Writing Guidelines for Solutions to Problems

Courses for David W. Lyons

Fall 2017

For all problems on writing assignments and for selected problems on exams, ***solutions to problems must be written using complete English sentences***. Complete solutions

- ***show key steps of calculations*** (if any),
- ***use appropriate vocabulary*** from the text and course notes for objects and ideas, and
- ***provide a clear, concise explanation to a peer*** (a fellow student in the same course).

Some solutions are short and can be written in one sentence, but even short solutions have at least one idea that should be expressed in writing.

Final results of calculations given with no supporting work receive no credit, even if the final result is correct.

Solutions are not journal style writing. To promote objectivity and avoid subjectivity, do not use the first person singular "I". To promote a lively style, avoid the past tense and the passive voice. Use the present tense and the active voice whenever possible. Use the pronouns "it" or "this" only when it is absolutely clear to both you and the reader what "it" or "this" refers to.

Use diagrams when a picture clarifies or illustrates your explanation.

Sample Problems and Solutions

Problem 1. Solve $x^2 + x - 2 = 0$.

Discussion (scratch work). The problem is to solve a quadratic expression for the unknown x. There are two methods: factoring; and using the quadratic formula.

Sample solution. Because $x^2 + x - 2 = (x + 2)(x - 1)$, we see the solutions are $x = -2, 1$.

Alternative solution. Using the quadratic formula with $a = 1$, $b = 1$, and

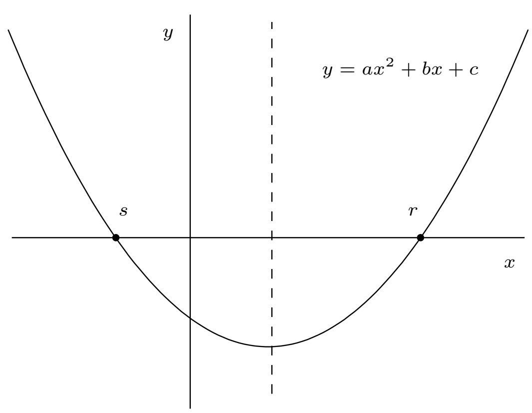
$c = -2$, we have

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{9}}{2} \\ &= \frac{-1 \pm 3}{2} \\ &= \frac{-4}{2}, \frac{2}{2} \\ &= -2, 1. \end{aligned}$$

Problem 2. Suppose you know that the equation $ax^2+bx+c=0$ has solutions $x = r, s$. Explain how you can use that knowledge to find the line of symmetry of the graph of $y = ax^2 + bx + c$.

Discussion (scratch work). The connection between solutions to an equation $f(x) = 0$ and the graph $y = f(x)$ is that if $f(a) = 0$, then $(a, 0)$ is a point on the graph, which means that $(a, 0)$ is an x-intercept of the graph. So we see that r, s are the x-intercepts. These have to be equally spaced to the right and left of the line of symmetry, so the line of symmetry must pass through the point on the x-axis half-way between r and s. The point half-way between two numbers is their average.

Sample solution. The solutions $x = r, s$ to the equation $ax^2 + bx + c = 0$ are the x-intercepts of a parabola $y = ax^2+bx+c$. The two x-intercepts are mirror reflections of one another across the line of symmetry, so the line of symmetry is half-way between them (see figure below). The half-way point along the x-axis between r and s is their average $(r + s)/2$, so the equation for the vertical line of symmetry is $x = \dfrac{r+s}{2}$.



Problem 3. Give a definition for the term ***irreducible quadratic polynomial***. Give an example, and say how you know your example is irreducible.

Discussion (scratch work). This problem asks for a definition and an example. The only work required is an explanation of why the example is in fact an example. This requires making the following connections.

- If a quadratic polynomial $f(x)$ factors as $f(x) = (ax+b)(cx+d)$ then we can solve $f(x) = 0$ by solving $ax + b = 0$ and $cx + d = 0$. This gives us one or two solutions.
- The solutions to $f(x) = 0$ are x-intercepts on the graph $y = f(x)$.
- So if a graph $y = f(x)$ has *no* x-intercepts, then it must *not* factor, and is therefore irreducible.

Sample solution. An ***irreducible quadratic polynomial*** is an expression of the form ax^2+bx+c that does not factor as a product of the form $(dx+e)(fx+g)$. An example is $x^2 + 1$. We know this is irreducible because if $x^2 + 1 = (dx + e)(fx + g)$, then the equation $x^2 + 1 = 0$ would have solutions $x = \frac{-e}{d}, \frac{-g}{f}$, so the parabola $y = x^2 + 1$ would have x-intercepts at $x = \frac{-e}{d}, \frac{-g}{f}$. But we know that $y = x^2+1$ has no x-intercepts because the y value $x^2+1 \geq 1 > 0$ is positive for every point on the graph.