mas162.04.15

Wednesday, April 15, 2020 9:55 AM

AGENDA

Leftovers: 8.4 alt. series error estimate ✓

anything else 8.1 - 8.4?

8.5 Ex 1, 2, 3

Lagrange error bound

8.4 Alt. Series Estimation Theorem

example $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \cdots + (-1)^{n+1}\frac{1}{n^2} + \cdots$

n=1 n=2

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$

This converges. Alt. Series Test.

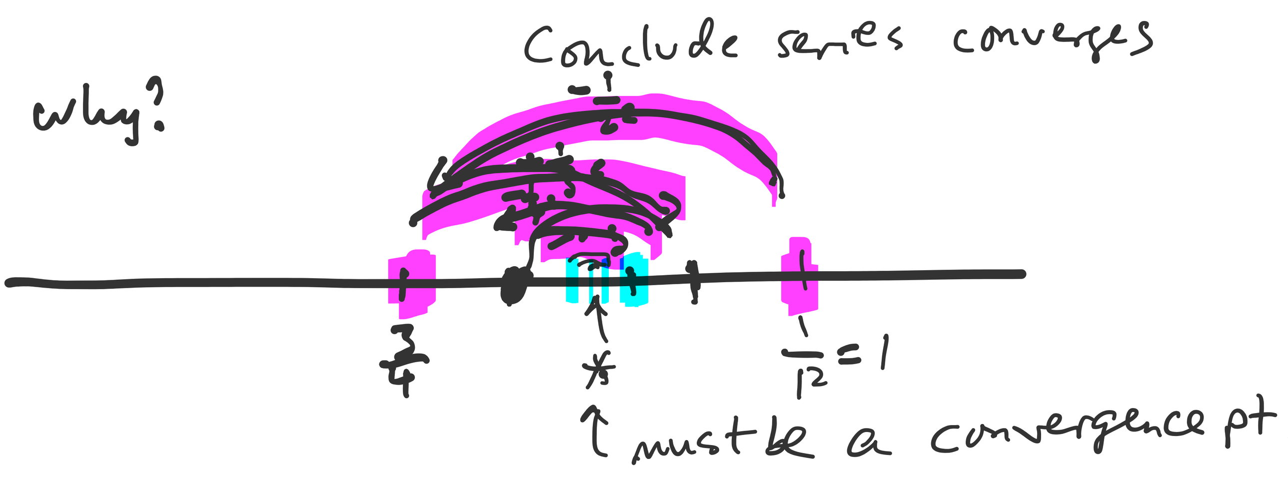
alternating? ✓

$\frac{1}{1^2} > \frac{1}{2^2} > \frac{1}{3^2} > \cdots$? ✓

$\frac{1}{n^2} \to 0$? ✓

Conclude series converges

why?



Q: $* = ?$ $\approx \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2}$

better $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2}$

$\boxed{|* - S_n| < \text{next hop size}} = \frac{1}{(n+1)^2}$ (in this example)

Q: Find how many terms needed in partial sum to approx $*$ with $\pm .001$.

Solve $.001 \geq \frac{1}{(n+1)^2}$ $A \le B$

$1000 \le (n+1)^2$ $\frac{1}{A} \ge \frac{1}{B}$

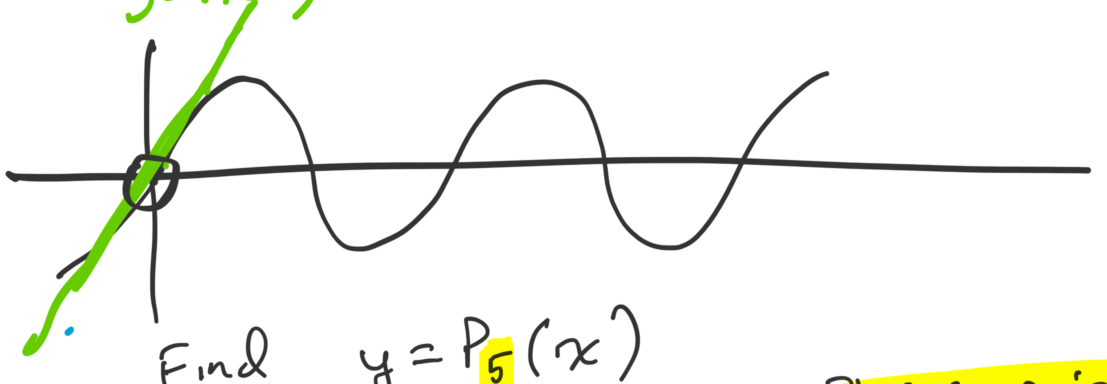
$\sqrt{1000} \le n+1$

$\sqrt{1000} + 1 \le n$

Example Ex 1

$f(x) = \sin(4x)$ $a = 0$

$y = P_1(x)$



Find $y = P_5(x)$

(1) $f(x) = \sin(4x)$

$f'(x) = 4\cos(4x)$

$f''(x) = -4^2 \sin(4x)$

$f^{(3)}(x) = -4^3 \cos(4x)$

$f^{(4)}(x) = 4^4 \sin(4x)$

$f^{(5)}(x) = 4^5 \cos(4x)$

$f^{(6)}(x) = -4^6 \sin(4x)$

trig fn "cycles"

pattern

(2) Plug $a=0$ in for x

$f(0) = 0$

$f'(0) = 4$

$f''(0) = 0$

$f^{(3)}(0) = -4^3$

$f^{(4)}(0) = 0$

$f^{(5)}(0) = +4^5$

0, -4^7, 0, $+4^9$

(3) Blue Box "Taylor Polynomials"

$P_5(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \frac{f^{(4)}(0)}{4!}(x-a)^4 + \frac{f^{(5)}(0)}{5!}(x-a)^5$

$\frac{f(a)}{0!}$ $\frac{f'(a)}{1!}$

$0! = 1$ $1! = 1$

weird? Definition!

$= 0 + 4^1 x^1 - \frac{4^3}{3!} x^3 + \frac{4^5}{5!} x^5$

$P_5(x) = 4x - \frac{4^3}{3!} x^3 + \frac{4^5}{5!} x^5$ Ans: 3rd box Ex 1

Point

$P_5(x) \approx \sin(4x)$ when $x \approx 0$

Q: how close?

$P_5(.1) \approx \sin(4(.1)) = \sin(.4)$

$|P_5(.1) - \sin(.4)|$ $\cancel{=}$ (how big is the diff, the error)

$|P_5(.1) - \sin(.4)| \le$ ____ worst case scenario

Lagrange Error Bound

$|P_5(.1) - \sin(.4)| \le M \frac{(.1)^6}{6!} \le \frac{4^6}{6!}(.1)^6 \approx .000006$

6×10^{-6}

$c = .1$

$a = 0$

$n = 5$

$\boxed{|P_n(c) - f(c)| \le M \frac{|c-a|^{n+1}}{(n+1)!}}$

$|f^{(6)}(x)| \le M$ can use $M = 4^6$

between 0, .1

for this problem

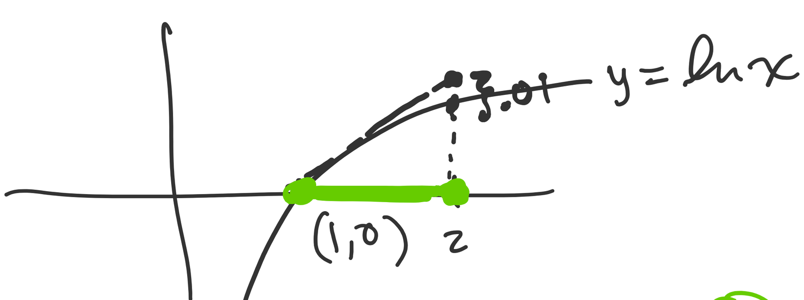
$|4^6 \sin(4x)| \le 4^6 \cdot 1 = M$

sin .4

7e. $f(x) = \ln x$ $a = 1$

$|P_n(2) - \ln 2| = ?$ wish ≤ 0.01

what n? How many terms?



(1) $f(x) = \ln x$

$f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = -x^{-2}$

$f'''(x) = 2x^{-3}$

$f^{(4)}(x) = -3 \cdot 2\, x^{-4}$ (3!)

$f^{(5)}(x) = (-4)(-3)(2)\, x^{-5}$ (4!)

$\boxed{f^{(n)}(x) = (-1)^{n+1}(n-1)!\, x^{-n}}$

(2) $f(1) = 0$

$f'(1) = 1$

$f''(1) = -1$

$f^{(3)}(1) = 2$

$f^{(4)}(1) = -3!$

$f^{(5)}(1) = 4!$

$f^{(n)}(1) = (-1)^{n+1}(n-1)!$

$|P_n(2) - \ln 2| \le M \frac{(2-1)^{n+1}}{(n+1)!} = \frac{n!}{(n+1)!} = \frac{1}{n+1}$

?

Solve $\frac{1}{n+1} \le .01$

$n+1 \ge 100$

$n = 101$

$|f^{(n+1)}(x)| \le M$

$1 \le x \le 2$

$|n!\, x^{-(n+1)}| \le M$

$= \left|\frac{n!}{x^{n+1}}\right| \le M$ worst

$= \left|\frac{n!}{1}\right| \le M$

Take $M = n!$