mas162.04.20

Friday, April 17, 2020 3:47 PM

Ch 8 Review

Exam 4 Ch 8

6 parts

1 complete sentence(s)

Canvas Assignment (same as Homework, not "Quiz")

opens 12:30 Fri 4/24

closes 2:30pm Fri ~~4~~/24

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given $y = f(x)$, find

$f(x) = a_0 + a_1(x-a) + a_2(x-a)^2 + \cdots$

which test to use?

depends on question

8.6 blue box summary "known" $(a=0)$

Taylor series any a is okay

Maclaurin series

next 2 easiest $\begin{cases} \sin(x) = 1\cdot x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots \\ \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \end{cases}$

next easiest $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

geometric $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ $\left(a + ar + ar^2 + \cdots = \frac{a}{1-r},\ |r|<1\right)$ $(a=1\ \ r=x)$

$f(x) = \sin(x)$	$f(0) = 0$	$f^{(0)}(0)/0! = 0$
$f'(x) = \cos(x)$	$f'(0) = 1$	$f'(0)/1! = 1$
$f''(x) = -\sin(x)$	$f''(0) = 0$	$f''(0)/2! = 0$
$f'''(x) = -\cos(x)$	$f'''(0) = -1$	$f'''(0)/3! = -\frac{1}{3!}$
$f^{(4)}(x) = \sin(x)$	$f^{(4)}(0) = 0$	$f^{(4)}(0)/4! = 0$
repeats	repeats	

$$\sum_{k=0} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

Find power series for $f(x) = e^{(x^2)}$

know $e^x = 1 + x + \frac{x^2}{2} + \cdots$ so $e^{(x^2)} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$

Find power series for $\int \cos(2x)\,dx$

know $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

① $\cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots$

2^2x^2

② ↓ term-by-term integration

$\int \cos(2x)\,dx = x - \frac{2^2x^3}{3\cdot 2!} + \frac{2^4x^5}{5\cdot 4!} - \frac{2^6x^7}{7\cdot 6!} + \cdots$

method 2 harder

$\int \cos(2x)\,dx = \frac{1}{2}\int \cos u\,dx = \frac{1}{2}\sin(2x)$

$u = 2x$

$du = 2dx$

$f(x) = \frac{1}{2}\sin 2x$ $f(0) = \sim$

$f'(x) = \sim$ $f'(0) = \sim$

$f''(x) = \sim$ $f''(0) = \sim$

Find the power series for $f(x) = \arctan x$ based at $x=0$ by starting with the geometric series $\frac{1}{1-x}$. Hint: $(\arctan x)' = \frac{1}{1+x^2}$

$\arctan' x = \frac{1}{1+x^2}$

sub $-x^2$ for x

$\frac{1}{1-(-x^2)} = \frac{1}{1+x^2}$

① sub. $\frac{1}{1-x} \rightsquigarrow \frac{1}{1+x^2}$

② term-by-term int.

① $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

sub $-x^2$ for x

$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \cdots$

$(-x^2)^2$ $(-x^2)^3$

② $\int \frac{1}{1+x^2}dx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

$\arctan x$

Maclaurin series for $\arctan x$

= Taylor series for $\arctan x$ based at $x=0$.

Q. Find the interval of convergence for

$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \pm \frac{x^{odd}}{\text{same odd}}$

use abs ratio test

$\text{term}_n = (\pm 1)\frac{x^{2n+1}}{2n+1}$

$y = 2n+1$

$n =$	1	2	3	4	
$2n+1 =$	3	5	7	9	...

$$\lim_{n\to\infty}\left|\frac{\text{term}_{n+1}}{\text{term}_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{x^{2(n+1)+1}}{2(n+1)+1}}{\frac{x^{2n+1}}{2n+1}}\right|$$

$2n+3$

$$= \lim_{n\to\infty}\left|\frac{x^{2n+3}}{x^{2n+1}}\right|\left|\frac{2n+1}{2n+3}\right| \to 1$$

$2n+3-(2n+1) = 2$

$$= \lim_{n\to\infty} x^2 = x^2$$

Conclusion $x^2 < 1$ is the interval of conv.

$-1 < x < 1$

Given some series, test for convergence/divergence

Study! 8.4.4 Summary of conv. tests.

- geometric
- Divergence Test
- Integral Test
- ~~Direct Comp Test~~
- Limit Comparison Test
- (Absolute) Ratio Test
- ~~Root Test~~
- Alternating Series Test

Practice Activity 8.4.7 (a)–(k)

$\sum_{k=1}^{\infty} \frac{\sqrt{k+2}}{k^2+2k+4}$ converges or diverge?

$a_k = \frac{\sqrt{k+2}}{k^2+2k+4} \approx \frac{\sqrt{k}}{k^2} = \frac{k^{1/2}}{k^2} = \frac{1}{k^{3/2}} \leftarrow p = 3/2$

Known $\sum_{k=1} \frac{1}{k^{3/2}}$ < conv because $p = 3/2 > 1$ / div

Idea so $\sum a_k$ should converge.

pre-work $a_k = \frac{\sqrt{k+2}}{k^2+2k+4} = \frac{\sqrt{k}\sqrt{1+\frac{2}{k}}}{k^2\left(1+\frac{2}{k}+\frac{4}{k^2}\right)} = \frac{1}{k^{3/2}}$ stuff that $\to 1$ as $k\to\infty$

Let $b_k = \frac{1}{k^{3/2}}$

We have $\lim_{k\to\infty}\frac{a_k}{b_k} = \frac{\frac{1}{k^{3/2}}\left(\text{stuff that}\to 1 \text{ as } k\to\infty\right)}{\frac{1}{k^{3/2}}} = 1.$

So by the Limit Comparison Test, $\sum a_k$, $\sum b_k$ either both conv or both div.

We know $\sum b_k$ conv $(p = 3/2 > 1)$ so $\sum a_k$ converges.