

Announcements

* Canvas Exam check
due 3/23 or 3/24
counts as a homework

* 7.6 #5 logistic solution
procedure is posted
in whiteboard slides
link

Exam Friday 12:30-2:30
3/27

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Review ch 7

Exam 3 8 parts

Office Hours

by appt.

7.5 Ex 4

"radioactive decay"

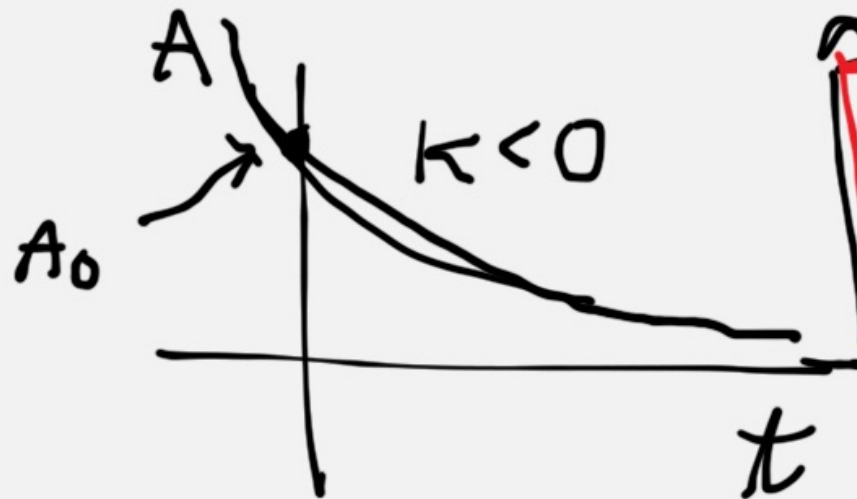
420 days \leftrightarrow decrease
by 30%

(a) half life?

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1. $A = A(t)$
= amount (or mass)
of material
at time t



$$\frac{dA}{dt} = kA$$

$$A(t) = A_0 e^{kt}$$
$$A_0 = A(0)$$

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$$(c) \quad t = 420$$

A goes from A_0

420 days \rightarrow .7 A_0 \leftarrow
later

$$A_0 - .3 A_0$$

$$\boxed{.7 A_0 = A(420) = A_0 e^{k \cdot 420}}$$

$$.7 = e^{k \cdot 420}$$

$$\ln(.7) = k(420)$$

$$k = \ln(.7)/420$$

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(b)

$$\underline{98} = \underline{100} e^{kt} \quad t = ?$$

$k = \text{known}$

$$.98 = e^{\frac{\ln(.7)}{420} t}$$
$$\ln(.98) = \frac{\ln(.7)}{420} t$$
$$t = \frac{420 \ln(.98)}{\ln(.7)}$$

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How to convert words to diff eqn.

"Basic exp"

$$\frac{dy}{dt} = ky$$

$k > 0$ (growth)
 $k < 0$ (decay)

Newton's law of cooling \rightarrow

$$\frac{dT}{dt} = k(T - K)$$

Logistic model \rightarrow

$$\frac{dP}{dt} = kP(N - P)$$

tank mixing \rightarrow

$$\frac{dx}{dt} = C_{in}r_{in} - C_{out}r_{out}$$

$C_{out} = \frac{x}{V}$ $x = x(t)$

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Solve


$$\frac{dx}{dt} = \left(3 - \frac{x}{7} \right) \quad x_0 = x(0) = \underline{7}$$

1. separate
 2. integrate
 3. solve


$$\int \left(3 - \frac{x}{7} \right) = \int dt \quad \parallel \quad \int \frac{-7du}{u}$$

$$= \underline{x + C}$$

$$\begin{aligned}
 u &= 3 - \frac{x}{7} \\
 du &= -\frac{1}{7} dx \\
 -7du &= \underline{dx}
 \end{aligned}$$

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$$\begin{aligned}
 & \text{LHS} \\
 & \rightarrow \int \frac{du}{u} = -\ln|u| \\
 & \text{LHS} = \text{RHS} \\
 & -\ln|u| = t + C \\
 & e^{\ln|u|} = e^{\left(-\frac{t+C}{7}\right)} \quad e^{-t/7} \\
 & |u| = e^{-\frac{t+C}{7}} = \underline{e^{-t/7} \cdot A} \\
 & \left|3 - \frac{x}{7}\right| = e^{-t/7} A \\
 & \left(\begin{array}{l} t=0 \\ x=7 \end{array}\right) \rightarrow 2 = A e^0 \Rightarrow A = 2
 \end{aligned}$$

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$$3 - \frac{x}{7} = 2e^{-t/7}$$



$$\frac{x}{7} = 3 - 2e^{-t/7}$$

$$x = \frac{7(3 - 2e^{-t/7})}{1}$$



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