MH1812 Tutorial Chapter 5: Combinatorics

Q1: A set menu proposes 2 choices of starters, 3 choices of main dishes, and 2 choices of desserts. How many possible set menus are available? **Solution**: $2 \times 3 \times 2 = 12$. Q2: • In a race with 30 runners where 8 trophies will be given to the top 8 runners (the trophies are distinct, there is a specific trophy for each place), in how many ways can this be done? • In how many ways can you solve the above problem if a certain person, say Jackson, must be one of the top 3 winners? **Solution**: Since the trophy for each place is distinct and fixed, we need to place the runners into the top 8 one by one: • $P(30,8) = 30 \times 29 \times 28 \times 27 \times 26 \times 25 \times 24 \times 23$. • $3 \times P(29,7) = 3 \times 29 \times 28 \times 27 \times 26 \times 25 \times 24 \times 23$. Q3: In how many ways can you pair up 8 boys and 8 girls? **Solution**: $P(8,8) = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. One can line up all boys, and try to pair the boys with girls one by one. П Q4: How many ternary strings of length 4 have zero ones? **Solution**: 2⁴. Since for each ternary bit, there are 2 choices after the exclusion of "1". Q5: How many permutations are there of the word "repetition"? **Solution**: There are distinct characters with number of occurrences: r(1), e(2), p(1),

 $10!/(1! \times 2! \times 1! \times 2! \times 2! \times 1! \times 1!)$

t(2), i(2), o(1), o(1). so the number of permutations is: