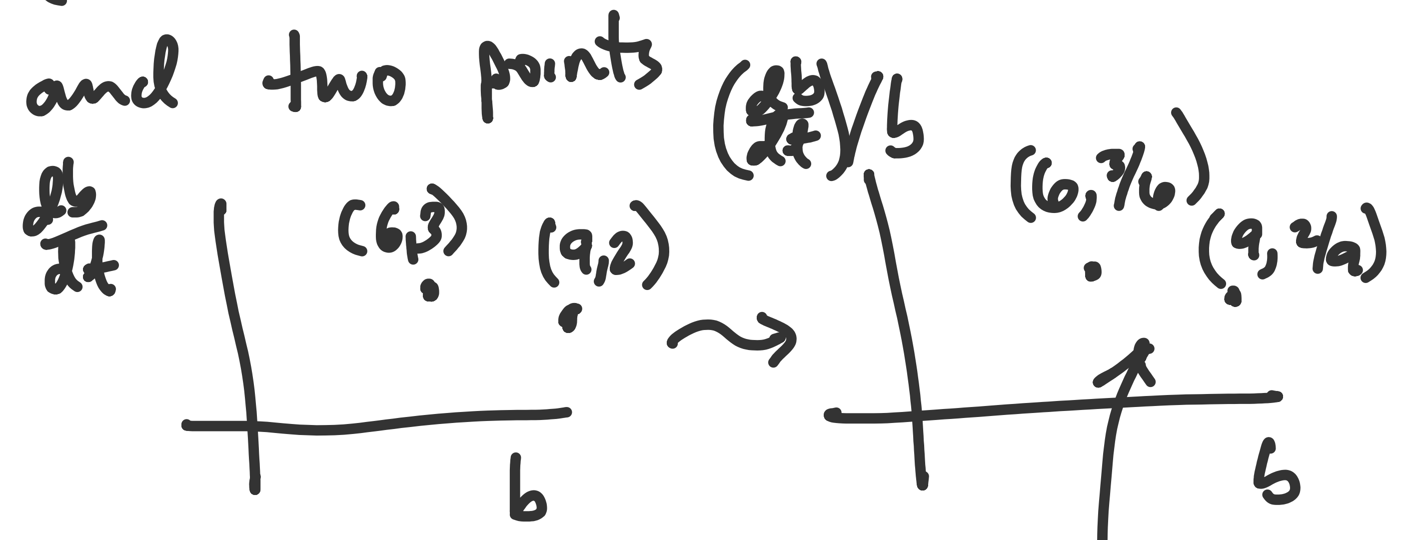
7.6 #6

Given $\dfrac{\left(\frac{db}{dt}\right)}{b} = Ab + B$

(some linear fn. of b)

and two points



Slope is $\dfrac{2/9 - 3/6}{9-3} = \dfrac{-5}{54}$

(a) use point-slope form

$$\left(\frac{db}{dt}\right)/b = -\frac{5}{54}(b-6) + \frac{1}{2}$$

some algebra

$$(*) \quad \frac{db}{dt} = \frac{5}{54}\, b\left(\frac{57}{5} - b\right).$$

(b) Carrying capacity $= \frac{57}{4}$
(you can see in (*))

(c) $\frac{db}{dt}$ is max at the vertex of the parabola $\frac{5}{54}\, b\left(\frac{57}{5} - b\right)$.
That happens half-way between the roots $0, 57/5$, which is $b = 57/10$.

(d) Plug in $k = \frac{5}{54}$, $N = \frac{57}{5}$, $P_0 = 1$
$P = .8\ (= .8 \cdot 1)$, solve for t

$$.8 = \frac{57/5}{\frac{57/5 - 1}{1} e^{-5/54 \cdot 57/5\, t} + 1}$$

7.6 #7

(a) $N = 10$ (just look at the D.E.)

(b) $\frac{dP}{dt} = .1P(10-P) - .2P$
$= .1P(8-P)$ some algebra

(c) new $N = 8$.

(d) use $k = .1$, $N = 8$, $P_0 = 10$ to find $P(1)$ in

$$P(t) = \frac{8}{\left(\frac{8-10}{10}\right) e^{(-.1)8t} + 1}.$$

(e) "within 10%" probably means $8 \pm .8$. Solve for t in

$$8.8 = \frac{8}{\left(\frac{8-10}{10}\right) e^{(-.1)(8t)} + 1}.$$