mas162.04.03

Friday, April 3, 2020 9:23 AM

Workshop = catch up 8.1, 8.2

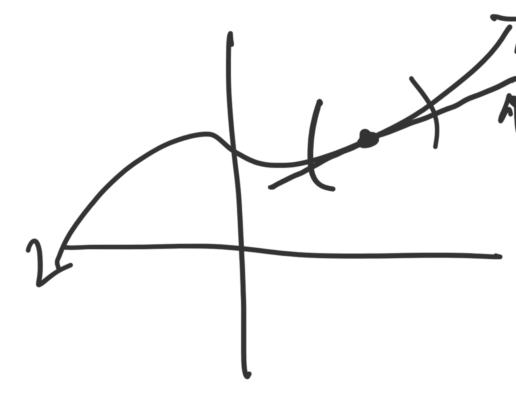
exam 3 grading in progress

No class Friday 4/10

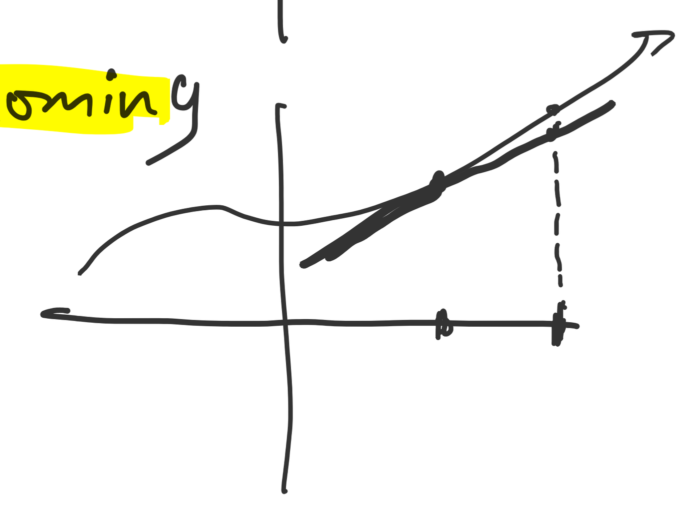
Monday 4/13

Main theme / Main point / Purpose for Ch8

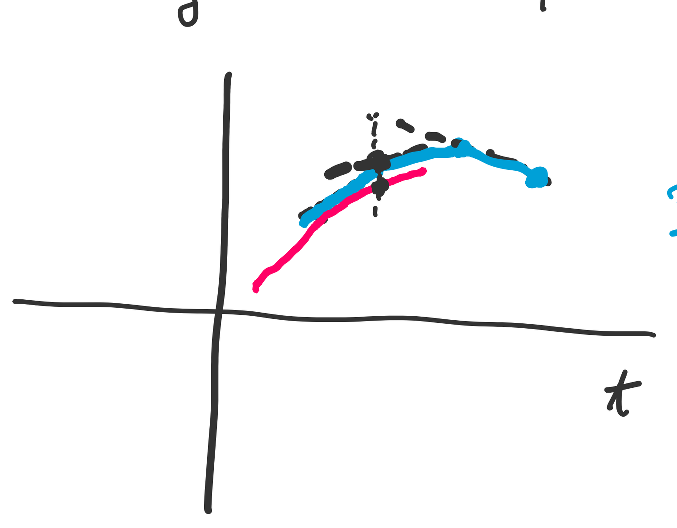
Calc 1



Shortcoming



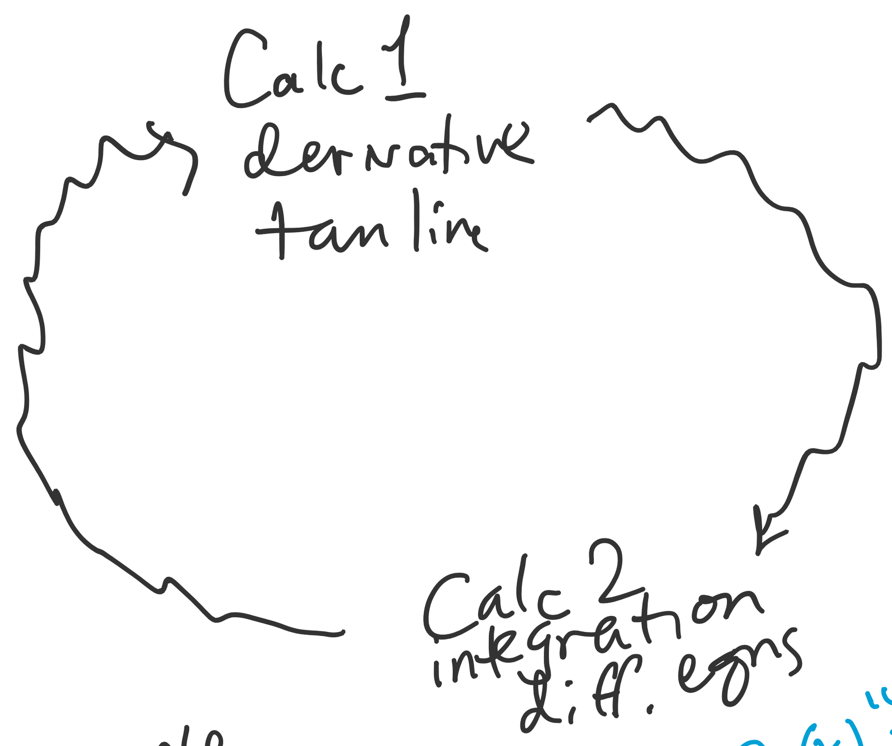
tangent is only a good approx near the point of focus



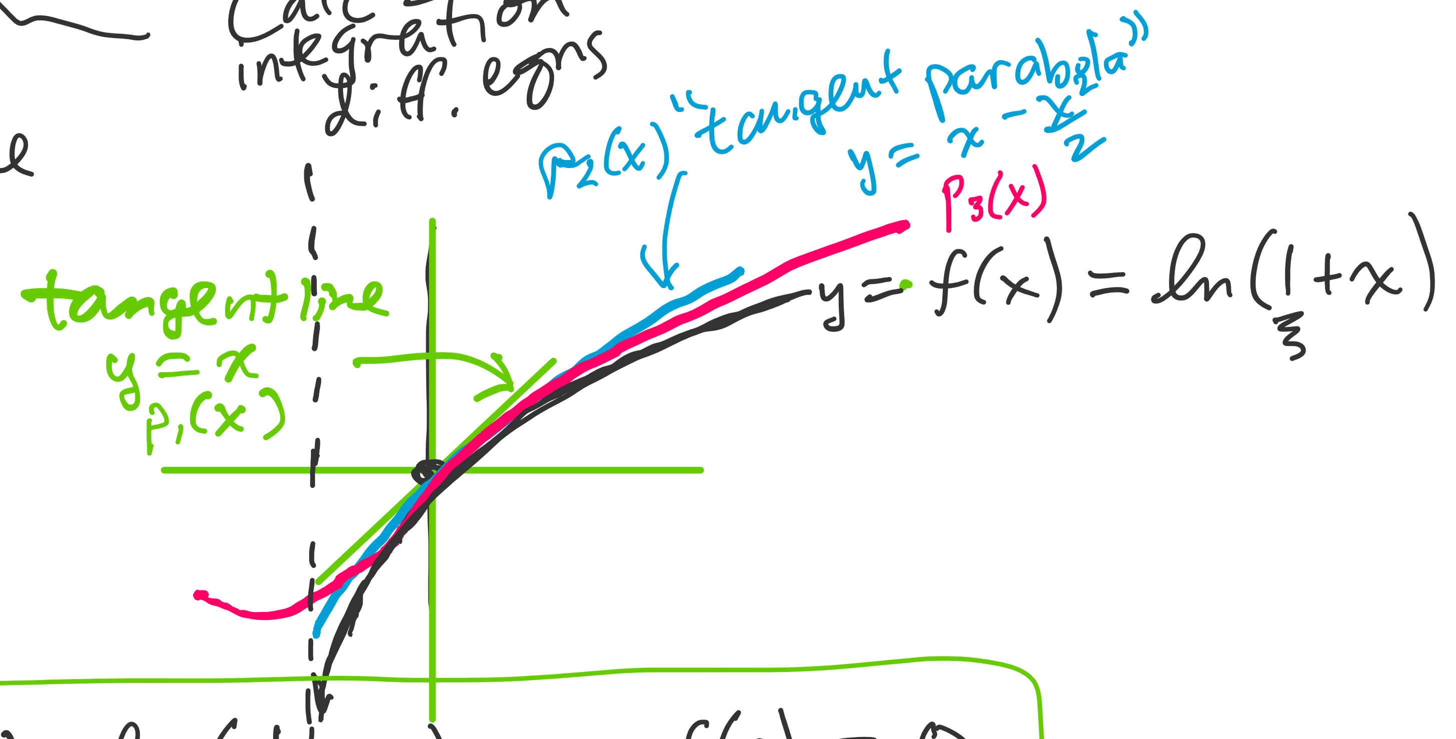
Given info $\frac{dy}{dt}=$ ~~~

Euler's method: $y=y(t)=$?

errors accumulate



example



$$\boxed{\begin{aligned} f(x) &= \ln(1+x) & f(0)&=0\\ f'(x) &= \frac{1}{1+x} = (1+x)^{-1} & f'(0)&=1\\ f''(x) &= (-1)(1+x)^{-2} & f''(0)&=-1\\ f'''(x) &= (-2)(-1)(1+x)^{-3} & f'''(0)&=2\\ f''''(x) &= (-3)(-2)(-1)(1+x)^{-4} &&\\ f'''''(x) &= (-4)(-3)(-2)(-1)(1+x)^{-5} && \end{aligned}}$$

$\pm 4!$

$f^{(\text{whatever derivative})}(x) = \pm(\text{factorial})(1+x)^{-(\text{whatever})}$

whatever -1

$P_1(x)=y=x$

$P_2(x)=y=x+ax^2$

$P_2''(x) = 2a$

$P_2(x) = x-\frac{x^2}{2}$

How to pick a? Best approx. make $P_2''(0)=f''(0)$

solve $P_2''(0)=-1$

$a=-\frac{1}{2}$

$P_3(x) = x-\frac{x^2}{2}+bx^3$

3 der: $0 \quad 0 \quad 3\cdot 2\cdot b$ ($3bx^2$ 1 der, $3\cdot 2\cdot b\cdot x$ 2 der, $3\cdot 2\cdot b$ 3 der)

$b=?$

$P_3'''(0)=f'''(0)$

$3\cdot 2\cdot b = 2$

$b=\frac{1}{3}$

$P_3(x) = x-\frac{x^2}{2}+\frac{x^3}{3}$

$P_4(x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$

$P_5(x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}$

$\lim_{n\to\infty} P_n(x)$

$$\boxed{p(x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots}$$

$P_1, P_2, P_3, P_4, \ldots$ are Taylor polynomials

$P_n \to P$ is Taylor series

$P_\infty(x)$ "infinite polynomial"

Hope as $n\to\infty$, $P\to f$

error $\to 0$

"perfect approx"

$$\ln(1+x) = \boxed{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots}$$

no longer approx

Possible! But

Plug in $x=1$

$\ln 2 = \ln(1+1) = \underbrace{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots}$

Ch 8 series: converges / diverges

converges? if so, to what

BUT!

Plug in $x=-1$

"$-\infty$" $=\ln 0 = \ln(1+-1) = \underbrace{-1-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}-\frac{1}{5}-\frac{1}{6}-\cdots}_{\text{diverges}}$

not defined