MAS 162 Calculus 2 Spring 2020

Ch 7: Matching Words to Differential Equations Summary

Forms of Differential Equations in 7.5 and 7.6

Word problems in 7.5 and 7.6 involve differential equations for $y=y(t)$ in one of these forms.

(1) $dy/dt = ay$

(2) $dy/dt = a(y-b)$

(3) $dy/dt = ay(b-y)$

With the substitution $u = y-b$, Equation (2) is transformed into (1) like this: let $u = y-b$, so $du/dt=dy/dt$. Then (2) becomes

(2') $du/dt = au$

which is the same form as (1).

Another form that appears is

(4) $dy/dt = g(t) - d(t)$

but (4) turns out to turn into (2) in all the examples in 7.5.

Matching Words to Equations (1)--(4)

Eqn. (1), "Basic" exponential growth or decay
Act. 7.5.1(a)(i), Act. 7.5.2(a), Act.7.5.3(a)(b), 7.5Ex.3,4

"a quantity y changes at a rate proportional to y"
$dy/dt = ay$

"a quantity y changes at a rate of p percent per year"
$dy/dt = (p/100)y$

"a quantity y changes at a rate of k units per time unit"
$dy/dt = ay$

"y grows" <--> $a>0$

"y decays" <--> $a<0$

Eqn. (2), Newton's Law of Cooling (or Warming)
7.1Ex4

"temperature y changes at a rate proportional to the difference between y and the ambient (constant) temperature"
$dy/dt = a(y-b)$

Eqn. (3), Logistic Equation
all of 7.6

"population growth follows a logistic model "
$dy/dt = ay(b-y)$

"relative change in population is a linear function of the population"
$(dy/dt)/y = a(b-y)$

Eqn. (4), "sum of growth term and decay term" (not an official name)
Act.7.5.1(a)(ii),(b), Act.7.5.2(b)--(f), Act.7.5.3(c)--(f), 7.5Ex1,2,5

"a quantity y changes at a rate with two contributing terms: a growth term $g(t)>0$ and a decay term $d(t)>0$"
$dy/dt = g(t) - d(t)$

(4a) continuous annuity

"g(t) is proportional to y and d(t) is constant"
$g(t) = ay$, $d(t) =$ const.

(4b) mixing a solution in a tank

$g(t) =$ (inflow concentration) * (inflow rate) = a constant
$d(t) =$ (outflow concentration) * (outflow rate) = (y/V)*(outflow rate)