Quantum Information and 4-Qubit States

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- RSA based on the difficulty of factoring large integers
- A quantum computer using Shor's algorithm would "break" the RSA cryptographic system

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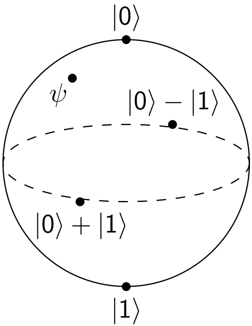
For example, a 1-qubit state can be written as $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ where α and β are any complex numbers.

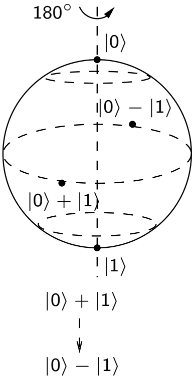
The Bloch Sphere

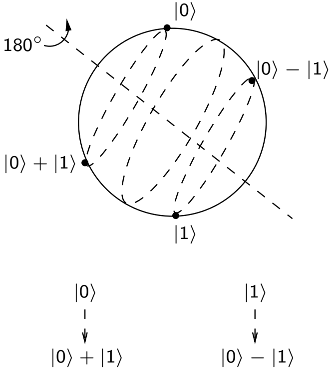


The state ψ of one qubit can be uniquely described by a point on the surface of a sphere. The poles of the sphere represent the classical 0 and 1 bits.

What can we do to 1-qubit states?

1-qubit systems evolve by rotations of the Bloch sphere, which can be achieved in the laboratory. Here are two examples.





4-Qubit States

A 4-qubit state is similar to a 1-qubit state, except that it is a linear combination of $|x\rangle$'s where x is any string of four 0's and 1's.

Examples:

$$
\begin{aligned}
|\psi\rangle &= a\,|0000\rangle + b\,|1111\rangle \\
|\phi\rangle &= a\,|0000\rangle + b\,|1010\rangle + c\,|1011\rangle + d\,|1111\rangle + e\,|1100\rangle \\
|\varphi\rangle &= a\,|1001\rangle \\
|\varsigma\rangle &= a\,|1000\rangle + b\,|0100\rangle + c\,|0010\rangle + d\,|0001\rangle
\end{aligned}
$$

Examples of Individual Qubit Rotations on a 4-qubit State

$$\psi = |0000\rangle + |1111\rangle$$

Apply
$|0\rangle \longmapsto |0\rangle$
$|1\rangle \longmapsto -|1\rangle$
to the first qubit

$$|0000\rangle - |1111\rangle$$

$$\psi = |0000\rangle + |1111\rangle$$

Apply
$|0\rangle \longmapsto |1\rangle$
$|1\rangle \longmapsto |0\rangle$
to the first qubit

$$|1000\rangle + |0111\rangle$$

Why are rotations of individual qubits important?

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$$\Big\downarrow$$

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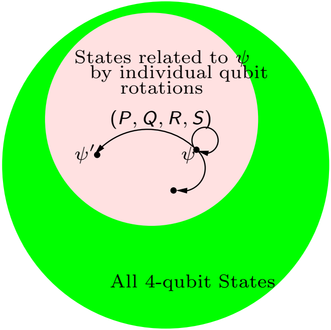
Quantum computing algorithms and quantum communications protocols utilize entanglement as an essential resource.

4-tuples of Individual Qubit Rotations

(P, Q, R, S) is a 4-tuple of individual qubit rotations. Each individual qubit rotation is applied to exactly one qubit in ψ.



We are interested in the inner pink circle on the Venn diagram because these states have the same entanglement type.

Stabilizers

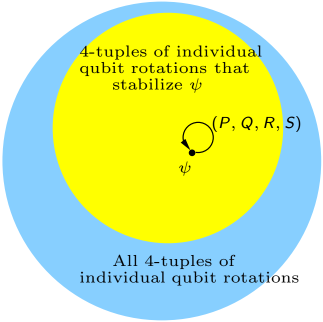
The 4-tuple (P, Q, R, S) is called a stabilizer if, when it is applied to ψ, the result is still ψ.

Example:

$$\psi = |0000\rangle + |1111\rangle$$

Apply
$|0\rangle \longmapsto |1\rangle$
$|1\rangle \longmapsto |0\rangle$
to all 4 qubits

$$|1111\rangle + |0000\rangle = \psi$$



We are interested in studying these because we know something about the structure of sets of stabilizers.

= set of stabilizing rotations for ψ

Main Research Idea

Stabilizers have structure in the form of Lie groups and Lie algebras, which we can use to study the states that go with them. The idea of stabilizer structure breaks into two problems:

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- Easy problem: Given a state, find its set of stabilizing rotations.
- Harder problem (but we can still do some): Given a set of stabilizing rotations, describe the states that have these as stabilizers.

My Results

Problem of Interest: What states are stabilized by applying any 4-tuple from the set of all (R, R, R, R) where R is an individual qubit rotation?

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Answer: States of the form

$$\begin{aligned} & a(|0011\rangle + |1100\rangle) \\ + \quad & b(|1001\rangle + |0110\rangle) \\ + \quad & c(|1010\rangle + |0101\rangle) \\ & a + b + c = 0 \end{aligned}$$

are the only states with these types of stabilizers.

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- Showing that a state of this form must have the desired stabilizer subalgebra

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- Writing and using computer code to find examples of states with the desired properties
- Determining the restrictions each stabilizer element put on the state
- Showing that a state of this form must have the desired stabilizer subalgebra
- Finding and manipulating invariants to determine the uniqueness of the state

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