mas162.04.29

Wednesday, April 29, 2020 10:53 AM

Review

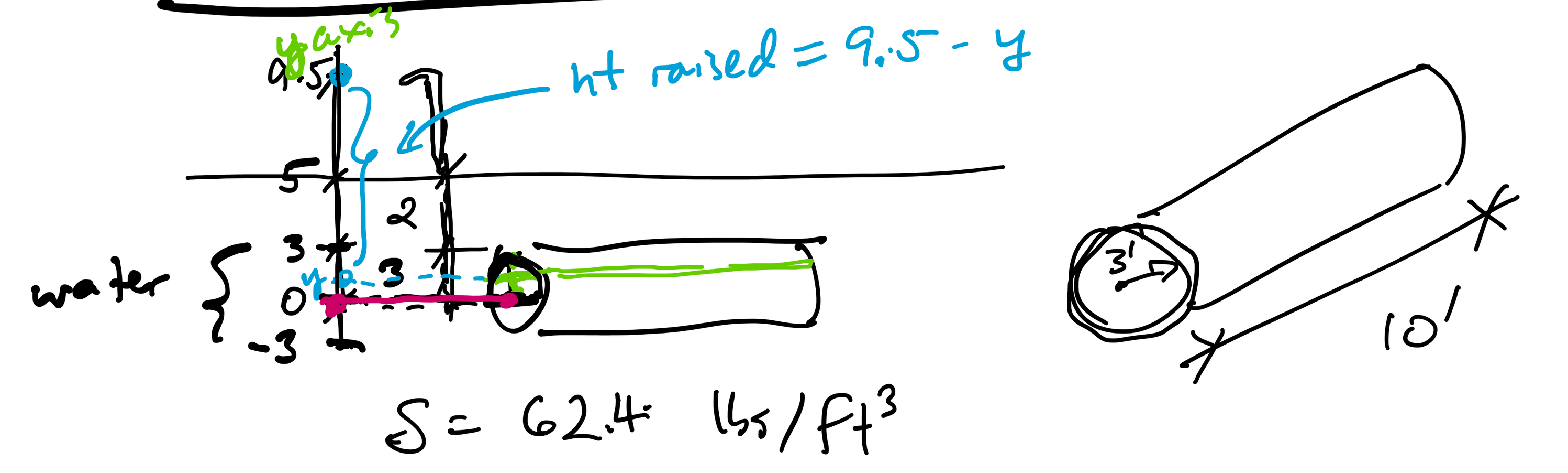
Final Exam
Wed 5/6
12 – 4 pm
noon
2-hr exam

Open office hour on zoom
Fri 1pm

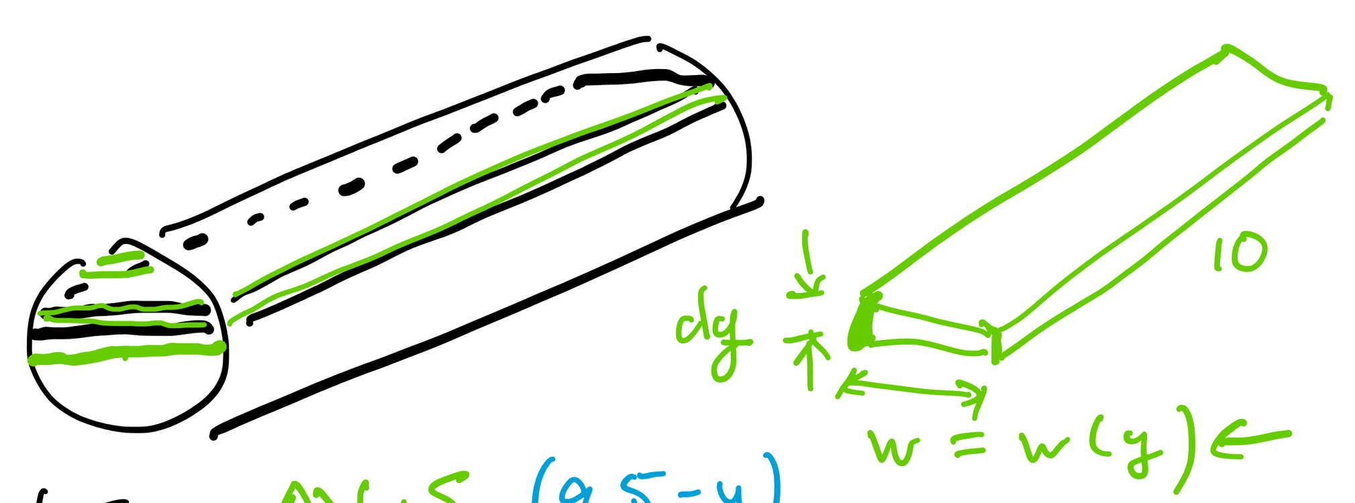
✓ 6.4 Ex 7 ✓ Act 3

7.6 Ex 4

Taylor Series



$\delta = 62.4$ lbs/ft^3



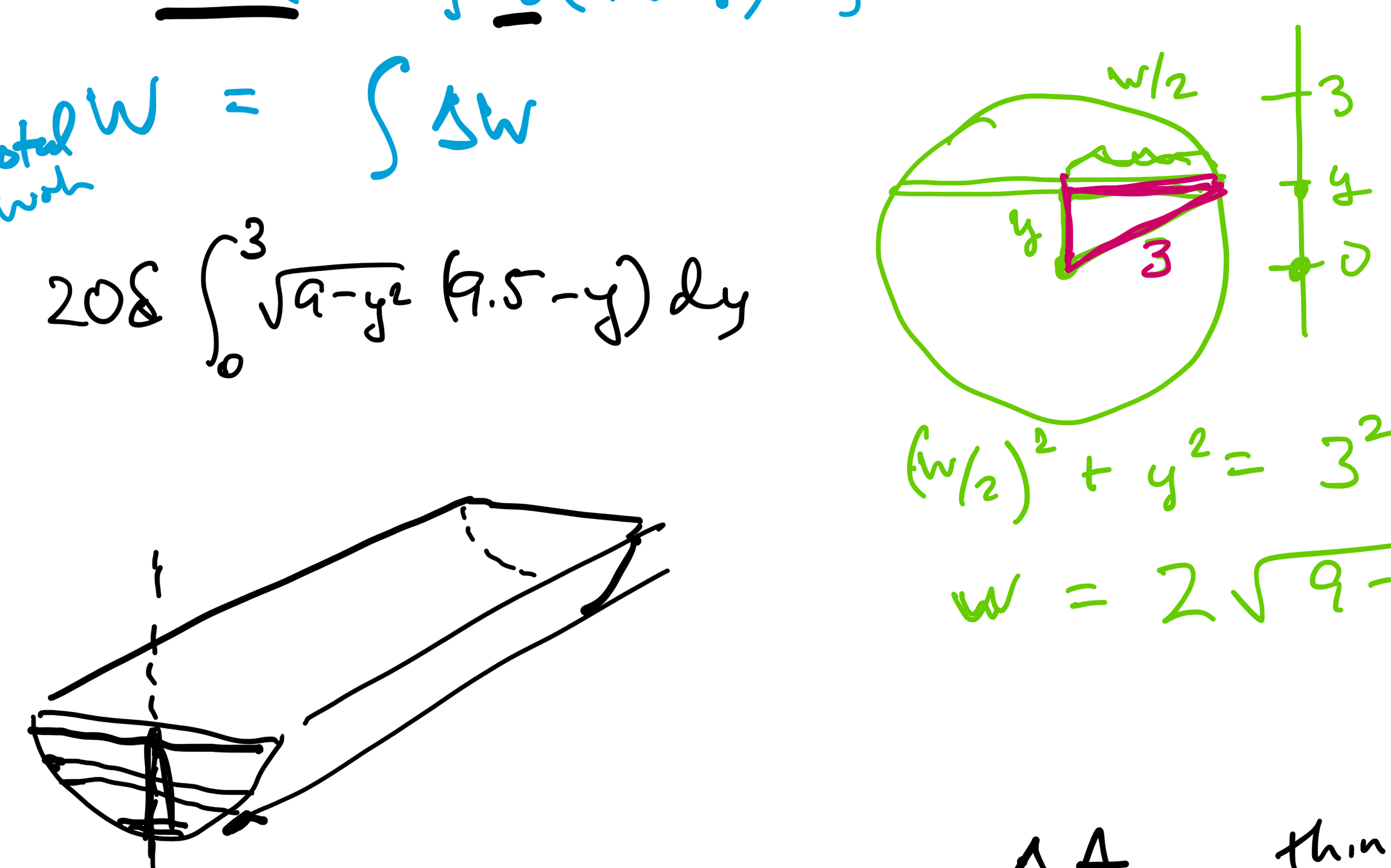
amt to work thin raise slab $\Delta W = \Delta V \cdot \delta \cdot (9.5-y)$ F (lbs)

$\Delta V = 10 w\, dy$, $w = w(y)$

$= 10 \cdot 2\sqrt{9-y^2}\, \delta\, (9.5-y)\, dy$

Ans: total work $W = \int \Delta W$

(a) $= 20\delta \int_0^3 \sqrt{9-y^2}\,(9.5-y)\,dy$



$(w/2)^2 + y^2 = 3^2$

$w = 2\sqrt{9-y^2}$

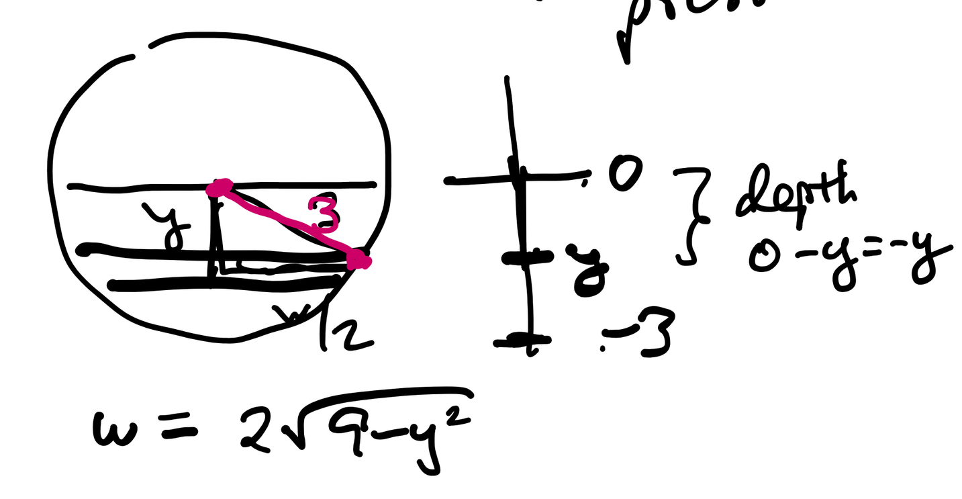
Pressure = Force / Area

$\Delta A = w \cdot dy$ thin horiz strip of ≈ const. pressure

Force = Pressure · Area (cleverness)

$\Delta F =$ depth · δ · ΔA

$\Delta F = (-y) \cdot \delta \cdot 2\sqrt{9-y^2}\, dy$



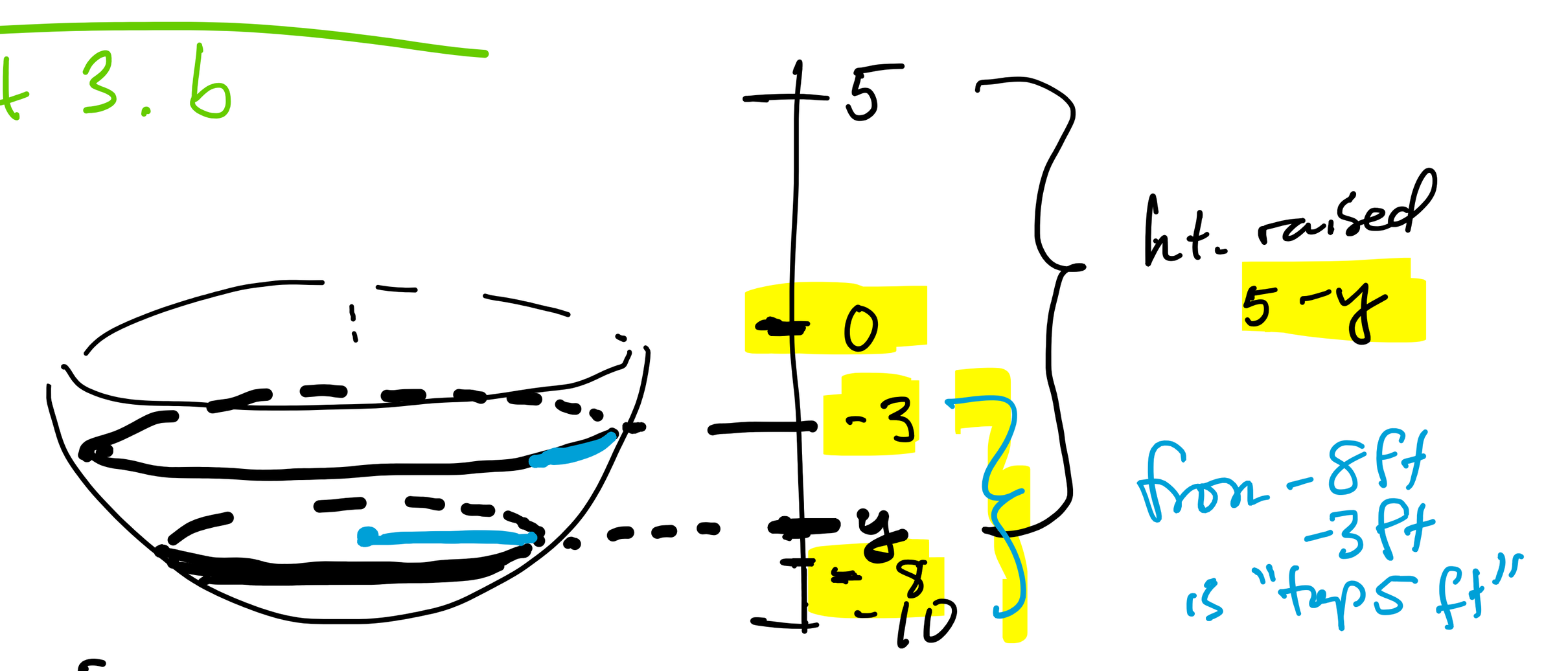
depth $0 - y = -y$

$w = 2\sqrt{9-y^2}$

total $F = \int \Delta F$

(b) $= -2\delta \int_{-3}^{0} y\sqrt{9-y^2}\, dy$

6.4 Act 3.b



dy $\Delta V = \pi r^2 dy = \pi(100-y^2)dy$

$r = r(y)$

$y^2 + r^2 = 10^2$

$r = \sqrt{100-y^2}$

$\Delta W = \Delta V \cdot \delta \cdot$ (ht raised)

F

$\pi(100-y^2)\,dy \quad 62.4 \quad (5-y)$

total wor $= 62.4\pi \int_{-8}^{-3} (100-y^2)(5-y)\, dy$

7.6 Ex 4

~~ex7 $\frac{dP}{dt} = .1P(10-P)$~~

Ex 4 $\frac{dP}{dt} = aP^2 - bP$

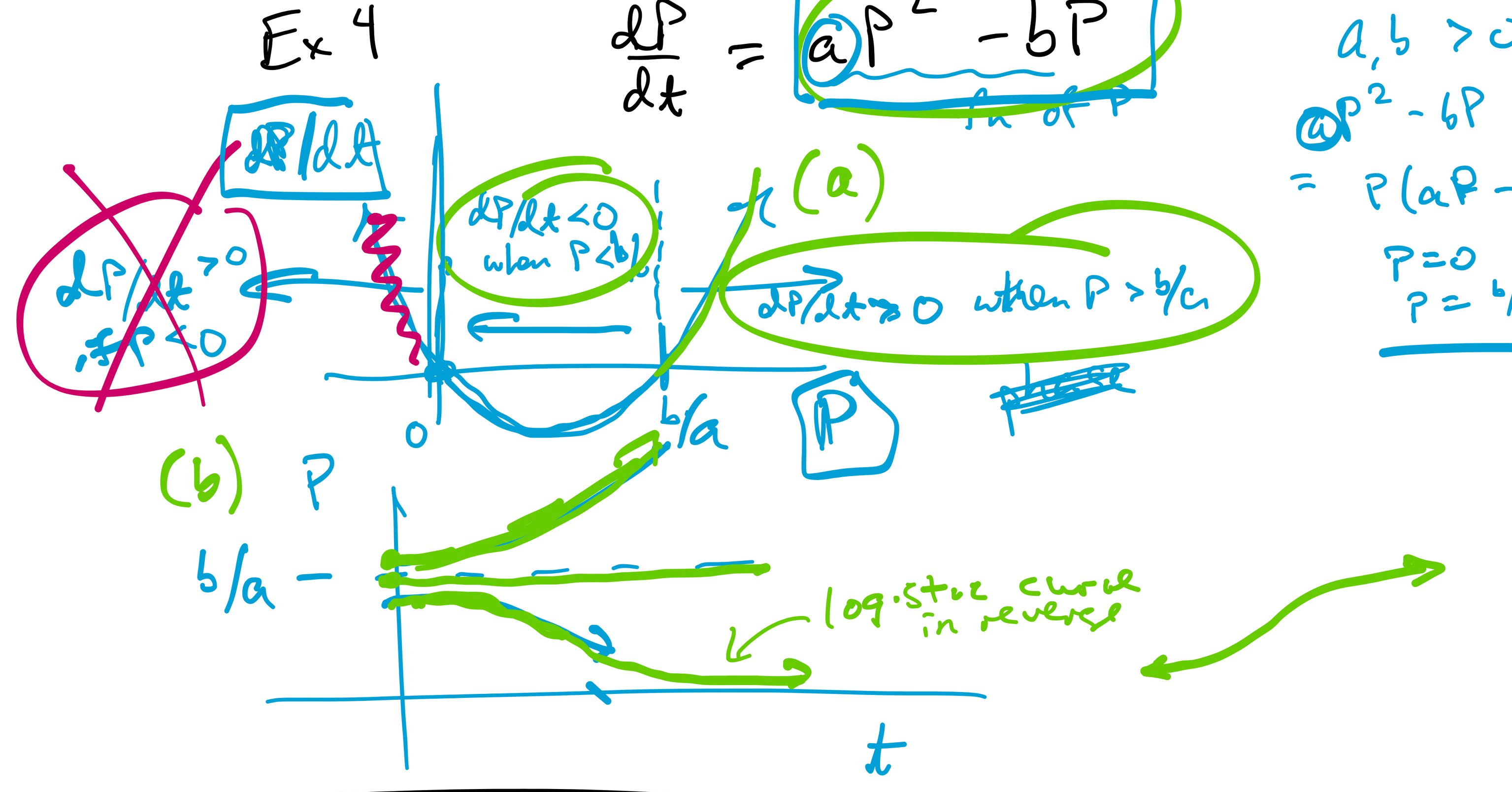
$a, b > 0$

$aP^2 - bP$

$= P(aP - b) = 0$

$P = 0$

$P = b/a$



Taylor Series

Problem types

- * Given $y = f(x) =$ ___
  Find Taylor Series for f.
- * Find interval of convergence for that series.
- * Starting with some known Taylor series for f find $f(x^2)$, $x^2 \cdot f(x)$ series

$\int f(x^2)\,dx$

$\int \sin(x^2)\,dx$ $[\sin(x^2)]'$

example $f(x)$ ~~$\int \ln(x^2)\,dx$~~

Find Taylor series for $f(x) = \ln x$ based at $x = 1$.

① Take derivs

② Plug in $x = a = 1$

$f(x) = \ln x$ $f(1) = 0$

$f'(x) = \frac{1}{x} = x^{-1}$ $f'(1) = 1$

k even $f''(x) = -x^{-2}$ sign − $f''(1) = -1$

odd $f'''(x) = (-2)(-1)x^{-3}$ + $f'''(1) = 2!$

even $f''''(x) = (-3)(-2)(-1)x^{-4}$ − $f''''(1) = -3!$

odd +

$f^{(k)}(x) = (-1)^{k+1}(k-1)!\, x^{-k}$

$f^{(k)}(1) = (-1)^{k+1}(k-1)!$

$f^{(k)}(x) = (-1)^{k+1}(k-1)!\, x^{-k}$ put $x = 1$

Taylor series for $\ln(x)$ at $x = 1$

$P(x) = \sum_{k=1}^{\infty} \frac{f^{(k)}(1)}{k!}(x-a)^k = \sum_{k=1}^{\infty} (-1)^{k+1}\frac{(k-1)!}{k!}(x-1)^k = \sum_{k=1}^{\infty} (-1)^{k+1}\frac{(x-1)^k}{k}$

recipe

simplifies

$\frac{1}{k} = \frac{(k-1)!}{k!} = \frac{(k-1)(k-2)(k-3)\cdots}{k(k-1)(k-2)(k-3)\cdots}$