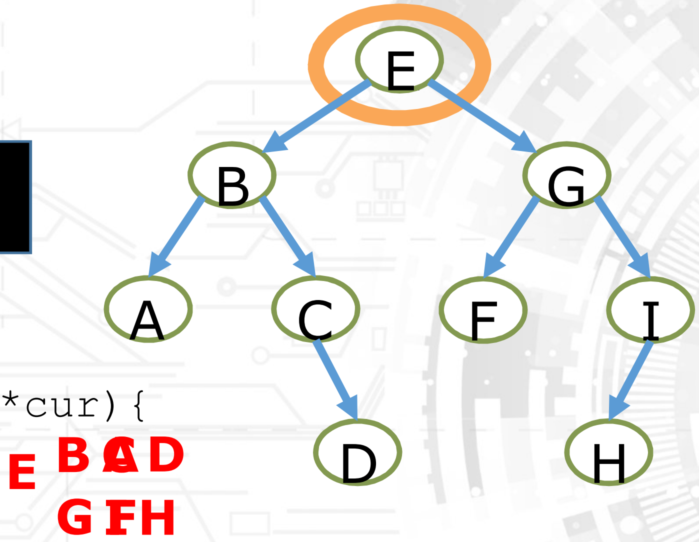
```
void TreeTraversal(BTNode *cur){
    if (cur == NULL)
        return;

    printf(“%c”,cur->item);

    TreeTraversal(cur->left); //Visit the left child node
    TreeTraversal(cur->right);//Visit the right child node
}
```

TREE TRAVERSAL PRE-ORDER: PRINT



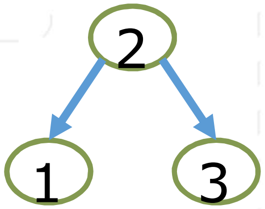
Output:

```
E B A C D G F I H
```



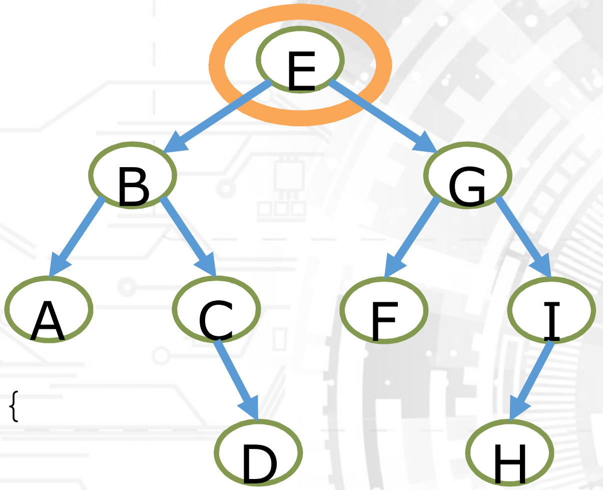
```
void TreeTraversal_pre(BTNode *cur){
    if (cur == NULL)
        return;

    printf("%c  ",cur->item);

    TreeTraversal_pre(cur->left); //Visit the left child node
    TreeTraversal_pre(cur->right);//Visit the right child node
}
```

TREE TRAVERSAL IN-ORDER: PRINT



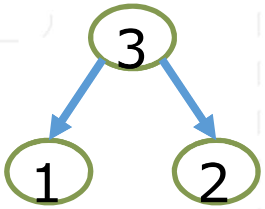
Output:

```
A  B  C  D  E  F  G  H  I
```



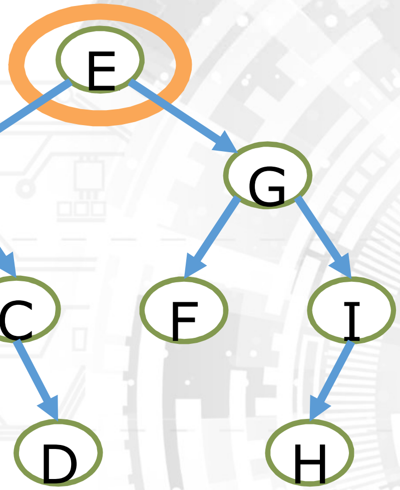
```
void TreeTraversal_in(BTNode *cur){

    if (cur == NULL)
        return;

    TreeTraversal_in(cur->left); //Visit the left child node
    printf(“%c  ”,cur->item);
    TreeTraversal_in(cur->right);//Visit the right child node
}
```

TREE TRAVERSAL POST-ORDER: PRINT



Output:

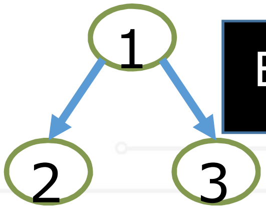
```
A D C B F H I G E
```



```
void TreeTraversal_post(BTNode *cur){
    if (cur == NULL)
        return;

   TreeTraversal_post(cur->left); //Visit the left child node
   TreeTraversal_post(cur->right);//Visit the right child node
   printf(“%c  ”,cur->item);
}
```

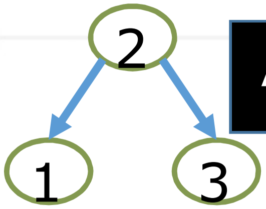
PRE-ORDER, IN-ORDER AND POST-ORDER

Pre-Order Traversal



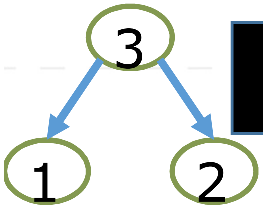
E B A C D G F I H

In-Order Traversal

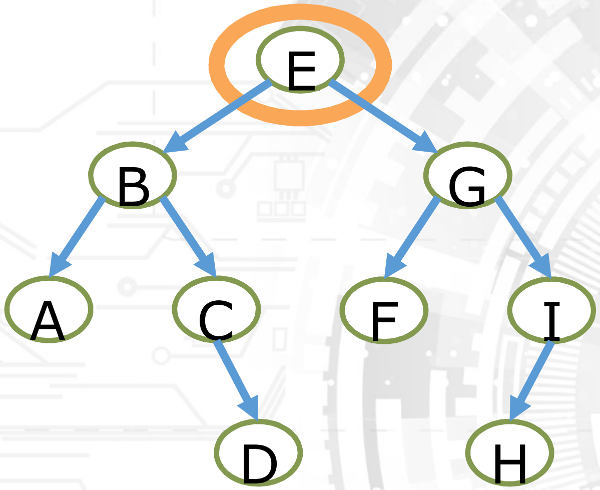


A B C D E F G H I

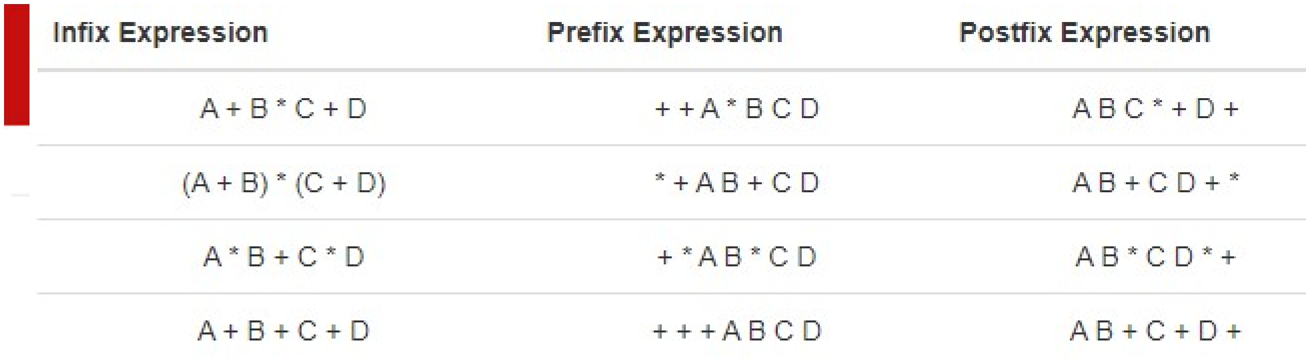
Post-Order Traversal



A D C B F H I G E

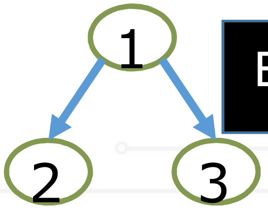


ST-ORDER



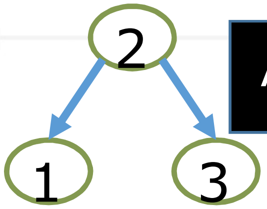
Infix Expression	Prefix Expression	Postfix Expression
A + B * C + D	+ + A * B C D	A B C * + D +
(A + B) * (C + D)	* + A B + C D	A B + C D + *
A * B + C * D	+ * A B * C D	A B * C D * +
A + B + C + D	+ + + A B C D	A B + C + D +

Pre-Order Traversal



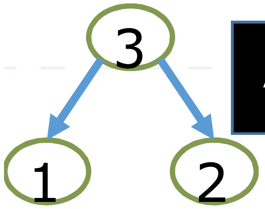
E B A C D G F I H

In-Order Traversal

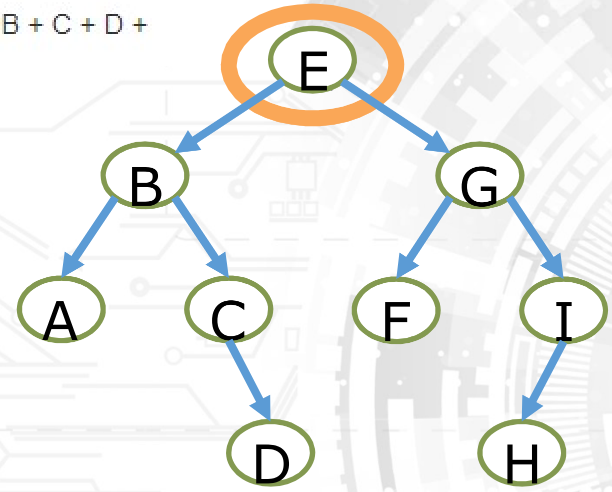


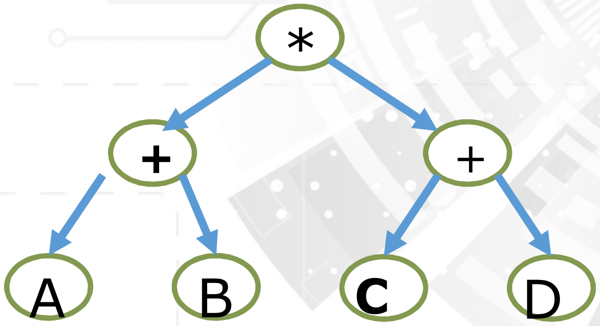
A B C D E F G H I

Post-Order Traversal



A D C B F H I G E





OUTLINE

- Non-linear data structures
- Tree data structure
  - Binary trees
- Implement binary tree nodes in C
- Binary Tree Traversal
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  - Pre-order
  - In-order
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  - **Count nodes in a binary tree**
  - Find grandchild nodes
  - Calculate height of every node
- Level-by-level traversal
- Preorder traversal with a stack

COUNT NODES IN A BINARY TREE

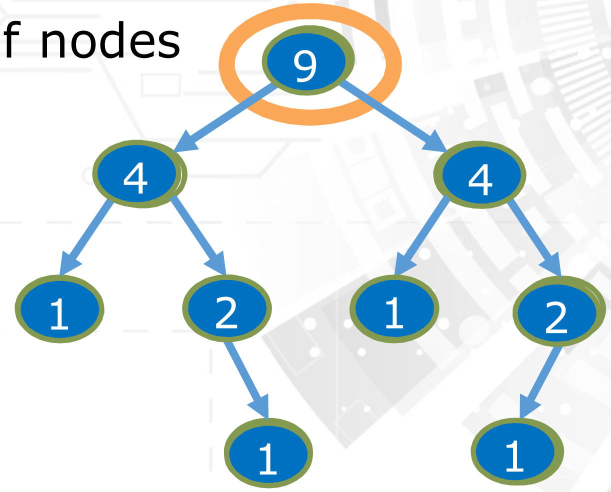
- Recursive definition:
  - Number of nodes in a tree

    = 1

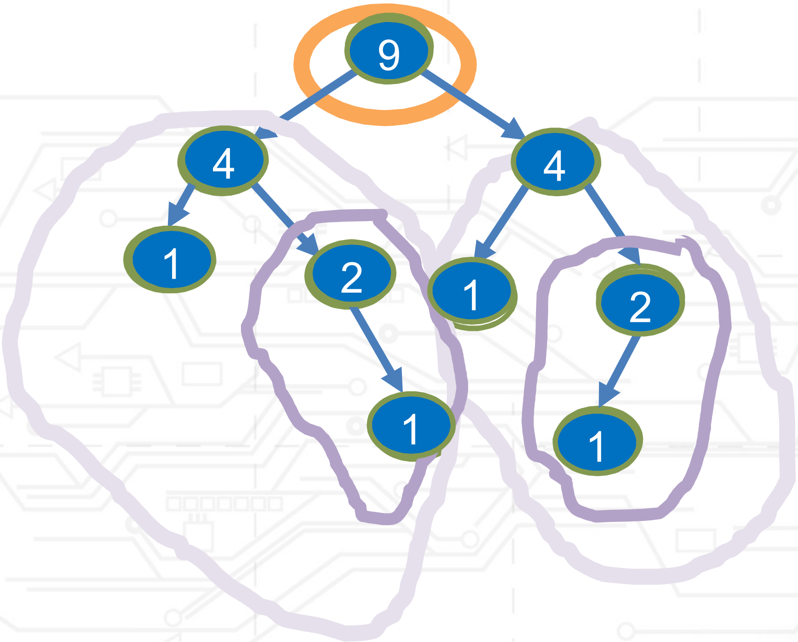
      + number of nodes in left subtree

      + number of nodes in right subtree

- Each node returns the number of nodes in its subtree



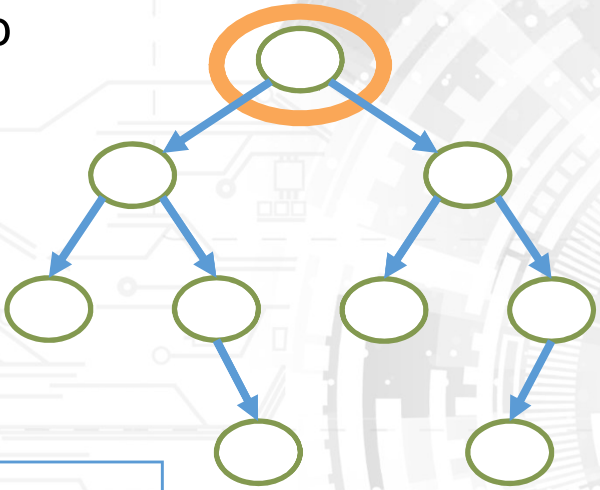
COUNT NODES IN A BINARY TREE



- Each node returns the number of nodes in its own subtree
- Leaf nodes return 1
  Information **propagates upwards** as TreeTraversal returns from visiting leaf nodes
- Which is the first/last count to be returned?

countNode()

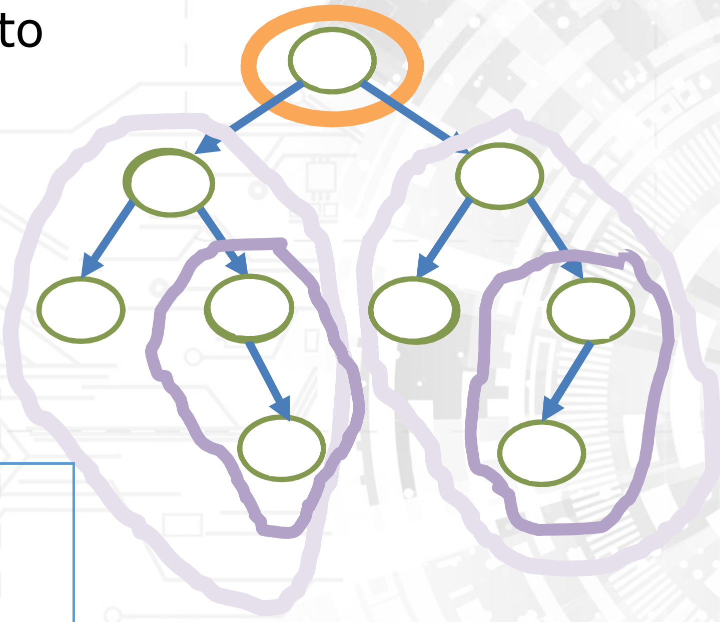
- Return the size of your subtree to your parent node
- Leaf nodes must return 1 to parent node
- Root node returns size of entire tree



```
void TreeTraversal(BTNode *cur){
    if (cur == NULL)
       return;
    //may do something with cur;
    TreeTraversal(cur->left);
    TreeTraversal(cur->right);
    //may do something with cur;
}
```

countNode()

- Return the size of your subtree to your parent node
- Leaf nodes must return 1 to parent node
- Root node returns size of entire tree



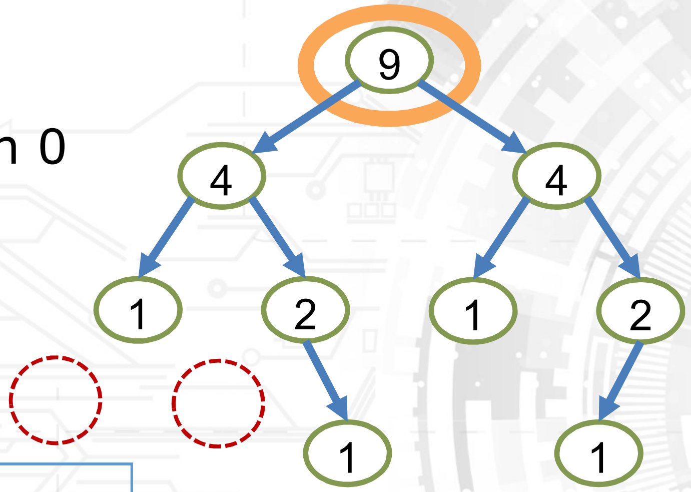
```
int countNode(BTNode *cur){

    if (cur == NULL)
        return ???;


    countNode(cur->left);
    countNode(cur->right);
    ??? //sum and get total;
}
```

countNode()

- Leaf nodes must return 1
  - “Null” nodes should return 0
- Leaf node returns 1 + 0 + 0

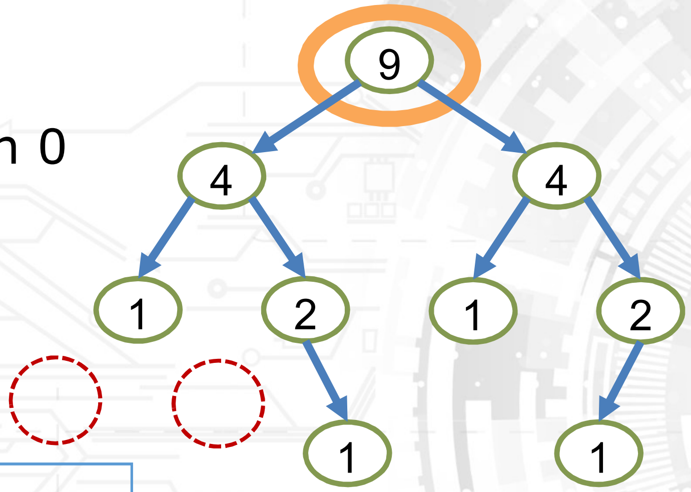


```
int countNode(BTNode *cur){
    if (cur == NULL)
       return 0;

 l =countNode(cur->left);
 r =countNode(cur->right);
    return l+r+1;
}
```

countNode()

- Leaf nodes must return 1
    - “Null” nodes should return 0
- Leaf node returns 1 + 0 + 0



```
int countNode(BTNode *cur){
    if (cur == NULL)
        return 0;

    return (countNode(cur->left)
          + countNode(cur->right)
          + 1);
}
```

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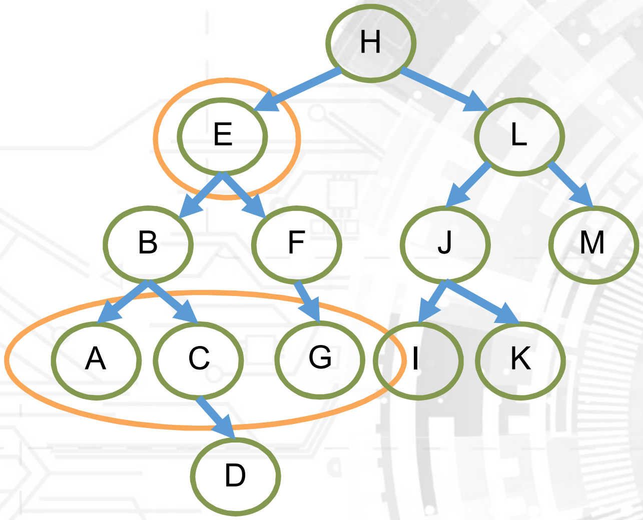
FIND GRANDCHILDREN

- Given a node X, find all the nodes that are X's grandchildren
- Given node E, we should return grandchild nodes A, C, and G
- What if we want to find k-level grandchildren?
  - **Need a way to keep track of how many levels down we've gone**

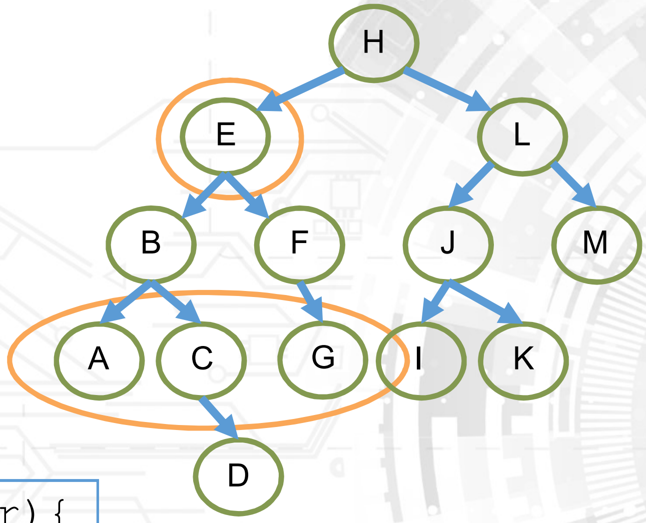


X->left->left
X->left->right
X->right->left
X->right->right

2-level grandchildren

FIND GRANDCHILDREN

- We want to go down k “levels”
- Use a counter to track how far down we’ve gone
- At each TreeTraversal(child), increment counter



```
void TreeTraversal(BTNode *cur){

    if (cur == NULL)
        return;

    // check counter

    TreeTraversal(cur->left);
    TreeTraversal(cur->right);
}
```

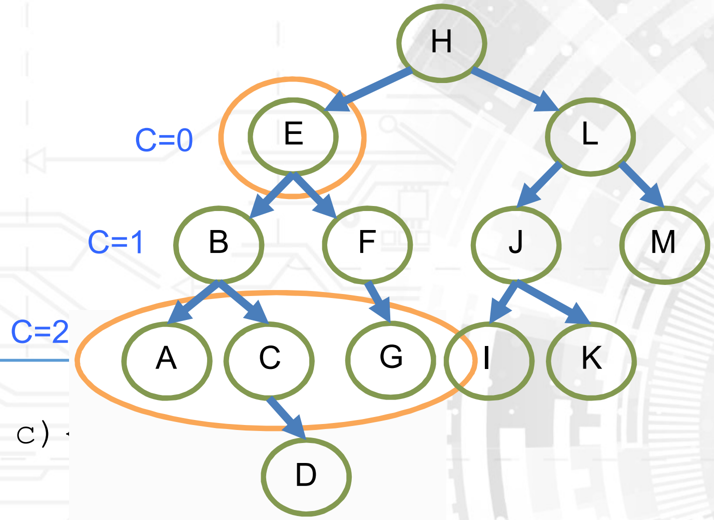
Do something with the current node’s data

Visit the left child node

Visit the right child node

FIND GRANDCHILDREN

```
void main( ){ …
     if (X == null)return;
     findgrandchildren(X,0);
}
```



```
void findgrandchildren(
                BTNode *cur, int c)
     if (cur == NULL) return;

     if (c == k){
        printf("%d ", cur->item);
        return;
     }
     if (c < k){
        findgrandchildren(cur->left, c+1);
        findgrandchildren(cur->right, c+1)};
}
```

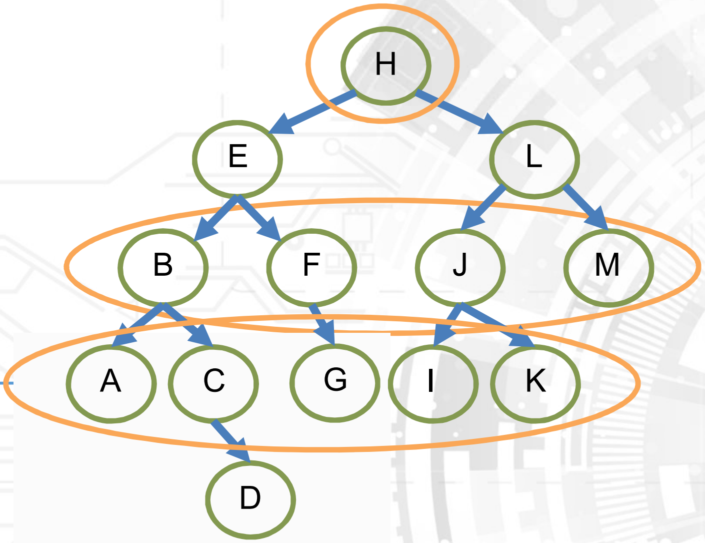
X= E, k = 2

FIND GRANDCHILDREN

```
void main( ){ …
     if (X = null)return;
     findgrandchildren(X,0);
}
```

```
void findgrandchildren(
                BTNode *cur, int c){
     if (cur == NULL) return;

     if (c == k){
        printf(“%d ”, cur->item);
        return;
     }
     if (c < k){
        findgrandchildren(cur->left, c+1);
        findgrandchildren(cur->right, c+1); }
}
```



if k=2, we call

findgrandchildren(H,0),

what is the output?

How about k=3?

How about

findgrandchildren(H,1)?

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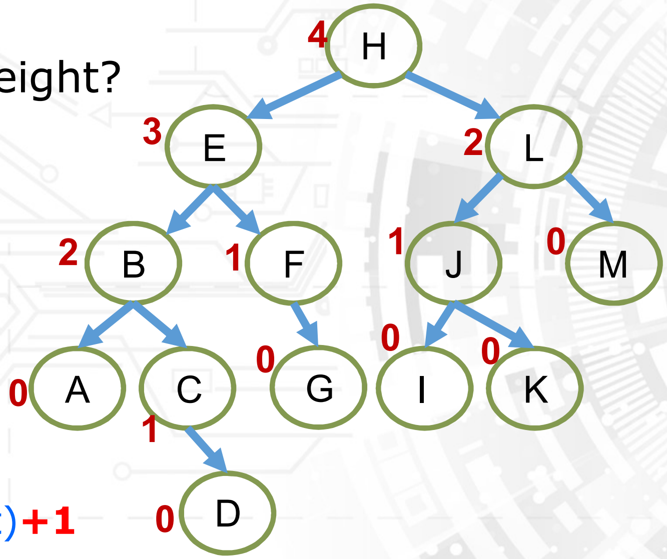
CALCULATE HEIGHT OF EVERY NODE

- Height of a node = number of links from that node to the deepest leaf node
- How does each node calculate its height?
  - What is the height of node D, C, H?



- We found:
  - leaf.height= 0
  - Non-leaf node X

X.height=**max**(X.left.height, X.right.height)**+1**

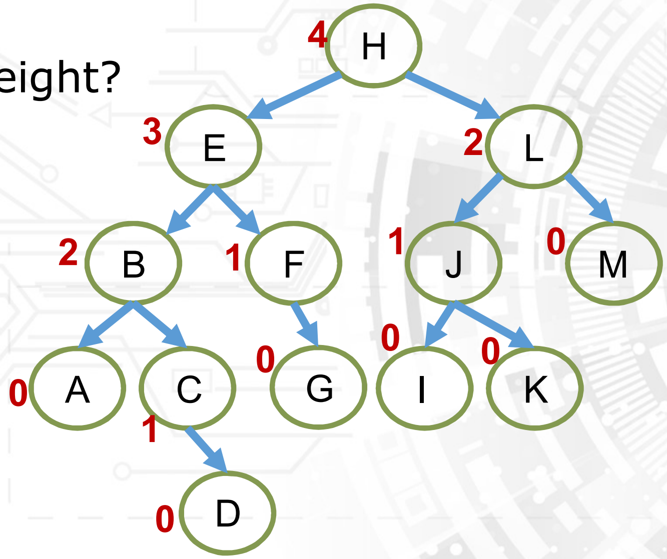
- Does information propagate upwards or downwards?

CALCULATE HEIGHT OF EVERY NODE

- Height of a node = number of links from that node to the deepest leaf node

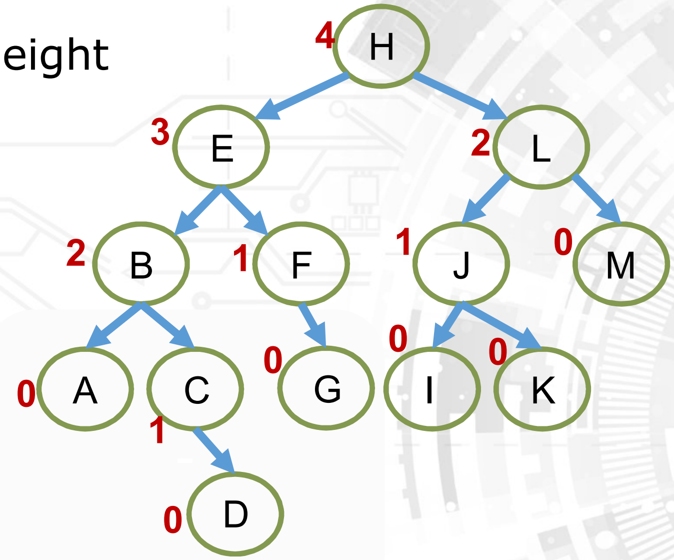
- How does each node calculate its height?
  - What is the height of node D, C, H?

- Go through entire tree: calculate and store height of each node in the item field



CALCULATE HEIGHT OF EVERY NODE

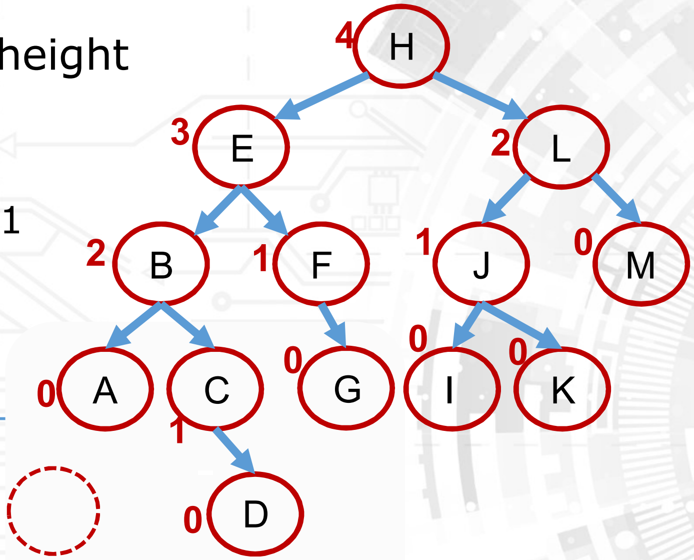
- We want each node to report its height
  - Leaf node must report 0



```
int TreeTraversal(BTNode *cur){

    if(cur == NULL)
       return   ;

    int l = TreeTraversal(cur->left);
    int r = TreeTraversal(cur->right);

    // do something here. Max( left, right)?

    return  ;

}
```

CALCULATE HEIGHT OF EVERY NODE

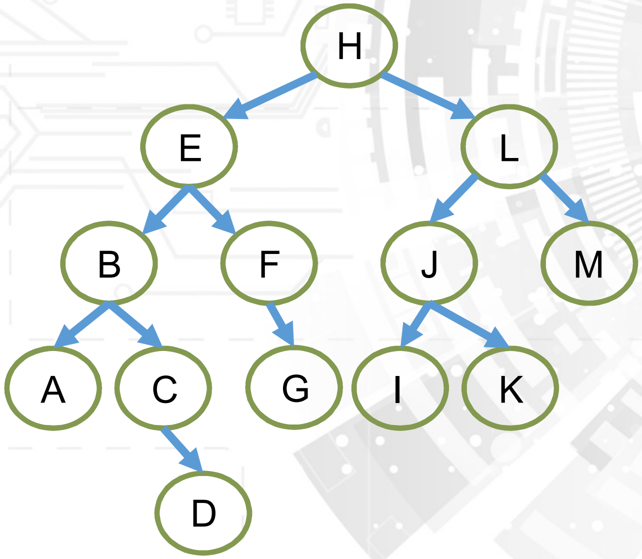
- We want each node to report its height
  - Leaf node must report 0
  - At “null” condition, must report -1



```
int TreeTraversal(BTNode *cur){

    if(cur == NULL)
       return -1;

    int l = TreeTraversal(cur->left);
    int r = TreeTraversal(cur->right);

    int c = max (l, r) + 1;

    return c;

}
```

QUESTIONS

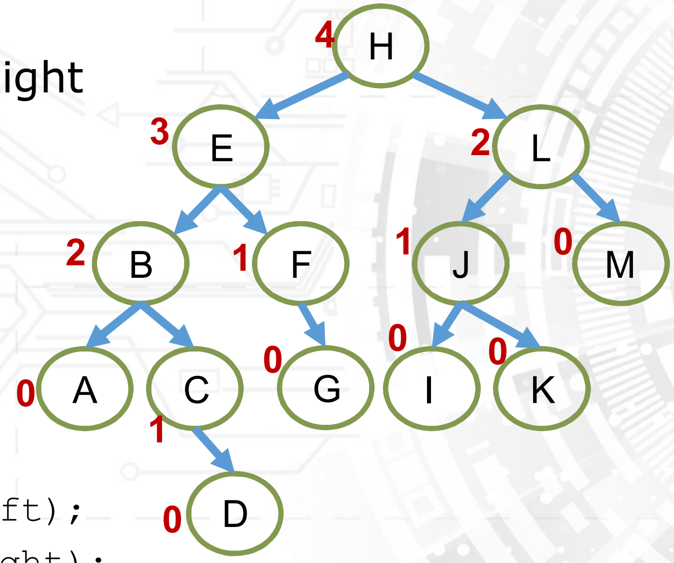
- Does the tree traversal order matter?
- Depth of a node = number of links from that node to the root node. How does each node calculate its depth?



CALCULATE HEIGHT OF EVERY NODE

- Height of a node = number of links from that node to the deepest leaf node
- We want each node to report its height
  - Leaf node must report 0
  - At "null" condition, must report -1

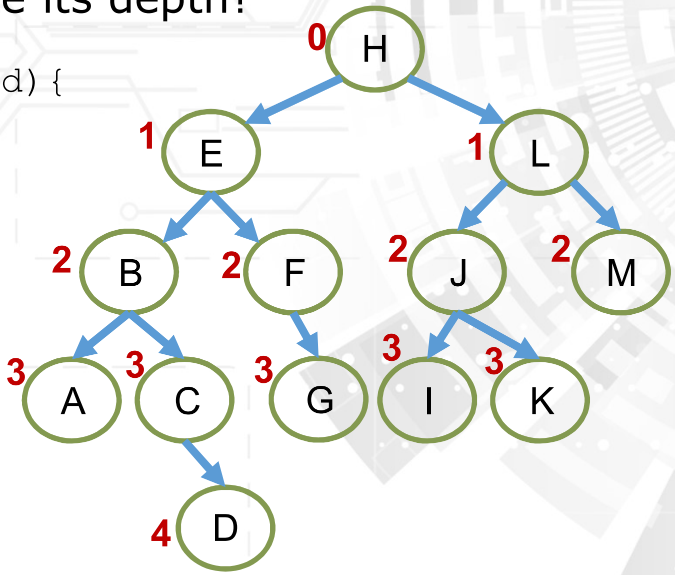
```
int TreeTraversal(BTNode *cur){
     if(cur == NULL)
         return -1;

     int l = TreeTraversal(cur->left);
     int r = TreeTraversal(cur->right);

     int c = max (l, r) + 1;

     return c;
}
```



QUESTIONS

- Does the tree traversal order matter?
- Height of a node = number of links from that node to the deepest leaf node
- Depth of a node = number of links from that node to the root node. How does each node calculate its depth?

```
void TreeTraversal(BTNode *cur, int d){
    if(cur == NULL)
        return;
    //print cur->item and d;
    TreeTraversal(cur->left, d+1);
    TreeTraversal(cur->right, d+1);
    return;
}
```



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LEVEL-BY-LEVEL: BREADTH-FIRST SEARCH

Depth-first search

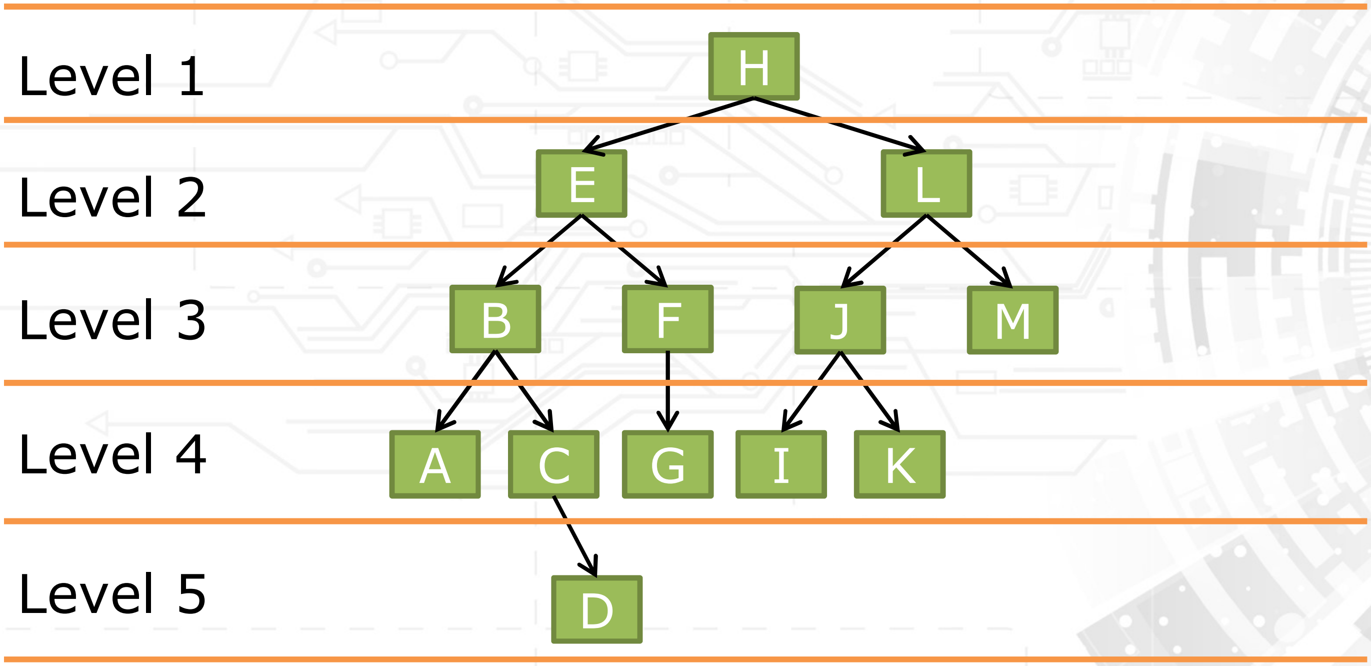
begins at the root and explores as far as possible along each branch before backtracking

E.g. the post-order traversal

Breadth-first search

begins at a root node and inspects all its children nodes. Then for each of those children nodes in turn, it inspects their children nodes, and so on.

LEVEL-BY-LEVEL TREE TRAVERSAL



Level 1

Level 2

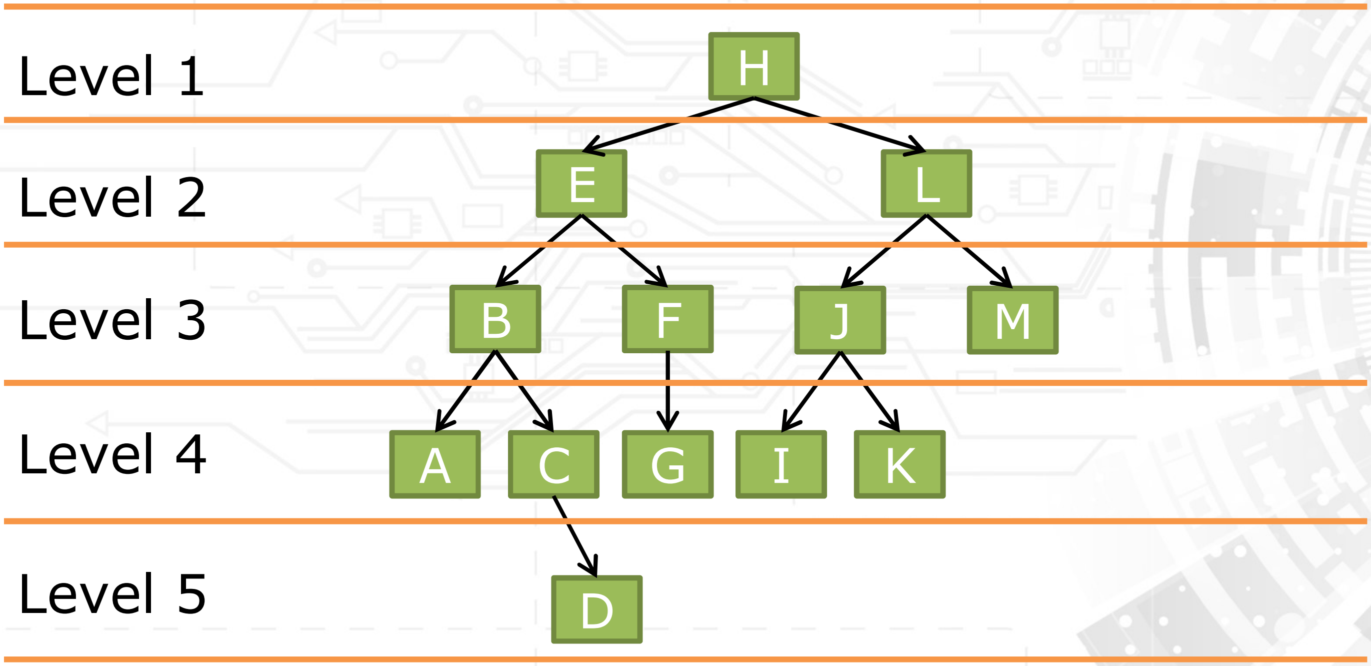
Level 3

Level 4

Level 5

LEVEL-BY-LEVEL TREE TRAVERSAL

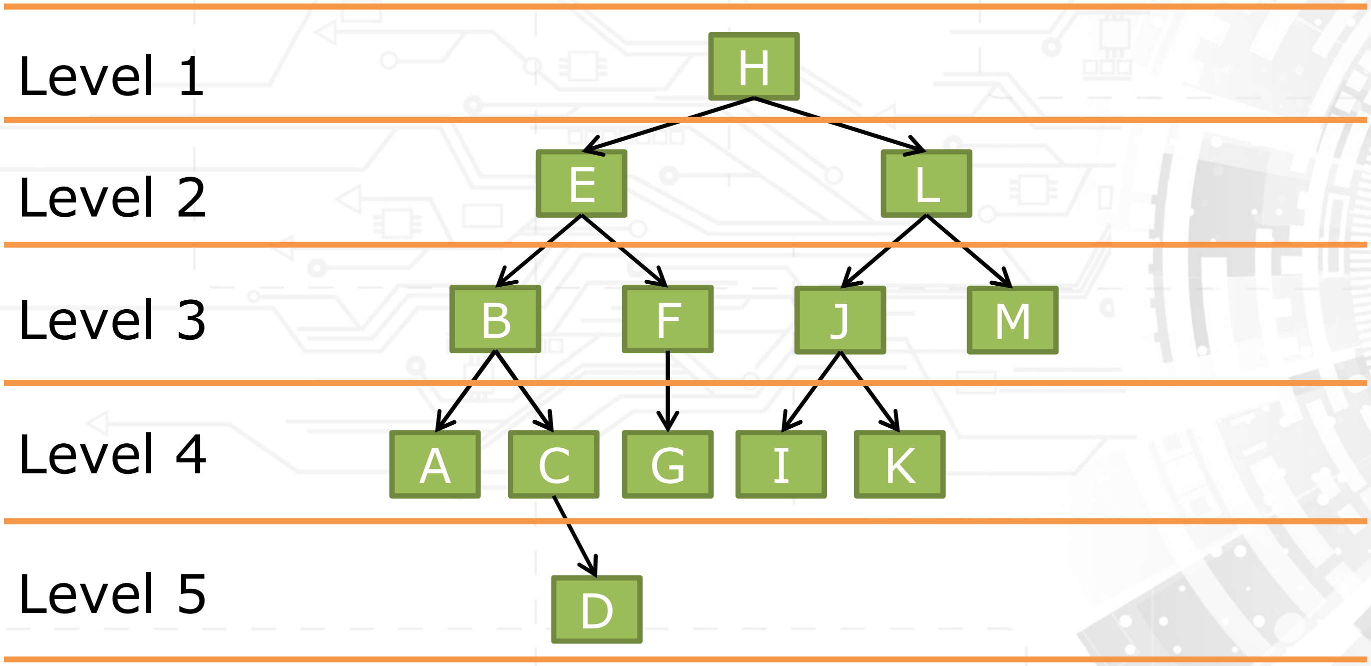
- Hint: Make use of another data structure



Nodes stored in order accessed in tree…

LEVEL-BY-LEVEL TREE TRAVERSAL

- Use a queue! Root node should be first



Level 1

Level 2

Level 3

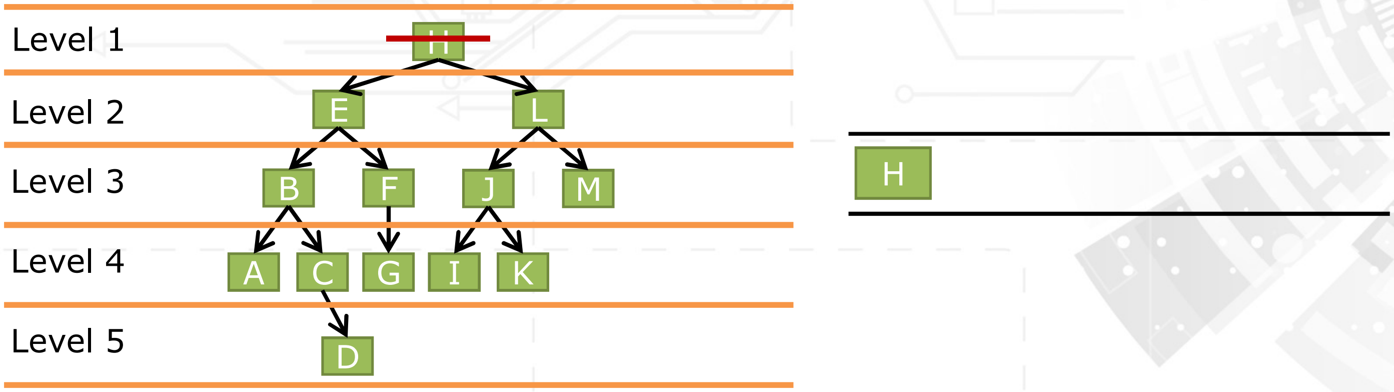
Level 4

Level 5

Nodes stored in order accessed in tree

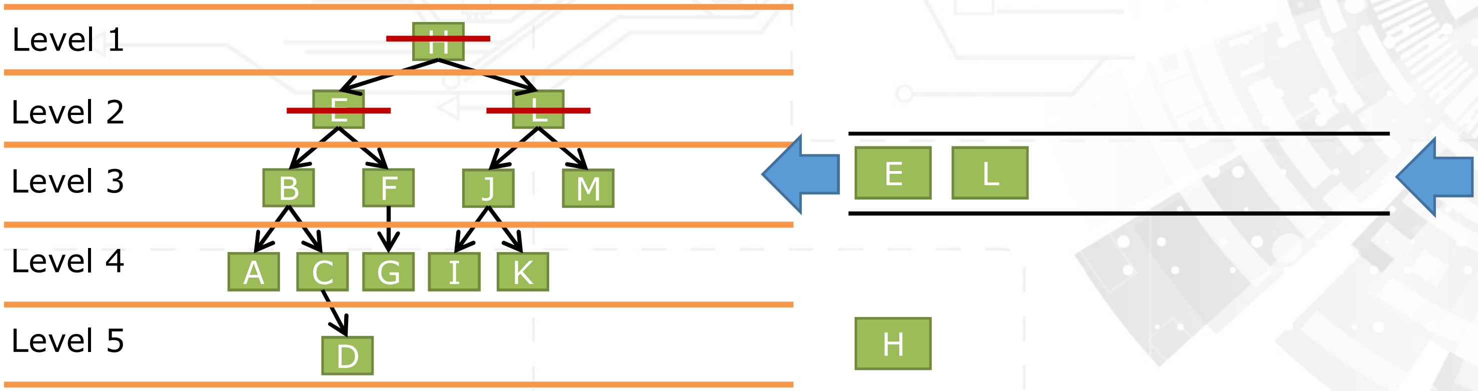
LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H



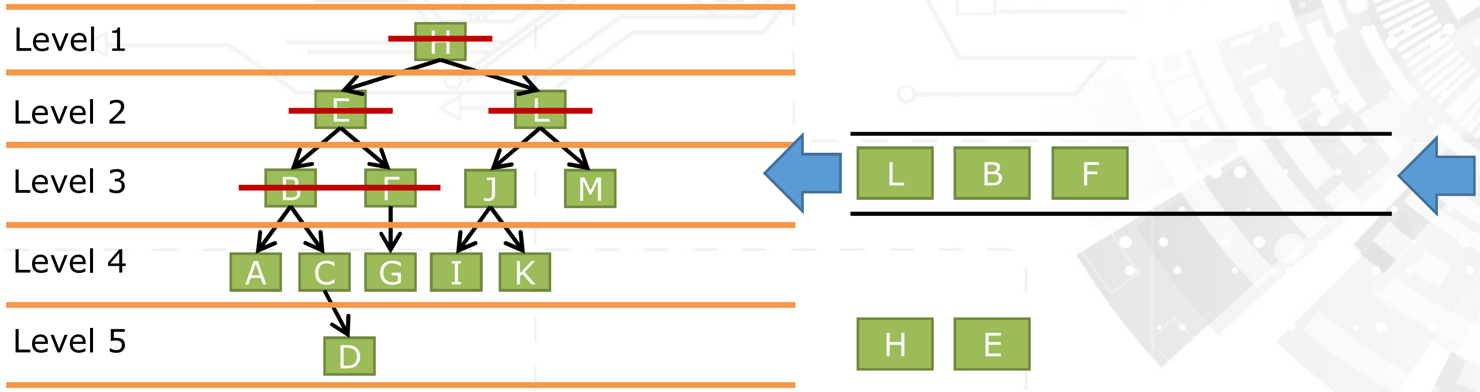
LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H’s children



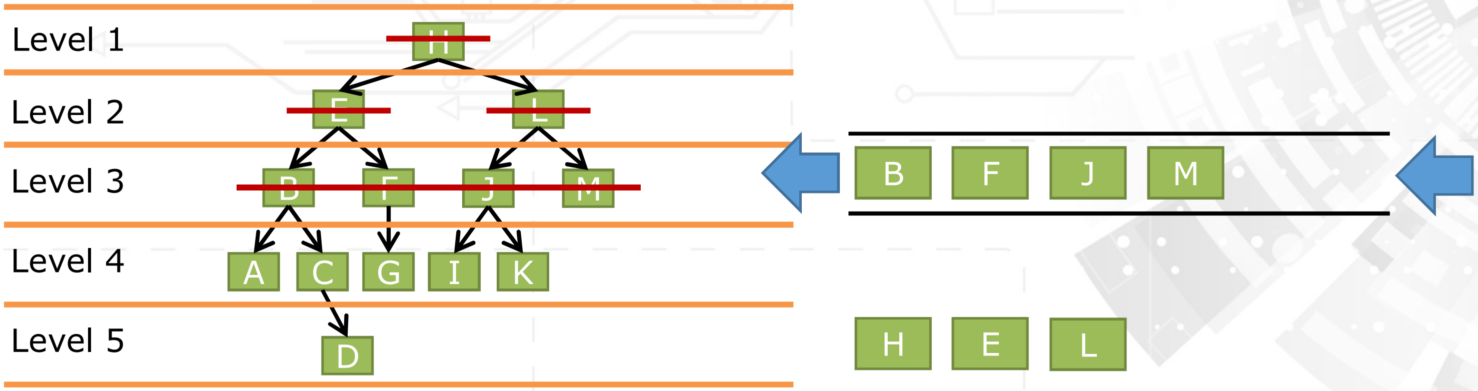
LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H’s children
- Dequeue E, and enqueue E’s children



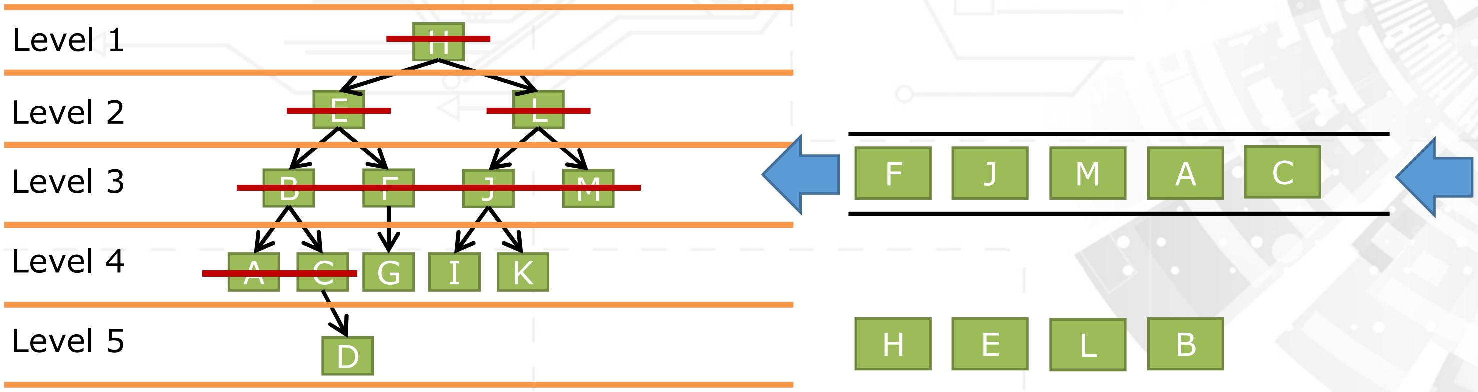
LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H's children
- Dequeue E, and enqueue E's children
- Dequeue L, and enqueue L's children



LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H’s children
- Dequeue E, and enqueue E’s children
- Dequeue L, and enqueue L’s children
- Dequeue B, and enqueue B’s children



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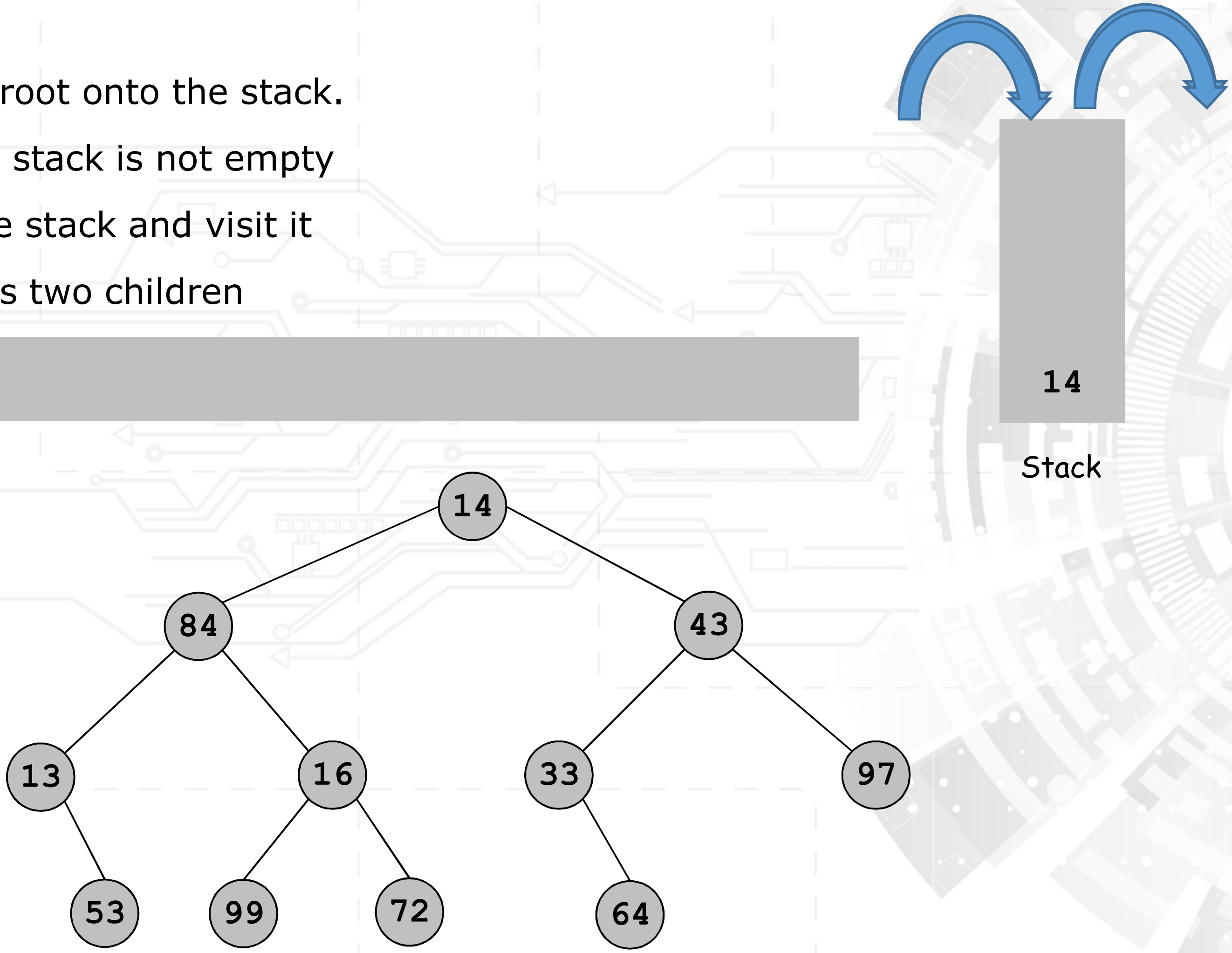
PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children



PREORDER TRAVERSAL WITH A STACK

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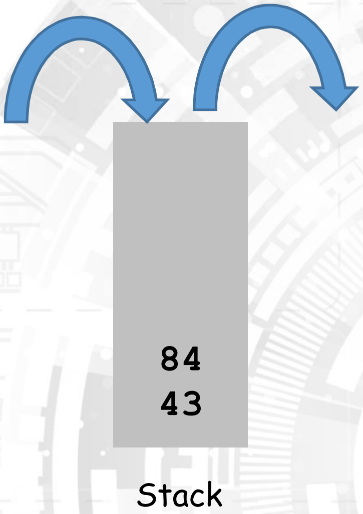
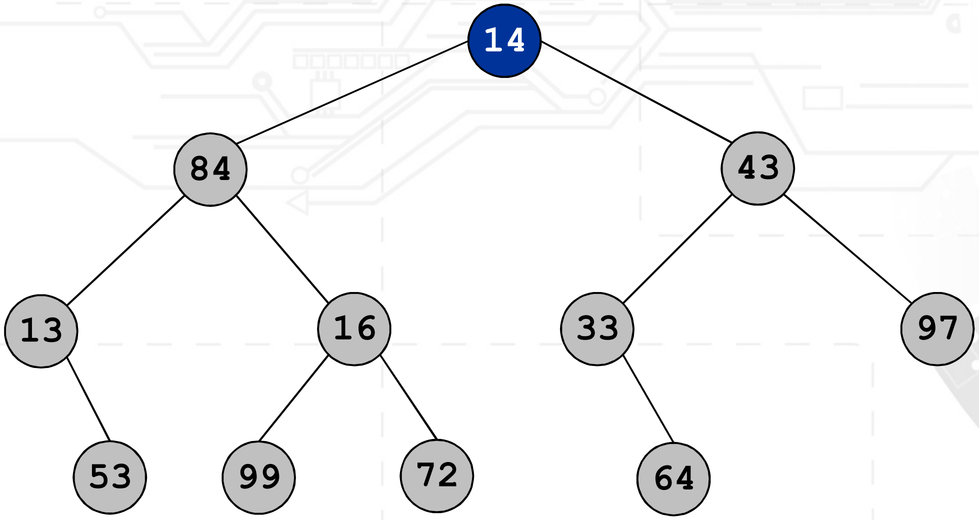
14





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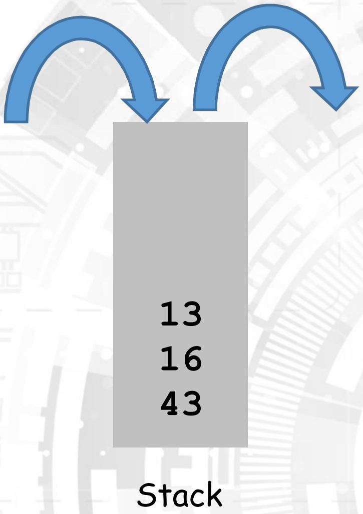
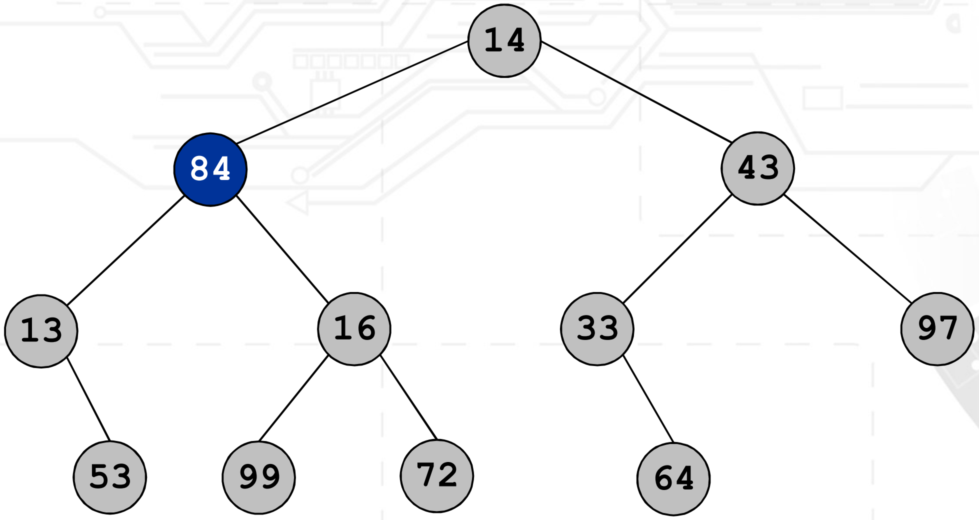
14 84





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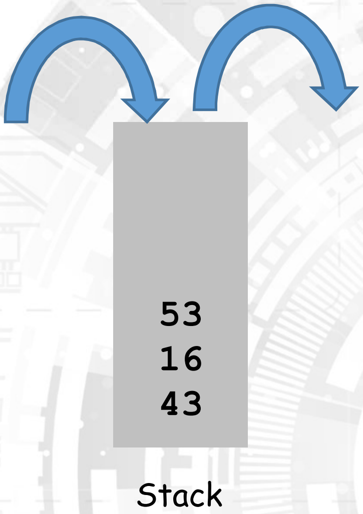
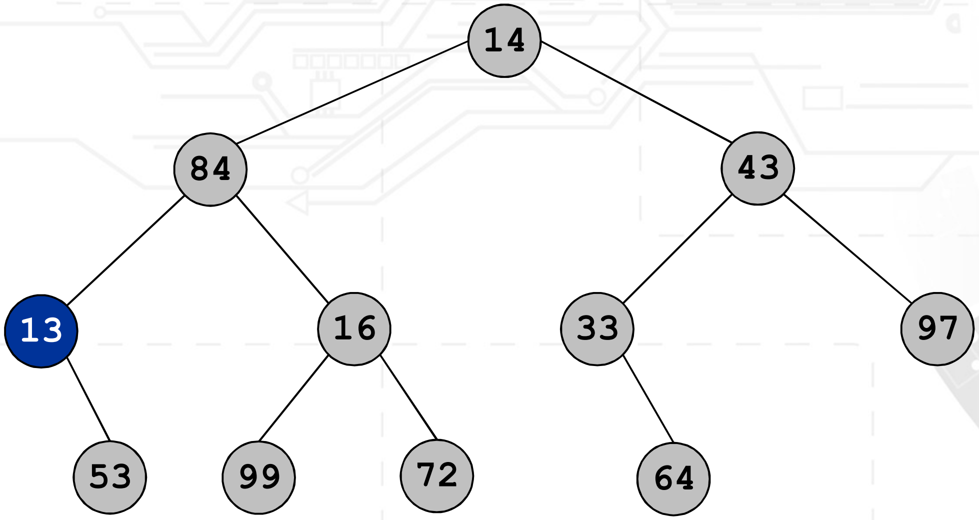
14 84 13





PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

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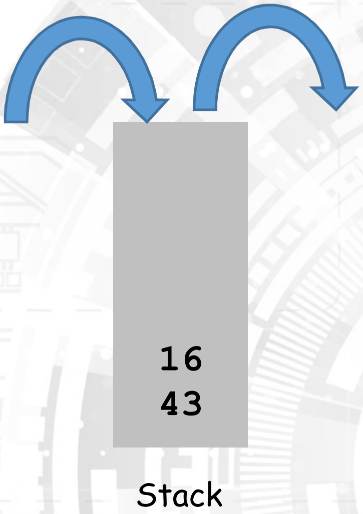
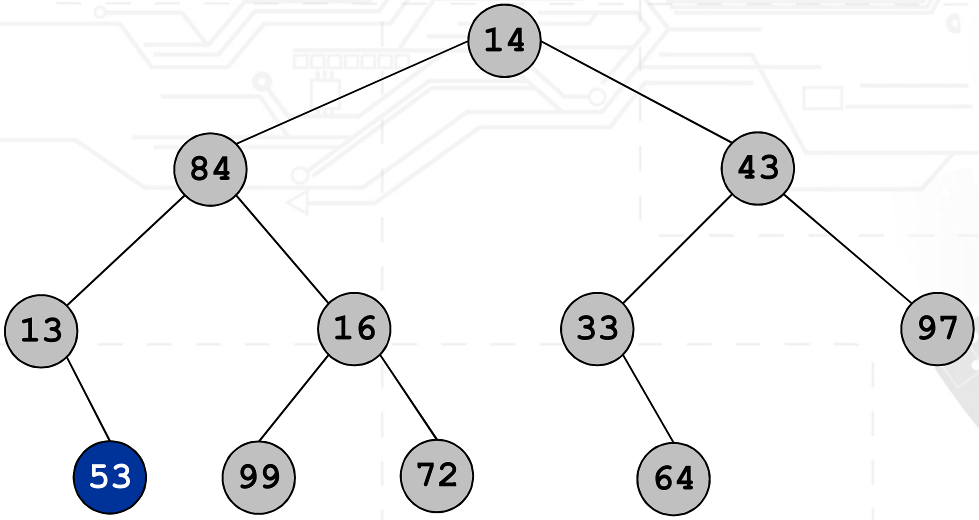
14 84 13 53





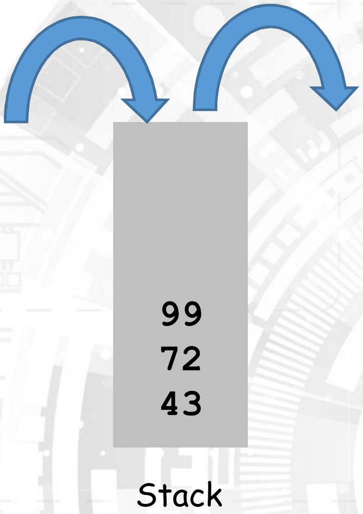
PREORDER TRAVERSAL WITH A STACK

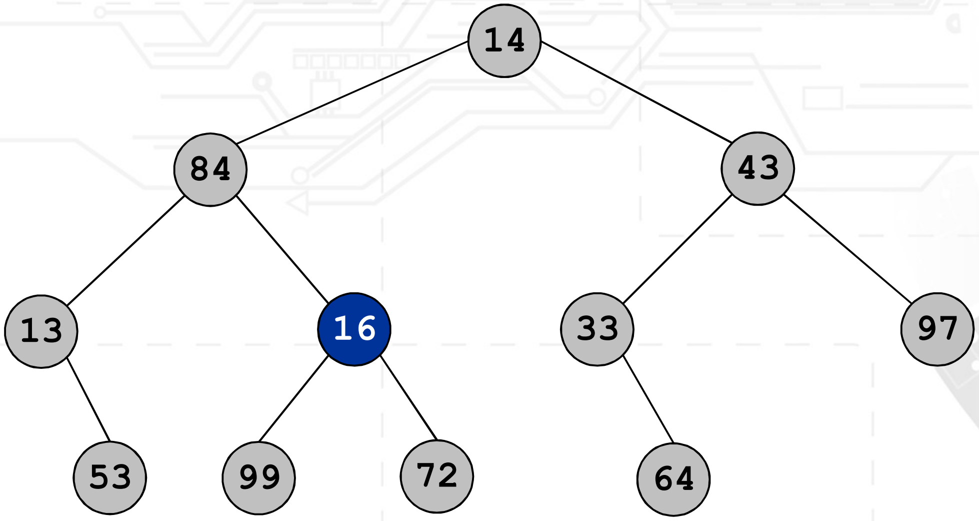
Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children

```
14 84 13 53 16
```





PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

While the stack is not empty

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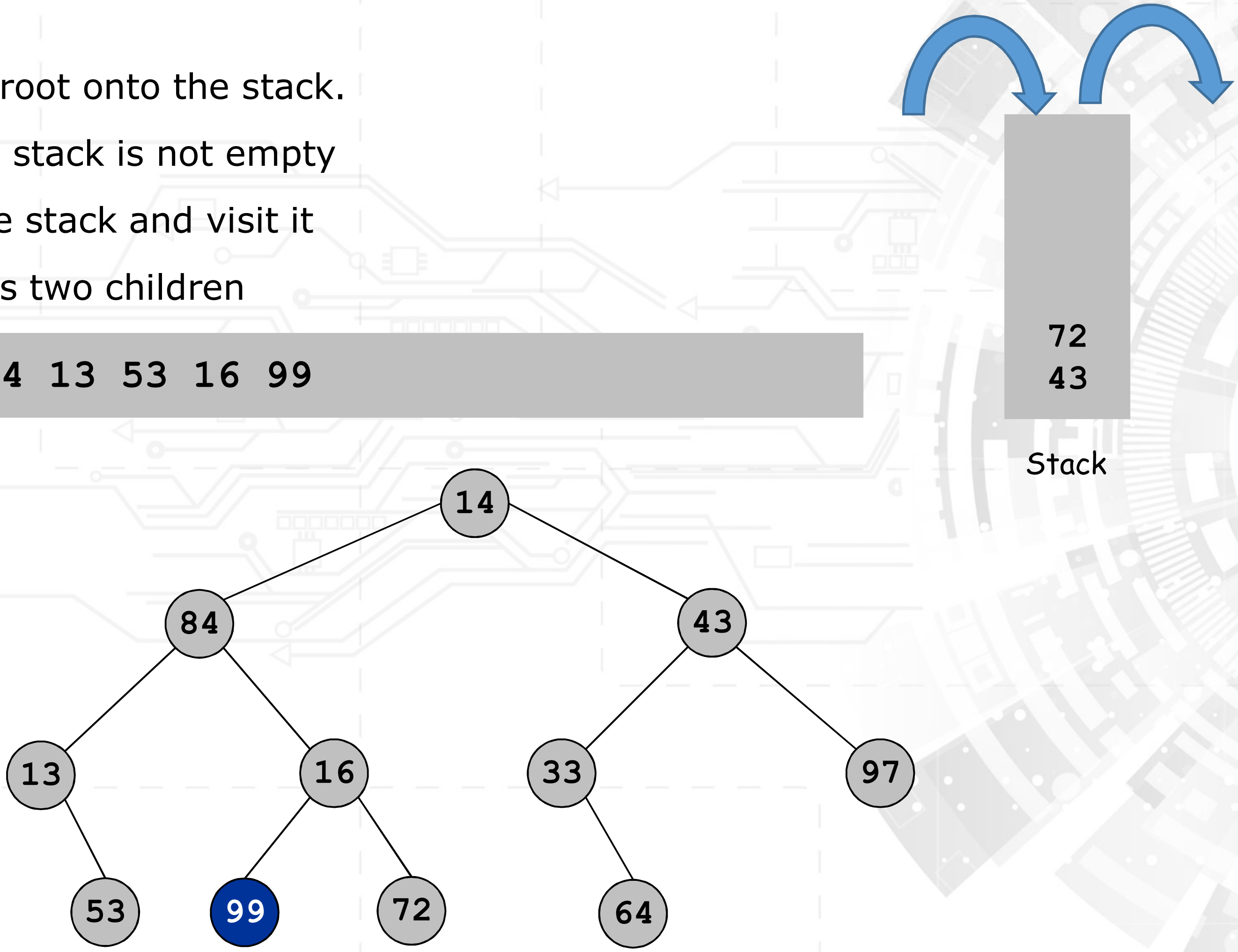
14 84 13 53 16 99



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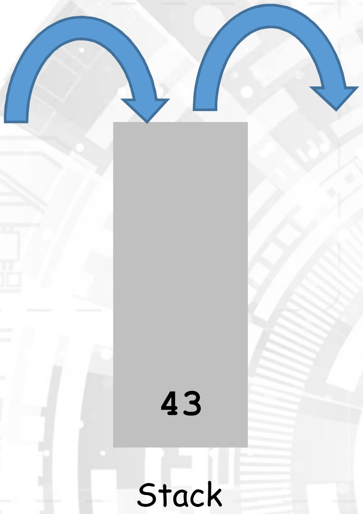
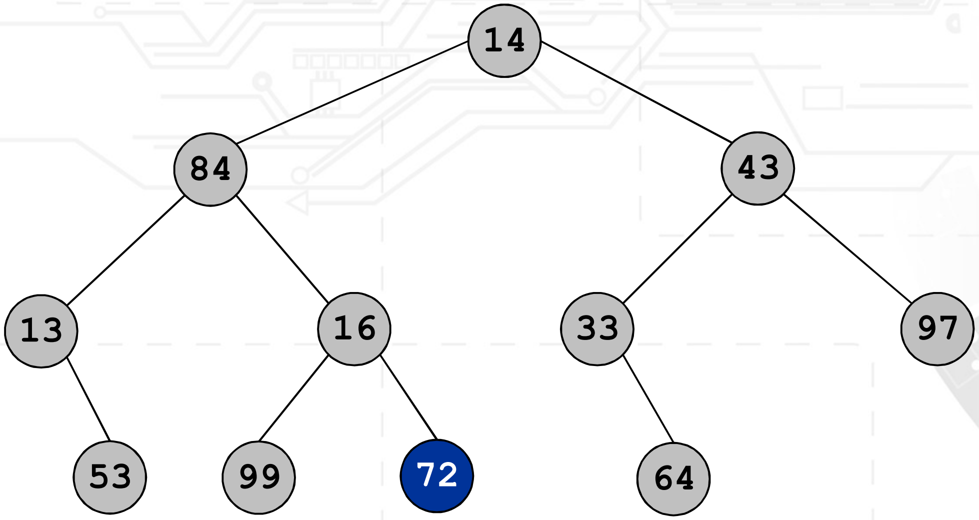
14 84 13 53 16 99 72





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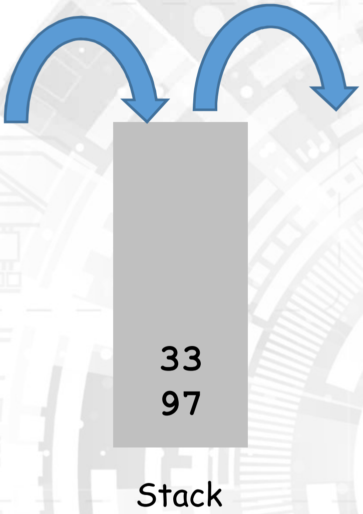

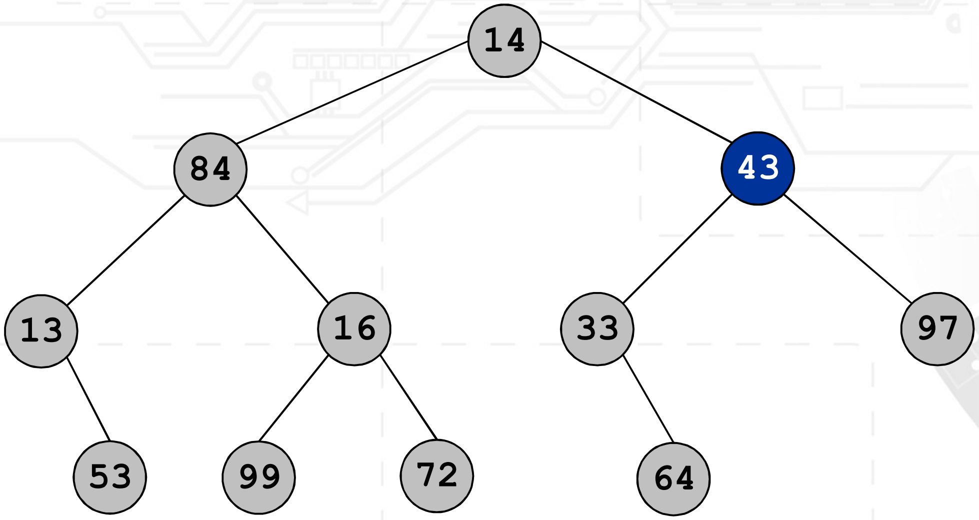

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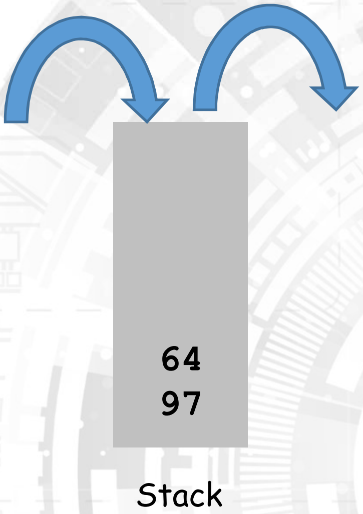
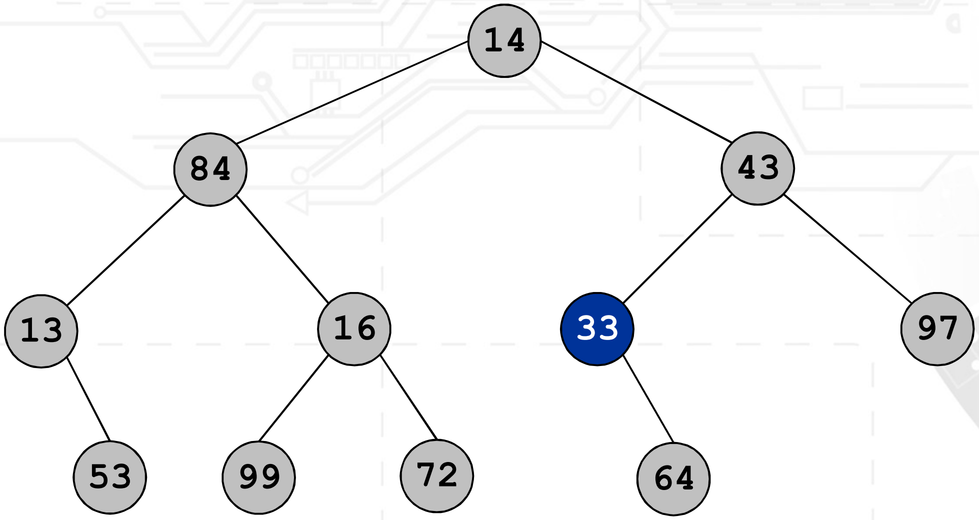
14 84 13 53 16 99 72 43 33



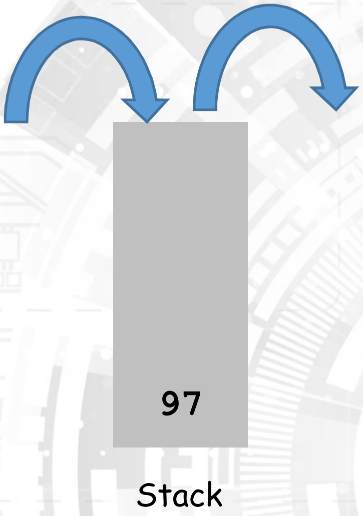


PREORDER TRAVERSAL WITH A STACK

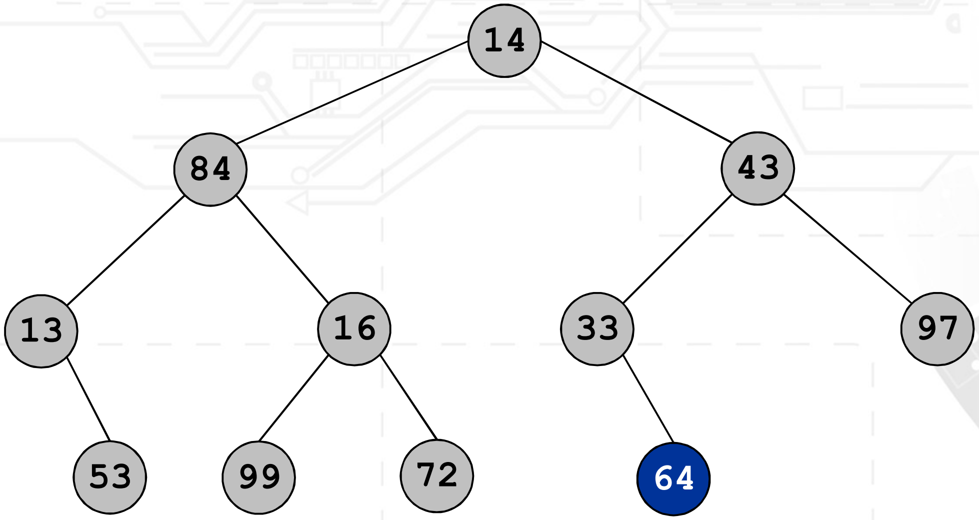
Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children



14 84 13 53 16 99 72 43 33 64



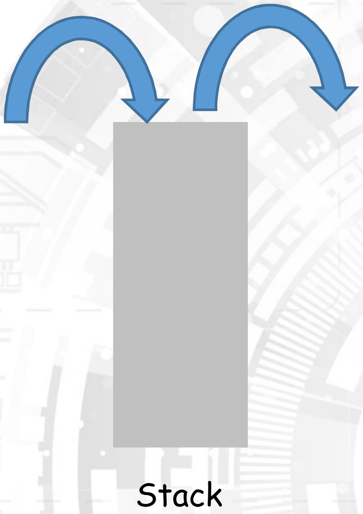
PREORDER TRAVERSAL WITH A STACK

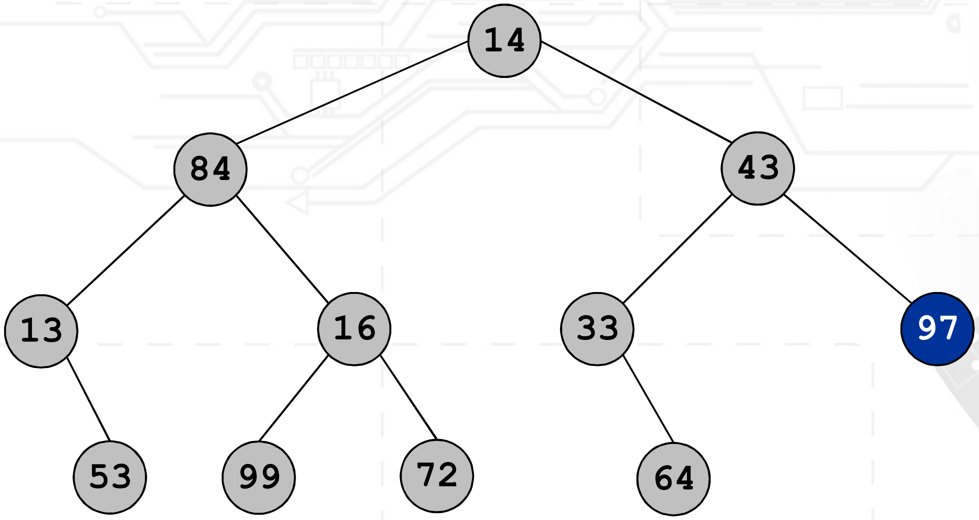
Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33 64 97





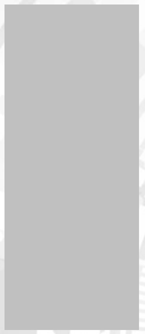
PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

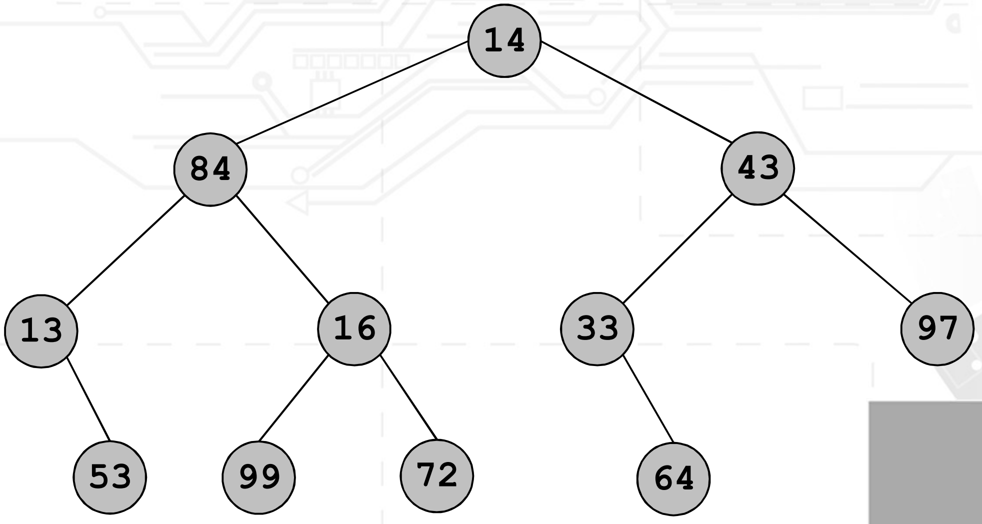
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33 64 97



Stack



Depth-first search

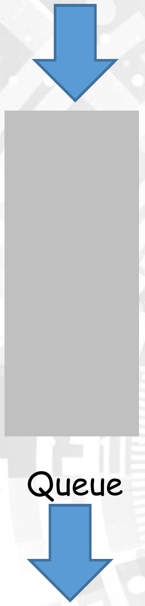
TRAVERSAL WITH A QUEUE

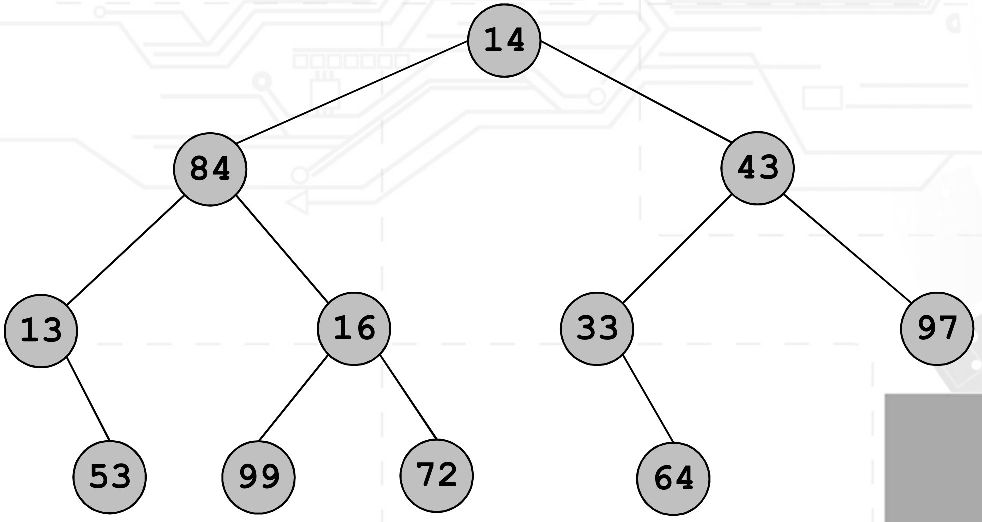
Enqueue the root onto the queue.

While the queue is not empty

- Dequeue the queue and visit it
- enqueue its two children

????





Breadth-first search

YOU SHOULD BE ABLE TO

- Binary tree Traverse:
  - Pre-order
  - In-order
  - Post-order
- Write recursive binary tree functions using the TreeTraversal template as a starting point
- Based on the traversal of the binary tree, do a lot of things: print, count numbers, count height/depth, find grandchildren,…, etc.