Writing Solutions

Courses for David W. Lyons

Fall 2011

For full credit on graded quizzes, exams, and writing assignments, ***solutions must be written using complete English sentences***. Complete solutions

- ***show key steps of calculations*** (if any),
- ***use appropriate vocabulary*** from the text and course notes for objects and ideas, and
- ***provide a clear, concise explanation to a peer*** (a fellow student in the same course).

Some solutions are short and can be written in one sentence, but even short solutions have at least one idea that should be expressed in writing.

Final results of calculuations given with no supporting work receive no credit, even if the final result is correct.

Solutions are not journal style writing. Do not use "I" and do not use the past tense. Use the present tense and the active voice rather than the passive voice when possible. Use the pronoun "it" only when it is absolutely clear to both you and the reader what "it" refers to.

Use diagrams when a picture clarifies or illustrates your explanation.

Sample Problems and Solutions

Problem 1. Solve $x^2 + x - 2 = 0$.

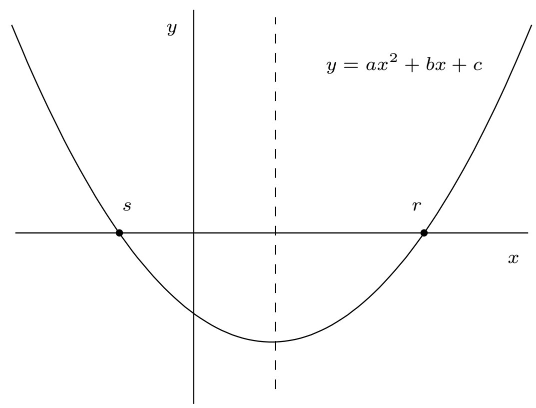
Solution. Since $x^2+x-2 = (x+2)(x-1)$, we see the solutions are $x = -2, 1$.

Alternative solution. Using the quadratic formula with $a = 1$, $b = 1$, and $c = -2$, we have

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{9}}{2} \\ &= \frac{-1 \pm 3}{2} \\ &= \frac{-4}{2}, \frac{2}{2} \\ &= -2, 1. \end{aligned}$$

Problem 2. Suppose you know that the equation $ax^2 + bx + c = 0$ has solutions $x = r, s$. Explain how you can use that knowledge to find the line of symmetry of the graph of $y = ax^2 + bx + c$.

Solution. The solutions $x = r, s$ to the equation $ax^2 + bx + c = 0$ are the x-intercepts of a parabola $y = ax^2 + bx + c$. The two x-intercepts are mirror reflections of one another across the line of symmetry, so the line of symmetry is half-way between them (see figure below). The half-way point along the x-axis between r and s is their average $(r + s)/2$, so the equation for the vertical line of symmetry is $x = \dfrac{r+s}{2}$.



Problem 3. Give a definition for the term ***irreducible quadratic polynomial***. Give an example, and say how you know your example is irreducible.

Solution. An ***irreducible quadratic polynomial*** is an expression of the form ax^2+bx+c that does not factor as a product of the form $(dx+e)(fx+g)$. An example is $x^2 + 1$. We know this is irreducible because if $x^2 + 1 = (dx+e)(fx+g)$, then the equation $x^2+1 = 0$ would have solutions $x = \frac{-e}{d}, \frac{-g}{f}$, so the parabola $y = x^2 + 1$ would have x-intercepts at $x = \frac{-e}{d}, \frac{-g}{f}$. But we know that $y = x^2 + 1$ has no x-intercepts because the y value $x^2 + 1 \geq 1 > 0$ is positive for every point on the graph.