mas162.03.30

Monday, March 30, 2020 9:01 AM

Exam 3 grading in progress.
See email announcements
OR Canvas announcements page.

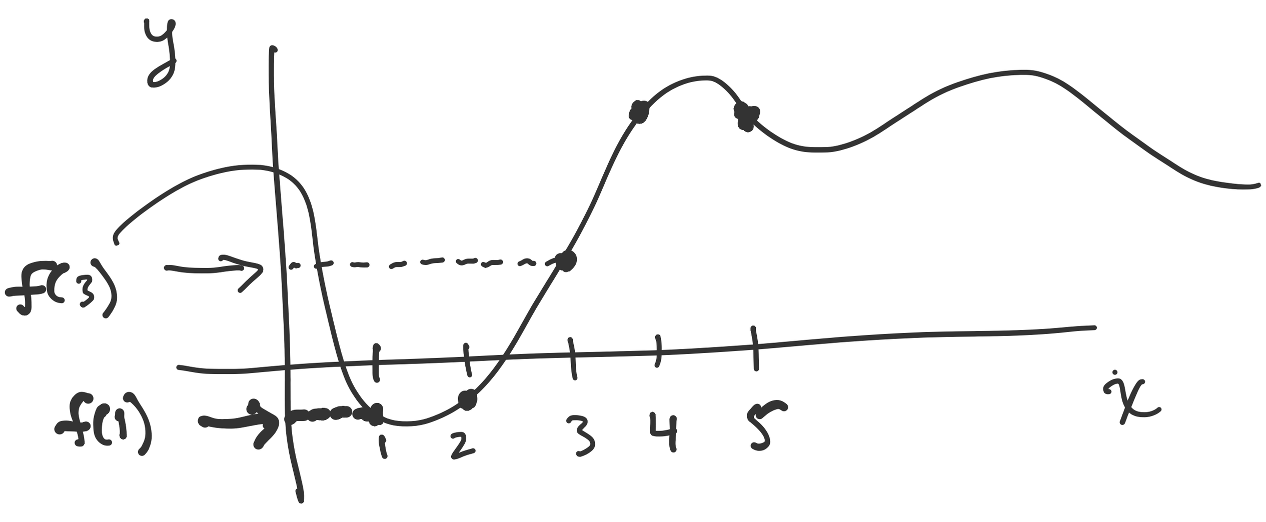
Requests. 8.1
✓ ~~Activity 4~~
✓ Ex. ~~1~~, ~~2~~, ~~6~~, ~~7~~

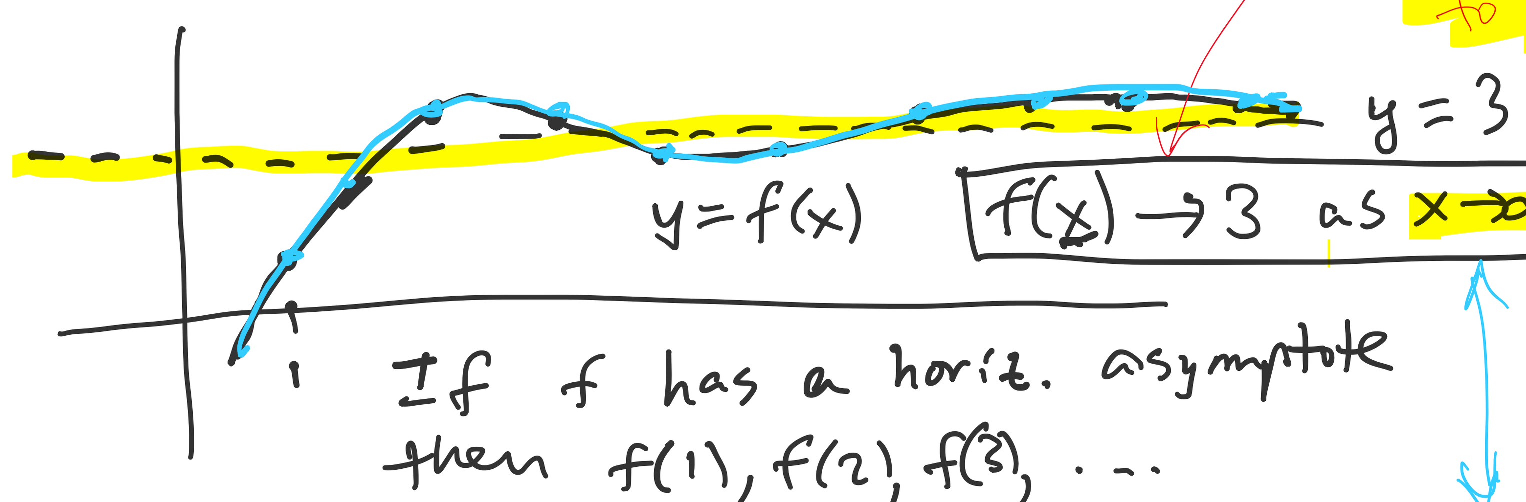
Comments on 8.1, visual for sequences and convergence

Start with $y = f(x)$
$\rightsquigarrow$ make a sequence

$\boxed{f(1), f(2), f(3), \dots}$ is a sequence

n	$f(n)$
1	$f(1)$
2	$f(2)$
3	$f(3)$
4	$f(4)$
5	⋮
⋮	⋮





"function f converges to 3"

$y = 3$

$y = f(x)$ $\quad \boxed{f(x) \to 3 \text{ as } x \to \infty}$

If f has a horiz. asymptote then $f(1), f(2), f(3), \dots$ converges to 3.

$\boxed{f(n) \to 3 \text{ as } n \to \infty}$

Activity 4

(a) $\left\{\frac{1+2n}{3n-2}\right\} = \frac{1+2\cdot1}{3\cdot1-2}, \frac{1+2\cdot2}{3\cdot2-2}, \frac{1+2\cdot3}{3\cdot3-2}, \dots$

why? $f(x) = \frac{1+2x}{3x-2}$ $\quad \boxed{\lim_{x\to\infty} f(x) = ? = \frac{2}{3}}$

$\approx \frac{2\cancel{x}}{3\cancel{x}} \to \frac{2}{3}$

Ans: $\left\{\frac{1+2n}{3n-2}\right\}$ converges to $\frac{2}{3}$.

(b) $f(x) = \frac{5+3^x}{10+2^x}$ $\quad \lim_{x\to\infty} f(x) = ?$ does not exist

$\longrightarrow \frac{5+3^x}{10+2^x} \approx \frac{3^x}{2^x} = \left(\frac{3}{2}\right)^x \to \infty$

Ans: the given sequence diverges

(c) $\left\{\frac{10^n}{n!}\right\}$ conv? div?

Factorial
$5! = 5\cdot4\cdot3\cdot2\cdot1$?
$n! = n(n-1)(n-2)\cdots3\cdot2\cdot1$
whole numbers $n \geq 1$

no good $\quad \boxed{f(x) = \frac{10^x}{x!}}$

$(3.2)! =$ "not defined" well ok there is

why factorial?

$(x^7)' = 7x^6$
$(x^7)'' = (7x^6)' = 7\cdot6x^5$
$(x^7)''' = \qquad = 7\cdot6\cdot5\,x^4$
$(x^7)'''' = \qquad = 7\cdot6\cdot5\cdot4x^3$
$(x^7)''''''' = (x^7)^{?} = 7!$ $\quad$ 7th deriv. $\quad = 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

(c) $\left\{\frac{10^n}{n!}\right\} = \frac{10^1}{1!}, \frac{10^2}{2\cdot1}, \frac{10^3}{3\cdot2\cdot1}, \frac{10^4}{4\cdot3\cdot2\cdot1}, \dots, \frac{10^{10}}{10\cdot9\cdot8\cdot7\cdots3\cdot2\cdot1} = \frac{10}{10}\cdot\frac{10}{9}\cdot\frac{10}{8}\cdots\frac{10}{1}$ $\quad \geq 1$

$\frac{10^{100}}{100!} = \frac{10}{100}\cdot\frac{10}{99}\cdot\frac{10}{98}\cdots\cdots\frac{10}{1}$

$\frac{10^n}{n!}$ leading factor $\quad \frac{10}{10000000}$ really small $\quad \frac{10^n}{n!} \searrow$ after $n > 10$

see $\frac{10^n}{n!} \to 0$ as $n \to \infty$
converges

Ex 1

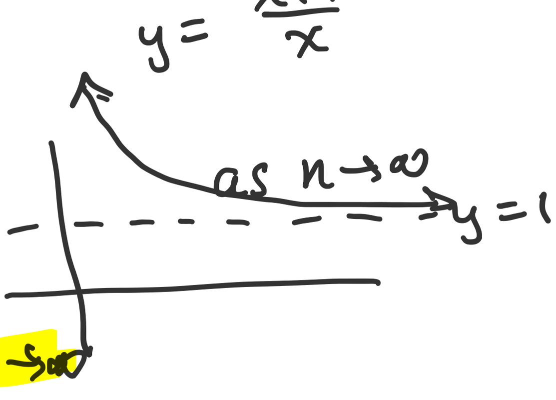
$1 + \frac{\cos(n)}{n}$

$n(n+1) - 1 \longrightarrow \infty$ (C) ans

$\frac{n+1}{n} \longrightarrow 1$ (E)

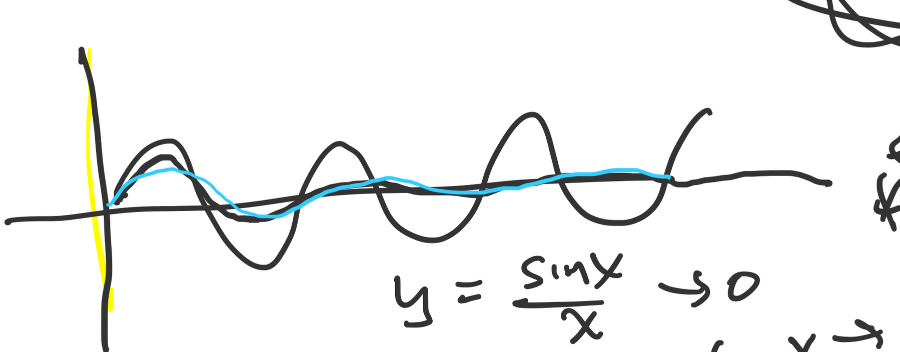
$\frac{\sin(n)}{n} \longrightarrow 0$ as $n\to\infty$ (A) as $n \to \infty$

$\left\{\frac{n\cdot\sin(n)}{n+1}\right\} \to$ does not conv. (B)



from Calc I $\quad \frac{\sin x}{x} \to 0$ as $x \to 0$

famous Calc I
& not obvious!
& takes work to justify



$y = \frac{\sin x}{x} \to 0$ as $x \to \infty$

$\frac{n\cdot\sin(n)}{n+1} = \left(\frac{n}{n+1}\right)\sin(n)$ $\quad \frac{n}{n+1} \to 1$ as $n \to \infty$

$y = \left(\frac{x}{x+1}\right)\sin x$

2.

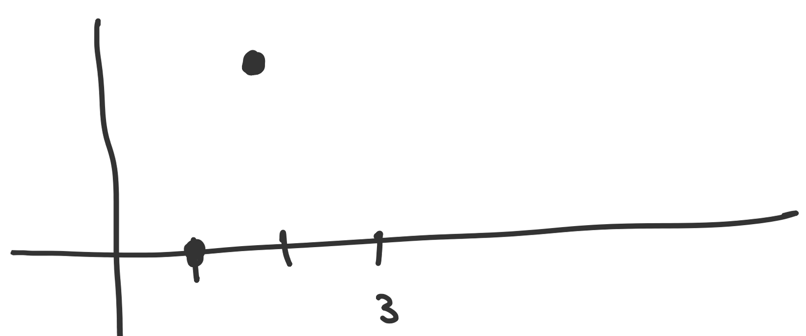
n	1	2	3	4	5	
$f(n)$ or s_n	0,	3,	8,	15,	24,	...

n	1	2	3	4	5
n^2	1	4	9	16	25

what's the function?

Ans: $s_n = n^2 - 1$

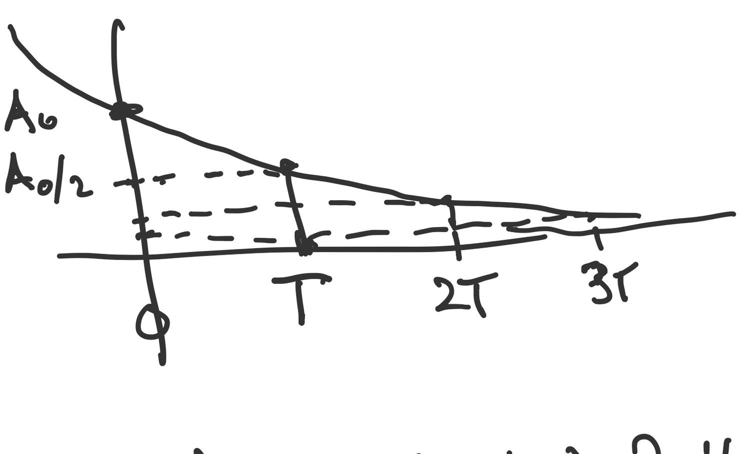
$f(x) = x^2 - 1$
$\{s_n\} = \{f(n)\}$



$\boxed{6.\ \text{OMIT}}$

7. Ch7 "half life" exponential decay

$\frac{dA}{dt} = kA \rightsquigarrow \boxed{A(t) = A_0 e^{kt}}, \ k < 0$



Sequence version

$A(t+5) = \frac{1}{2}A(t)$ $\quad t = 1, 2, 3, 4, \dots$

$\boxed{A_n = A_0 e^{kn}}$