mas162.04.22

Wednesday, April 22, 2020 10:47 AM

# Ch 8 Review, cont'd

Exam 4 on Ch 8 Friday 4/24
12:30 – 2:30

Canvas Assignment
(not Quiz)
6 parts (1 with sentence(s))

Act 8.4.7 (a) – (k)
a, c

8.4 Ex 5, 8

8.5.6 8a, b

find interval of convergence

Known p-series

diverge (a) $\sum_{k=3}^{\infty} \frac{2}{\sqrt{k-2}}$ $\qquad \frac{2}{\sqrt{k-2}} \sim \frac{2}{\sqrt{k}}$, $k \to \infty$

$\sum_{k=3}^{\infty} \frac{1}{\sqrt{k}} = \sum_{k=3} \frac{1}{k^{1/2}}$ diverge, $p = 1/2 < 1$

diverges (c) $\sum_{k=0}^{\infty} \frac{2k^2+1}{k^3+k+1}$ $\qquad \frac{2k^2+1}{k^3+k+1} \sim \frac{2k^2}{k^3} = \frac{2}{k}$

$\sum_{k=1}^{\infty} \frac{1}{k^p}$ conv $p > 1$, div $p \leq 1$

$\sum \frac{1}{k}$ harmonic $p=1$ diverges

(a) $\frac{2}{\sqrt{k-2}} = \frac{2}{\sqrt{k}\sqrt{1-\frac{2}{k}}} \sim \frac{2}{\sqrt{k}}$ as $k \to \infty$.

Let $a_k = \frac{2}{\sqrt{k-2}}$

$b_k = \frac{2}{\sqrt{k}}$

$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{\frac{2}{\sqrt{k}}\left(\frac{1}{\sqrt{1-\frac{2}{k}}}\right)}{\frac{2}{\sqrt{k}}} = 1.$

Since $\lim_{k\to\infty} \frac{a_k}{b_k} = 1$, the Limit Comparison Test says $\sum a_k$, $\sum b_k$ (known p-series) either both converge or both diverge. We know $\sum b_k$ diverges $(p = \frac{1}{2} < 1)$ so $\sum a_k$ also diverges.

Act 8.4.7

(a) Limit Comp. Test $\sum \frac{1}{\sqrt{k}}$ $\quad b_k = \frac{1}{\sqrt{k}} \Rightarrow \sum a_k$ div

(b) Divergence Test $\frac{k}{1+2k} \to \frac{1}{2} \neq 0 \Rightarrow \sum a_k$ div

(c) Lim. Comp. Test $\sum \frac{1}{k}$ $\quad b_k = \frac{1}{k} \Rightarrow \sum a_k$ div

(d) Ratio Test (factorials) conv

(e) Geometric! $r = \frac{2}{5} < 1$

(f) Lim. Comp. Test $\sum \frac{1}{k^2}$ $\quad b_k = \frac{1}{k^2} \Rightarrow \sum a_k$ conv

(g) Geometric! $r = \frac{3}{7} < 1$

(h) SKIP

(i) Alt. Series Test

(j) Integral Test

(k) done already

try Ratio Test $\lim = 1$, inconclusive
all other tests inconclusive
$\int \frac{1}{x \ln x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$
$\int \frac{1}{u}\,du$

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8.4 Ex 5, 8

5 Taylor series for arctan x (8.6)

$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$ (tricky cool)

$\frac{\pi}{4} = \arctan(1) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$

(a) Show series conv.

$a_k = \frac{1}{2k+1}$ $\quad$ series is $\sum_{k=1}^{\infty} (-1)^k a_k$

✓ $a_1 > a_2 > a_3$ $\quad 1 > \frac{1}{3} > \frac{1}{5} > \frac{1}{7} \dots$ $\qquad$ ✓ $a_n \to 0$ $\quad \frac{1}{2k+1} \to 0$ as $n \to \infty$

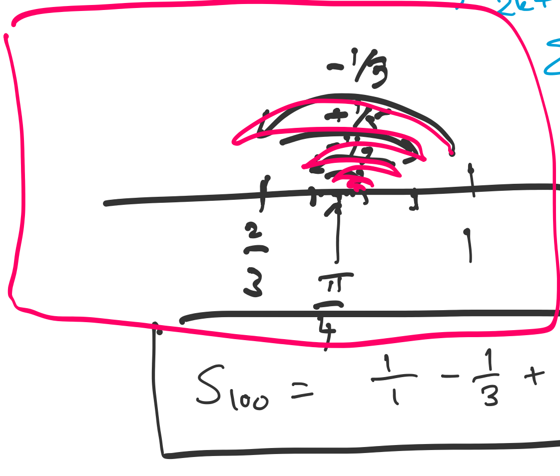
By Alt. Series test, series conv.

Show conv. conditionally

$\sum_{k=1} \frac{1}{2k+1} = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots$

$\frac{1}{2k+1} \approx \frac{1}{2}\,\frac{1}{k}$ $\quad$ Lim. Comp. Test

$\sum a_k \sim \frac{1}{2}\sum \frac{1}{k}$ harmonic diverges




$S_{100} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \dots + \frac{1}{2\cdot100+1} - \frac{1}{2\cdot101+1}$

term #100 $\qquad$ term #101

$|S_n - \text{conv pt}| < |n+1\text{st step}|$ $\quad S_{101}$

(b) $\left|S_{100} - \frac{\pi}{4}\right| < \left|\text{next step}\right| = \left|\text{term \#101}\right| = \frac{1}{203} \sim .005$

(c) what if you want $\pm .0000000001$

$\left|S_n - \frac{\pi}{4}\right| < \frac{1}{2\cdot n+1} < 10^{-10}$

Solve for n $\qquad 2n+1 > 10^{10}$

$n > \frac{10^{10}-1}{2} \sim 5$ billion

8.4 #8
SKIP OMIT CUT
IGNORE

8.5.6 Ex a, b

Given $f(x) = 1x^3 - 2x^2 + 3x - 1$

Find 3rd order Taylor poly for $f$ at $x=0$.

$P(x) = f(x)$ how?

$f(x) = x^3 - 2x^2 + 3x - 1$ $\qquad f(0) = -1$

$f'(x) = 3x^2 - 4x + 3$ $\qquad f'(0) = 3$

$f''(x) = 6x - 4$ $\qquad f''(0) = -4$

$f'''(x) = 6$ $\qquad f'''(0) = 6$

$f^{(4)}(x) = 0$

$P_3(x) = \frac{-1}{0!} + \frac{3}{1!}x + \frac{-4}{2!}x^2 + \frac{6}{3!}x^3 = f(x)$

$0! = 1! = 1$

$P_{100}(x) = -1 + 3x - 4x^2 + x^3 = P_3(x) = f(x)$

8.5 8a 8b

$f(x) = \sin(x^2)$

Find $P_4(x)$ based at $x=0$ for $f(x)$.

$f(x) = \sin(x^2)$

$f'(x) = \cos(x^2)\cdot 2x$

$f''(x) = (-\sin(x^2)\cdot 2x)\cdot 2x + \cos(x)^2\cdot 2$

$f'''(x)$ product + chain rule → more terms

$f^{(4)}(x)$ even worse

YUK! terrible

Alternative!

Know! $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

Get cheap $\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

$P_5(x) = P_4(x) = x^2 = P_2(x)$

$P_6(x) = x^2 - \frac{x^6}{3!}$

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Given! $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

Find interval of convergence.

$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$ $\qquad (-1)^{n+1}\frac{x^{4n-2}}{(2n-1)!}$

shorter, better example to appear!

| term counter $n$ | exp. of $x$ | fact. |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 6 | 3 |
| 3 | 10 | 5 |
| 4 | 14 | 7 |
| $n$ | $4n-2$ | $2n-1$ |

(+1 between successive $n$; +4 between successive exponents)

exp of x: (1,2), (2,4), (3,...) ; $n$

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Better "find the interval of convergence" example. Added after class meeting

Q: Find the interval of convergence for the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{4n-2}}{(2n-1)!}$.

A: Procedure: Evaluate the limit $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

where $a_n = (-1)^{n+1}\frac{x^{4n-2}}{(2n-1)!}$.

We have $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\left(\frac{x^{4(n+1)-2}}{[2(n+1)-1]!}\right)}{\left(\frac{x^{4n-2}}{(2n-1)!}\right)}\right|$

$= \lim_{n\to\infty}\left|\frac{x^{4n+2}}{x^{4n-2}}\right|\frac{(2n-1)!}{(2n+1)!}$

$= \lim_{n\to\infty} |x^4| \frac{1}{(2n+1)(2n)}$

$= |x^4| \lim_{n\to\infty}\frac{1}{(2n+1)(2n)} = 0.$

Since $0 < 1$, we conclude $\sum a_n$ converges for all values of $x$. So the interval of convergence is the whole real line.

Comments

$(-1)^n$ can be dropped because of abs. value signs

$\frac{(2n-1)!}{(2n+1)!} = \frac{\cancel{(2n-1)}\cancel{(2n-2)}\cdots\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{(2n+1)(2n)\cancel{(2n-1)}\cancel{(2n-2)}\cdots\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}$

$|x^4|$ is a constant in this limit as $x \to \infty$, so factors out of the limit

Intuitively, this limit says $\sum a_n$ is like a geometric series with $r = 0$, so it should converge (no matter what $x$ is).