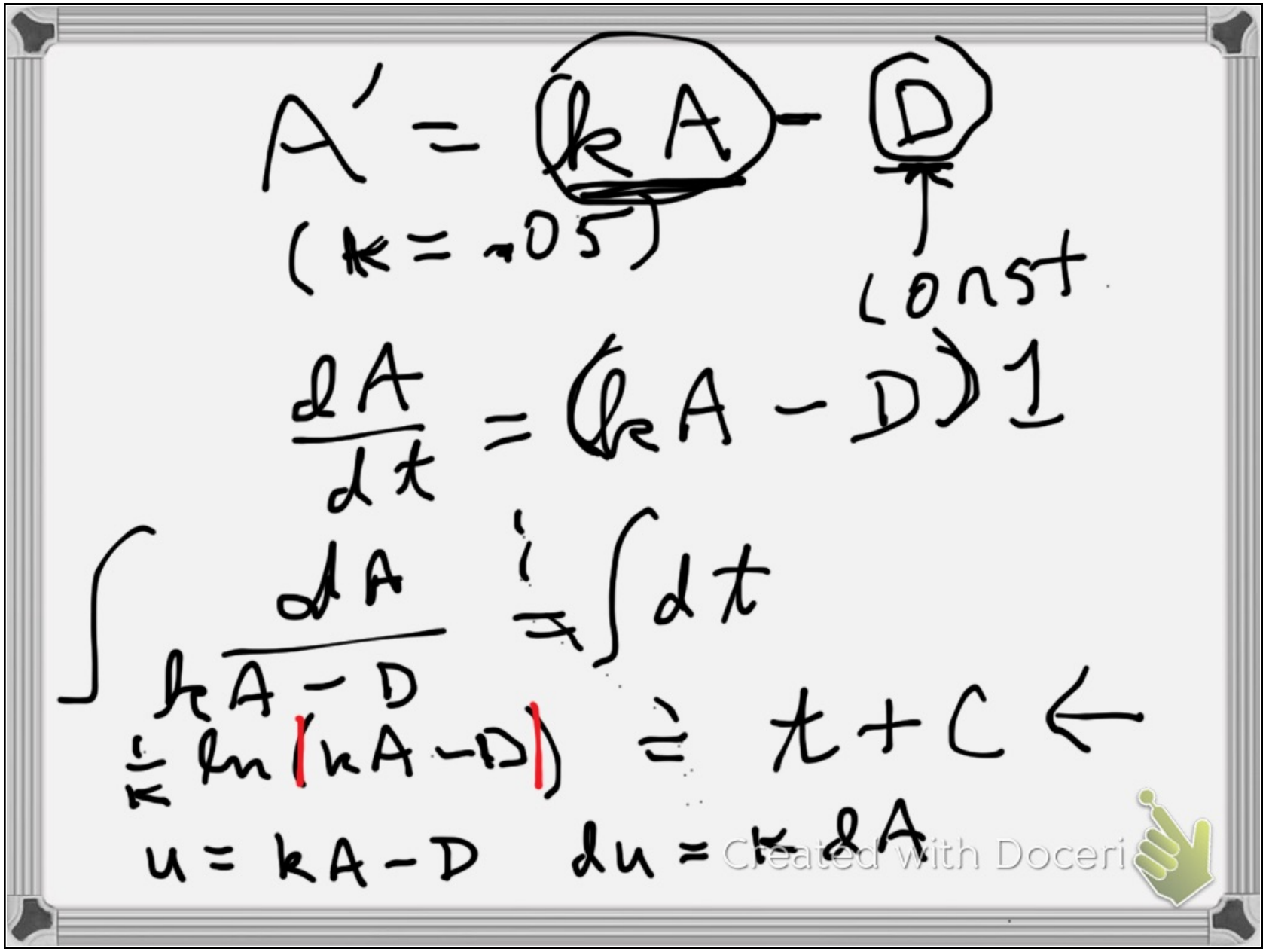
grows by 5% per year

$A = A(t)$

$\rightarrow \boxed{A'(t) = .05\,A(t)}$

Act 7 5 2

(b) $\boxed{A' = .05A - 10000}$


A' = kA - D
(k = .05)
const
dA/dt = (kA - D)1
∫ dA / (kA - D) = ∫ dt
(1/k) ln|kA - D| = t + C
u = kA - D
du = k dA
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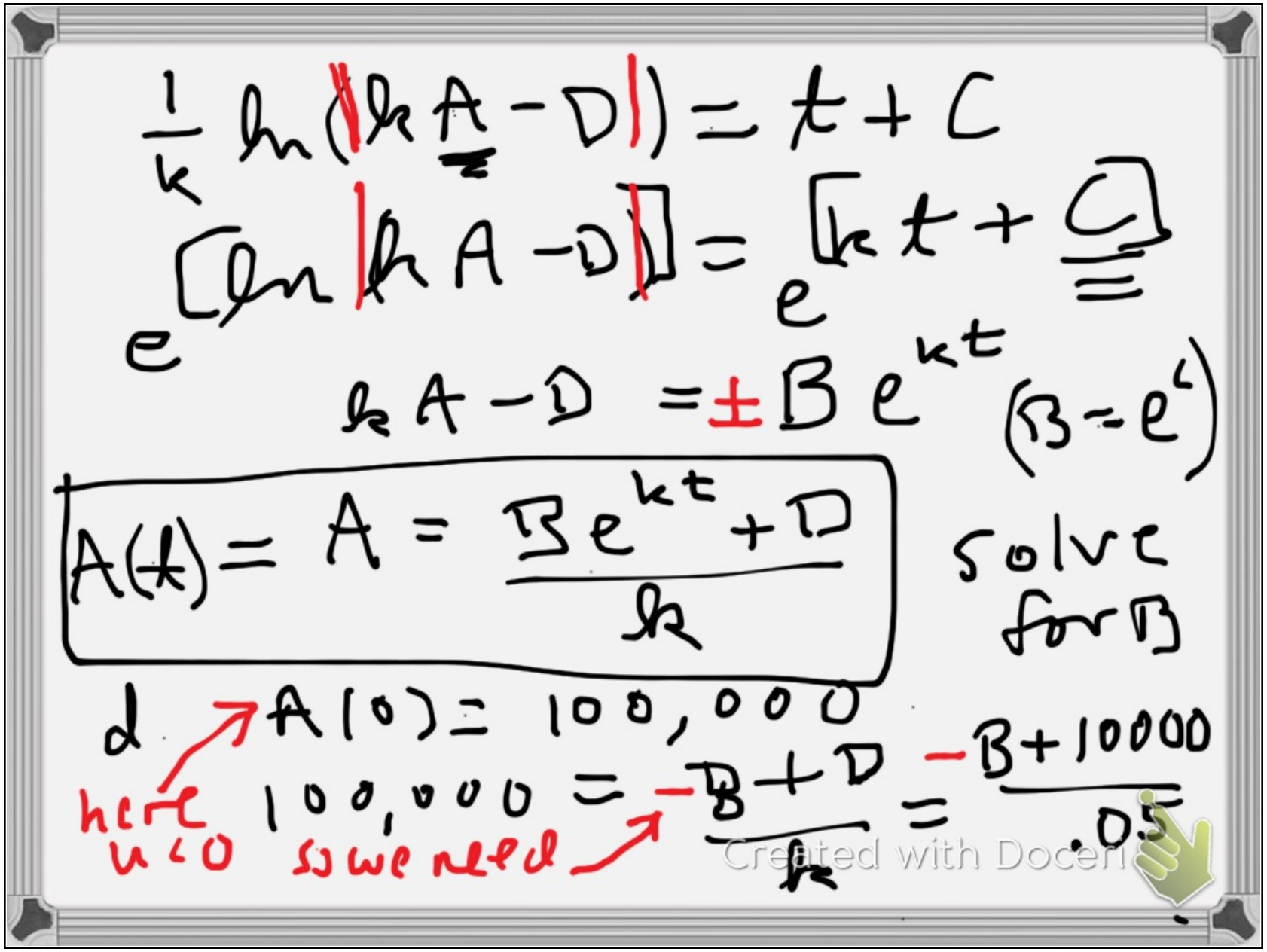
$\frac{1}{k}\ln(|kA-D|) = t + C$
$e^{[\ln|kA-D|]} = e^{[kt+C]}$
$kA - D = \pm B e^{kt}$ $(B = e^C)$
$A(t) = A = \frac{Be^{kt}+D}{k}$
solve for B
if $A(0) = 100{,}000$
here $u < 0$ so we need
$100{,}000 = \frac{-B+D}{k} = \frac{-B+10000}{.05}$
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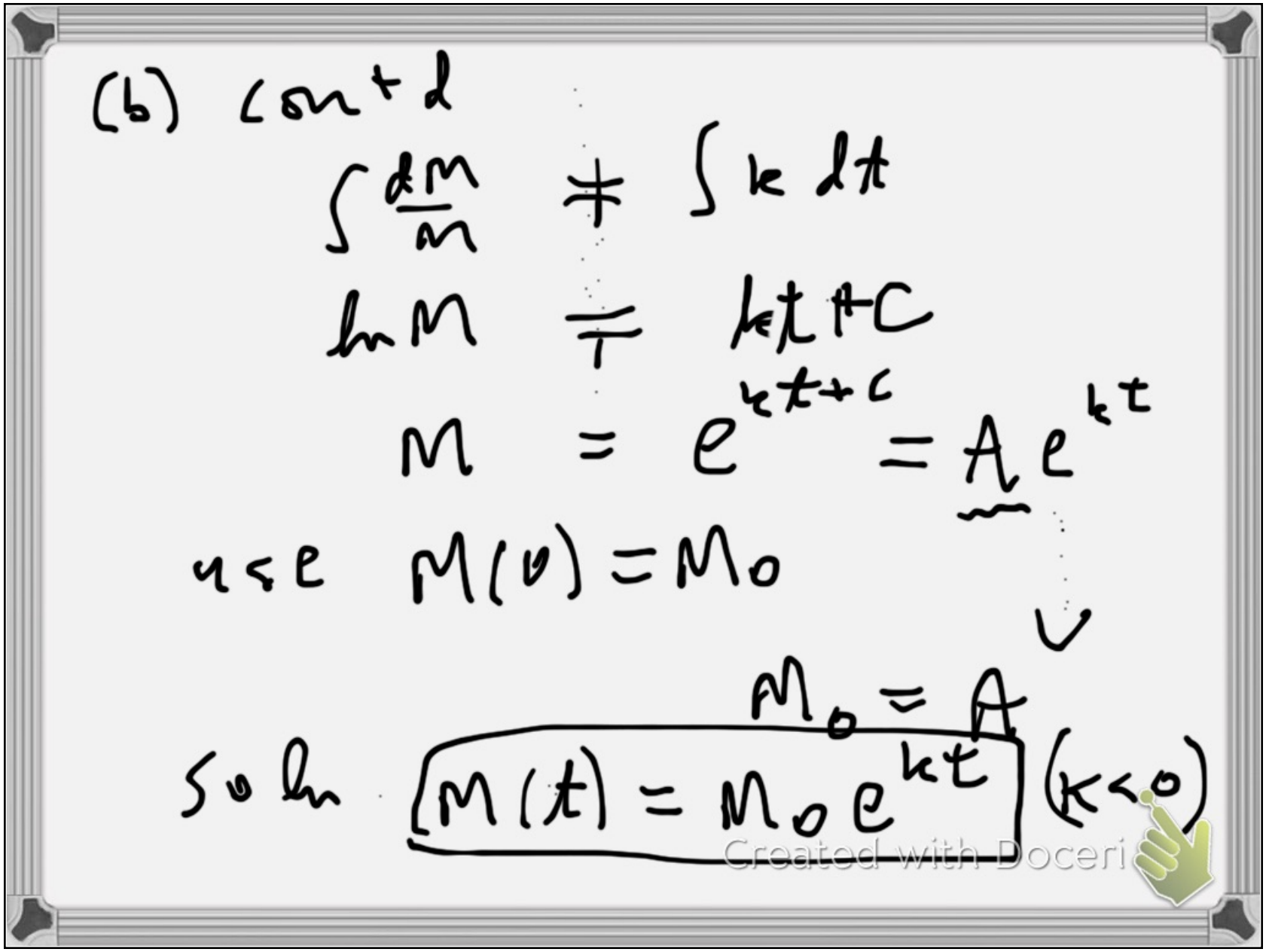
7.5.3 Act

$M' = kM$

(a) $\boxed{M'(t) = kM(t)}$


(b) $M(0) = M_0$

$\rightarrow \int \frac{M'}{M} = \int k\,dt$ $\left\{ \frac{dM}{M} = \int k\,dt \right.$


(b) cont'd
$\int \frac{dM}{M} = \int k\,dt$
$\ln M = kt + C$
$M = e^{kt+C} = Ae^{kt}$
$t = 0 \quad M(0) = M_0$
$M_0 = A$
So $M(t) = M_0 e^{kt}$ $(k<0)$
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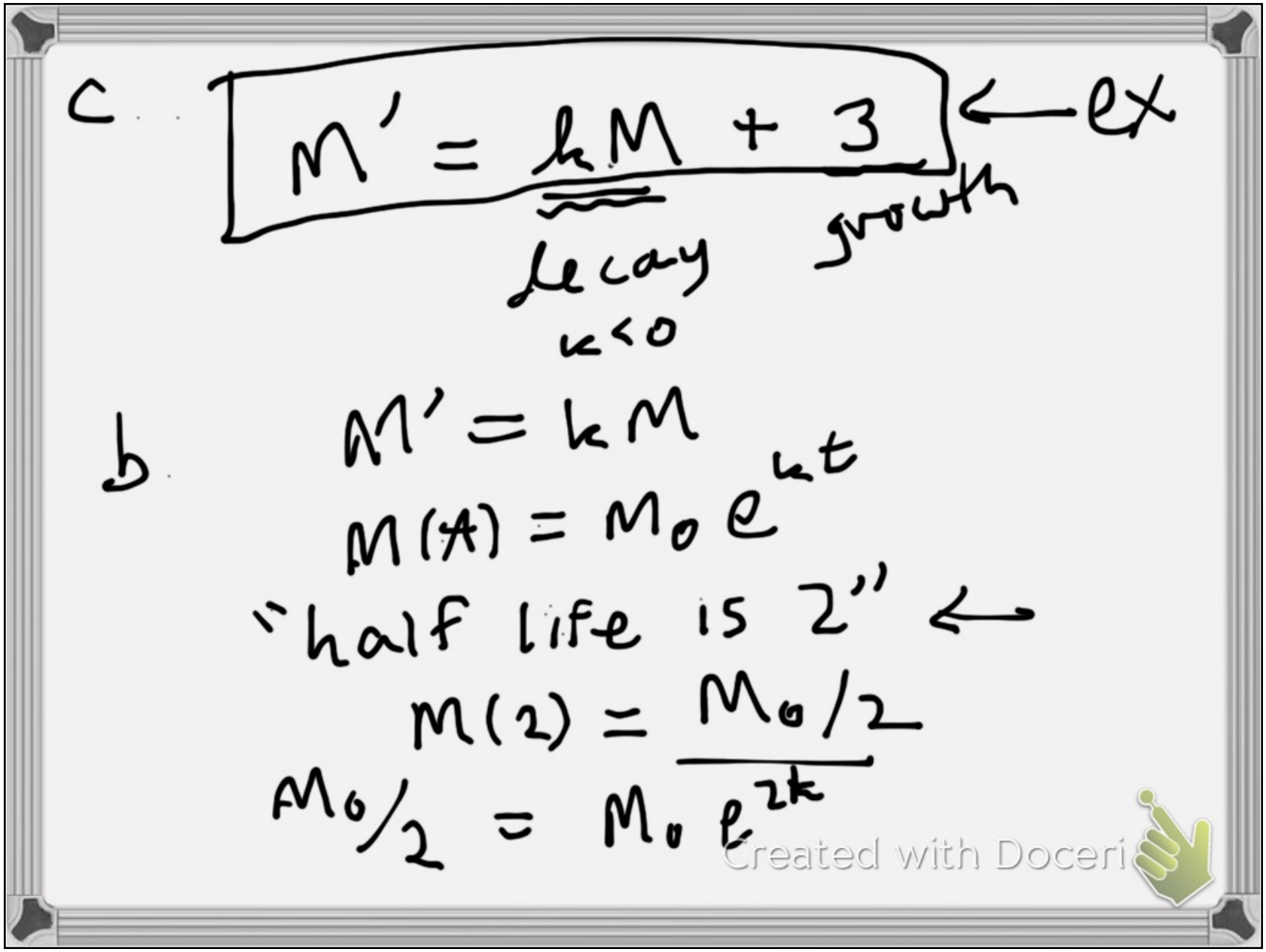
c
$M' = kM + 3$
ex
decay
growth
$k<0$
b
$M' = kM$
$M(t) = M_0 e^{kt}$
"half life is 2"
$M(2) = M_0/2$
$M_0/2 = M_0 e^{2k}$
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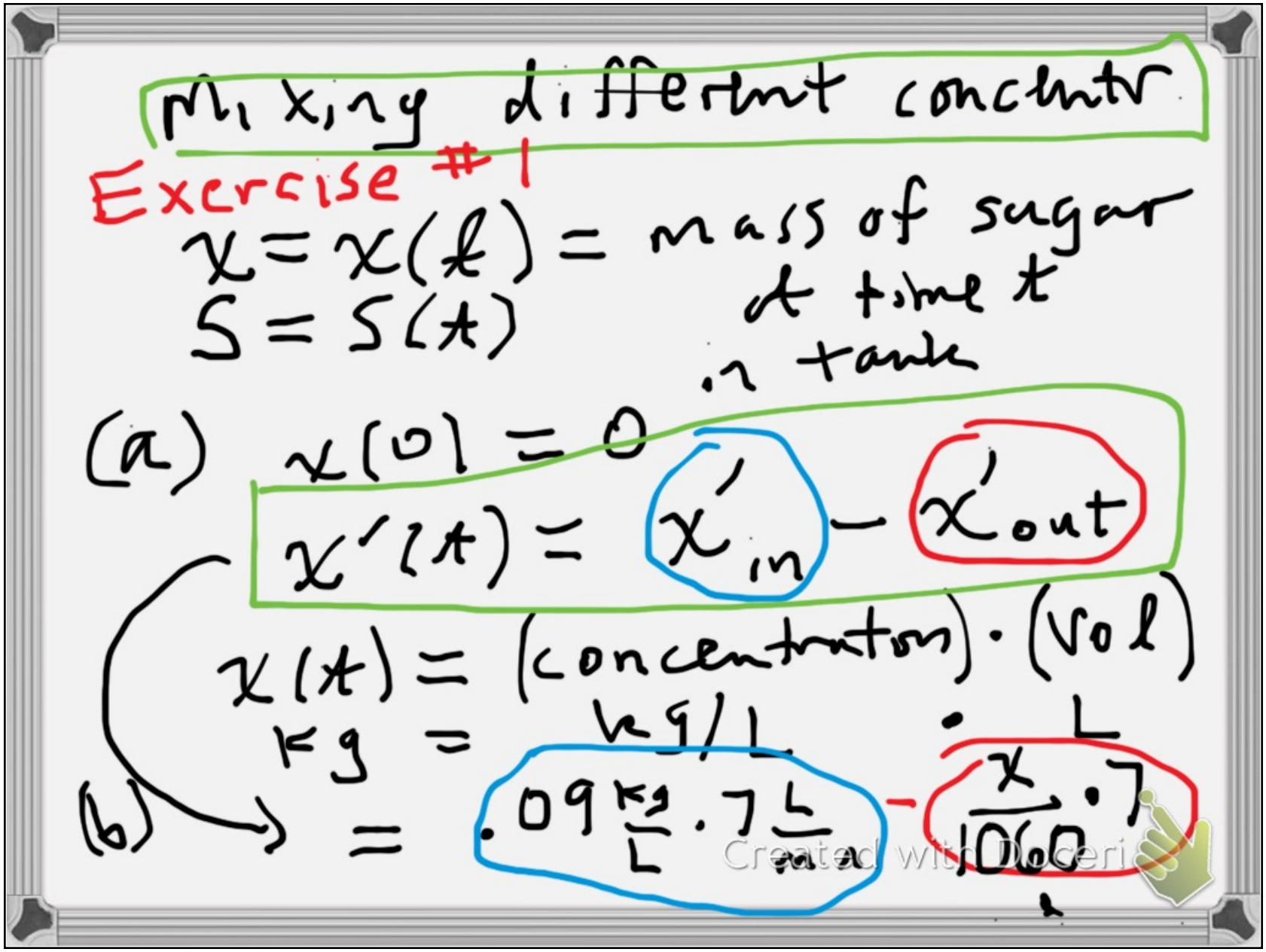
$$\frac{M_0}{2} = M_0 e^{2k}$$

$$\ln\left(\frac{1}{2}\right) = 2k$$

$$k = \ln\left(\frac{1}{2}\right)/2 \approx \_\_\_$$


Mixing different concntr
Exercise #1
x = x(t) = mass of sugar at time t in tank
S = S(t)
(a) x(0) = 0
x'(t) = x'_in − x'_out
x(t) = (concentration) · (vol)
kg = kg/L · L
(b) = .09 kg/L · .7 L − x/1000 · 7 L
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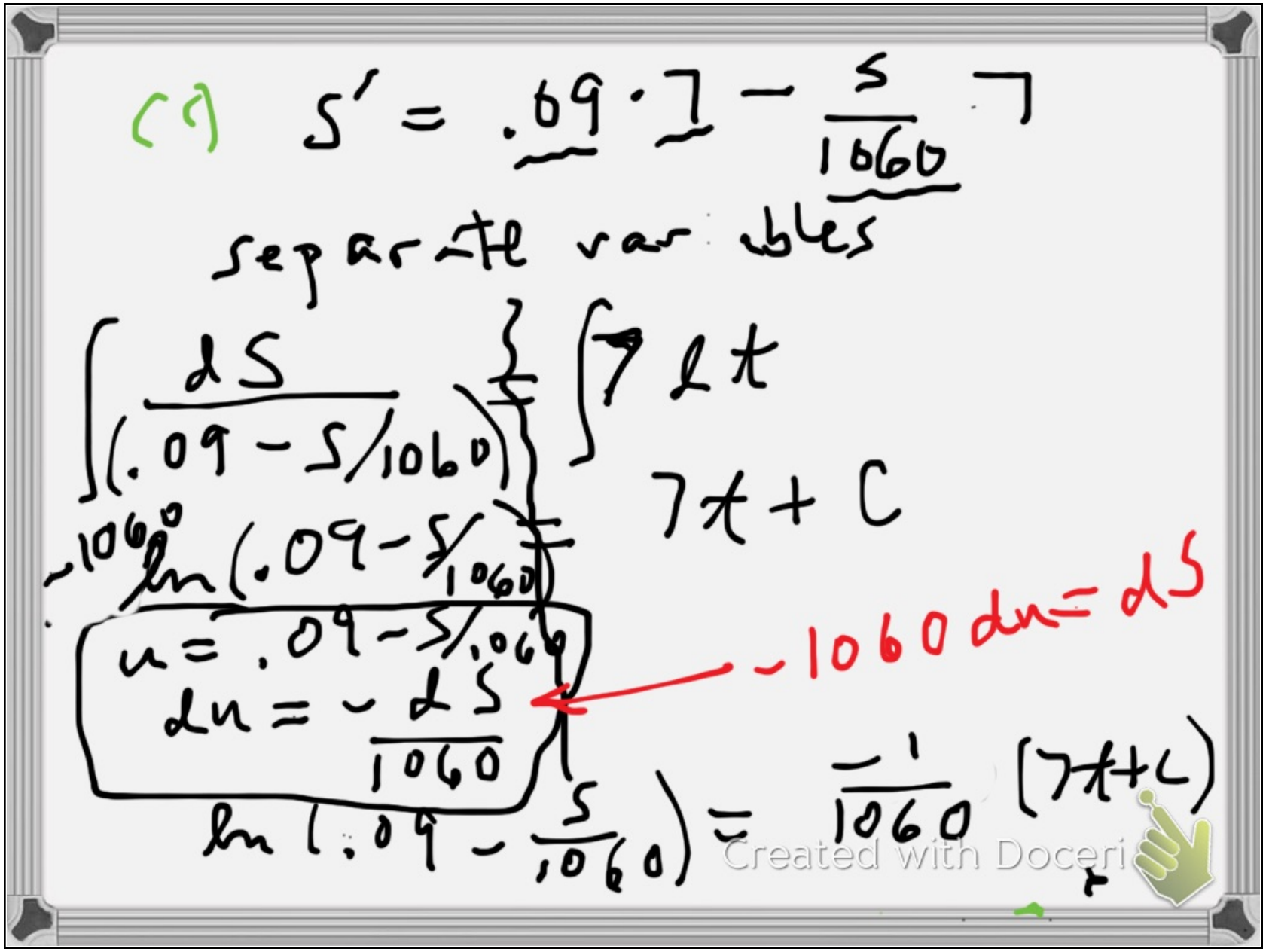
(9) $S' = .09 \cdot 7 - \frac{S}{1060}$
separable variables
$\int \frac{dS}{(.09 - S/1060)} = \int 7\,dt$
$7t + C$
$-1060 \ln(.09 - S/1060) = 7t + C$
$u = .09 - S/1060$
$du = -\frac{dS}{1060}$
$-1060\,du = dS$
$\ln(.09 - \frac{S}{1060}) = \frac{-1}{1060}(7t + C)$
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$$.09 - \frac{S}{1060} = e^{\left(-\frac{1}{1060} \cdot 7t\right)} \cdot A$$

could solve for S

$S(0) = 0$ solve for A

$$.09 = A$$

$$S(t) = \boxed{.09 - \frac{S}{1060} = .09\, e^{\left(\frac{-7t}{1060}\right)}}$$

$$S = (1060)(.09)\left(1 - e^{\frac{-7t}{1060}}\right)$$