mas162.04.06

Monday, April 6, 2020 10:18 AM

8.3 convergence of series

Ex 1, 2, 3, 5 Act 2c, 3c

Comments / Overview

Sequence $a_1, a_2, a_3, \dots$ (1.3+)

Series $a_1 + a_2 + a_3 + \dots$ (sum possibly infinite)

Theme / Skills

pattern finding

limits of infinite sums < convergence / divergence

Key examples / facts so far

geometric series

$a + ar + ar^2 + ar^3 + \dots$ < converges to $\frac{a}{1-r}$ if $|r|<1$ / diverges if $|r| \geq 1$

sequence of summands: $a, ar, ar^2, ar^3, \dots$

finite geometric sum

$$\underbrace{a + ar + ar^2 + \dots + ar^{n-1}}_{n \text{ summands}} = a\left(\frac{1-r^n}{1-r}\right)$$

Harmonic sequence

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

p=1 Harmonic series

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ < conv.? / div? not obvious! Why?

p sequence / series

$1, \frac{1}{2^p}, \frac{1}{3^p}, \frac{1}{4^p}, \dots$

ex p=2 $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{25}, \dots$

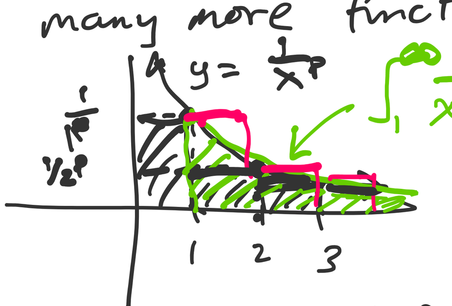
p=2 $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{25} + \dots$ < conv? if $p>1$ / diverges? if $p \leq 1$

because $\int_1^\infty \frac{1}{x^p}\,dx$ < conv if $p>1$ / div if $p \leq 1$

How to see conv/div of p series

many more functions.

$f(x) = \frac{1}{x^p}$, $p>0$ $(x>0)$



Area = $\frac{1}{n^p}$

$\frac{1}{n^p}$ tall

1 unit wide

$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ ← sum of areas shaded boxes

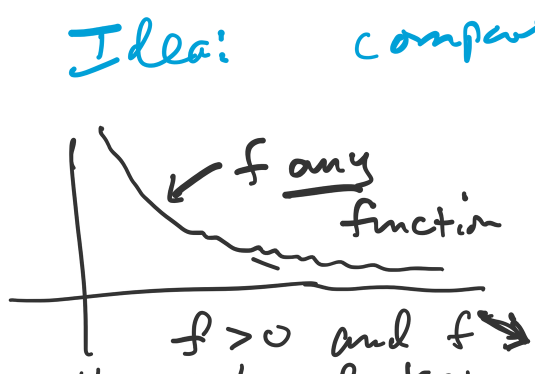
Idea: compare

$\frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} \leq \boxed{\int_1^n \frac{1}{x^p}\,dp} \leq \frac{1}{1^p} + \frac{1}{2^p} + \dots + \frac{1}{n^p}$

green

If integral $\int_1^\infty$ is finite then is finite

If integral $\int_1^\infty$ diverges then diverges



$f>0$ and $f \searrow$ then integral test applies

Activity 8.3.2 c.

$\sum_{k=1}^{9} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \dots + \frac{1}{9^2}$

(warm up)

$\frac{1}{1^2}$

$\frac{1}{1^2} + \frac{1}{2^2}$

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}$

$\vdots$

$\frac{1}{1^2} + \frac{1}{2^2} + \dots + \frac{1}{10^2}$ (looks like it is converging)

8.3.3 c

partial sum of series $\sum_{k=1}^{\infty} a_k$

$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \dots$

n-th partial sum $\sum_{k=1}^{n} a_k = (\cancel{a_1} + \cancel{a_2} + \cancel{a_3} + \cancel{\dots} + a_n)$

(n-1)st partial sum $-(\cancel{a_1} + \cancel{a_2} + \cancel{\dots} + \cancel{a_{n-1}})$

difference Ans: a_n

Ex 1. $A_n = \frac{90}{9^n}$ $f(x) = 90 \cdot 9^{-x} = \frac{90}{9^x}$

$\{A_n\} = \frac{90}{9^1}, \frac{90}{9^2}, \frac{90}{9^3}, \dots$ ($\cdot\frac{1}{9}$, $\cdot\frac{1}{9}$, $\cdot\frac{1}{9}$) geometric? $r = \frac{1}{9} < 1$

(a) $\sum_{n=1}^{\infty} A_n$ < conv / div. conv to $\frac{\frac{90}{9}}{1-\frac{1}{9}} = \frac{10}{\frac{8}{9}} = \frac{90}{8}$

OR integral test

$\int_1^\infty 90 \cdot 9^{-x}\,dx = \text{number}$

(b) $\{A_n\}$ < conv / div $\frac{90}{9^n} \to ?$ as $n \to \infty$

Yes $\frac{90}{9^n} \to 0$ as $n \to \infty$

Ex 2

$S_4 = ? = \sum_{i=1}^{4} \frac{2}{i+7} = \frac{2}{1+7} + \frac{2}{2+7} + \frac{2}{3+7} + \frac{2}{4+7}$

$S_8 = \frac{2}{1+7} + \dots + \frac{2}{8+7}$

Ex 3

$\sum_{n=1}^{\infty} \frac{3n}{6n+13}$ < conv / div $\approx \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots = \infty$

notice $\frac{3n}{6n+13} \to ?\ \frac{1}{2}$ as $n \to \infty$

$\frac{3n}{6n+13} \approx \frac{3}{6} = \frac{1}{2}$

Why? does the series diverge?

Ans: $\frac{3n}{6n+13} \to \frac{1}{2}$ as $n \to \infty$ $\frac{1}{2} \neq 0$

(The Divergence Test)

If $a_n \not\to 0$ as $n \to \infty$ then $\sum_{n=1}^{\infty} a_n$ diverges.

~~Some~~ many texts also called "n^{th} term test"

Ex 5

Ratio Test: basic idea / intuition

Know $a + ar + ar^2 + ar^3 + \dots$ ($\cdot r$, $\cdot r$, $\cdot r$) < conv if $|r|<1$ / div if $|r| \geq 1$

$\frac{ar^3}{ar^2} = r$

Given unknown?? $a_1 + a_2 + a_3 + a_4 + \dots$ < conv? / div?

$\frac{a_2}{a_1}, \frac{a_3}{a_2}, \frac{a_4}{a_3}, \dots, \frac{a_{n+1}}{a_n}, \dots$ (if geom, every fraction $= r$)

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$
- might exist and <1 → conv.
- might exist and >1 → div
- might not exist div
- might exist and $=1$???

Given $\left\{\frac{b^n}{n!}\right\}$ b const

$\frac{b^1}{1!}, \frac{b^2}{2!}, \frac{b^3}{3!}, \dots$

$\sum_{n=1}^{\infty} \frac{b^n}{n!}$

series $\frac{b^1}{1!} + \frac{b^2}{2!} + \frac{b^3}{3!} + \dots$ shrinking to 0

Try ratio test: ① $\frac{(n+1)^{st} \text{ term}}{n^{th} \text{ term}} = \frac{\left(\frac{b^{n+1}}{(n+1)!}\right)}{\left(\frac{b^n}{n!}\right)} = \frac{b^{n+1}}{(n+1)!} \cdot \frac{n!}{b^n}$

$= \frac{b^{n+1}}{b^n} \cdot \frac{n!}{(n+1)!}$

$= b \cdot \frac{1}{n+1} = \frac{b}{n+1}$

$\frac{\left(\frac{A}{B}\right)}{\left(\frac{C}{D}\right)} = \frac{A}{B} \cdot \frac{D}{C}$

$\frac{n!}{(n+1)!} = \frac{\cancel{n} \cdot \cancel{(n-1)}(\cancel{n-2}) \cdots \cancel{3 \cdot 2 \cdot 1}}{(n+1)\cancel{n}(\cancel{n-1}) \cdots \cancel{3 \cdot 2 \cdot 1}} = \frac{1}{n+1}$

② $\lim_{n\to\infty} \frac{b}{n+1} = 0 < 1$ (b const)

Conclusion: ?

The series $\sum_{n=1}^{\infty} \frac{b^n}{n!}$ converges by the Ratio Test.

8.4 next

lot of material

but yes time