open office hours 05.01

Friday, May 1, 2020 11:27 AM

5.4 Ex 7

which int. technique?

Euler's method

7.5 ~~8~~ Ex 8

5.4 Ex 7

neither (5)
u-sub (1)
parts (2)
both (3)

(a) $u = x^3$ $du = 3x^2\,dx$ $\frac{1}{3}\cos u\,du$

$\frac{1}{3}du = x^2\,dx$

(b) $\int x^3 \cos(x^2)\,dx$ → $\frac{1}{3}\int u\cos u\,du = \frac{1}{3}[uv - \int v\,du]$

try $u = x^2$ $du = 3x^2\,dx$

$u = w$ $dv = \cos w\,dw$ $= \frac{1}{3}[w\sin w - \int \sin w\,dw]$

$du = dw$ $v = \sin w$ easy

$\int u\,dv = uv - \int v\,du$ $= \frac{1}{3}[w\sin w + \cos w]$

guess $\int w\cdot\cos w\,dw$ $u = w$ $dv = \cos w\,dw$ $du = dw$ $v = \sin w$

(c) $\int x \ln(x^2)\,dx = \frac{1}{2}\int \ln w\,dw$ $u = \ln x$ $dv = dw$ $du = \frac{1}{x}dx$ $v = w$

$(\ln x)' = \frac{1}{x}$ $\int \ln x\,dx = ?$

$u = x^2$ $dw = 2x\,dx$ $\frac{1}{2}dw = x\,dx$

first example $\int xe^x\,dx$ (u, dv)

second example $\int \ln x\,dx$ $u = \ln x$ $dv = dx$ $x\ln x - x$

(d) $\int \sin(x^4)\,dx$ try $u =$ ✗ true $u =$ — $dv =$ — ✗

(e) $\int x^3 \sin(x^4)\,dx = \frac{1}{4}\int \sin u\,du$

$u = x^4$ $du = 4x^3\,dx$ $\frac{1}{4}du = x^3\,dx$

(f) $\int x^3\cdot x^2 \sin(x^5)\,dx$ → $\int w\sin w\,dw$ just like (b) above

~~Euler's method~~ Not on final exam

7.5 Ex 8

~~words~~ whiteboard slides >>

matching_words_diffeqs.pdf

"the rate at which a baby gains weight"

$W = W(t)$ = weight of a baby at time t

$W'(t) = k\cdot W$

"is proportional to" $\equiv [A = kB]$

A is proportional to B

$A \propto B$

(a) $W' = kW$ → $W\cdot W' = k$

$\frac{d}{dt}W(t) = kW(t)$ $\int W\,W'_{dt} = \int k\,dt$

$\int w\,dw = \frac{w^2}{2}$ $\int W(t)\cdot W'(t)\,dt = \int k\,dt$

$u = W$, $du = W'dt$

$W(0) = 8$, $W(1) = 9$, $W(12) = ?$ $\frac{w^2}{2} = kt + C$ $= kt + C$

$\frac{8^2}{2} = k\cdot 0 + C = 32$

$\frac{9^2}{2} = k\cdot 1 + 32$

$k = \frac{81}{2} - \frac{64}{2} = \frac{17}{2}$

81 64 17

$\frac{w^2}{2} = \frac{17}{2}t + 32$ check $t=0$ $w=8$ $t=1$ $w=9$

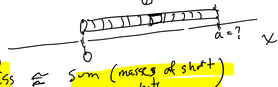
$W(12) = ?$ $\frac{W(12)^2}{2} = \frac{17}{2}\cdot 12 + 32$ → $w =$ —

$w = \sqrt{17t + 64}$ CRUDE

6.3 Ex 5a, d

~~Center of mass~~ not on final exam Ch 6

$\rho = \rho(x) = 10e^{-0.1x}$



total mass ≈ Sum (masses of short bits)

≈ sum [(density)(length of short bit)]

$30 = \text{total mass} = \int_0^a \rho(x)\,dx$

$30 = \int_0^a 10e^{-.1x}\,dx$ $u = -.1x$ $du = -.1\,dx$ $-10\,du = dx$

$= -100\int_{\text{from}}^{\text{to}} e^u\,du$

$= -100e^u\Big|_{\text{from}}^{\text{to}}$

$= -100e^{-.1x}\Big|_0^a$

$30 = -100(e^{-.1a} - 1)$

$-.3 + 1 = e^{-.1a}$

$.7 = e^{-.1a}$

$\ln(.7) = -.1a$

$-10\ln(.7) = a$

Partial Fractions

Find $\int \frac{dx}{(3x+1)(2x-5)} = -\frac{3}{17}\int \frac{1}{3x+1}\,dx + \frac{8}{17}\int \frac{1}{2x-5}\,dx$ $u = 3x+1$ $u = 2x-5$

$= -\frac{3}{17}\ln|3x+1| + \frac{8}{17}\ln|2x-5| + C$

$= \frac{1}{17}\ln\left|\frac{2x-5}{3x+1}\right| + C$

$\int \frac{1}{(3x+1)(2x-5)}\,dx = \int \frac{A}{3x+1}\,dx + \int \frac{B}{2x-5}\,dx$

"w/ common denominator" $3x+1$ $2x-5$

put it by $(3x+1)(2x-5)$

$1 = A(2x-5) + B(3x+1)$

$2x-5=0$ $2x=5$ $x=5/2$

plug in magic values

$1 = A\cdot 0 + B(3\cdot\frac{5}{2}+1)$ $\frac{15}{2}+\frac{2}{2}=\frac{17}{2}$

$1 = B\left(\frac{17}{2}\right)$

$B = \frac{2}{17}$

$3x+1=0$ $x = -1/3$

$1 = A(2(-\frac{1}{3})-5) + B\cdot 0$

$A\left(-\frac{2}{3}-\frac{15}{3}\right)$

$1 = A\left(-\frac{17}{3}\right)$

$A = -\frac{3}{17}$