Instructions

- Due at the beginning of class Friday 25 January.

1. Give proofs of the following facts that establish that the set $U(n)$, defined in Example 11 on p.46, is a group. Let $n > 1$ be a natural number.
   (a) Show that $U(n)$ is closed under multiplication modulo n.
   (b) Show that the positive integer a has a multiplicative inverse modulo n if and only if a and n are relatively prime.
2. Let G be a group with the property that for any x, y, z in the group, $xy = zx$ implies $y = z$. Show that G is Abelian.
3. Prove that in a group, $(ab)^2 = a^2b^2$ if and only if $ab = ba$.