mas162.04.01

Wednesday, April 1, 2020 9:29 AM

* 8.1 leftovers?
* 8.2 * vocabulary and a few comments
  * discussion?

Request list:
applications for geometric series?
Ex 1, 2, 3, 4b
Act 2

Vocab IMPORTANT

sequence: list of numbers $S_1, S_2, S_3, \ldots$

series: sum $S_1 + S_2 + S_3 + \cdots + S_n$ finite series

$S_1 + S_2 + S_3 + \cdots$ infinite series

defn, involves limit

Last time: sequences arise from functions

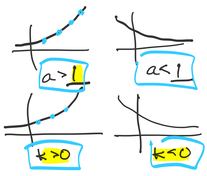
Given $y = f(x)$, make sequence $f(1), f(2), f(3), \ldots$

Two important functions

linear $f(x) = ax + b$

exponential $f(x) = Ca^x$

$f(t) = Ce^{kt}$



arithmetic sequence

sequence made from linear function

$a\cdot 1 + b,\ a\cdot 2 + b,\ a\cdot 3 + b, \ldots$ (+a, +a, +a)

repeatedly adding a

sequence made from exponential function

$Ca^1, Ca^2, Ca^3, Ca^4, \ldots$ (·a, ·a, ·a)

repeatedly mult by a

called geometric sequence

Act 2 Derivation of finite geom. series formula

$$a + ar^1 + ar^2 + ar^3 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

if $|r|<1$: $r^n \to 0$ as $n\to\infty$, so $\frac{a}{1-r}$ as $n\to\infty$

S_n = sum of first n terms of a geom. seq.

$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$

$-rS_n = -ar - ar^2 - ar^3 - \cdots - ar^{n-1} - ar^n$

"shift and cancel"

$S_n - rS_n = a - ar^n$

factor S_n | factor a

$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

Ex 1

$-1, -3.5, -12.25, \_\_\_$ (geometric)

$\cdot 3.5$ $\cdot 3.5$ $\cdot 3.5$

Ans: $-12.25(3.5)$

Infinite geometric sum formula

$$a + ar + ar^2 + \cdots = \begin{cases} \frac{a}{1-r} & \text{if } |r|<1 \\ \text{diverges} & \text{if } |r|\geq 1 \end{cases}$$

ex. $a=1$, $r=2$: $1 + 2 + 4 + 8 + \cdots = \infty$

series diverges

Ex 2

$2 + \frac{2}{7} + \frac{2}{7^2} + \cdots = \frac{7}{3}$ < number, diverges

$\cdot\frac{1}{7}$ $\cdot\frac{1}{7}$ $r = \frac{1}{7}$

$a = 2$

Sum is $\frac{2}{1-\frac{1}{7}}$ (formula) $= \frac{2}{\left(\frac{6}{7}\right)} = \frac{14}{6} = \frac{7}{3}$

Ex 3

$$\sum_{n=1}^{\infty}\left(\frac{3^n + 8^n}{12^n}\right)$$

$\frac{3^1+8^1}{12^1} + \frac{3^2+8^2}{12^2} + \frac{3^3+8^3}{12^3} + \cdots$

no consistent const. multiplier ⇒ not geometric

$\frac{3}{12} + \frac{3^2}{12^2} + \frac{3^3}{12^3} + \cdots$ } geom. $a = \frac{3}{12} = \frac{1}{4}$, $r = \frac{3}{12} = \frac{1}{4}$

$+ \frac{8}{12} + \frac{8^2}{12^2} + \frac{8^3}{12^3} + \cdots$ } geom. $a = \frac{8}{12} = \frac{2}{3}$, $r = \frac{8}{12} = \frac{2}{3}$

$= \frac{1/4}{1-1/4} + \frac{2/3}{1-2/3}$

Ex 4b.

$$\sum_{n=4}^{17}\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6 + \cdots + \left(\frac{1}{2}\right)^{17}$$

$\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$

(1) $r = \frac{1}{2}$

(2) $a = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

(3) $n = \_\_\_?$ = # terms in sum

$= \frac{a(1-r^n)}{1-r}$

Ans: $= \frac{\frac{1}{16}\left(1-\left(\frac{1}{2}\right)^{14}\right)}{\left(1-\frac{1}{2}\right)}$.

4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

how many?

$17 - 4 + 1 = 14$

"off-by-one" error