Differentiation Formulas

1. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$
2. $(kf(x))' = kf'(x)$
3. $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
4. $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
5. $(f(g(x)))' = f'(g(x)) \cdot g'(x)$
6. $\dfrac{d}{dx}(x^n) = nx^{n-1}$
7. $\dfrac{d}{dx}(e^x) = e^x$
8. $\dfrac{d}{dx}(a^x) = a^x \ln a \quad (a > 0)$
9. $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$
10. $\dfrac{d}{dx}(\sin x) = \cos x$
11. $\dfrac{d}{dx}(\cos x) = -\sin x$
12. $\dfrac{d}{dx}(\tan x) = \dfrac{1}{\cos^2 x}$
13. $\dfrac{d}{dx}(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}$
14. $\dfrac{d}{dx}(\arctan x) = \dfrac{1}{1+x^2}$

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\displaystyle\int a^x\,dx = \frac{1}{\ln a}a^x + C, \quad a > 0$
4. $\displaystyle\int \ln x\,dx = x\ln x - x + C$
5. $\displaystyle\int \sin x\,dx = -\cos x + C$
6. $\displaystyle\int \cos x\,dx = \sin x + C$
7. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\displaystyle\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$
9. $\displaystyle\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$
10. $\displaystyle\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$
11. $\displaystyle\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$
12. $\displaystyle\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, $\sin x$

13. $\displaystyle\int x^n\ln x\,dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1$
14. $$\begin{aligned}\int p(x)e^{ax}\,dx &= \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)e^{ax}\,dx \\ &= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots \\ &\quad (+ - + - \ldots)\end{aligned}$$

(signs alternate)

15. $\displaystyle\int p(x)\sin ax\,dx = -\frac{1}{a}p(x)\cos ax + \frac{1}{a}\int p'(x)\cos ax\,dx$

$\displaystyle\quad = -\frac{1}{a}p(x)\cos ax + \frac{1}{a^2}p'(x)\sin ax + \frac{1}{a^3}p''(x)\cos ax - \cdots$

$(- + + - - + + \ldots)$

(signs alternate in pairs after first term)

16. $\displaystyle\int p(x)\cos ax\,dx = \frac{1}{a}p(x)\sin ax - \frac{1}{a}\int p'(x)\sin ax\,dx$

$\displaystyle\quad = \frac{1}{a}p(x)\sin ax + \frac{1}{a^2}p'(x)\cos ax - \frac{1}{a^3}p''(x)\sin ax - \cdots$

$(+ + - - + + - - \ldots)$ (signs alternate in pairs)

IV. Integer Powers of sin x and cos x

17. $\displaystyle\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx,$ n positive

18. $\displaystyle\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx,$ n positive

19. $\displaystyle\int \frac{1}{\sin^m x}\,dx = \frac{-1}{m-1}\frac{\cos x}{\sin^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}x}\,dx,$ $m \neq 1$, m positive

20. $\displaystyle\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{(\cos x)-1}{(\cos x)+1}\right| + C$

21. $\displaystyle\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}x}\,dx,$ $m \neq 1$, m positive

22. $\displaystyle\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{(\sin x)+1}{(\sin x)-1}\right| + C$

23. $\displaystyle\int \sin^m x\cos^n x\,dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and positive, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. If both m and n are even and negative, substitute $w = \tan x$, which converts the integrand to a rational function that can be integrated by the method of partial fractions.

V. Quadratic in the Denominator

24. $\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C,$ $a \neq 0$

25. $\displaystyle\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}\arctan\frac{x}{a} + C,$ $a \neq 0$

26. $\displaystyle\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + C,$ $a \neq b$

27. $\displaystyle\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C,$ $a \neq b$

VI. Integrands Involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $a > 0$

28. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C$

29. $\displaystyle\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

30. $\displaystyle\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$

31. $\displaystyle\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2 - a^2}}\,dx\right) + C$