mas162.04.17

Thursday, April 16, 2020 3:05 PM

* Exam 4 Friday 4/24 $\boxed{12{:}30\text{pm} - 2{:}30\text{pm}}$
  Ch 8
  6 parts total 50-minute
* 8.1 – 8.5 leftover discussion?
* 8.6 discussion?
  Ex 1, 2
  Act ~~2~~ ~~c, d, b~~
  Act 5

8.6 Act 2

(a) $\boxed{\sum_{k=1}^{\infty} \frac{(x-1)^k}{3k}} = \frac{x-1}{3} + \frac{(x-1)^2}{6} + \frac{(x-1)^3}{9} + \cdots$

$f(1.5) = \underbrace{\frac{\frac{1}{2}}{3} + \frac{\left(\frac{1}{2}\right)^2}{6} + \frac{\left(\frac{1}{2}\right)^3}{9} + \cdots}_{\text{approx take some finite partial sum}} = ?$ ← often Hard question

$f(3) = \frac{2}{3} + \frac{2^2}{6} + \frac{2^3}{9} + \frac{2^4}{12} + \cdots$ diverges?

3 is not okay for x

1, is okay for x

"interval of convergence" ≡ domain of okay x-values
≡ all the values of x for which the series converges

example 8.6.3
before Act 8.6.2

Method for determining interval of convergence
Apply (absolute) Ratio Test

Let $a_k = \frac{(x-1)^k}{3k}$

Take $\lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = \lim_{k\to\infty} \left| \frac{\frac{(x-1)^{k+1}}{3(k+1)}}{\frac{(x-1)^k}{3k}} \right|$ $\qquad \frac{\left(\frac{A}{B}\right)}{\left(\frac{C}{D}\right)} = \frac{A}{B}\cdot\frac{D}{C}$

$= \lim_{k\to\infty} \left| \frac{3k}{3(k+1)} \cdot \frac{(x-1)^{k+1}}{(x-1)^k} \right|$ $\qquad \frac{3k}{3(k+1)} \approx 1$

$= \lim_{k\to\infty} |x-1| = |x-1|$

Series conv when $\boxed{|x-1| < 1}$

Ans (a) $\boxed{0 < x < 2}$

See 8.4 summary

Suppose $\lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = r$,
if $r > 1$ → the series diverges
if $r < 1$ → the series converges
if $r = 1$ → test is inconclusive

(b) $a_k = k x^k$

$\lim_{k\to\infty} \frac{|(k+1)x^{k+1}|}{|k x^k|} = \lim_{k\to\infty} \frac{k+1}{k} \left| \frac{x^{k+1}}{x^k} \right| = |x|$

Ans: $|x| < 1$

$-1 < x < 1$

(c) $\lim_{k\to\infty} \frac{\left| \frac{(k+1)^2 (x+1)^{k+1}}{4^{k+1}} \right|}{\left| \frac{k^2 (x+1)^k}{4^k} \right|}$ → $\lim_{k\to\infty} \left| \frac{4^k}{4^{k+1}} \frac{(k+1)^2}{k^2} \frac{(x+1)^{k+1}}{(x+1)^k} \right|$

$= \frac{1}{4}|x+1|$

Ans: $\frac{1}{4}|x+1| < 1$

$|x+1| < 4$

$|x-(-1)| < 4$

$-5 < x < 3$

(a) Does $\sum_{k=1}^{\infty} \frac{(x-1)^k}{3k}$ converge for $x = 2$?

Ans: No

$\sum_{k=1}^{\infty} \frac{(2-1)^k}{3k} = \sum_{k=1}^{\infty} \frac{1}{3k} = \frac{1}{3} \boxed{\sum_{k=1}^{\infty} \frac{1}{k}}$

harmonic diverges!

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

Act 5

$\ln(1+x) = \_ + \_x + \_x^2 + \_x^3 + \_x^4 + \cdots$

Find the Taylor series for $\ln(1+x)$ based at 0

(1) Method (8.5)

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$

$f''(x) = -(1+x)^{-2}$

$f^{(3)}(x) = +2(1+x)^{-3}$

$f^{(4)}(x) = -3\cdot 2\cdot(1+x)^{-4}$

$f^{(k)}(x) = (-1)^{k+1}(k-1)!(1+x)^{-k}$

(2)

$f(0) = 0$

$f'(0) = 1$

$f''(0) = -1$

$f^{(3)}(0) = 2$

$-3!$

$4!$

$\vdots$

$f^{(k)}(0) = (-1)^{k+1}(k-1)!$

8.4 recipe for Taylor Series $\quad \frac{f^{(k)}(0)}{k!} x^k$

$\ln(1+x) = 0 + (x) + \frac{-1}{2!}(x)^2 + \frac{2(x)^3}{3!} - \cdots \frac{(k-1)!}{k!}(x)^k$

$\boxed{\ln(x+1) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}}$

Method 8.6

$a = 1$, $r = -x$

$[\ln(1+x)]' = \boxed{\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots}$

$a = 1$, $r = x$ $\quad \boxed{\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots}$

$\boxed{\text{Known! } \frac{a}{1-r} = a + ar + ar^2 + ar^3 + \cdots}$

$\ln(1+x) = \int (1 - x + x^2 - x^3 + \cdots)\, dx$ 8.6

$\boxed{\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots}$

see Example 8.6.5

Ex 1

$\frac{5}{1-6x} =$ $\quad a = 5$, $r = 6x$, plug in

$\frac{a}{1-r} = a + ar + ar^2 + ar^3 + \cdots$

Ex 2

$(\arctan x)' = \frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 + \cdots$ ← geometric

$\frac{a}{1-r}$ $\quad a = 1$, $r = -x^2$

$\arctan x = \int (1 - x^2 + x^4 - x^6 + x^8 + \cdots)\, dx$

$\boxed{\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots}$

$\arctan(3x) = (3x) - \frac{(3x)^3}{3} + \frac{(3x)^5}{5} - \frac{(3x)^7}{7} + \cdots$

$\boxed{5x \arctan(3x) = (5x)(3x) - \frac{5x(3x)^3}{3} + \frac{5x(3x)^5}{5} - \cdots}$

Ex 3 Recommended.