mas162.04.27

Monday, April 27, 2020 12:47 PM

Review ch 5, 6, 7, 8

equal balance

~ 2 x length 50-minute

~ 100-minute

Idea
Wed 5/6
~~12:30 - 3:00~~
12:00 - 4:00

Study ① Vocabulary / Facts
② Work problems

Ch 6 density - mass - ~~center of mass~~ not on final

Applications of integration

area (of 2-D region)

~~arc length~~ not on final

?? volume (solids of revolution)

(variable) density in some solid

less dense
more dense

work (lifting mass against gravity)

☆ $F = m \cdot g$ $W = F \cdot d$

hydrostatic force

☆ force ↔ depth ↔ areas of slices

force = pressure · area

Area

Given $y = f(x)$, $y = g(x)$

not given: (a,b), (c,d)

① solve $f(x) = g(x)$
solutions $x=a, c$
plug $f(a) = g(a) = b$
$f(c) = g(c) = d$

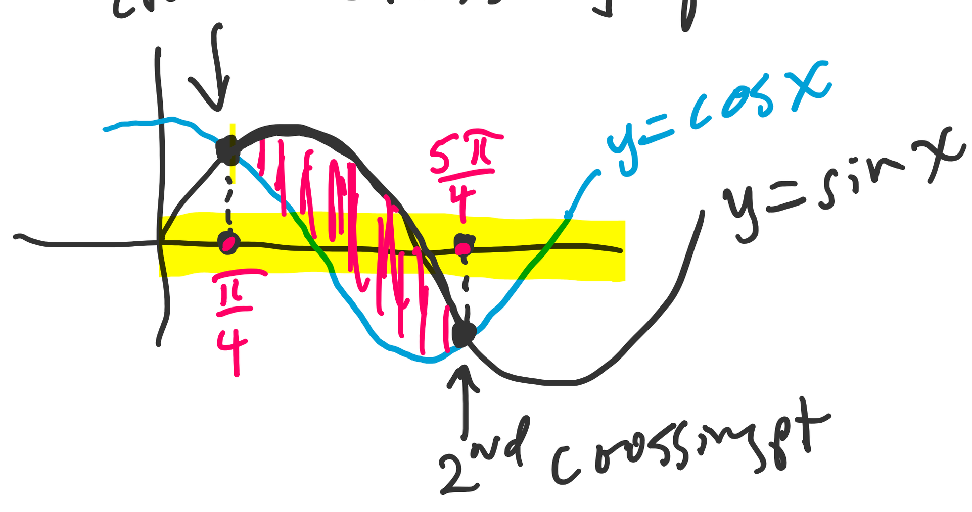
might be all that's required

"Set up, but do not evaluate, an integral for the area..."

Ans: $\int_a^c (g(x) - f(x))\,dx$ = number = area
top − bottom
evaluate

② find antideriv. for $g(x) - f(x)$
then plug in a, c, subtract

Example: Set up and evaluate an integral for the area between the curves $y = \sin x$ and $y = \cos x$ in the region between their first two crossing points where $x \geq 0$.



① $\sin x = \cos x$
y-coord x-coord

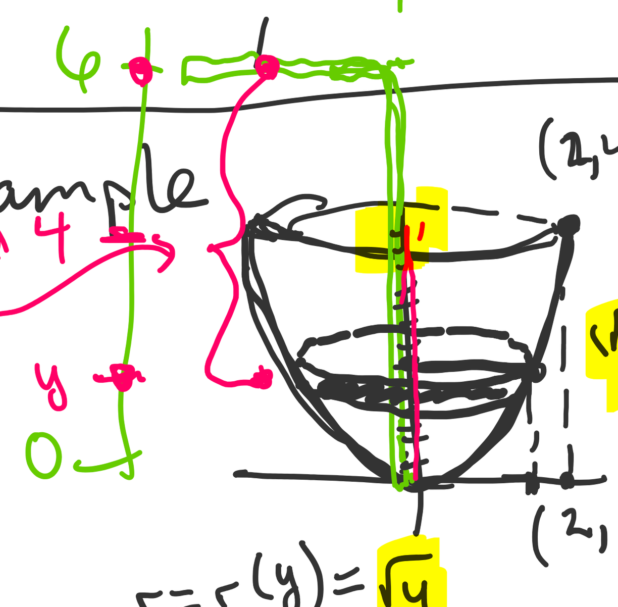
$135° = \frac{5\pi}{4}$ $45° = \frac{\pi}{4}$

② $\int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx = (-\cos x - \sin x)\Big|_{\pi/4}^{5\pi/4}$

$= \sqrt{2}$

$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Example



$y = x^2$
$\sqrt{y} = x$

Bowl: shell (surface) made by spinning $y = x^2$ around y-axis. Filled with liquid of density δ (mass units)/(vol unit). Accel. of gravity is g (dist units)/(time unit)². Find total work done pumping all liquid to height of y=6.

$r = r(y) = \sqrt{y}$

slice liquid into thin horiz disks

$dy = \Delta y$

$\Delta V = \pi r^2 dy$

$= \pi(\sqrt{y})^2 dy \cdot \delta \cdot g \cdot (6-y)$

$= \Delta Vol \cdot dens \cdot g \cdot d$

$= \Delta m \cdot g \cdot d$ $\Delta F = \Delta m \cdot g$

$\Delta W = \Delta F \cdot d$

$\Delta W = \pi \delta g\, y(6-y)\,dy$

total work $= \int \Delta W = \pi\delta g \int_0^4 y(6-y)\,dy$

$= \pi\delta g \left(\frac{6y^2}{2} - \frac{y^3}{3}\right)\Big|_0^4$

$= \pi\delta g \left(3 \cdot 16 - \frac{64}{3}\right)$