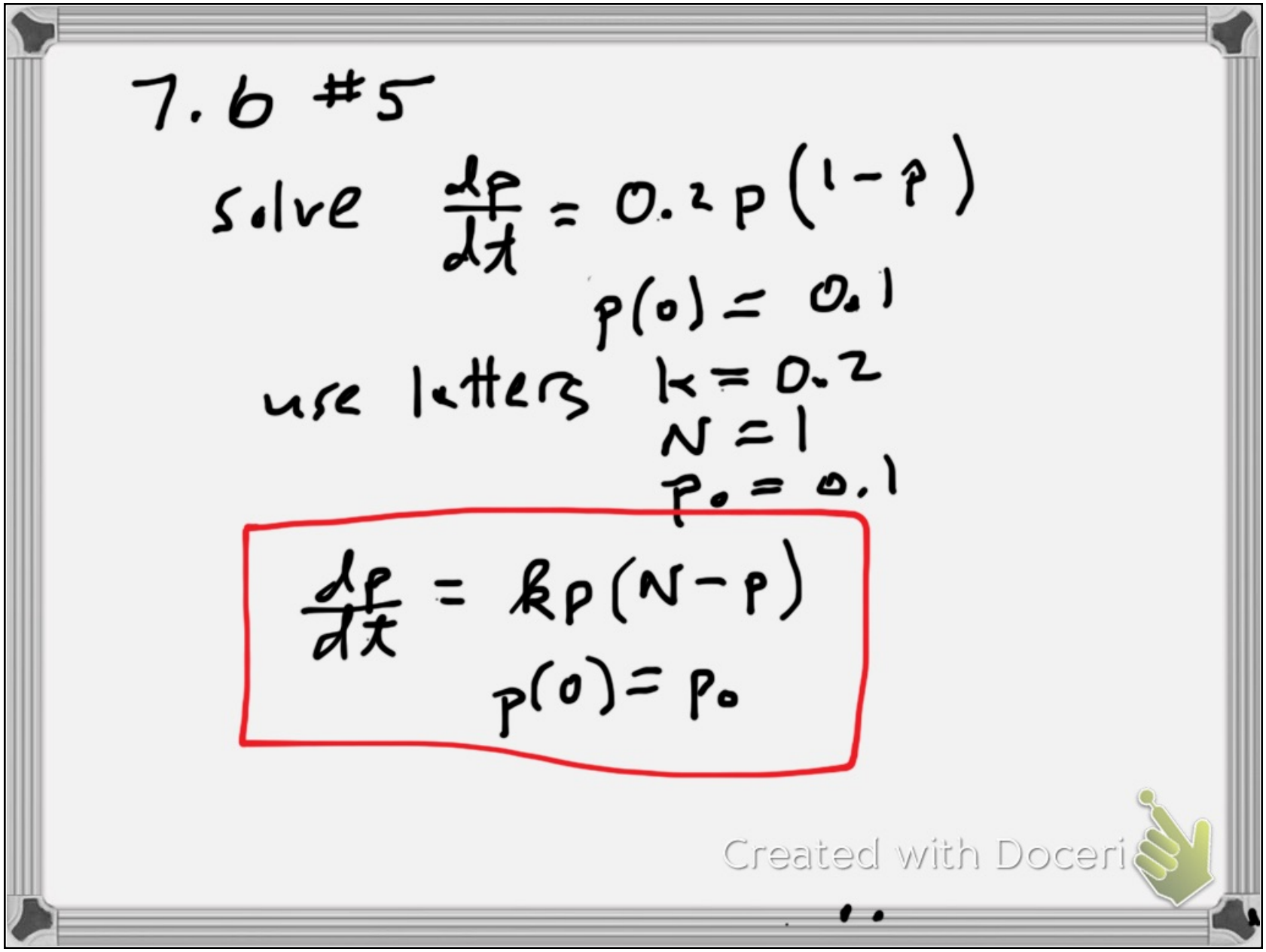
7.6 #5
solve $\frac{dP}{dt} = 0.2P(1-P)$
$P(0) = 0.1$
use letters $k = 0.2$
$N = 1$
$P_0 = 0.1$
$\frac{dP}{dt} = kP(N-P)$
$P(0) = P_0$
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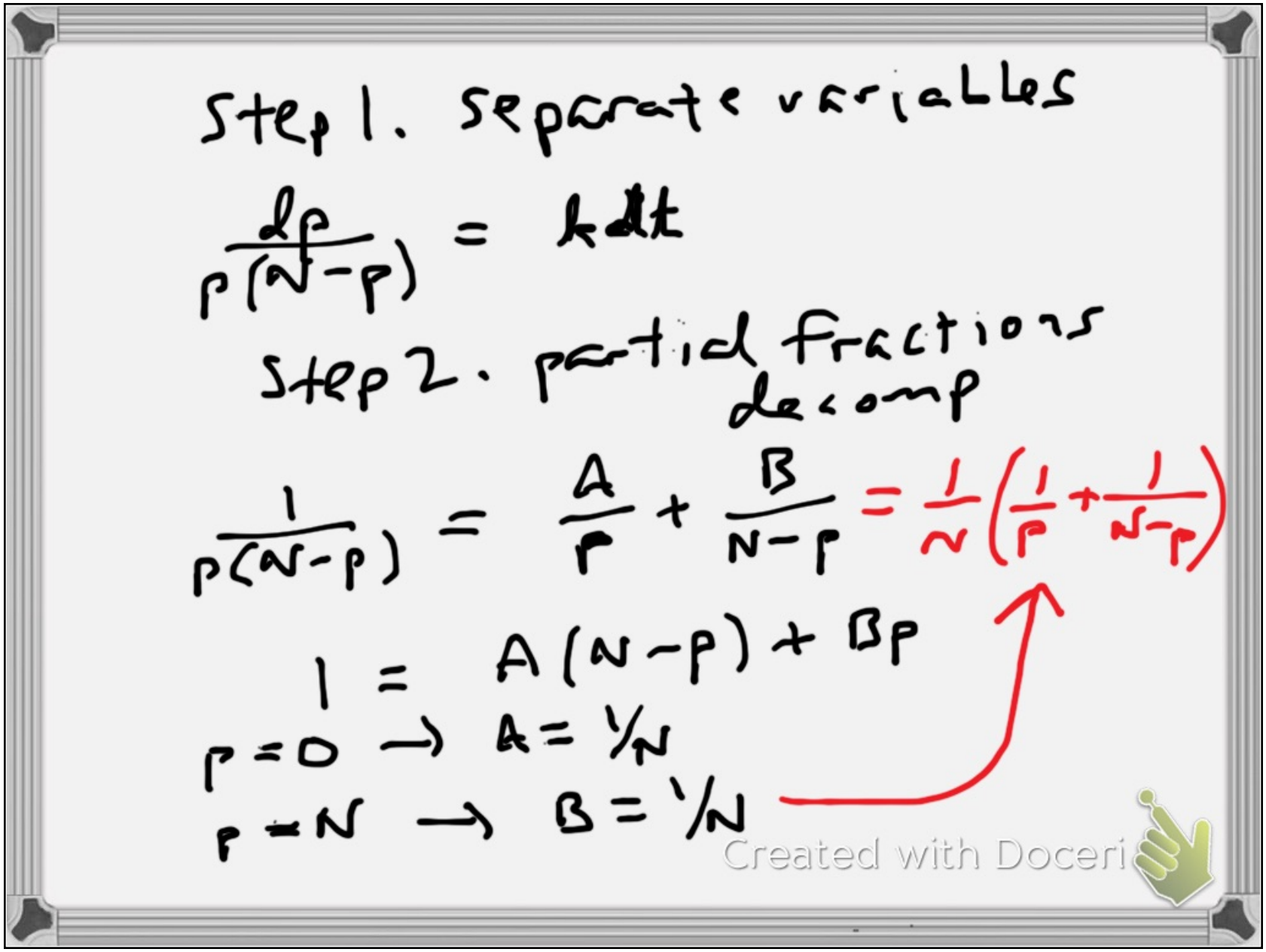
Step 1. Separate variables
$\frac{dP}{P(N-P)} = k\,dt$
Step 2. partial fractions decomp
$\frac{1}{P(N-P)} = \frac{A}{P} + \frac{B}{N-P} = \frac{1}{N}\left(\frac{1}{P} + \frac{1}{N-P}\right)$
$1 = A(N-P) + BP$
$P = 0 \rightarrow A = 1/N$
$P = N \rightarrow B = 1/N$
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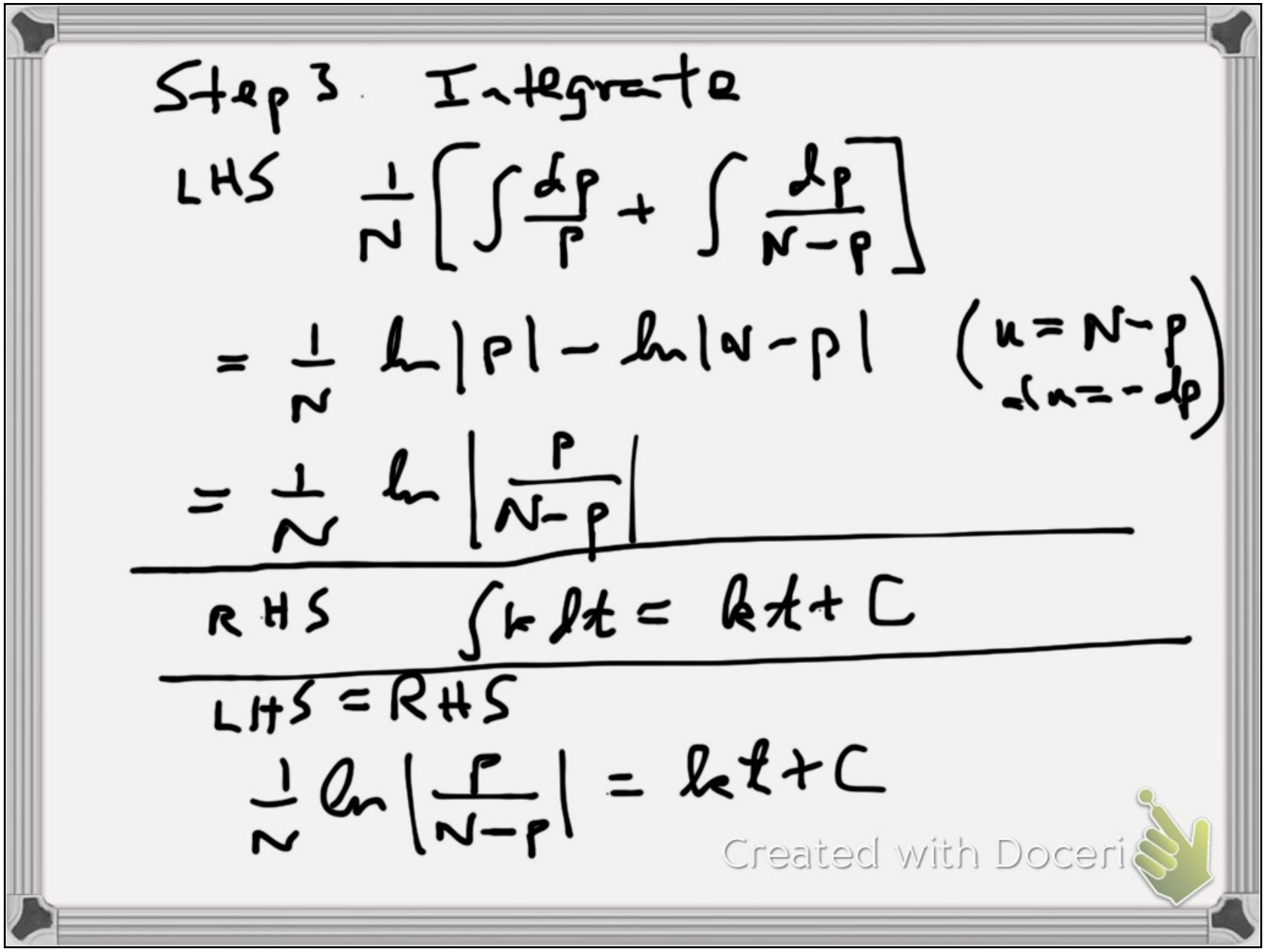
Step 3 Integrate
LHS $\frac{1}{N}\left[\int \frac{dp}{p} + \int \frac{dp}{N-p}\right]$
$= \frac{1}{N} \ln|p| - \ln|N-p|$ $(u = N-p$ $du = -dp)$
$= \frac{1}{N} \ln\left|\frac{p}{N-p}\right|$
RHS $\int k\,dt = kt + C$
LHS = RHS
$\frac{1}{N} \ln\left|\frac{p}{N-p}\right| = kt + C$
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Step 4 solve for P

$$\ln\left|\frac{P}{N-P}\right| = Nkt + C$$

$$\left|\frac{P}{N-P}\right| = Ae^{Nkt}$$

(let A take care of abs val)

$$\frac{P}{N-P} = Ae^{Nkt}$$

Step 4a Solve for A

put $t=0$, $P=P_0$

$$A = \frac{P_0}{N-P_0}$$

Back to Step 4

$$\frac{P}{N-P} = \frac{P_0}{N-P_0} e^{Nkt}$$

$$P = \frac{P_0}{N-P_0} e^{Nkt} (N-P)$$

$$P\left(1 + \frac{P_0}{N-P_0} e^{Nkt}\right) = \frac{P_0}{N-P_0} N e^{Nkt}$$

$$P = \frac{\left(\frac{P_0}{N-P_0}\right) e^{Nkt}}{1 + \left(\frac{P_0}{N-P_0}\right) e^{Nkt}} \quad \text{(continued)}$$

Step 4 cont'd

mult top and bottom of last fraction by $\frac{N-P_0}{P_0} e^{-Nkt}$

$$P = \frac{N}{\frac{N-P_0}{P_0} e^{-Nkt} + 1}$$

Done