mas162.04.08

Wednesday, April 8, 2020 12:35 PM

8.4

(2) Activity 8.4.7 especially — WILL POST

No class meetings Friday 4/10, Monday 4/13

New material remaining

8.5 & 8.6

Rev days — Exam Ch 8

Rev days — Final Exam

(1) Act 2, 36

(3) Ex 1, 2, 3, 6

✓ Activity 2

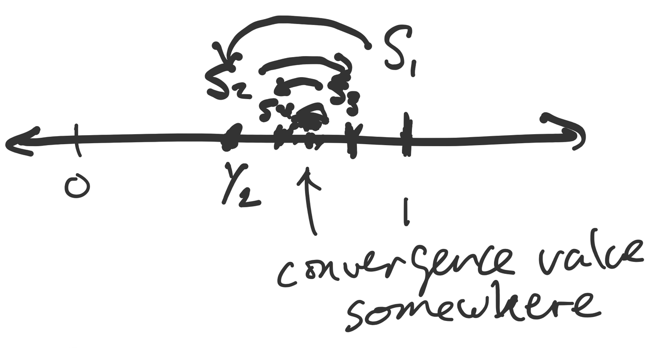
Harmonic series ≡ p-series p=1

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ (diverges) why?

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ (converges)

alternating harmonic series

n	partial sum
1	1
2	$1-\frac{1}{2}$
3	$1-\frac{1}{2}+\frac{1}{3}$
4	$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$



Activity 36

$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\,2k}{k+5}$ < conv? / div.?

"$(-1)^{k+1}$ alternator" Do divergence test

$= +1\left(\frac{2}{1+5}\right) -1\left(\frac{4}{2+5}\right) +1\left(\frac{6}{3+5}\right) -1(\text{etc})$

Since $\lim_{k\to\infty} \frac{2k}{k+5} = \boxed{2}$, the given series diverges by the Divergence Test.

$\frac{2k}{k+5} \sim \frac{2k}{k} = 2$ when k is big

Act. 8.4.7 (a) — (k)

(Summary 8.4.4 of all convergence tests) add your own example(s): 1 that converges, 1 that diverges

Recall integration

* u-sub

next int. by parts

suggests partial fractions

Convergence Tests

* geometric? is $|r|<1$ or $|r|\ge 1$?

* Divergence Test

Integral (if it looks like something you can integrate)

Limit Comparison

Ratio

Root

Alt. Series Test

8.4.7 (a) — (k)

which are geometric? $\sum_{n=1}^{\infty} ar^n$

(e) $\sum_{k=1}^{\infty} \frac{2^k}{5^k} = \sum_{k=1}^{\infty}\left(\frac{2}{5}\right)^k$ $\quad \frac{2^k}{5^k} = \left(\frac{2}{5}\right)^k$ $\quad r = 2/5 < 1 \Rightarrow$ conv.

(h) No $\quad \sum_{k=2}^{\infty} \frac{1}{k^k} = \frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\frac{1}{5^5}$ $\quad \cdot\frac{4}{9} \quad \cdot\frac{9}{16} \quad \cdot\frac{16}{25}$

(g) Yes

(g) $\sum_{k=2}^{\infty} \frac{3^{k-1}}{7^k} = \frac{3}{7^2}+\frac{3^2}{7^3}+\frac{3^3}{7^4}+\cdots$ $\quad \cdot 3/7 \quad \cdot 3/7 \quad \cdot 3/7$

yes geometric $r = 3/7 < 1 \Rightarrow$ conv.

Try Div. Test next

who fails? $\sum_{k=1}^{\infty} a_k \to \lim_{k\to\infty} a_k = \begin{cases} 0 \Rightarrow \text{no conclusion} \\ \text{not } 0 \Rightarrow \sum a_k \text{ diverges} \end{cases}$

summand

(b) $\sum_{k=1}^{\infty} \frac{k}{1+2k}$ $\quad$ Since $\lim_{k\to\infty} \frac{k}{1+2k} = \frac{1}{2}$, the series $\sum \frac{k}{1+2k}$ diverges by the Divergence Test.

$\frac{k}{1+2k} \approx \frac{k}{2k} = \frac{1}{2}$ not 0

(c) $\sum_{k=0}^{\infty} \underbrace{\frac{2k^2+1}{k^3+k+1}}_{a_k}$ $\quad$ not geometric

$\frac{2k^2+1}{k^3+k+1} \approx \frac{2k^2}{k^3} = \frac{2}{k} \to 0$

$\lim_{k\to\infty} a_k = 0$ Div. Test inconclusive

$\sum_{k=0} a_k \approx \sum_{k=0}^{\infty} \frac{2}{k} = 2\sum_{k=0} \frac{1}{k}$

Guess: $\sum a_k$ diverges because it is $\approx \sum \frac{1}{k}$

Ha! Harmonic $\sum_{n=1} \frac{1}{n^p}$ p=1 diverges

Let $a_k = \frac{2k^2+1}{k^3+k+1} = \frac{k^2}{k^3}\cdot\frac{(2+\frac{1}{k^2})}{(1+\frac{1}{k^2}+\frac{1}{k^3})} = \frac{1}{k}\cdot\frac{2+\frac{1}{k^2}}{1+\frac{1}{k^2}+\frac{1}{k^3}}$.

goes to 2 as $k\to\infty$

Let $b_k = \boxed{\frac{1}{k}}$

$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{(2+\frac{1}{k^2})}{(1+\frac{1}{k^2}+\frac{1}{k^3})} = 2.$

$\frac{a_k}{b_k} \approx 2$ $\quad a_k \approx 2b_k$

Since $\sum b_k$ diverges (Harmonic, p=1),

we conclude that $\sum a_k$ also diverges

by the Limit Comparison Test.

Similar: (a) $\frac{2}{\sqrt{k}-2} \approx \frac{2}{\sqrt{k}} \approx 2\frac{1}{k^{1/2}}$ $\quad p = 1/2 < 1$ $\quad \sum \frac{1}{\sqrt{k}}$ < conv / div

use $b_k = \frac{1}{k^{1/2}}$

$\sum_{k=1}^{\infty} \frac{1}{k^p}$ < conv if $p>1$ / div if $p\le 1$

use integral test

(f) $\frac{k^3-1}{k^5+1} \approx \frac{k^3}{k^5} = \frac{1}{k^2}$ $\quad \sum \frac{1}{k^2}$ p=2, converges

$b_k = \frac{1}{k^2}$

Ex 1

(a) $\sum \frac{(-1)^n}{3n} = -\frac{1}{3}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\right)$

conv. by Alt. Series Test

conv < absolutely ✗ / conditionally

Q: $\sum |a_k|$ < conv / div

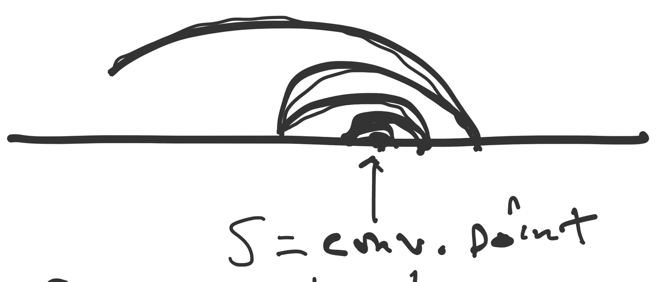
$= \frac{1}{3}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\right)$

div. ✗ (p=1)

(b) is abs. conv.

geometric $r = -\frac{1}{3}$

$\left|-\frac{1}{3}\right| < 1$



Ex 6 OMIT

S = conv. point

To be continued: Approx fact about Conv. Alt. Series

Next time $\quad \sum a_n(-1)^n \quad |S_n - S| \le a_{n+1}$

how far is nth partial sum from S